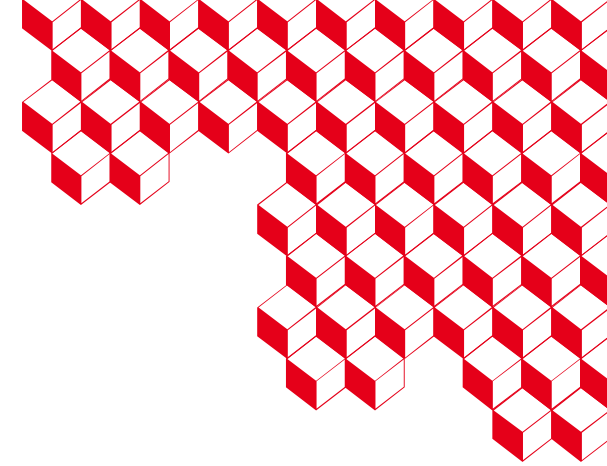




list



# Evaluation of two boundary integral formulations for the eddy current nondestructive testing of metal structures

Edouard Demaldent<sup>1</sup>, Marc Bakry<sup>1</sup>, Adrien Merlini<sup>2</sup>, Francesco P. Andriulli<sup>3</sup>, Marc Bonnet<sup>4</sup>

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<sup>2</sup> IMT Atlantique, France

<sup>3</sup> Politecnico de Torino, Italia

<sup>4</sup> POEMS (CNRS, INRIA, ENSTA Paris), France



WAVES 2024

16th International Conference on  
Mathematical and Numerical Aspects of Wave Propagation

30 June - 5 July 2024  
Harnack House, Berlin, Germany





# **1** Eddy current testing & Motivations

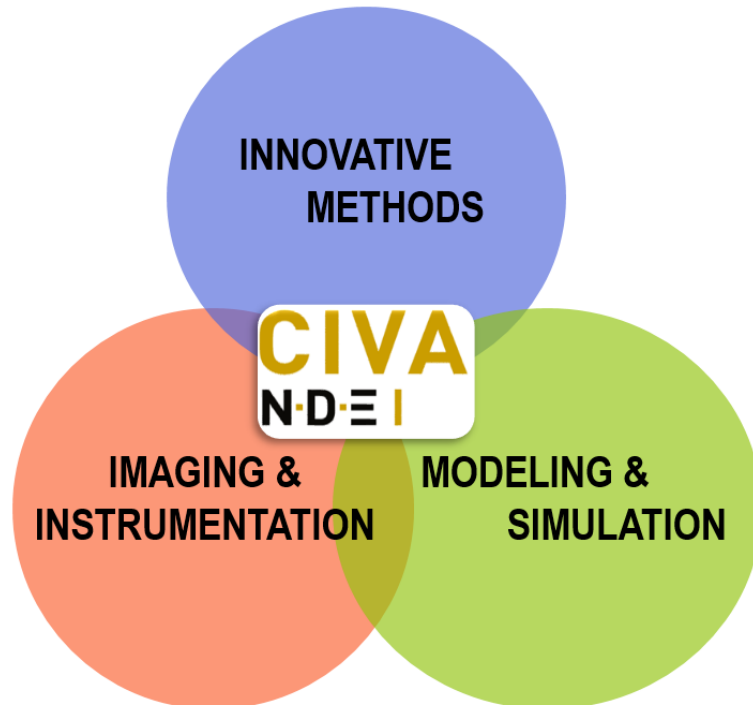
**2** Boundary integral formalisms

**3** Preliminary results

# Non Destructive Testing at CEA List

## NDT - CEA List

- ~ 110 staff members including 25 PhD & post-doc
- Strong academic & industrial partnership
- Three-fold R&D activity



## Simulation

- CIVA software designed for NDT practitioners
- Analytical, numerical & hybrid modeling approaches
- Addressing the main NDT modalities

**CIVA**  
NDEI

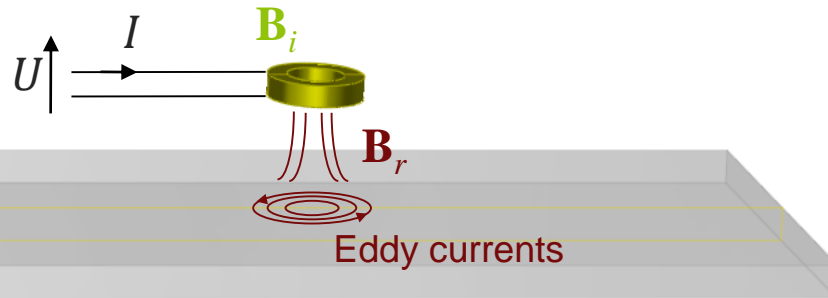
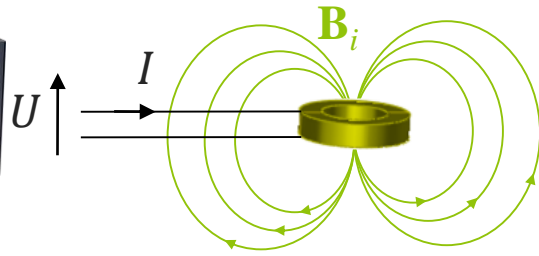
300 companies in 40 countries



# Eddy current testing



Impedance analyzer



1. Injecting a current  $I$  into a coil and creating a magnetic field

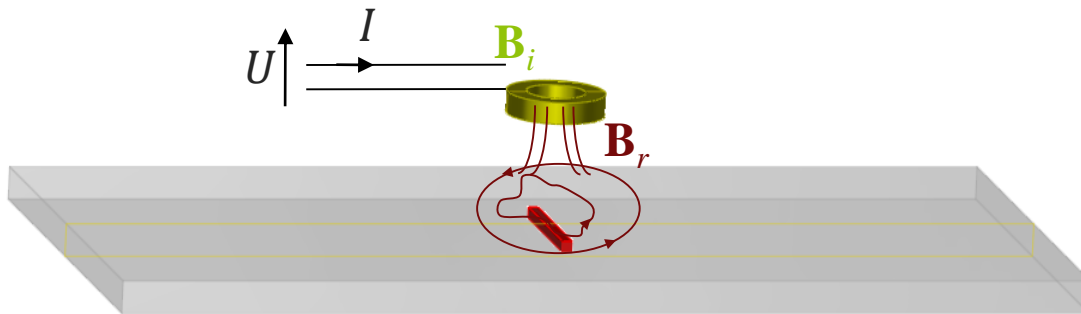
$$U = Z.I \text{ with } Z = jL_0\omega$$

2. In a conductive material, (eddy) currents are induced and a magnetic reaction field is created

$$Z = R_1 + jL_1\omega$$

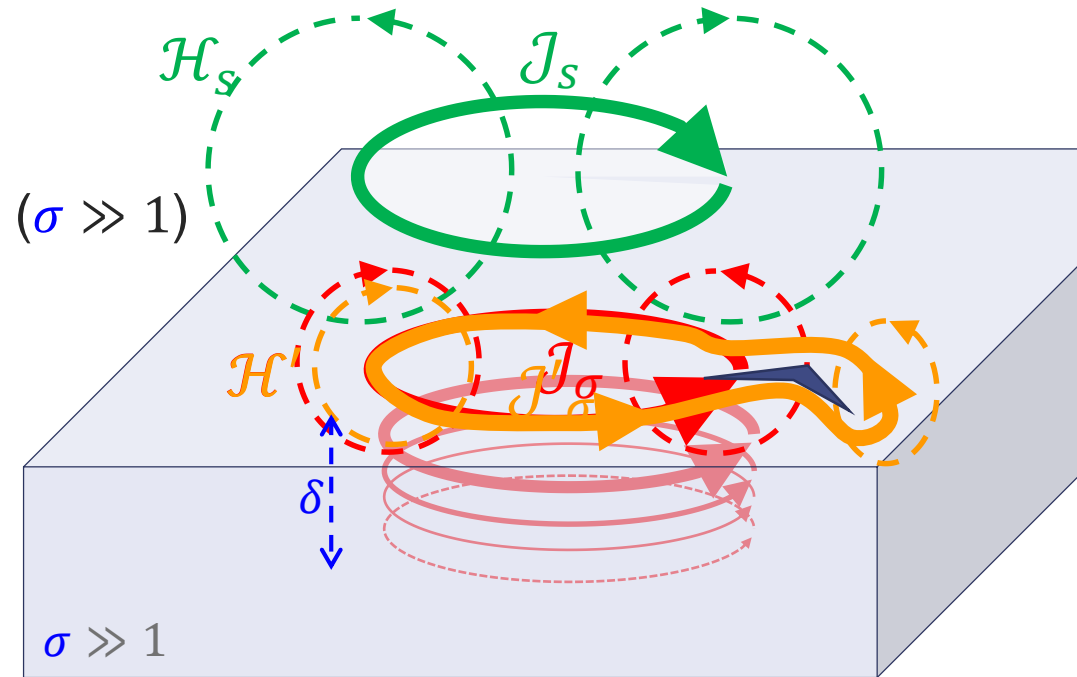
3. In the presence of a flaw, eddy currents and the reaction field are modified

