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## Orbital angular momentum of azimuthal spin waves

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In the context of a growing interdisciplinary interest in the angular momentum of wave fields, the spin-wave case has yet to be fully explored, with the extensively studied notion of spin transport being only part of the broader picture. Here we report experimental evidence for nonzero magnon orbital angular momentum inside magnetic disk by resolving the frequency splitting between magnon modes with counterrotating wavefronts and thereby avoiding formation of azimuthal standing waves. This requires an unambiguous formulation of spin and orbital angular momenta for spin waves, which we provide in full generality taking advantage of a systematic application of quantum field theory techniques as detailed in a companion paper [T. Valet *et al.*, *Phys. Rev. B* **113**, 104437 (2026)]. The results unequivocally establish magnetic dipole-dipole interactions as a magnetic-field controllable spin-orbit interaction for magnons. Our findings open a new research direction, leveraging the spectroscopic readability of angular momentum for azimuthal spin waves and other related systems.

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There has been an increasing realization in recent decades of the fundamental importance of the angular momentum (AM) carried by wave fields [1,2], which can be separated into spin (SAM) and orbital (OAM) components in certain cases. The latter is a universal feature of waves in uniform continuum media represented by helical or rotational wavefronts, and can potentially encode a large amount of information for mode multiplexed communication channels or multilevel registers of quantum states. Theoretical and experimental investigations of wave AM are already well developed for electromagnetic waves [3–5], plasma waves [6], fluid waves [7,8] or elastic waves [9]. Acknowledging wave AM in solid state media should be termed pseudo-AM [10,11], we omit the prefix following the common practice of the magnetism community. In recent years, experiments have found AM transfer between spin waves (SWs) and optical vortices [12–16], or elastic waves [17–22]. Most of these works, however, focused on SAM of SWs, leaving their OAM experimentally unresolved. Although Kerr and Brillouin light scattering (BLS) microscopy successfully visualized the azimuthal wavefronts in magnetic vortex states [23–26], they did not discuss any associated OAM. The situation may also be placed in the context of the general difficulty in directly observing rotating wavefronts [27].

In this Letter, we report on a spectroscopic measurement of SW eigenstates with nonvanishing OAM in a normally magnetized disk, paving the way for powerful detection schemes of general wave OAM through the aforementioned transduction of SWs to electromagnetic and elastic waves. This leverages on the lifted degeneracy between the SW modes

with counterrotating wavefronts, which originates from a type of spin-orbit interaction (SOI) that communicates the broken time-reversal symmetry associated with the equilibrium magnetization to SW textures. While a handful of theoretical works discuss AM of magnetization dynamics, none of them address the crucial role of SOI. They also have limited scopes, either specifying certain textures [28–31] and geometries [32–37] or not discussing azimuthal SW eigenstates [11,38–41]. We shall also mention recent theories of OAM [42,43] and SOI [44] across the magnon Brillouin zone, which are distinct topics from long wavelength SWs in mesoscopic ferromagnets. As a prerequisite to the spectroscopic detection of AM eigenstates, therefore, we developed a general formulation of magnon AM that clarifies its relation to azimuthal SW eigenstates in any axisymmetric geometry, whose details are exposed in a companion paper [45]. We identify the dynamical dipole-dipole interaction (DDI) to be the generic SOI for SWs, and assign correct AM indices to various spectral lines, which was not done properly in the previous studies [15,30,33]. Agreement with the experiment in a fully saturated state is quantitative, and establishes a spectroscopic measurement of the SW SOI and assignment of OAM quantum number. Our findings lay a foundation for reading OAM states not only for SWs but also for phonons or photons that can hybridize with SWs, and thereby open a new research direction in the study of general wave AM.

