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# Comparison of the method of classes and the quadrature of moment for the modelling of Neodymium Oxalate Precipitation

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## Introduction

### Oxalic precipitation :

- to deal with radioactive waste and recover the actinides

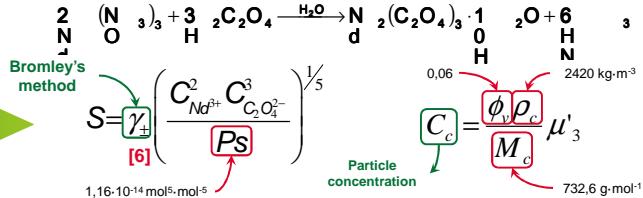
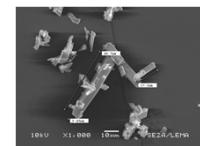
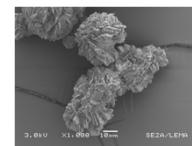
Ianthanides → To facilitate the development of experimental methods

### Modelling approach:

- Experimentation → Thermodynamics + kinetics & numerical methods

[1] [2] [7] [4] [5]

$\text{Nd}_2(\text{C}_2\text{O}_4)_3 \cdot 10 \text{ H}_2\text{O}$  obtained for precipitation different conditions



## Thermodynamics et kinetic laws

- Supersaturation ratio ( $S$ ) → The driving force of the precipitation process
- Kinetic laws of nucleation, cristal growth and agglomeration =  $f(S)$



## Crystal growth

$$G = 2,9 \cdot 10^{-6} \exp \left[ -\frac{14000}{RT} \right] (P_s)^{1/5} (S-1) \quad 293K < T < 333K$$

[2] [3]

## Homogeneous primary nucleation

$$R_N = 3 \cdot 10^{31} \exp \left[ -\frac{67600}{RT} \right] \exp \left[ -\frac{187}{(\ln S)^2} \right] \quad S > 50 \quad 293K < T < 333K$$

[1] [8] [9]

## Agglomeration

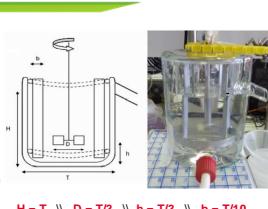
$$\begin{aligned} \beta &= 2,55 \cdot 10^{-7} \cdot I^{-0,70} \cdot S \cdot \dot{\gamma}^{-0,24} \exp \left[ -\frac{40900}{RT} \right] \\ S > 61 & \quad 293K < T < 333K \\ 45s^{-1} < \dot{\gamma} < 1024s^{-1} & \quad 600 \text{ mol} \cdot \text{m}^{-3} < I < 2100 \text{ mol} \cdot \text{m}^{-3} \\ \dot{\gamma} &= \sqrt{\frac{E}{V}} \quad \bar{e} = \frac{Np \cdot N^3 \cdot D^5}{V} \end{aligned}$$

## Experimental Study

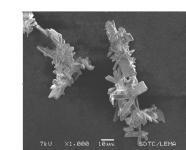
### MSMPR

#### Continuous precipitation until steady state

- $V = 200 \text{ ml}$
- Temperature =  $20^\circ \text{ C}$
- four stainless steel baffles
- stainless steel four  $45^\circ$  pitched blade turbine →  $N_p = 1,5$



MEB observations of Loose agglomerates



### Continuous experiments

- Short mean residence time ≈ 1 min → high  $S$
- Constant agglomeration kernel :  $\beta$
- Scanning Electron Microscopy → crystal size distributions

	N tr/min	C <sub>Nd</sub> mol·m <sup>-3</sup>	C <sub>Ox</sub> mol·m <sup>-3</sup>	S <sup>-1</sup>	L <sub>4,3</sub> μm
Run 1	1000	142,2	213,7	362	65
Run 2	2000	142,2	213,7	1024	41

## Two population balances

### Method of classes

$$\begin{aligned} \frac{dN'_k}{dt} + \frac{N'_k}{\tau} &= 0^k R_N + \frac{1}{\Delta L_k} \left[ F(G_{k+\frac{1}{2}}) - F(G_{k-\frac{1}{2}}) \right] \\ \text{Nucleation} & \quad \text{Growth} \\ \text{Source} & \quad \text{Source} \\ \frac{dN_k}{dt} + \frac{N_k}{\tau} &= \frac{N'_k}{\tau} + B_k - D_k \quad \begin{array}{l} \text{■ 3rd order scheme of Koren [4]} \\ \text{■ scheme of Litster [5]} \end{array} \end{aligned}$$

### Elementary particles

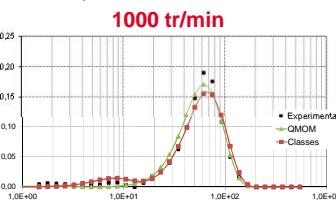
### Loose agglomerates

### Quadrature of Moments

$$\begin{aligned} \frac{d\mu'_k}{dt} + \frac{\mu'_k}{\tau} &= 0^k R_N + k G \mu'_{k-1} \quad \text{Nucleation} \quad \text{Growth} \\ \text{Source} & \quad \text{Source} \\ \frac{d\mu_k}{dt} + \frac{\mu_k}{\tau} &= \frac{\mu'_k}{\tau} + \frac{\beta}{2} \sum_{i=1}^n w_i \sum_{j=1}^n w_j (L_i^3 + L_j^3)^{k/3} - \frac{\beta}{2} \sum_{i=1}^n w_i L_i^k \sum_{j=1}^n w_j \quad \begin{array}{l} \text{Birth} \\ \text{Death} \end{array} \end{aligned} \quad [10]$$

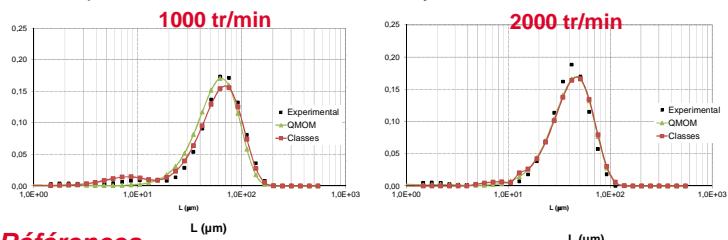
## Main results

### Comparison of volume fractions at 5 τ



### Crystal size distributions → Experimental vs. Predicted

### Comparison of volume fractions at steady state



## Conclusions

- Kinetic laws ( $R_N$ ,  $G$  et  $\beta$ ) and loose agglomerates from experimental runs
- Two population balance models : solved by the Method of classes and QMOM
- Both methods compared well with experimental data during transient and at steady state
- QMOM required much less computational effort and is preferentially used with the reconstruction method detailed in [10]

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