$$Z = (R_1 + \Delta R) + j(L_1 + \Delta L)\omega$$



# Eddy current testing

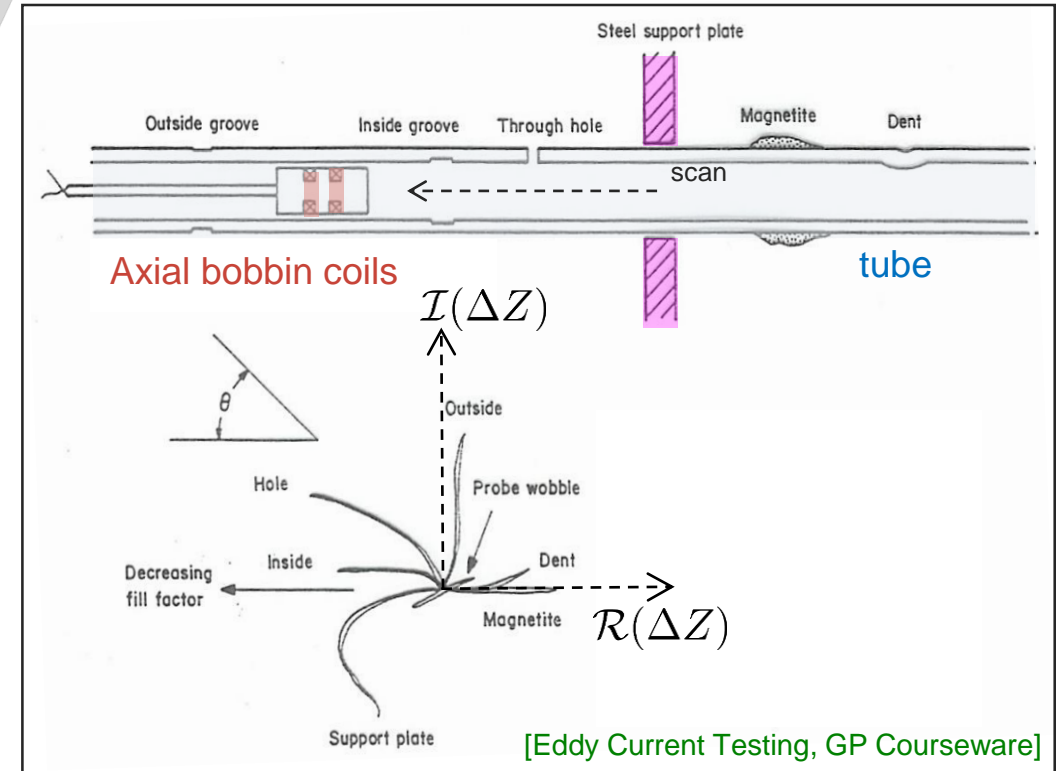
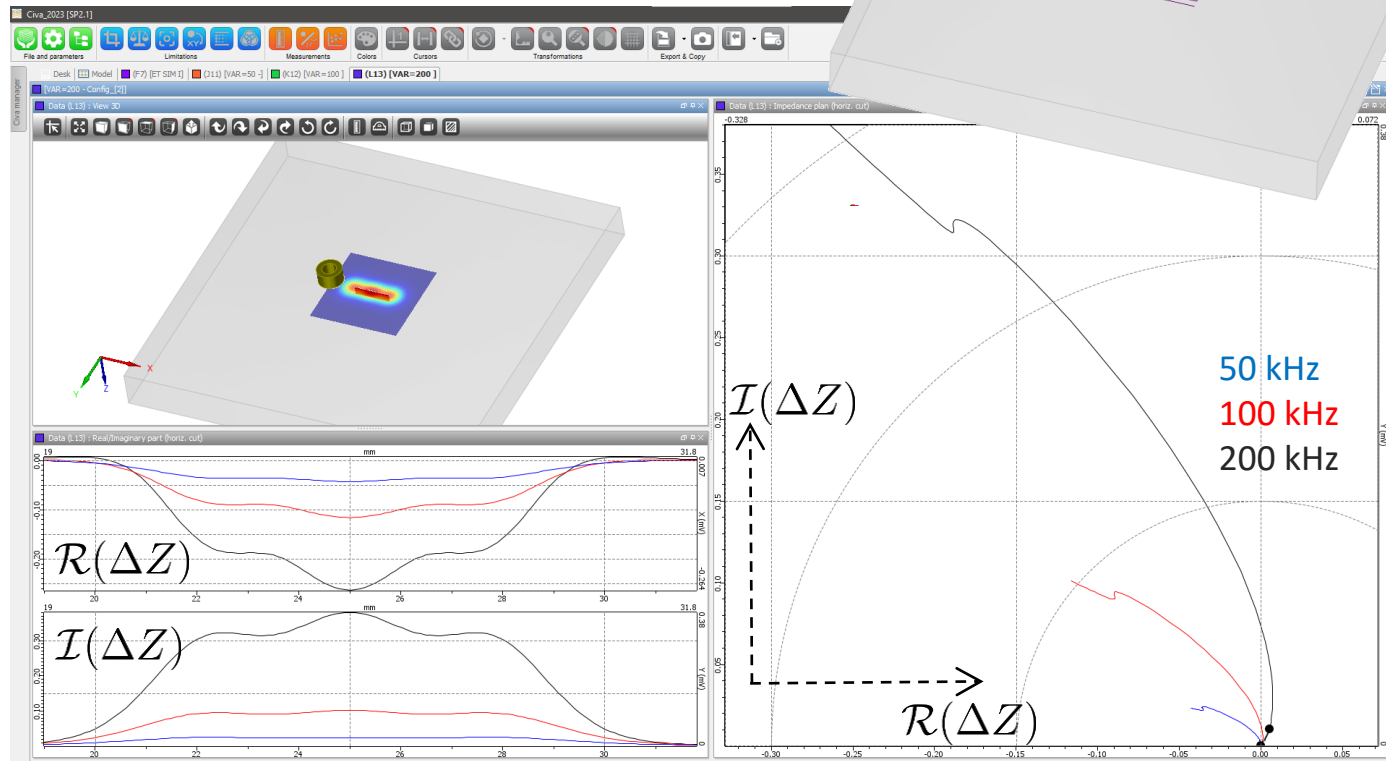
- **Input.** Closed loop of a sinusoidal source current  $J_s$
- **Compute.** Eddy currents  $J_\sigma = \sigma \mathcal{E}$  in a conductive medium ( $\sigma \gg 1$ )
- $J_\sigma$  decreases exponentially w.r.t. the skin depth  $\delta$
- Low frequency to penetrate the conductive medium
- A defect disturbs the eddy currents  $J'_\sigma$
- **Output.** Impedance is given by applying a reciprocity identity [Auld, 81]



Conductivity $\sigma = 1 \text{ MS/m}$	} $\delta = 1/\sqrt{\pi\mu f\sigma} \approx 1.6 \text{ mm}$
Frequency $f = 100 \text{ kHz}$	
$\lambda_0 \approx 3 \text{ km}, \kappa_0^2 = \omega^2 \epsilon_0 \mu_0 \approx 4 \times 10^{-6}$	
$\lambda_1 \approx 7 \text{ mm}, \kappa_1^2 = \kappa_0^2 + i\omega\mu\sigma \approx 4 \times 10^{-6} + i 8 \times 10^{+5}$	

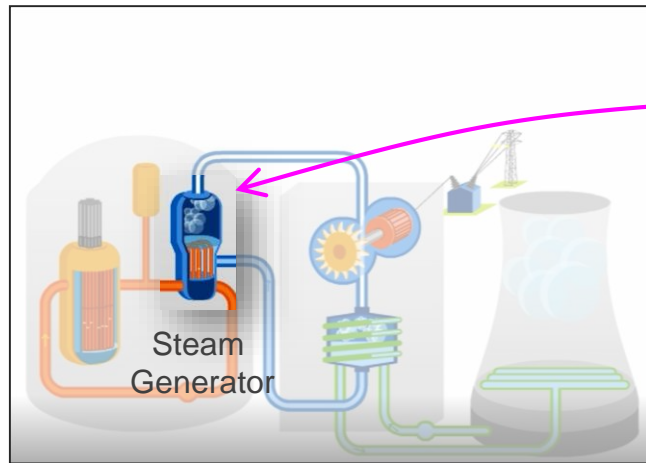
$$I^2 Z = \int_{\Gamma} \mathcal{H}_s \cdot (\mathcal{E} \times \mathbf{n}) + \mathcal{E}_s \cdot (\mathcal{H} \times \mathbf{n}) \, dx.$$