Figure 1 shows the measured SW spectra near the saturation field  $H_{\text{sat}}$  [46]. The spectroscopy is conducted by a magnetic resonance force microscope (MRFM) [47], an instrument that employs a soft cantilever with a magnetic nanosphere at its apex to mechanically detect the magnetization dynamics in the sample placed underneath. It probes the variation of the magnetization along the local equilibrium axis, enabling the detection of modes with nonuniform spatial

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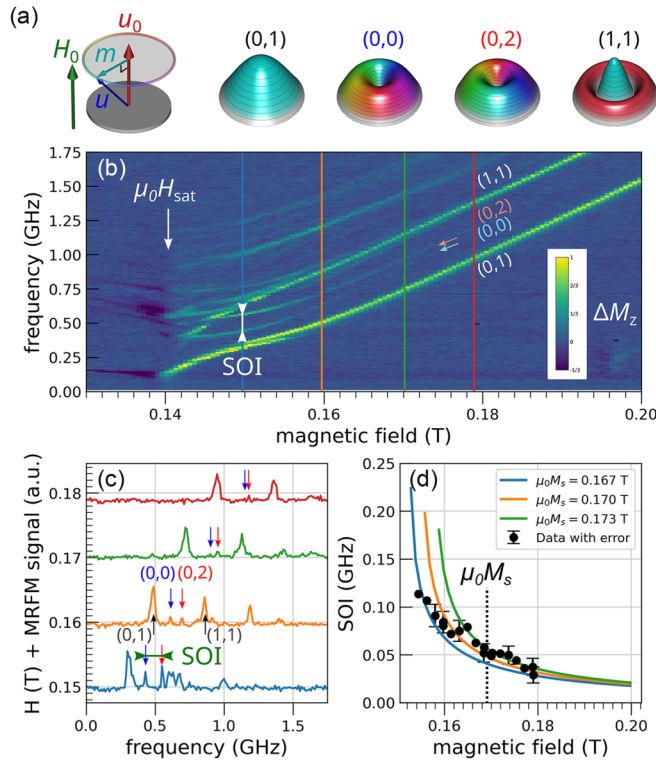


FIG. 1. (a) Graphical representation of the precession pattern of  $m(x)$  for SW modes labeled  $(n_R, n_J)$ . (b) MRFM (Magnetic resonance force microscopy) spectroscopy as a function of normal magnetic field and frequency on a YIG disk. (c) Line cuts at field values indicated by vertical lines of respective colors in (b). The split between the  $(0,2)$  and  $(0,0)$  peaks defines the SOI. (d) Magnetic field dependence of the SOI. The dots are the experimental points, while the solid lines are theoretical predictions calculated by Eq. (6) for three different values of  $\mu_0 M_s = 0.167, 0.170$ , and  $0.173$  T corresponding to its uncertainty range.

profiles, which are difficult to characterize with conventional probes. The sample is a yttrium iron garnet (YIG) microdisk, which has been patterned by Ar etching from a thin film grown by liquid phase epitaxy (LPE), with a diameter of  $1 \mu\text{m}$  and a thickness of  $55 \text{ nm}$  determined from the splitting between the Kittel mode and the first higher-order thickness standing spin-wave mode (exchange dominated) observed by BLS, assuming that the exchange length in YIG  $\lambda_{\text{ex}} = 15 \text{ nm}$  [48]. Subsequently, a radio-frequency antenna is deposited on its surface. The apparatus is situated between the poles of an electromagnet that produces a large magnetic field  $H_0$ . In our setup, a slight misalignment exists in the tilt angle ( $< 1^\circ$ ) between the applied magnetic field and the normal of the disk [46]. As will be shown below, this tilt seems sufficient to excite nonvanishing OAM SW modes. Deviation of the field from the normal introduces a rigid shift of the whole spectrum for  $H_0 \gtrsim M_s$  [46] and hence should have no impact on analyzing the frequency separation between modes. More details about the experimental setup and the YIG disk can be found in the Supplemental Material [46] (see also references [49–55] therein).

