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A FFT-BASED VARIATIONAL PROCEDURE FOR HOMOGENIZATION OF MAXWELLIAN VISCOELASTIC COMPOSITES

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ABSTRACT

Homogenization of non-linear composites is a difficult task, generally addressed with two types of approaches at the microstructure scale: mean-field schemes (MFS), or full-field computation. At the structure scale, MFS are sometimes preferred due to their low computational cost, although their accuracy need to be improved. In our approach, we propose a homogenized behaviour as fast as MFS for the integration, and whose accuracy is close to the exact solution.

Variational approaches are powerful tools often used in homogenization. In a purely linear elastic framework, the homogenized stiffness minimizes the Helmholtz potential over the kinematic strain fields. Also, it minimizes the Hashin-Shtrikman functional over unconstrained polarization fields. In the framework of generalized standard materials, the solution minimizes at each time step the incremental potential [1]. This incremental potential was recently rewritten by Idiart [2] within the framework of maxwellian viscoelasticity by means of kinematic and divergence-free fields as:

$$\mathbf{J}(\bar{\boldsymbol{\varepsilon}}, \boldsymbol{\alpha}_n, \Delta t) = \min_{\bar{\boldsymbol{\alpha}}} \min_{\boldsymbol{\alpha}_s \in \mathcal{S}(\mathbf{C}_0)} \left\{ \min_{\mathbf{e} \in \mathcal{K}(\bar{\boldsymbol{\varepsilon}} - \bar{\boldsymbol{\alpha}})} \langle w(\mathbf{e} - \boldsymbol{\alpha}_s) \rangle + \Delta t \min_{\dot{\boldsymbol{\alpha}}_c \in \mathcal{K}(\bar{\boldsymbol{\alpha}})} \langle \varphi(\dot{\boldsymbol{\alpha}}_c + \dot{\boldsymbol{\alpha}}_s) \rangle \right\} \quad (1)$$

where $\dot{\boldsymbol{\alpha}} = \frac{\boldsymbol{\alpha} - \boldsymbol{\alpha}_n}{\Delta t}$, and $\mathcal{S}(\mathbf{C}_0)$ is the set of fields $\boldsymbol{\alpha}_s$ such that $\text{div}(\mathbf{C}_0 : \boldsymbol{\alpha}_s) = 0$, with $\langle \boldsymbol{\alpha}_s \rangle = 0$; and $\mathcal{K}(\bar{\boldsymbol{\varepsilon}})$ the set of fields \mathbf{e} such that $\mathbf{e} = \bar{\boldsymbol{\varepsilon}} + \text{grad}^s \boldsymbol{\xi}$ ($\boldsymbol{\xi}$ being a displacement field which checks the boundary conditions). Besides, the potentials are given by $w(\boldsymbol{\varepsilon}) = \frac{1}{2} \boldsymbol{\varepsilon} : \mathbf{C} : \boldsymbol{\varepsilon}$, and $\varphi(\dot{\boldsymbol{\alpha}}) = \frac{1}{2} \dot{\boldsymbol{\alpha}} : \mathbf{M} : \dot{\boldsymbol{\alpha}}$.

The set of fields in $\mathcal{K}(\bar{\boldsymbol{\varepsilon}})$ can be generated by means of the Green operator:

$$\mathbf{e} = \bar{\boldsymbol{\varepsilon}} + \boldsymbol{\Gamma}_0(\boldsymbol{\tau}) \quad (2)$$

where $\boldsymbol{\Gamma}_0$ is the Green operator related to a reference medium whose elasticity is \mathbf{C}_0 , and $\boldsymbol{\tau}$ is an unconstrained polarization field. Similarly, the set $\mathcal{S}(\mathbf{C}_0)$ can be generated by means of the stress Green operator:

$$\boldsymbol{\alpha}_s = \mathbf{S}_0 : \boldsymbol{\Delta}_0(\mathbf{a}) - \langle \mathbf{a} \rangle \quad (3)$$

where $\boldsymbol{\Delta}_0 = \mathbf{C}_0 - \mathbf{C}_0 \boldsymbol{\Gamma}_0 \mathbf{C}_0$ and \mathbf{a} is an unconstrained eigenstrain field.

The first set of compatible fields was already explored analytically by Torquato [3] for spherical particles within the framework of conductivity. However, for a general morphology, this set is rather difficult to describe, analytically.

Here, we propose to explore the sets \mathcal{S} and \mathcal{K} within a numerical framework. Indeed, Green operators are easily computed by means of Fast Fourier Transform. These operators generate kinematically/statically admissible fields: one example of such a field belonging to $\mathcal{K}(\bar{\boldsymbol{\varepsilon}})$ is:

$$\mathbf{e} = \bar{\boldsymbol{\varepsilon}} + \boldsymbol{\Gamma}_0(\chi \mathbf{e}_i \otimes \mathbf{e}_j) \bar{\tau}_{ij} \quad (4)$$

where χ is the characteristic function of one phase, and $\bar{\tau}_{ij}$ is the unknown to be optimized by the variational procedure. These fields, called *modes*, can be computed off-line, and they do not depend

on the material parameters. Hence, comparing to NTFA approach [4], computation of the modes is immediate, and do not change if the material changes. After this computation, the integration of the behaviour is very fast, allowing to go to the structure scale easily.

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