# Eddy current testing

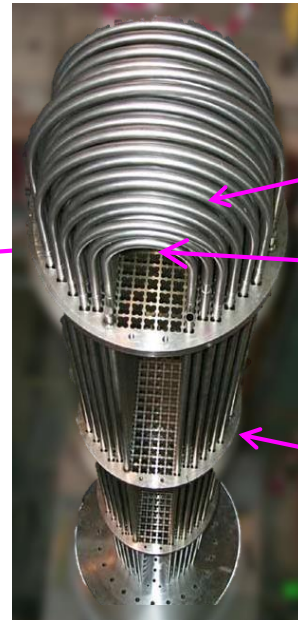


# Ex.1 Steam Generator tube inspection

- NDT-oriented module with parametric description



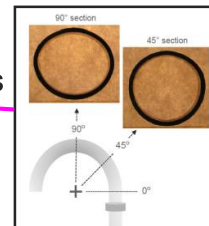
Pressurized water reactor



SG tubes

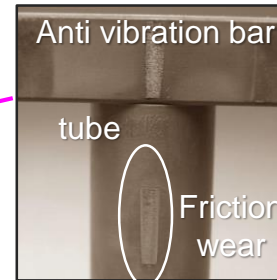
Anti-Vibration Bar

Small-radius bent tubes



[Bakhtiari et al. EPRI 2012]

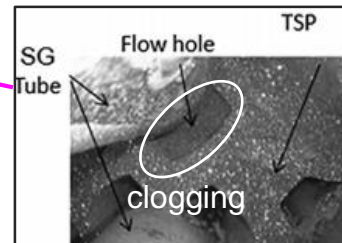
Tube Support Plate (TSP)



Anti vibration bar

tube

Friction wear

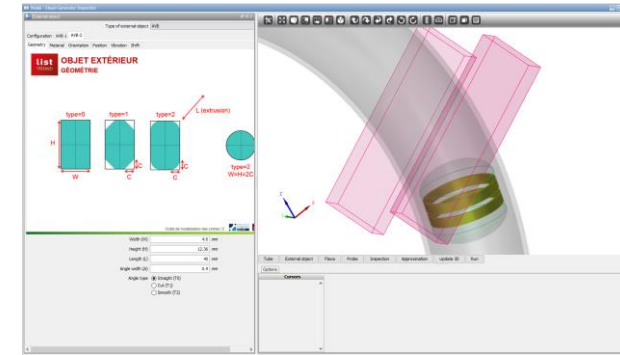


SG Tube

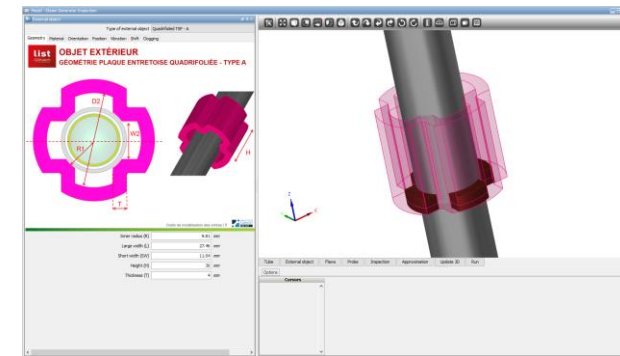
Flow hole

TSP

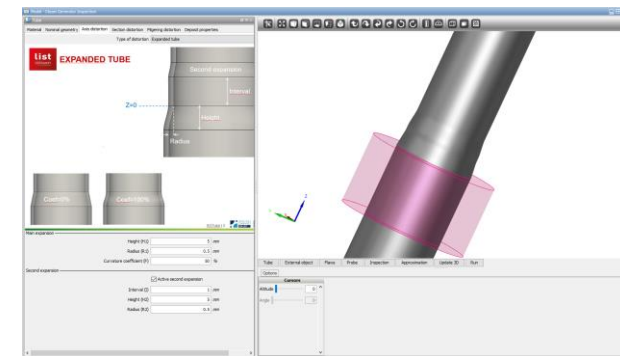
clogging



Friction wear under AVB



Tube support plate clogging



Expansion transition zone

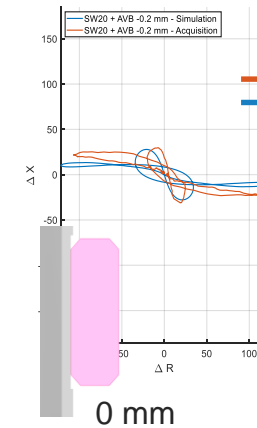
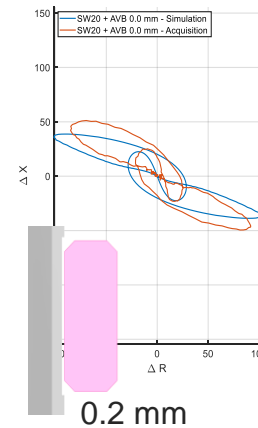
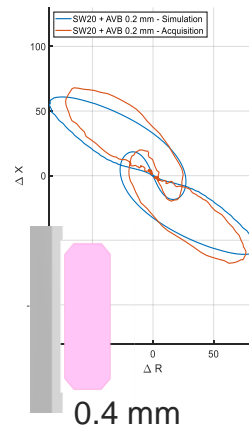
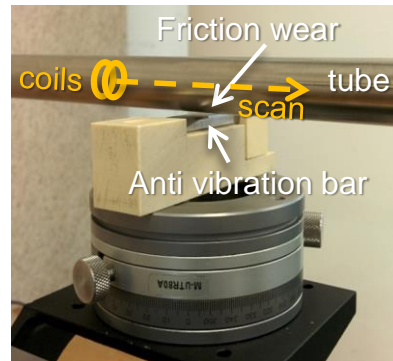


# Ex.1 Steam Generator tube inspection

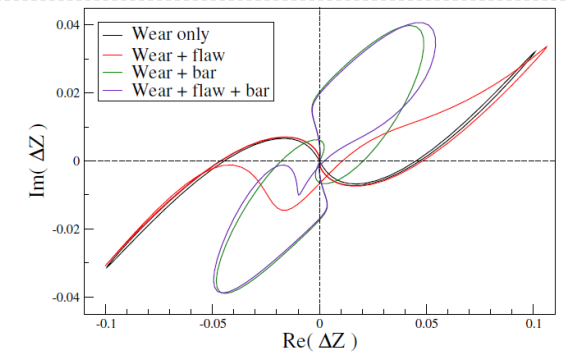
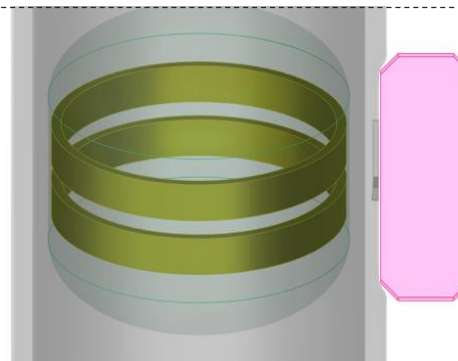
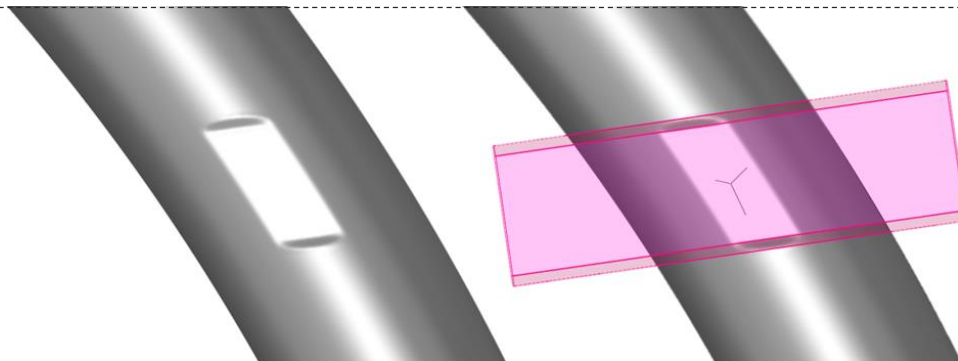
- NDT-oriented module with parametric description
- Experimental validation, sensitivity studies, optimization (IRSN, EDF...)

[NDE in Nuclear 2023, Cofrend 23, ECNDT 23, WCNDT 24...]

**IRSN**  
INSTITUT DE RADIOPROTECTION  
ET DE SÛRETÉ NUCLÉAIRE



Experiment  
Simulation





# Ex.1 Steam Generator tube inspection

- NDT-oriented module with parametric description
- Experimental validation, sensitivity studies, optimization (IRSN, EDF...)
- Based on a robust BEM formalism for Eddy Current (EC) simulation

[NDE in Nuclear 2023, Cofrend 23, ECNDT 23, WCNDT 24...]

**1st kind direct BIE + Helmholtz decomposition + Asymptotic expansion**



3/4 reduction of the number of unknowns – numerical noise reduction

Parametric meshing (almost structured) – HO compatible (mesh & basis functions)

Access to LU factorization on a standard PC (ooc block-LU)

Limited but robust vs iterative solvers, full scan simulation in a few minutes to one hour ...