Figure 1(b) is a density plot of the variation of the MRFM signal as a function of  $H_0$  and the excitation frequency

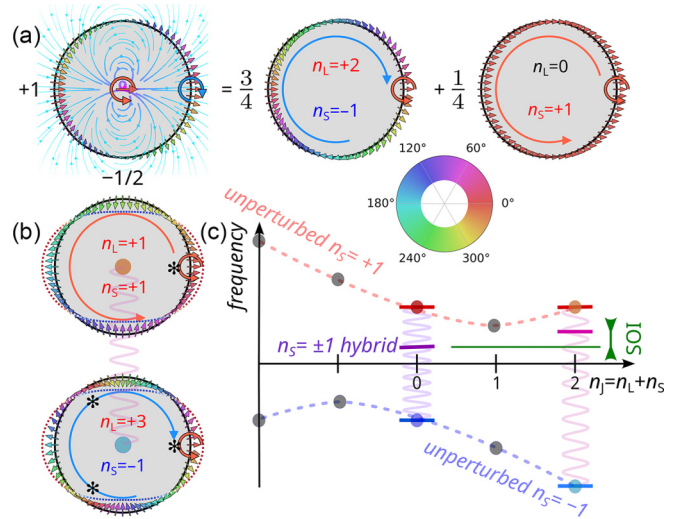


FIG. 2. (a) Illustration of the azimuthal pattern of the dynamical DDI. The stray magnetic field profile at a fixed radius from the counterclockwise rotating magnetic dipole that generates it is a 3:1 superposition of the  $(n_L = +2, n_S = -1)$  and  $(n_L = 0, n_S = +1)$  in the OAM eigenmode basis. (b) Coupling (wavy line) of the two OAM eigenmodes with  $n_J = 2$  via dynamical DDI. The asterisk helps count  $n_L$ : the repetition of a given orientation (here at  $0^\circ$ , cf. color wheel). (c) Comparison of the dynamical DDI coupling for two states of opposite OAM  $n_L = \pm 1$ . The red (blue) dotted line shows the dispersion of the  $n_R = 0$  eigenstates of  $\hat{O}_d$  for  $n_S = +1$  ( $n_S = -1$ ). The unperturbed  $(n_L = \pm 1, n_S = +1)$  eigenstates undergo energy lowering through hybridization mediated by  $\hat{O}_d^{(-)}$  with  $n_S = -1$  states of different  $|n_L|$ . This gives rise to two distinct elliptically precessing states (magenta levels) depending on the polarity of  $n_L$ . The frequency shifts are inversely proportional to the gap between the  $n_S = \pm 1$  branches, resulting in a SOI splitting that increases when the applied magnetic field approaches saturation.

$\omega/(2\pi)$ . The spectral peaks are labeled by radial index  $n_R \in \mathbb{Z}^+$  and azimuthal index  $n_J \in \mathbb{Z}$ , to be discussed in detail shortly. In this plot, the most intense signals arise from the Kittel mode  $(n_R, n_J) = (0, 1)$ , and its first radial harmonic  $(1, 1)$ . Expected precession patterns of their dynamical magnetization  $m(x)$  are sketched in Fig. 1(a) where the height indicates the amplitude and the color the azimuthal orientation (cf. colorwheel in Fig. 2). Because of their spatially uniform precession along the azimuthal direction, these are naturally the modes coupled strongest to the antenna. By linearly fitting the variation of the  $(0, 1)$  mode frequency with the external magnetic field between  $0.3$  and  $1$  T, we determine the values of the gyromagnetic ratio  $|\gamma| = (1.77 \pm 0.01) \times 10^{11} \text{ rad s}^{-1} \text{ T}^{-1}$ . The saturation magnetization  $\mu_0 M_s = 0.170 \pm 0.003 \text{ T}$  ( $\mu_0$  is the vacuum permeability) is extracted from the ferromagnetic resonance of the extended thin film with the magnetic field applied in plane [46]. The obtained value is in agreement with previous studies made on LPE grown YIG thin films [48]. Between these landmark eigenstates, we also observe two additional small peaks which can be uniquely assigned to the modes  $(0, 0)$  and  $(0, 2)$ . No other modes are expected in this frequency range given our disk dimensions. These two eigenstates correspond to SW modes with counterrotating wavefronts, as illustrated in Fig. 1(a). Their split in frequency

increases as  $H_0$  approaches  $H_{\text{sat}}$  from above. We will establish below that this is due to dynamical DDI, whose influence increases with decreasing  $H_0$ . It introduces an ellipticity in the spin precession of the homogeneously magnetized disk and acts as a SOI for SW. The field-controllable SOI can be made much larger than the linewidth by tuning  $H_0$  closer to  $H_{\text{sat}}$ , making the split readily observable.

