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► To cite this version:

Alexis Touzalin, Emanuele Arcese, Sébastien Pernet. A boundary integral equation method based on virtual element approximation spaces for 3D electromagnetic scattering. WAVES 2024 - 16th International Conference on Mathematical and Numerical Aspects of Wave Propagation, Jun 2024, Berlin, Germany. pp.125-126, 10.17617/3.MBE4AA . cea-04951065

HAL Id: cea-04951065

<https://cea.hal.science/cea-04951065v1>

Submitted on 17 Feb 2025

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A boundary integral equation method based on virtual element approximation spaces for 3D electromagnetic scattering

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Abstract

We develop a new Galerkin approximation of boundary integral equations of ElectroMagnetic (EM) scattering based on a Virtual Element Method (VEM), which allows polygonal meshes. This aims at improving the performance and versatility of the classical Boundary Element Method (BEM) solvers in applications involving mesh refinement techniques. In this work, we present the construction, the theoretical analysis, the implementation of this virtual boundary element approach, as well as the numerical experiments highlighting its effectiveness on Radar Cross Section (RCS) benchmarks.

Keywords: Maxwell equations, Boundary integral operators, Virtual elements, Convergence analysis, Polygonal meshes.

1 Introduction

The BEM, relying on Galerkin approximation by means of the lowest-order Raviart-Thomas (RT) finite elements, have proven their efficiency for the solution of boundary integral equations describing the EM scattering of time-harmonic waves by electrically large structures. However, these methods, which only necessitate a surface mesh of the physical domain, can show a lack of flexibility in meshing complex, multiscale structures. Indeed, the standard RT finite elements setting does not allow hanging nodes that may appear during adaptive mesh refinement to capture local EM details. This mesh conformity requirement can significantly limit the performance of classical BEM.

To relax the nature of the allowed meshes, authors in [1] devised a discontinuous Galerkin approximation of boundary integral operators, but at the expense of an increase in the number of unknowns at the nonmatching-grid interfaces, as well as the complexity of the weak formulation. In this work, as an alternative, we propose a new conforming Galerkin approximation of boundary integral equations, based

on the newly introduced virtual elements [2], namely, the natural div-conforming virtual element counterparts of RT elements. The resulting discretization improves the flexibility in dealing with ever-growing complex geometries by supporting polygonal surface meshes.

2 Virtual boundary element method

For the sake of simplicity, we present the discretization of the Electric Field Integral Equation (EFIE) for perfectly conducting bodies.

Let Γ be the nonsmooth boundary of the closed domain $\Omega \subset \mathbb{R}^3$, \mathbf{n} its outward unit normal and \mathbf{E}^I the electric field of the plane wave illuminating Ω , with $\kappa > 0$ being its wavenumber. Let consider a polygonal mesh \mathcal{T}_h that approximates Γ (h denotes the mesh size), we derive the virtual element-based discretization of the EFIE (here termed V-BEM) by seeking an approximation \mathbf{J}_h of the only unknown \mathbf{J} , being the electric current living on Γ , in the div-conforming lowest-order serendipity virtual space \mathcal{V}_h [2] defined on \mathcal{T}_h . The virtual basis functions spanning this space are not analytically known, but their divergence and their L^2 -orthogonal projection $\pi_h^0: \mathcal{V}_h \rightarrow (\mathbb{P}_1(\mathcal{T}_h))^3$ such that $\pi_h^0 \mathbf{u} \cdot \mathbf{n} = 0$ can be explicitly computed from the only knowledge of the degrees of freedom (dofs) related to the virtual functions. As a result, the V-BEM weak formulation reads as: Find $\mathbf{J}_h \in \mathcal{V}_h$ such that $\forall \mathbf{J}'_h \in \mathcal{V}_h$,

$$a_h(\mathbf{J}_h, \mathbf{J}'_h) = f_h(\mathbf{J}'_h),$$

where the sesquilinear form is

$$a_h(\mathbf{J}_h, \mathbf{J}'_h) = \langle \mathbf{S}_\kappa \pi_h^0 \mathbf{J}_h, \pi_h^0 \mathbf{J}'_h \rangle - \frac{1}{\kappa^2} \langle S_\kappa \nabla_\Gamma \cdot \mathbf{J}_h, \nabla_\Gamma \cdot \mathbf{J}'_h \rangle,$$

and the right-hand side (rhs) is

$$f_h(\mathbf{J}'_h) = -\frac{1}{i\kappa Z_0} \langle \mathbf{n} \times \mathbf{E}^I \times \mathbf{n}, \pi_h^0 \mathbf{J}'_h \rangle,$$

where \mathbf{S}_κ and S_κ are the vectorial and scalar single layer boundary integral operators of the

Helmholtz equation [3], respectively, $\langle \cdot, \cdot \rangle$ is the classical duality pairing and Z_0 is the impedance in vacuum.