*...Good enough for this application !*

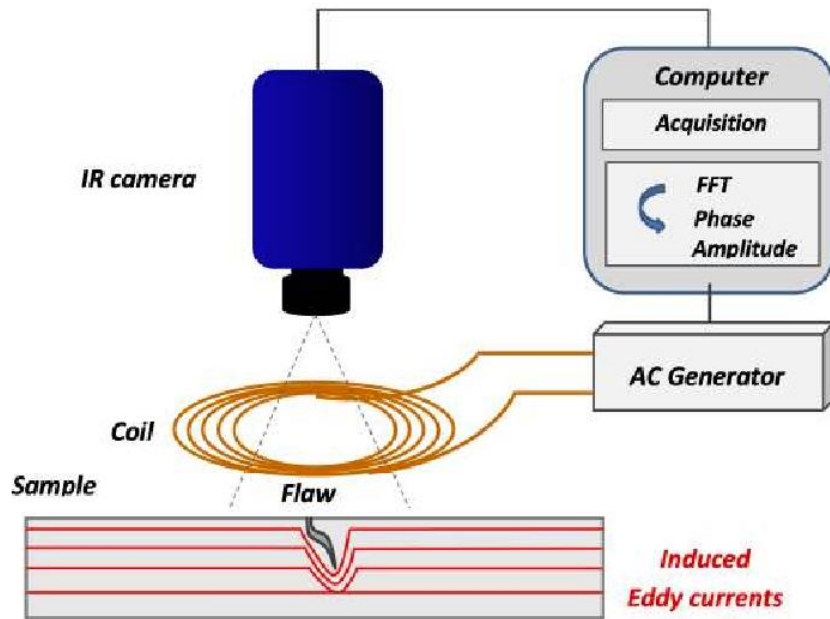
**2nd kind BEM - EC:** Mayergoyz 1982, Kalaichelvan 1987, Schmindlin 2001, Smajic 2007, Bao 2020-21...

**1st kind BEM - EC:** Hipmair 2007, Bonnet & Demaldent CAMWA 2019 & 2023

**1st kind BEM:** Chhim 2020 (rescaling & preconditioning)

# Ex.2 Induction heating for thermography inspection

- Induction thermography for the detection of small cracks near the surface of metallic parts



Induction thermography principle

[P. Bouteille *et al.*, ECNDT 2014]

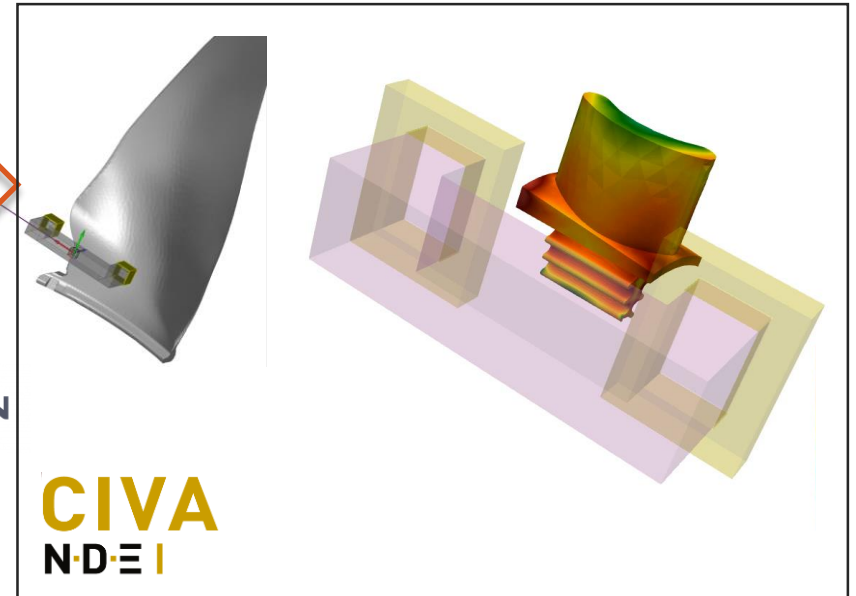
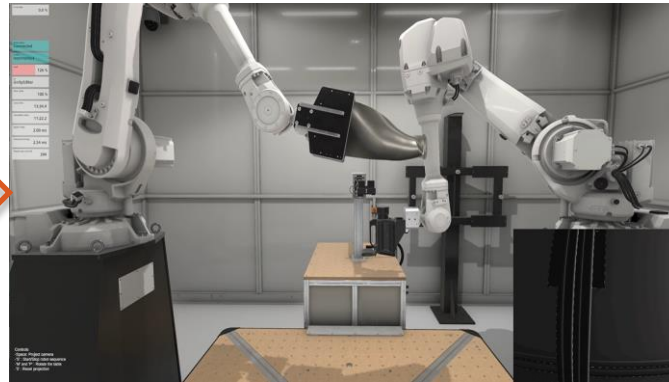
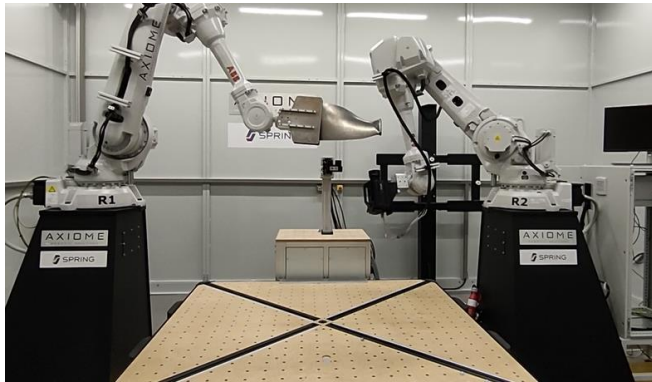
Excitation at short times using electromagnetic induction (eddy currents)  
Computation of the dissipated power volume density  $P$  (Joule effect)



$$P = \int_{\Omega} \sigma |\mathcal{E}|^2 dV.$$

# Ex.2 Induction heating for thermography inspection

- Induction thermography for the detection of small cracks near the surface of metallic parts
- Simulation to optimize the induction patterns through the design of inductors and trajectories



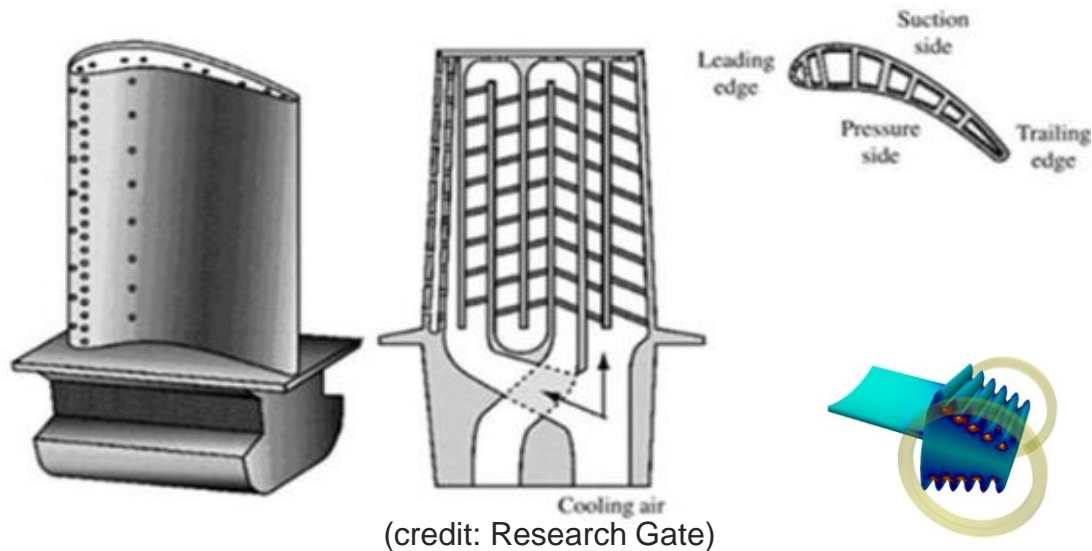
Experimental platform  (and its digital twin) installed at CEA LIST 

Pre-processing the LU factorization allows reasonable simulation times  
Revised CAD files & meshing libraries ; trivial DDM on large specimen...

Simulation of EC induction heating  
(Boundary Element Method)

# Ex.2 Induction heating for thermography inspection

- Induction thermography for the detection of small cracks near the surface of metallic parts
- Simulation to optimize the induction patterns through the design of inductors and trajectories
- Practical limitations: managing an arbitrarily complex CAD with numerous tunnels



Explicit Helmholtz decomposition ;  
Nb of unknowns  $\nearrow$  (conditioning)

Alternative formalism:

[Chim, Merlini, Rahmouni, Guzman, Andriulli, *Eddy current modeling in multiply connected regions via a full-wave solver based on the quasi-Helmholtz projectors*, IEEE 2020]

This contribution: introduction, adaptation then evaluation on NDT toy cases.

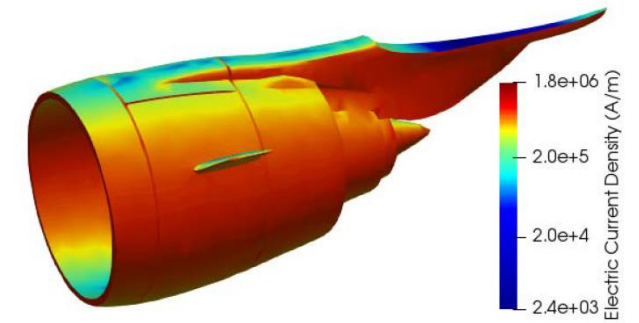


FIGURE 16. Jet engine outer shell of length 0.9 m with 9196 elements and  $\sigma_1 = 2 \times 10^7 \text{ Sm}^{-1}$ ; Electric current density norm given an exciting plane wave of frequency  $f = 10^{-40} \text{ Hz}$  along  $-\hat{z}$  and with  $E_0 = 1 \text{ Vm}^{-1}$ .