To substantiate the claims made in the previous paragraph, we set up a theoretical model, relate SW eigenstates to OAM, and compute their spectrum. Consider a ferromagnet occupying a region  $\Omega$ , whose magnetization is given by  $\mathbf{M}(\mathbf{x}, t) = M_s \mathbf{u}(\mathbf{x}, t)$ , with  $\mathbf{u}(\mathbf{x}, t)$  a time-dependent unit vector field [Fig. 1(a)]. We assume the existence of a textured equilibrium  $\mathbf{u}_0(\mathbf{x})$  and define the dimensionless SW field  $\mathbf{m} = \mathbf{u} - \mathbf{u}_0$ . The linear dynamics of  $\mathbf{m}$  is determined by minimizing the action  $\mathcal{S} = \mu_0 M_s^2 \int dt \int_{\Omega} d^3x \mathcal{L} \equiv \mu_0 M_s^2 V \int dt \langle \mathcal{L} \rangle_{\Omega}$ , where  $V$  is the finite volume of  $\Omega$  and quantities between  $\langle$  and  $\rangle_{\Omega}$  denoting spatial averaging, and the normalized Lagrangian density  $\mathcal{L}$  is given by [38,45]

$$\mathcal{L} = \frac{1}{2} \frac{(\mathbf{u}_0 \times \mathbf{m})}{\omega_M} \cdot \partial_t \mathbf{m} - \mathcal{U} - \kappa \mathbf{u}_0 \cdot \mathbf{m}. \quad (1)$$

The first term plays the role of a kinetic energy density, with  $\omega_M = |\gamma| \mu_0 M_s$  a natural frequency scale. As for the ‘‘potential’’ energy density [56,57], we choose for concreteness  $\mathcal{U} = \frac{1}{2} h_0 \mathbf{m} \cdot \mathbf{m} + \frac{1}{2} \lambda_{\text{exc}}^2 \nabla \mathbf{m} : \nabla \mathbf{m} - \frac{1}{2} h_a (\mathbf{e}_a \cdot \mathbf{m})^2 + \mathcal{U}_d$ , where  $h_0 \mathbf{u}_0$  is the equilibrium effective field,  $\lambda_{\text{exc}}$  the exchange length,  $h_a$  the anisotropy field along the unit vector  $\mathbf{e}_a$ , and  $\mathcal{U}_d = -\frac{1}{2} (\nabla \phi)^2 + \mathbf{m} \cdot \nabla \phi$  the dynamical DDI contribution with  $\phi$  the magnetic scalar potential. The fields  $h_0$ ,  $h_a$ , and  $-\nabla \phi$  shall be understood as having been normalized by  $\mu_0 M_s$ . Finally,  $\kappa(\mathbf{x}, t)$  is a Lagrange multiplier. AM is the conserved quantity associated with a rotation symmetry of the Lagrangian. Let  $\delta r_J$  be the infinitesimal rotation by an angle  $\delta\theta$ , around a certain  $Oz$  axis. It admits a decomposition  $\delta r_J = \delta r_L \circ \delta r_S$  where the orbital  $\delta r_L$  and spin  $\delta r_S$  parts act on  $\mathbf{x}$  and  $\mathbf{m}$ , respectively. We introduce a global cylindrical coordinate system  $(O; r, \theta, z)$  with local unit vector basis  $\{\mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_z\}$ . The action of  $\delta r_J$  on  $\mathbf{m}(\mathbf{x})$  is then given by  $\delta r_J : \mathbf{m} \rightarrow \mathbf{m} + \delta \mathbf{m}_J$  and  $\frac{\delta \mathbf{m}_J}{\delta \theta} \equiv \frac{\delta \mathbf{m}_L}{\delta \theta} + \frac{\delta \mathbf{m}_S}{\delta \theta} = -\partial_\theta \mathbf{m} + \mathbf{e}_z \times \mathbf{m}$ . If we assume the Lagrangian to be invariant under  $\delta r_J$ ,  $\delta r_L$  or  $\delta r_S$ , an application of the Noether’s theorem [58] yields, respectively, as globally conserved quantities, the projections along  $Oz$  of the volume integrated AM,  $J^z$ , OAM,  $L^z$ , and SAM,  $S^z$ , as [45]