3 Theoretical and Implementation issues

The particularity of this V-BEM formulation comes from the L^2 -projector that leads to an approximate sesquilinear form a_h and an approximate rhs f_h , which is one of the principal analysis difficulties. On the basis of the classical analysis methodology (Hodge decompositions) of the EFIE [3] and for a family of polygonal meshes satisfying classical VEM regularity assumptions, we have established the well-posedness of the formulation by proving a uniform discrete *inf-sup* condition:

$$\exists C > 0, h_0 > 0 \text{ tel que } \forall h \in (0, h_0),$$

$$\inf_{\mathbf{J}_h \in \mathcal{V}_h} \sup_{\mathbf{J}'_h \in \mathcal{V}_h} \frac{|a_h(\mathbf{J}_h, \mathbf{J}'_h)|}{\|\mathbf{J}_h\|_{\mathbf{W}} \|\mathbf{J}'_h\|_{\mathbf{W}}} \geq C.$$

Moreover, the *a priori* error analysis reveals the quasi-optimal convergence of the scheme in the natural norm of $\mathbf{W} = \mathbf{H}^{-1/2}(\text{div}_\Gamma, \Gamma)$. As a result, we get same convergence rates as the RT discretization.

From the implementation point of view, the way of assembling the Galerkin matrix is the same as the one used in RT finite elements setting. Nevertheless, a difficulty comes from the evaluation of the elementary singular integrals on polygons. To address this issue, we have employed a strategy that consists in partitioning each polygon into a minimum number of triangles in order to exploit classical numerical singularity extraction techniques. Finally, the resulting linear system is solved using dense direct methods with a fully MPI-type parallelism based on LAPACK library.

4 Numerical Results

We numerically analyze the proposed V-BEM scheme taking as reference approach the BEM. Firstly, we consider the EM wave scattering by a sphere in order to investigate the mesh convergence in the $\mathbf{H}(\text{div}_\Gamma, \Gamma)$ -norm ($\kappa = 10.5 \text{ m}^{-1}$). To that end, we use triangular meshes for the BEM, meshes mixing triangles and n -sided polygons (*V-BEM_a*) and n -sided polygonal meshes (*V-BEM_b*), with $n > 3$. We remark that both schemes exhibit the same convergence rates (fig. 1). Afterwards, we demonstrate the effective-

ness of using the V-BEM approach on a cone-sphere with gap benchmark along with a surface mesh that is refined near the gap to capture its RCS signature (fig. 2) ($\kappa = 62.9 \text{ m}^{-1}$). Compared to the BEM that handles a dense triangular mesh with no hanging nodes, the V-BEM shows a clear performance gain for an equivalent accuracy (tab. 1).

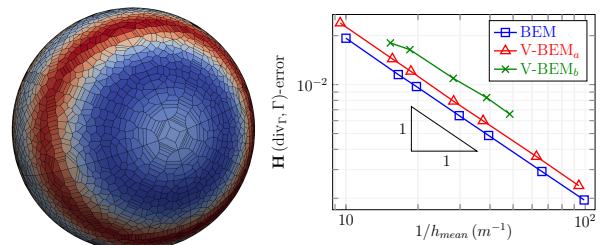


Figure 1: Distribution of \mathbf{J}_h (A.m^{-1}) real part (V-BEM_b, left) and relative error on \mathbf{J} vs. the average mesh size h_{mean} .

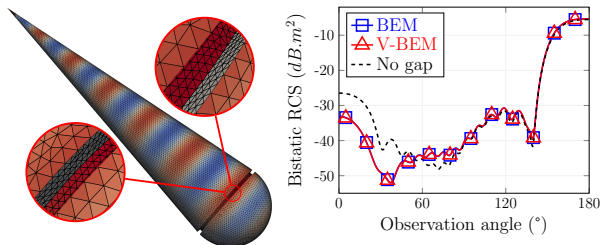


Figure 2: Distribution of \mathbf{J}_h (A.m^{-1}) real part (V-BEM, left) and the RCS of the cone-sphere.

	BEM	V-BEM	Gains
h_{mean} (m)	$\pi/(15\kappa)$	$\pi/(13\kappa)$	
# dofs	61662	48640	-21%
Memory fp (Gb)	28.3	17.6	-38%
CPU time (s)	269	192	-29%

Table 1: Performances stats.

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