**1** Eddy current testing & Motivations

**2** **Boundary integral formalisms**

**3** Preliminary results

# Configuration

3D-bounded object excited by time-harmonic EM fields created by a current density  $\mathbf{J}_s$

Maxwell transmission problem:

$$\begin{aligned} (\mathbf{rot rot} - \kappa_0^2) \mathbf{E}_0 &= i\omega\mu_0 \mathbf{J}_s & \text{in } \Omega_0, & \quad \gamma_{\times}^{-} \mathbf{E}_1 - \gamma_{\times}^{+} \mathbf{E}_0 = \mathbf{0} & \text{on } \Gamma, \\ (\mathbf{rot rot} - \kappa_1^2) \mathbf{E}_1 &= \mathbf{0} & \text{in } \Omega_1, & \quad \mu_r^{-1} \gamma_N^{-} \mathbf{E}_1 - \gamma_N^{+} \mathbf{E}_0 = \mathbf{0} & \text{on } \Gamma, \end{aligned}$$

+ Silver-Müller condition at  $\infty$ ,

with  $\gamma_{\times}^{\pm} \mathbf{u} := \gamma^{\pm} \mathbf{u} \times \mathbf{n}$ ,  $\gamma_N^{\pm} \mathbf{u} := \gamma^{\pm} \mathbf{rot} \mathbf{u}$ .

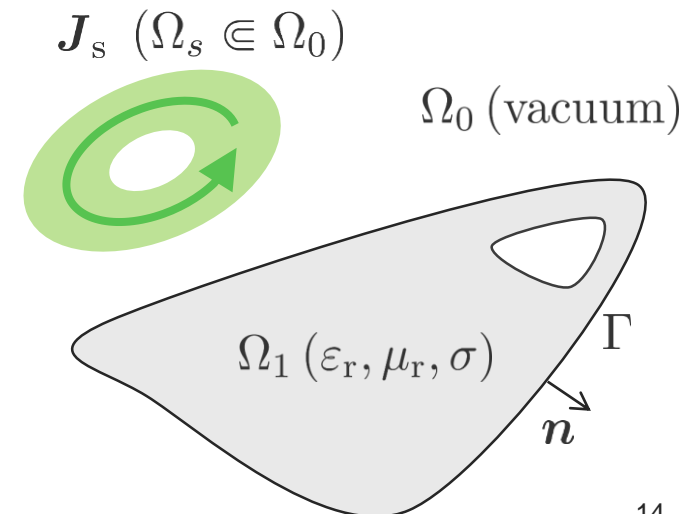
and  $\kappa_0^2 = \omega^2 \varepsilon_0 \mu_0$ ,  $\kappa_1^2 = \kappa_0^2 \varepsilon_r \mu_r$  with  $\varepsilon_r := \varepsilon_r^d + i \frac{\sigma}{\omega \varepsilon_0}$ ,

The incident field is given by the *Biot & Savart law*  $\mathbf{E}^{\text{inc}}(\mathbf{x}) = i\omega\mu_0 \Phi[\mathbf{J}_s]$ ,

$$\Phi[\mathbf{J}] := \int_{\mathbb{R}^3} G(\mathbf{x} - \mathbf{x}'; \kappa_0) \mathbf{J}(\mathbf{x}') dV(\mathbf{x}'), \quad G(\mathbf{z}; \kappa) = \frac{e^{i\kappa|\mathbf{z}|}}{4\pi|\mathbf{z}|}, \quad \mathbf{z} \in \mathbb{R}^3 \setminus \{\mathbf{0}\}.$$

## Assumption

$$\text{div } \mathbf{J}_s = 0 \quad \text{in } \Omega_s, \quad \mathbf{J}_s \cdot \mathbf{n} = 0 \quad \text{on } \partial\Omega_s.$$





# Maxwell BIE

Maxwell boundary integral operators from  $\mathcal{V} \rightarrow \mathcal{V} := \{v \in H_{\parallel}^{-1/2}(\Gamma) : \operatorname{div}_S v \in H^{-1/2}(\Gamma)\}$ ,

$$\mathbf{S}_\ell := \frac{1}{2}(\gamma_{\times}^+ + \gamma_{\times}^-)\mathcal{S}_\ell, \quad \mathbf{D}_\ell := \frac{1}{2}(\gamma_{\times}^+ + \gamma_{\times}^-)\mathcal{D}_\ell, \quad \mathbf{A}_\ell := \frac{1}{2}(\gamma_{\times}^+ + \gamma_{\times}^-)\mathcal{A}_\ell, \quad \mathbf{K}_\ell := \frac{1}{2}(\gamma_{\times}^+ + \gamma_{\times}^-)\nabla\mathcal{A}_\ell \circ \operatorname{div}_S,$$

with  $\mathcal{S}_\ell[u] = \mathcal{A}_\ell[u] + \kappa_\ell^{-2}\nabla\mathcal{A}_\ell[\operatorname{div}_S u]$  ( $\kappa_\ell \neq 0$ ),  $\mathcal{D}_\ell[u] = L_c \operatorname{rot} \mathcal{A}_\ell[u]$ ,

where  $\mathcal{A}_\ell[u](x) := \int_{\Gamma} G(x - x'; \kappa_\ell)u(x') \, dS(x')$ .

Surface unknowns densities  $\gamma_N^+ \mathbf{E}_0 = i\omega\mu_0 \mathbf{J}$ ,  $\gamma_N^- \mathbf{E}_1 = i\omega\mu_0\mu_r \mathbf{J}$ ,  $\gamma_{\times}^+ \mathbf{E}_0 = \gamma_{\times}^- \mathbf{E} = i\omega\mu_0 L_c \mathbf{M}$ .

$L_c$  is a characteristic length (e.g. coil diameter) allowing all potentials to have the same unit.

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Surface unknowns densities  $\gamma_N^+ E_0 = i\omega\mu_0 J$ ,  $\gamma_N^- E_1 = i\omega\mu_0\mu_r J$ ,  $\gamma_{\times}^+ E_0 = \gamma_{\times}^- E = i\omega\mu_0 L_c M$ .

Governing system of boundary integral equations **PMCHWT** (Poggio-Miller-Chang-Harrington-Wu-Tsai):

$$(\mathbf{Z}_0 + \mathbf{Z}_1) \mathbf{X} = \mathbf{Y},$$

$$\mathbf{Z}_\ell = \begin{bmatrix} \mu_{r\ell} \mathbf{S}_\ell & \mathbf{D}_\ell \\ \mathbf{D}_\ell & \mu_{r\ell}^{-1} (\kappa_\ell L_c)^2 \mathbf{S}_\ell \end{bmatrix}, \quad \mathbf{X} = \{J, M\}^T, \quad \mathbf{Y} = \{\gamma_{\times}^+ \Phi[J_s], L_c \gamma_N^+ \Phi[J_s]\}^T.$$

# Maxwell BIE

Maxwell boundary integral operators from  $\mathcal{V} \rightarrow \mathcal{V} := \{v \in H_{\parallel}^{-1/2}(\Gamma) : \text{div}_S v \in H^{-1/2}(\Gamma)\}$ ,

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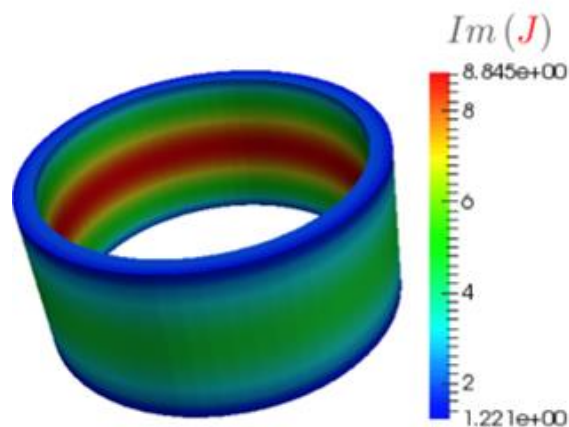
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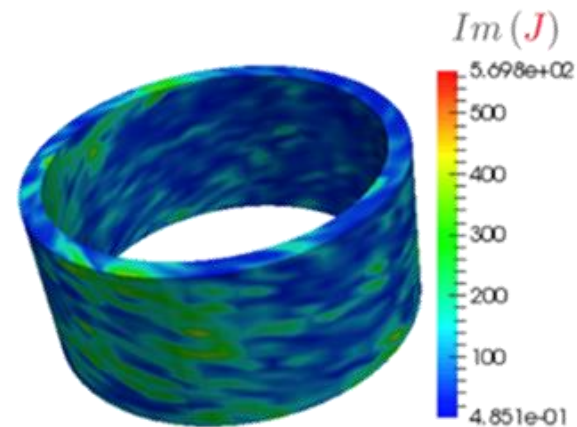
Surface unknowns densities  $\gamma_N^+ E_0 = i\omega\mu_0 J$ ,  $\gamma_N^- E_1 = i\omega\mu_0\mu_r J$ ,  $\gamma_{\times}^+ E_0 = \gamma_{\times}^- E = i\omega\mu_0 L_c M$ .