$$J^z = L^z + S^z, \quad (2a)$$

$$L^z = -\mathcal{J}_M \int_{\Omega} d^3x (\mathbf{u}_0 \times \mathbf{m}) \cdot \partial_\theta \mathbf{m}, \quad (2b)$$

$$S^z = +\mathcal{J}_M \int_{\Omega} d^3x (\mathbf{u}_0 \times \mathbf{m}) \cdot (\mathbf{e}_z \times \mathbf{m}), \quad (2c)$$

in which  $\mathcal{J}_M = M_s / (2|\gamma|)$  emerges as a natural scale of AM density for the linear SWs.

We next clarify the relation between the AM densities and azimuthal SW eigenmodes. The Euler-Lagrange equation for  $\mathbf{m}$  associated with  $\mathcal{S}$  reads as  $\partial_t \mathbf{m} = \omega_M \mathbf{u}_0 \times \frac{\delta \mathcal{U}}{\delta \mathbf{m}}$ , under the constraint  $\mathbf{u}_0 \cdot \mathbf{m} = 0$ , which we recognize as the linearization of the Landau-Lifshitz equation [59]. The Fourier transform in time  $\mathbf{m}(\mathbf{x}, t) = \text{Re}[\tilde{\mathbf{m}}(\mathbf{x})e^{-i\omega t}]$  with  $\omega \in \mathbb{R}$  and  $\tilde{\mathbf{m}} \in \mathbb{C}^3 / \{\mathbf{u}_0 \cdot \tilde{\mathbf{m}} = 0\}$  yields an eigenvalue problem  $\omega \tilde{\mathbf{m}} = \hat{\mathcal{O}} \tilde{\mathbf{m}}$ , where the

concrete expression of the integrodifferential operator  $\hat{\mathcal{O}} = \mathbf{u}_0 \times \frac{\delta^2 \mathcal{U}}{\delta \mathbf{m} \otimes \delta \mathbf{m}}$  is given in Eq. (55) of the accompanying paper [45]. The resultant SW eigenmodes, labeled by a stand-in mode index  $\nu$  to be specified by boundary conditions, satisfy two general properties [47,60,61]: (i) if  $\{\omega_\nu, \tilde{\mathbf{m}}_\nu\}$  is a SW eigenpair, then  $\{-\omega_\nu, \tilde{\mathbf{m}}_\nu^*\}$  is one as well, representing the same physical state, and (ii) the SW eigenmodes satisfy the orthogonality relation [47]

$$-\frac{\mathbf{i}}{2} \frac{\langle (\mathbf{u}_0 \times \tilde{\mathbf{m}}_\nu^*) \cdot \tilde{\mathbf{m}}_{\nu'} \rangle_{\Omega}}{\text{sgn}(\omega_\nu)} = A_\nu^2 \delta_{\nu, \nu'}, \quad (3)$$

