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Evaluation of two boundary integral formulations for the Eddy current nondestructive testing of metal structures

Edouard Demaldent^{1,*}, Marc Bakry¹, Adrien Merlini², Francesco P. Andriulli³, Marc Bonnet⁴

¹Université Paris-Saclay, CEA List, F-91120 Palaiseau, France

²IMT Atlantique, France

³Politecnico de Torino, Italia

⁴POEMS (CNRS, INRIA, ENSTA Paris), France

*Email: edouard.demaldent@cea.fr

Abstract

We investigate two boundary integral formulations for the resolution of the Maxwell equations in the Eddy Current (EC) regime in a context of nondestructive testing (NdT). The first one, based on an approximation of the Maxwell equations, requires a *loop-star* decomposition of the surface currents and the global *loops* are constructed manually for non-simply connected domains. The second formulation is stabilized by using quasi-Helmholtz projectors, thus avoiding the definition of global *loops*.

Keywords: boundary integral equations, eddy current simulations, nondestructive testing, conditioning

1 Introduction

In the framework of nondestructive evaluation of metal structures, the harmonic Maxwell equations must be solved in the frequency range from 1 kHz to 1 MHz and conductivities of the order of 1 MS. It allows the evaluation of the impedance variation of the excitation coils for multiple flaw configurations and one can compute a quantitative information on the measured signals when one or more flaws are present. Multiple methods like the Finite Element Method exist but the boundary integral methods make the construction of the model and the management of the electromagnetic sources easier as only the interface between the different media must be meshed. The well-known PMCHWT formulation [6] suffers from the "low-frequency breakdown" in the EC regime and the surface current (\mathbf{J}, \mathbf{M}) are decomposed in *loop* and *star* components ("**LT**"). At the CEA-List, an asymptotic approximation of "**LT**" in the EC regime [1] is used (named "**EC**"). It is an issue on non-simply connect surfaces as the approximation basis must be completed with so-called

globals loops. While this operation is trivial on structured meshes, it is hard to have a *black-box* algorithm for general meshes. To circumvent this issue, we evaluate another formulation [2,3] (named "**QH**") based on quasi-Helmholtz projectors.

2 Boundary integrals in the EC regime

"**EC**" is an asymptotic approximation of "**LT**" following two parameters $\gamma \ll 1$ characterizing the ratio between frequency and conductivity and $\xi = \mathcal{O}(1)$ characterizing a reference length of the inspected area with respect to the skin depth. Moreover, the excitation field should be divergence free. The unknown J^T (*star* component of \mathbf{J}) can be discarded when only the impedance of the exciting coils is needed thus reducing from a 4×4 blocks system to a 3×3 system whose multiple blocks are zero. This formulation is particularly attractive for direct solvers.

The formulation [2] is based on the analysis of the behaviour of the operators when the conductivity σ is fixed and the frequency goes to 0. It has been extended to other regimes and features a preconditioner for densely meshed geometries [4]. As presented in [3,4], its application domain is *different* from "**EC**". It is based on two *quasi-Helmholtz* projectors on the subspaces *loop* and *star*, and their complement on the primal and dual Buffa-Christiansen space [7]. The projectors on the *loop* spaces project *implicitly* on the global *loops* which must not be constructed anymore. These projectors are then normalized by constants bringing in the asymptotic behaviour of the blocks of the "**LT**" system and the behaviour of the excitation. The final system of equations is therefore optimized for the current frequency regime [3].

There exists other formulations based on a

double-layer operator integral formulations [5]. However, preliminary results show that a greater number of unknowns is required to achieve the same accuracy and they are not taken into account in this study.

3 Numerical study of the conditioning

We propose a *first academic application* which does not correspond to a realistic NdT context. We consider a tore with radius $R = 1$ and cross-section radius $r = 0.3$. Its conductivity is $\sigma = 10^6$ (steel-like) and the relative permeability is $\mu_r = 1$. The excitation is a coil with radius $R_c = 5$. The geometry is a structured mesh with 512 quads so that the whole matrix is stored in the RAM. In this configuration, two global *loops* are required for "LT" and "EC". The conditioning of the system matrix \mathbf{A} is computed as

$$R_{\text{cond}} = \|\mathbf{A}\|_2 \cdot \|\mathbf{A}^{-1}\|_2 \quad (1)$$

for different values of the frequency. A diagonal preconditioner is applied to *all* formulations. The results are in TABLE 1.

Table 1: Condition number

f (Hz)	LT	EC	QH
10^{-6}	$1.182 \cdot 10^6$	$7.118 \cdot 10^5$	$1.844 \cdot 10^4$
10^{-2}	$1.489 \cdot 10^4$	$8.931 \cdot 10^3$	$1.702 \cdot 10^4$
10^0	$7.771 \cdot 10^3$	$3.453 \cdot 10^3$	$3.145 \cdot 10^3$
10^2	$9.44 \cdot 10^3$	$3.601 \cdot 10^3$	$1.607 \cdot 10^4$
10^4	$9.208 \cdot 10^3$	$4.088 \cdot 10^3$	$1.169 \cdot 10^6$

For $f = 10^{-6}$ Hz, the best condition number is obtained with "QH" as expected. Moreover, the impedance variation (not represented here) diverges for "EC" from the reference "LT" because the frequency is too low ($\xi \ll 1$). In the gap $[10^{-2}, 10^2]$ Hz, it is excellent for all three formulations. For higher frequencies, the condition number for "QH" worsen but the impedance is correct. This is expected behaviour as the preconditioner given in [3] must be adapted.

4 Conclusion

The "QH" formulation allows the implicit management of the global *loop* functions. On the other side, the "EC" formulation enables the computation of dense problems up to 10^5 unknowns on a regular computer but is limited by the complexity of the geometries. Therefore, "QH" is promising for complex studies once the

preconditioner is adapted. Future work will be carried out to apply the "QH" formulation to realistic NdT configurations and to automatize the construction of the global *loops*.

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