The single-layer operator suffers from a *low-frequency breakdown* when  $\kappa_l \searrow 0$ .



Reference



Spurious current  
(Low frequency breakdown)

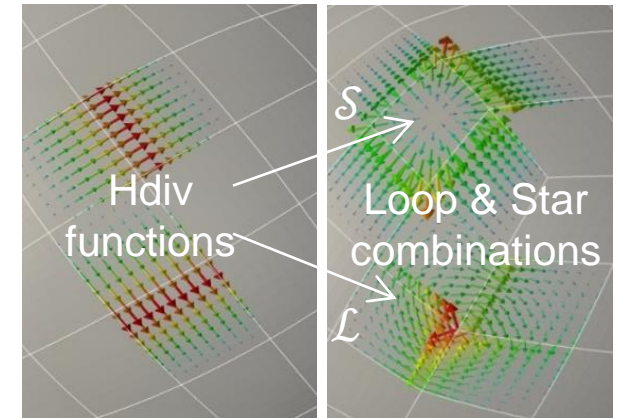
# Helmholtz-Hodge decomposition

Helmholtz-Hodge decomposition of the space  $\mathcal{V} := \{v \in H_{\parallel}^{-1/2}(\Gamma) : \text{div}_S v \in H^{-1/2}(\Gamma)\}$ ,

$$\mathcal{V} = \mathcal{V}_L \oplus \mathcal{V}_S, \quad \mathcal{V}_L := \{u \in \mathcal{V} : \text{div}_S u = 0\}, \quad \mathbf{J} = \mathbf{J}_L + \mathbf{J}_S, \quad \mathbf{M} = \mathbf{M}_L + \mathbf{M}_S$$

$$\mathbf{Z}_\ell = \begin{bmatrix} \mu_{rl} \mathbf{A}_\ell & \mu_{rl} \mathbf{A}_\ell & & \mathbf{D}_{l*} & & \mathbf{D}_\ell \\ \mu_{rl} \mathbf{A}_\ell & \mu_{rl} \mathbf{S}_\ell & & \mathbf{D}_\ell & & \mathbf{D}_\ell \\ & & & & \mu_{rl}^{-1} (\kappa_\ell L_c)^2 \mathbf{A}_\ell & \mu_{rl}^{-1} (\kappa_\ell L_c)^2 \mathbf{A}_\ell \\ \mathbf{D}_{l*} & \mathbf{D}_\ell & & & & \\ & \mathbf{D}_\ell & & & \mu_{rl}^{-1} (\kappa_\ell L_c)^2 \mathbf{A}_\ell & \mu_{rl}^{-1} (\kappa_\ell L_c)^2 \mathbf{S}_\ell \end{bmatrix},$$

$$\mathbf{X} = \{\mathbf{J}_L, \mathbf{J}_S, \mathbf{M}_L, \mathbf{M}_S\}^T.$$



Discrete decomposition: Loop-Tree/Star combination of Hdiv functions

$\mathcal{V}_L = \text{rot}_S H^{1/2}(\Gamma)$  in a simply-connected body.



Circumvented *low frequency breakdown* but unbalanced system (execrable conditioning)

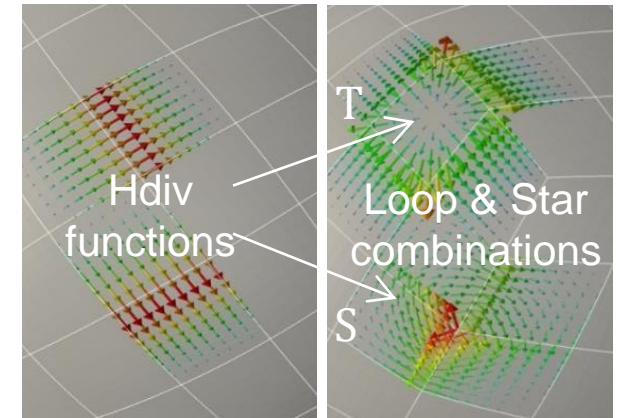
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$$\mathbf{X} = \{\mathbf{J}_L, \mathbf{J}_S, \mathbf{M}_L, \mathbf{M}_S\}^T.$$



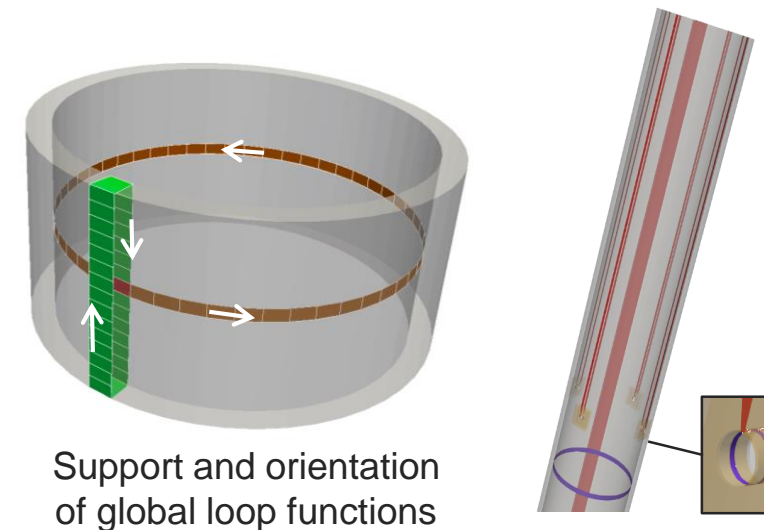
Discrete decomposition: Loop-Tree/Star combination of Hdiv functions

$\mathcal{V}_L = \text{rot}_S H^{1/2}(\Gamma)$  in a simply-connected body.

$\mathcal{V}_L = \text{rot}_S H^{1/2}(\Gamma) \oplus \mathcal{V}_H$  otherwise where  $\mathcal{V}_H$  is a finite-dimensional space of *global loop harmonic* functions (2 per tunnel/handle)



Non-trivial on arbitrarily complex geometries



Support and orientation of global loop functions

# Asymptotic framework

## Assumption

$$\sigma = \eta^{-1} \xi^2 \sigma_{\text{ref}}.$$

We consider the low frequency & high conductivity case:  $\eta := \kappa_0 L_c = o(1)$ ,

Where  $\xi = \sqrt{2/\mu_r} L_c / \delta$  is close to the inverse normalized skin depth  $\delta = \sqrt{2/\omega \sigma \mu_0 \mu_r}$ . ( $\sigma_{\text{ref}} := \frac{1}{L_c} \sqrt{\varepsilon_0 / \mu_0}$ )

The integral operator matrix and the RHS depend on  $\eta$  through the wavenumbers  $\kappa_l$ :

$$\kappa_0 L_c = \eta,$$

$$\kappa_1 L_c = \sqrt{\mu_r (i\xi^2 + \varepsilon_r^d \eta^2)} = q + O(\eta^2), \quad q := \xi \sqrt{i\mu_r}.$$

(does not vanish as  $\eta$  tends to 0)

$$G(\mathbf{z}; \kappa_0) = \underbrace{G_0(\mathbf{z})}_{\text{static kernel}} + \underbrace{\eta G_1}_{\text{Constant w.r.t. } z} + O(\eta^2),$$

$$L_c \nabla G(\mathbf{z}; \kappa_0) = \underbrace{H_0(\mathbf{z})}_{\text{static kernel}} \mathbf{z} + O(\eta^2).$$

$$G(\mathbf{z}; \kappa_1) = G^c(\mathbf{z}) + O(\eta^2), \quad L_c \nabla G(\mathbf{z}; \kappa_1) = H^c(\mathbf{z}) + O(\eta^2),$$

$$\text{with } G^c(\mathbf{z}) = \frac{e^{iq|\mathbf{z}|/L_c}}{4\pi|\mathbf{z}|}, \quad H^c(\mathbf{z}) = \frac{e^{iq|\mathbf{z}|/L_c}}{4\pi|\mathbf{z}|^3} (iq|\mathbf{z}| - L_c) \mathbf{z}.$$

« eddy current » kernel



# Useful properties of potentials and integral operators

## Lemma 1

Let  $\mathbf{A}_{(m)}$ ,  $\mathbf{K}_{(m)}$  and  $\mathbf{D}_{(m)}$  be the integral operators with  $G$  replaced by  $G_m$ .

- (a)  $\mathbf{A}_{(1)}\phi_L = 0$  for any  $\phi_L \in \mathcal{V}_L$ .
- (b)  $\mathbf{D}_{(1)}\phi = 0$  and  $\mathbf{K}_{(1)}\phi = 0$  for any  $\phi \in \mathcal{V}$ .
- (c)  $\langle \tilde{\phi}_L, \mathbf{A}_{(1)}\phi \rangle_x = 0$  for any  $\tilde{\phi}_L \in \mathcal{V}_L$  and  $\phi \in \mathcal{V}$ .
- (d)  $\langle \tilde{\phi}_L, \mathbf{D}_{(0)}\phi_L \rangle_x = 0$  for any  $\tilde{\phi}_L$  and  $\phi_L \in \mathcal{V}_L$  if  $\Gamma$  is simply-connected.