with  $A_\nu > 0$  being *defined* as the natural norm of the mode. If the action is invariant under  $\delta r_J$ , the eigenmodes can be chosen as eigenfunctions of the associated infinitesimal generator, i.e.,  $\mathbf{m}_{\nu, n_J} = \text{Re}[\tilde{\mathbf{m}}_{\nu, n_J}(r, z)e^{i(n_J \theta - \omega_{\nu, n_J} t)}]$  with  $\tilde{\mathbf{m}}_{\nu, n_J} \equiv \tilde{m}_{\nu, n_J}^r(r, z)\mathbf{e}_r + \tilde{m}_{\nu, n_J}^\theta(r, z)\mathbf{e}_\theta + \tilde{m}_{\nu, n_J}^z(r, z)\mathbf{e}_z$  and  $n_J \in \mathbb{Z}$  being an eigenvalue of  $\delta r_J$ , since  $[(\mathbf{e}_z \times \cdot) - \partial_\theta] \tilde{\mathbf{m}}_{\nu, n_J} = -in_J \tilde{\mathbf{m}}_{\nu, n_J}$ . Substituting this waveform into Eq. (2a) gives

$$J_{\nu, n_J}^z = \text{sgn}(\omega_{\nu, n_J}) n_J V \mathcal{J}_M A_{\nu, n_J}^2. \quad (4)$$

If the action is additionally invariant under  $\delta r_L$  and  $\delta r_S$ , the eigenmodes can be eigenfunctions of both associated infinitesimal generators, i.e.,  $\mathbf{m}_{\nu, n_L, n_S}^0 = m_{\nu, n_L, n_S}^0(r, z) \text{Re}[(\mathbf{e}_r + \mathbf{i}n_S \mathbf{e}_\theta)e^{i(n_J \theta - n_S \omega_{\nu, n_L}^0 t)}]$ , with the corresponding eigenvalues  $n_L = n_J - n_S$ ,  $n_L \in \mathbb{Z}$ ,  $n_S = \pm 1$ , and  $\omega_{\nu, n_L}^0 > 0$ , as  $\mathbf{e}_z \times (\mathbf{e}_r \pm \mathbf{i} \mathbf{e}_\theta) = \mp \mathbf{i}(\mathbf{e}_r \pm \mathbf{i} \mathbf{e}_\theta)$ . Then Eqs. (2b) and (2c) give  $L_{\nu, n_J}^z = n_S n_L V \mathcal{J}_M A_{\nu, n_J}^2$  and  $S_{\nu, n_J}^z = V \mathcal{J}_M A_{\nu, n_J}^2$ . The canonical quantization for constrained systems [45,62,63] establishes  $V \mathcal{J}_M A_{\nu, n_J}^2 = \hbar$  for single magnon states, confirming the quantization of conserved angular momenta per magnon.

Let us elaborate on the physical meaning of the angular momentum labels  $n_J$  and  $n_L$ . The Lagrangian is generically invariant under  $\delta r_J$  if (i) the region  $\Omega$ , (ii) the equilibrium texture  $\mathbf{u}_0$ , and (iii) all the magnetic material properties are invariant under  $\delta r_J$ . This defines a broad class of axisymmetric ferromagnets, such as YIG spheres [12,32] or mesoscopic disks in one of their known axisymmetric equilibria [55,64–66]. According to Eq. (4),  $n_J$  is a measure of AM up to the choice  $\text{sgn}(\omega_{\nu, n_J}) = \pm 1$ . In all concrete calculations, we use the positive sign convention, by which  $n_J > 0$  represents positive AM. OAM can be associated to the integer label  $n_L$  *only if* the Lagrangian is invariant under  $\delta r_L$ , which occurs if, in addition to (i)–(iii), we also have (iv)  $\mathbf{u}_0 = \mathbf{e}_a = \mathbf{e}_z$ , and (v) a negligible dynamical DDI [38,39].<sup>1</sup> Hereafter, we further specialize the problem to a sufficiently thin disk so that  $\mathbf{m}$  can be considered independent of  $z$ . Then, under (i)–(v), the equation  $\omega \mathbf{m} = \hat{\mathcal{O}} \mathbf{m}$  reduces to

$$\omega_{n_R, n_L, n_S}^0 m_{n_R, n_L}^0 = n_S \left[ h_0 + \lambda_{\text{exc}}^2 \left( \frac{n_L^2}{r^2} - \nabla_\perp^2 \right) \right] m_{n_R, n_L}^0, \quad (5)$$