## Lemma 2


Let  $\mathbf{J}_s$  be such that  $\operatorname{div}\mathbf{J}_s$  and  $\mathbf{J}_s \cdot \mathbf{n} = 0$  on  $\partial\Omega_s$ . Let  $\Phi_{(m)}$  be the volume potential obtained by replacing  $G(\cdot; \kappa_0)$  with  $G_m$ . Then:

- (a)  $\Phi_{(1)}[\mathbf{J}_s] = 0$ .
- (b)  $\langle \tilde{\phi}_L, \gamma_N^+ \Phi_{(0)}[\mathbf{J}_s] \rangle_x = 0$  for any  $\tilde{\phi}_L \in \mathcal{V}_L$  if  $\Gamma$  is simply-connected.

# Eddy Current BIE

Leading orders of the *PMCHWT* system:  $\mathbf{Z}_1 = O\left(\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}\right)$ ,  $\mathbf{Z}_0 = O\left(\begin{bmatrix} 1 & 1 & \eta^2 & 1 \\ 1 & \eta^{-2} & 1 & 1 \\ \eta^2 & 1 & \eta^2 & \eta^2 \\ 1 & 1 & \eta^2 & 1 \end{bmatrix}\right)$ ,  $\mathbf{Y} = O\left(\begin{bmatrix} 1 \\ 1 \\ \eta^2 \\ 1 \end{bmatrix}\right)$ .

Rescaled (Maxwell) *PMCHWT* problem:  $\mathbf{Z}^\eta \mathbf{X}^\eta = \mathbf{Y}^\eta$ ,

where  $\mathbf{Z}^\eta = (\mathbf{Z}_0 + \mathbf{Z}_1) \Sigma_\eta$ ,  $\mathbf{Y}^\eta = \mathbf{Y}$ ,  $\mathbf{X} = \Sigma^\eta \mathbf{X}^\eta$ ,  $\Sigma^\eta = \begin{bmatrix} 1 & & & \\ & \eta^2 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$ .  Stabilized conditioning

Eddy current *PMCHWT* problem:  $\mathbf{Z}^{\text{EC}} \mathbf{X}^{\text{EC}} = \mathbf{Y}^{\text{EC}}$ ,

Coincides with [\[Hiptmair, 07\]](#)

where  $\mathbf{Z}^\eta = \mathbf{Z}^{\text{EC}} + O(\eta^2) = (\mathbf{Z}_1^{\text{EC}} + \mathbf{Z}_0^{\text{EC}}) + O(\eta^2)$ ,  $\mathbf{Y}^\eta = \mathbf{Y}^{\text{EC}} + O(\eta^2)$ .

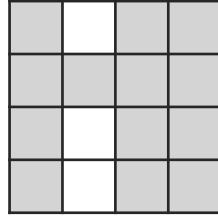
**Th.** [\[Bonnet & Demaldent, 19\]](#)

$$\mathbf{X}^\eta = \mathbf{X}^{\text{EC}} + O(\eta^2).$$

$$\mathbf{Z}_1^{\text{EC}} = \begin{bmatrix} \mu_{ra} \mathbf{A}_C & 0 & \mathbf{D}_C & \mathbf{D}_C \\ \mu_{ra} \mathbf{A}_C & 0 & \mathbf{D}_C & \mathbf{D}_C \\ \mathbf{D}_C - \mathbf{D} & 0 & \mu_r^{-1} q^2 \mathbf{A}_C & \mu_r^{-1} q^2 \mathbf{A}_C \\ \mathbf{D}_C & 0 & \mu_r^{-1} q^2 \mathbf{A}_C & \mu_r^{-1} (q^2 \mathbf{A}_C + \mathbf{K}_C) \end{bmatrix}, \quad \mathbf{Z}_0^{\text{EC}} = \begin{bmatrix} \mathbf{A}_{(0)} & 0 & \mathbf{D}_{(0)}^* & \mathbf{D}_{(0)} \\ \mathbf{A}_{(0)} & \mathbf{K}_{(0)} & \mathbf{D}_{(0)} & \mathbf{D}_{(0)} \\ \mathbf{D}_{(0)}^* & 0 & 0 & 0 \\ \mathbf{D}_{(0)} & 0 & 0 & \mathbf{K}_{(0)} \end{bmatrix}.$$

# Eddy Current BIE

Block wise solution:



Stage 1:  $(\mathbf{J}_L^{\text{EC}}, \mathbf{M}_L^{\text{EC}}, \mathbf{M}_S^{\text{EC}})$

Stage 2:  $(\mathbf{J}_S^{\text{EC}})$

Post-processing:

$$\begin{aligned}
 \mathbf{E} &= \eta (\mathbf{E}^{\text{EC}} + O(\eta^2)), & \text{with} & & \mathbf{E}^{\text{EC}}, \mathbf{H}^{\text{EC}} &\sim (\mathbf{J}_L^{\text{EC}}, \mathbf{M}_L^{\text{EC}}, \mathbf{M}_S^{\text{EC}}) \text{ in } \Omega_1, \\
 \mathbf{H} &= \mathbf{H}^{\text{EC}} + O(\eta^2), & & & \mathbf{H}^{\text{EC}} &\sim (\mathbf{J}_L^{\text{EC}}, \mathbf{M}_S^{\text{EC}}) \text{ in } \Omega_0, \\
 \Delta Z &= \eta (\Delta Z^{\text{EC}} + O(\eta^2)), & & & \Delta Z^{\text{EC}} &\sim (\mathbf{J}_L^{\text{EC}}, \mathbf{M}_S^{\text{EC}}).
 \end{aligned}$$

Stage 2 is useless for practical NDT  $\Rightarrow$  substantial reduction in the linear system to be solved.

# Matrix system (Recap)

(Homogenized  $L_c$ ) PMCHWT:  $\mathbf{Z} \mathbf{X} = \mathbf{Y}$

$$\mathbf{Z} := \langle \widetilde{\mathbf{X}}, \mathbf{Z} \mathbf{X} \rangle_x, \quad \mathbf{Y} := \langle \widetilde{\mathbf{X}}, \mathbf{Y} \rangle_x, \quad \langle \mathbf{u}, \mathbf{v} \rangle_x := \int_{\Gamma} (\mathbf{n} \times \mathbf{v}) \cdot \mathbf{u} \, d\mathbf{S}.$$

Rescaled, loop-star PMCHWT:  $\mathbf{Z}^\eta \mathbf{X}^\eta = \mathbf{Y}^\eta$

$$\begin{aligned} \mathbf{Z}^\eta &= \mathbf{L}^\eta \mathbf{Z} \mathbf{R}^\eta, \\ \mathbf{Y}^\eta &= \mathbf{L}^\eta \mathbf{Y}, \\ \mathbf{X} &= \mathbf{R}^\eta \mathbf{X}^\eta, \end{aligned} \quad \mathbf{L}^\eta = \begin{bmatrix} [\mathcal{L}] & 0 \\ \mathcal{S} & [\mathcal{L}] \\ 0 & \mathcal{S} \end{bmatrix}, \quad \mathbf{R}^\eta = \begin{bmatrix} [\mathcal{L}] & 0 \\ \eta^2 \mathcal{S} & [\mathcal{L}] \\ 0 & \mathcal{S} \end{bmatrix}^\top.$$

# Matrix system (Recap)

Standard PMCHWT:

$$\hat{\mathbf{Z}} \hat{\mathbf{X}} = \hat{\mathbf{Y}}$$

$$\mathbf{Z} := \langle \tilde{\mathbf{X}}, \mathbf{Z} \mathbf{X} \rangle_{\times}, \quad \mathbf{Y} := \langle \tilde{\mathbf{X}}, \mathbf{Y} \rangle_{\times}, \quad \langle \mathbf{u}, \mathbf{v} \rangle_{\times} := \int_{\Gamma} (\mathbf{n} \times \mathbf{v}) \cdot \mathbf{u} \, d\mathbf{S}.$$

Rescaled, loop-star PMCHWT:

$$\mathbf{Z}^{\eta} \mathbf{X}^{\eta} = \mathbf{Y}^{\eta}$$

$$\begin{aligned} \mathbf{Z}^{\eta} &= \hat{\mathbf{L}}^{\eta} \hat{\mathbf{Z}} \hat{\mathbf{R}}^{\eta}, \\ \mathbf{Y}^{\eta} &= \hat{\mathbf{L}}^{\eta} \hat{\mathbf{Y}}, \\ \hat{\mathbf{X}} &= \hat{\mathbf{R}}^{\eta} \mathbf{X}^{\eta}, \end{aligned} \quad \hat{\mathbf{L}}^{\eta} = \begin{bmatrix} \begin{bmatrix} \mathcal{L} \\ \mathcal{S} \end{bmatrix} & 0 \\ 0 & \begin{bmatrix} \mathcal{L} \\ \mathcal{S} \end{bmatrix} \end{bmatrix} \begin{bmatrix} (i\omega\mu_0)^{-1} & 0 \\ 0 & L_c \end{bmatrix}, \quad \hat{\mathbf{R}}^{\eta} = \begin{bmatrix} 1 & 0 \\ 0 & i\omega\mu_0 L_c \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \mathcal{L} \\ \eta^2 \mathcal{S} \end{bmatrix} & 0 \\ 0 & \begin{bmatrix} \mathcal{L} \\ \mathcal{S} \end{bmatrix} \end{bmatrix}^{\text{T}}.$$