in which  $\nabla_\perp^2 \equiv \partial_r^2 + (1/r)\partial_r$  and  $n_R$  is the associated radial index. It immediately follows that  $\omega_{n_R, n_L, n_S}^0 = n_S \omega_{n_R, |n_L|}^0$  with  $\omega_{n_R, |n_L|}^0 > 0$ , while the corresponding eigenvector amplitudes can be expressed as  $\tilde{\mathbf{m}}_{n_R, n_L, n_S}^0 = m_{n_R, |n_L|}^0(r, z)(\mathbf{e}_r + \mathbf{i}n_S \mathbf{e}_\theta)$ .

<sup>1</sup>It shall be noted that a negligible dynamical DDI is a *sufficient* condition for magnons to carry separately conserved and quantized SAM and OAM, but *not* a necessary condition.

Similarly to  $n_J$ ,  $n_L$  quantifies OAM only up to the sign of eigenfrequency, or equivalently the eigenvalue  $n_S$  of  $\delta r_S$  that carries no physical significance. In Fig. 2(b), the AM and OAM of the  $n_J = 2$  SW eigenstates derived from Eq. (5) are shown schematically. The dotted line highlights in red and blue the number of oscillatory cycles  $|n_J|$  of the radial component of  $\mathbf{m}$  around the disk circumference. It differs by one unit from  $|n_L|$ , which counts the number of revolutions of  $\mathbf{m}$ , encoded in the cyclic color wheel. For  $n_L \neq 0$  and fixed  $n_S$ , the sign of  $n_L$  indicates whether the direction of rotation of the Cartesian component wavefront (large circular arrow) is identical or reversed compared to the direction of local precession (small circular arrow), i.e., whether the OAM is parallel or antiparallel to the SAM.

We now introduce the general notion of magnon SOI. Consider a system where (i)–(v) are satisfied and fix  $n_L \in \mathbb{Z}$ . The magnons come in degenerate pairs  $n_J = \pm n_L + n_S$  with identical radial profile and opposite values of the OAM  $\propto \pm n_S n_L$ . The SOI manifests itself through the lift of this degeneracy, and is quantified by the frequency splittings between magnons with opposite OAM, i.e.,  $\text{SOI}_{n_R, n_L} = (\omega_{n_R, 1+n_L} - \omega_{n_R, 1-n_L})$  under the convention  $n_S = +1$ . In the present case, the splitting is induced by dynamical DDI. When neglecting the dynamical DDI, all the SW eigenmodes of the disk are circularly polarized with right-handed precession, due to the time-reversal symmetry breaking associated with the equilibrium magnetization as shown in Fig. 2(b). However, they induce a dynamic dipolar field with both right- and left-handed circularly polarized components in the disk plane [Fig. 2(a)]. More quantitatively, the linear operator  $\hat{O}$  can be split as  $\hat{O} = \hat{O}_0 + \hat{O}_d^{(+)} + \hat{O}_d^{(-)}$  with  $\hat{O}_0$  corresponding to the equilibrium effective field and exchange contributions.  $\hat{O}_d^{(\pm)}$  are defined to be parts of the dynamical DDI that commute and anticommute with  $\delta r_S$ , respectively. Therefore, represented in the basis  $\{\tilde{\mathbf{m}}_{n_R, n_L, n_S}^0, n_L \in \mathbb{Z}, n_S = \pm 1\}$ ,  $\hat{O}_0 + \hat{O}_d^{(+)}$  leaves the  $n_S = \pm 1$  subspaces invariant, while  $\hat{O}_d^{(-)}$  swaps the two and is the sole source of SOI, with  $\hat{O}_d^{(\pm)} \mathbf{m}_{n_R, n_L, n_S}^0$  proportional to right- and left-handed components of the dynamical stray field. They both conserve  $n_J = n_L + n_S$ , hence,  $\hat{O}_d^{(-)}$  uniquely couples  $n_L$  with  $n_S = +1$  to  $n_L + 2$  with  $n_S = -1$ , breaking the  $\pm n_L$  spectral symmetry as illustrated in Fig. 2(c) where the eigenvalues  $\omega_{n_R, n_L, n_S}^0$  of  $\hat{O}_0$ , i.e., “unperturbed” frequency spectrum, are plotted for the  $n_R = 0$  branch with the dotted lines being a guide for the eyes. In some intermediate field range  $h_0 \gtrsim 1$ , we can try to obtain a semiquantitative insight into the emergence of the SOI by considering the dynamical DDI as a perturbation. Focusing on the  $n_R = 0$  branch, and up to second order in the dynamical DDI, we obtain [45]

$$\text{SOI}_{0, n_L} = \sum_{n_R} \left[ \frac{\Gamma_{\langle 0 | n_R \rangle, n_L}^-}{2\omega_K + \Delta_{\langle 0 | n_R \rangle, n_L}^-} - \frac{\Gamma_{\langle 0 | n_R \rangle, n_L}^+}{2\omega_K + \Delta_{\langle 0 | n_R \rangle, n_L}^+} \right], \quad (6)$$

where  $\Gamma_{\langle 0 | n_R \rangle, n_L}^\pm = \langle m_{n_R, |2 \pm n_L|}^0 \hat{O}_d^{(\pm)} m_{0, |n_L|}^0 \rangle_\Omega$  and  $\Delta_{\langle 0 | n_R \rangle, n_L}^\pm = \omega_{0, |n_L|}^0 + \omega_{n_R, |2 \pm n_L|}^0 - 2\omega_K$  with  $\omega_K$  being the frequency of the quasiform Kittel mode for the disk. By direct inspection of Eq. (6) we obtain the confirmation of a field-controllable SOI, with a leading asymptotic term inversely proportional to  $\omega_K$ . The splitting between  $(n_L = \pm 1, n_S = +1)$  through the hybridization with  $(n_L = 2 \pm 1, n_S = -1)$  is illustrated in

Fig. 2(c). The eigenfrequencies and eigenmodes of  $\hat{O}_0$  can be obtained semianalytically from a Galerkin projection of the corresponding spectral boundary value problem over a truncated basis of the analytical exchange SWs, while explicit integral forms of the matrix elements of  $\hat{O}_d^\pm$  can be obtained on the same basis [45]. This in principle enables a numerical evaluation of the approximate SOI frequency split (6). However, it is found more straightforward, and more accurate over the whole field range, to perform a Galerkin projection of the full spectral boundary value problem associated with  $\hat{O}$  [45]. The SOI frequency split can then be directly obtained from the numerically estimated exchange-dipole SW frequencies. This is the method that has been used to generate the continuous line in Fig. 1(d).

We now compare the semianalytical prediction with the experiment. In Fig. 1(d), we plot the variation of the experimentally determined SOI splitting as a function of the magnetic field strength  $H_0$ . The superimposed solid lines are the theoretically computed [45]  $\text{SOI}_{0,1}$  obtained for three values of  $\mu_0 M_s = 0.167, 0.170$ , and  $0.173$  T corresponding to the uncertainty in the independent measurement of  $M_s$ . We find quantitative agreement between the data and the model for field strengths  $H_0 \geq M_s$ . Moreover, the observed experimental splitting exhibits a clear field dependence, which rules out the possibility of an accidental fixed pinning of comparable strength. The deviations observed for  $H_0 < M_s$  can be attributed to the details of how the magnetization projection along the symmetry axis evolves near the compensation field, particularly in the presence of a slight misalignment [46].

In summary, we provide here a comprehensive semianalytical picture for identifying the pseudo-OAM of azimuthal spin waves and its practical implementation. The method relies on the fact in magnetic systems the SOI is controllable by the magnetic field and can be made very large. Our findings open a unique avenue in the study of wave OAM by providing a robust detection mechanism not only for SWs but also potentially for photons and phonons through hybridization with magnons. The latter in particular offers a transduction of SAM to a mechanical torque, which could in principle be harnessed to rotate mesoscopic elastic bodies.

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*Data availability.* The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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