Eddy current BIE (leading order + system reduction):  $\mathbf{Z}^{\text{EC}} \mathbf{X}^{\text{EC}} = \mathbf{Y}^{\text{EC}}$

$$\begin{aligned} \mathbf{Z}^{\text{EC}} &= \mathbf{L}^{\text{EC}} \mathbf{Z}_{(0)}^{\eta} \mathbf{R}^{\text{EC}}, \\ \mathbf{Y}^{\text{EC}} &= \mathbf{L}^{\text{EC}} \mathbf{Y}_{(0)}^{\eta}, \\ \mathbf{X}_{(0)}^{\eta} &= \mathbf{R}^{\text{EC}} \mathbf{X}^{\text{EC}}, \end{aligned} \quad \mathbf{L}^{\text{EC}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{R}^{\text{EC}} = (\mathbf{L}^{\text{EC}})^{\text{T}}.$$



Non-trivial Helmholtz decomposition on arbitrarily complex multi-connected geometries.

# Quasi-Helmholtz (QH) BIE

Standard PMCHWT:

$$\widehat{\mathbf{Z}} \widehat{\mathbf{X}} = \widehat{\mathbf{Y}}$$

$$\mathbf{Z} := \langle \widetilde{\mathbf{X}}, \mathbf{Z} \mathbf{X} \rangle_{\times}, \quad \mathbf{Y} := \langle \widetilde{\mathbf{X}}, \mathbf{Y} \rangle_{\times}, \quad \langle \mathbf{u}, \mathbf{v} \rangle_{\times} := \int_{\Gamma} (\mathbf{n} \times \mathbf{v}) \cdot \mathbf{u} \, d\mathbf{S}.$$

QH projectors - PMCHWT:

$$\mathbf{Z}^{\text{QH}} \mathbf{X}^{\text{QH}} = \mathbf{Y}^{\text{QH}}$$

$$\mathbf{Z}^{\text{QH}} = \mathbf{L}^{\text{QH}} \widehat{\mathbf{Z}} \mathbf{R}^{\text{QH}},$$

$$\mathbf{Y}^{\text{QH}} = \mathbf{L}^{\text{QH}} \widehat{\mathbf{Y}},$$

$$\widehat{\mathbf{X}} = \mathbf{R}^{\text{QH}} \mathbf{X}^{\text{QH}},$$

$$\mathbf{L}^{\text{QH}} = \begin{bmatrix} M_1 & 0 \\ 0 & \mathbb{M}_2 \mathbb{G}^{-1} \end{bmatrix} \quad \mathbf{R}^{\text{QH}} = \begin{bmatrix} M_3 & 0 \\ 0 & M_4 \end{bmatrix}$$

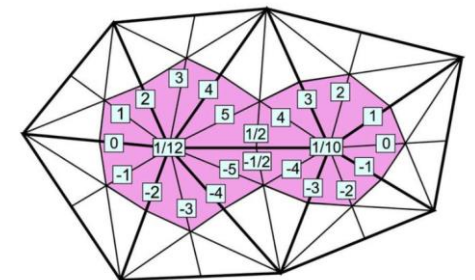
$$M_i = \alpha_i \Pi_S + \beta_i (\mathbf{I} - \Pi_S), \quad i = 1, 3, 4,$$

$$\mathbb{M}_2 = \alpha_2 \mathbb{P}_S + \beta_2 (\mathbb{I} - \mathbb{P}_S).$$

$\Pi_S = \mathcal{S}^T (\mathcal{S} \mathcal{S}^T)^{-1} \mathcal{S}$  : projection matrix on the non div-free « star » subspace,

$\mathbb{G} = \langle \phi^{\text{bc}}, \phi \rangle_{\times}$  : projection matrix on the dual space [Buffa-Christiansen, 07],

$\mathbb{P}_S = \mathcal{L}^T (\mathcal{L} \mathcal{L}^T)^{-1} \mathcal{L}$  : projection matrix on the non div-free « dual star » subspace,  
(here  $\mathcal{L}$  contains only *local loop* combinations)



Credits: [Cools, Andriulli, De Zutter, Michielssen, 2011]



No explicit calculation of *global loop* harmonic functions !



# Quasi-Helmholtz (QH) BIE

Standard PMCHWT:

$$\widehat{\mathbf{Z}} \widehat{\mathbf{X}} = \widehat{\mathbf{Y}}$$

$$\mathbf{Z} := \langle \widetilde{\mathbf{X}}, \mathbf{Z} \mathbf{X} \rangle_{\times}, \quad \mathbf{Y} := \langle \widetilde{\mathbf{X}}, \mathbf{Y} \rangle_{\times}, \quad \langle \mathbf{u}, \mathbf{v} \rangle_{\times} := \int_{\Gamma} (\mathbf{n} \times \mathbf{v}) \cdot \mathbf{u} \, d\mathbf{S}.$$

QH projectors - PMCHWT:

$$\mathbf{Z}^{\text{QH}} \mathbf{X}^{\text{QH}} = \mathbf{Y}^{\text{QH}}$$

$$\mathbf{Z}^{\text{QH}} = \mathbf{L}^{\text{QH}} \widehat{\mathbf{Z}} \mathbf{R}^{\text{QH}},$$

$$\mathbf{Y}^{\text{QH}} = \mathbf{L}^{\text{QH}} \widehat{\mathbf{Y}},$$

$$\widehat{\mathbf{X}} = \mathbf{R}^{\text{QH}} \mathbf{X}^{\text{QH}},$$

$$\mathbf{L}^{\text{QH}} = \begin{bmatrix} M_1 & 0 \\ 0 & \mathbb{M}_2 \mathbb{G}^{-1} \end{bmatrix} \quad \mathbf{R}^{\text{QH}} = \begin{bmatrix} M_3 & 0 \\ 0 & M_4 \end{bmatrix}$$

$$M_i = \alpha_i \Pi_S + \beta_i (\mathbf{I} - \Pi_S), \quad i = 1, 3, 4,$$

$$\mathbb{M}_2 = \alpha_2 \mathbb{P}_S + \beta_2 (\mathbb{I} - \mathbb{P}_S).$$

Manual implementation of cancelled integral terms

## Lemma 3

(a)  $(\mathbf{I} - \Pi_S) \mathbf{K} = \mathbf{K} (\mathbf{I} - \Pi_S) = 0.$

(b)  $\mathbb{P}_S \mathbb{G}^{-1} \mathbf{K} = \mathbf{0}.$

(c)  $\mathbb{P}_S \mathbb{G}^{-1} \mathbf{D}_{(0)} (\mathbf{I} - \Pi_S) = 0.$

with

$\mathbf{K}$  the div-div operator,

$\mathbf{D}_{(0)}$  the static double layer.

# Quasi-Helmholtz (QH) BIE

Standard PMCHWT:

$$\widehat{\mathbf{Z}} \widehat{\mathbf{X}} = \widehat{\mathbf{Y}}$$

$$\mathbf{Z} := \langle \widetilde{\mathbf{X}}, \mathbf{Z} \mathbf{X} \rangle_{\times}, \quad \mathbf{Y} := \langle \widetilde{\mathbf{X}}, \mathbf{Y} \rangle_{\times}, \quad \langle \mathbf{u}, \mathbf{v} \rangle_{\times} := \int_{\Gamma} (\mathbf{n} \times \mathbf{v}) \cdot \mathbf{u} \, d\mathbf{S}.$$

QH projectors - PMCHWT:

$$\mathbf{Z}^{\text{QH}} \mathbf{X}^{\text{QH}} = \mathbf{Y}^{\text{QH}}$$

$$\mathbf{Z}^{\text{QH}} = \mathbf{L}^{\text{QH}} \widehat{\mathbf{Z}} \mathbf{R}^{\text{QH}},$$

$$\mathbf{Y}^{\text{QH}} = \mathbf{L}^{\text{QH}} \widehat{\mathbf{Y}},$$

$$\widehat{\mathbf{X}} = \mathbf{R}^{\text{QH}} \mathbf{X}^{\text{QH}},$$

$$\mathbf{L}^{\text{QH}} = \begin{bmatrix} M_1 & 0 \\ 0 & \mathbb{M}_2 \mathbb{G}^{-1} \end{bmatrix} \quad \mathbf{R}^{\text{QH}} = \begin{bmatrix} M_3 & 0 \\ 0 & M_4 \end{bmatrix}$$

$$M_i = \alpha_i \Pi_{\mathbf{S}} + \beta_i (\mathbf{I} - \Pi_{\mathbf{S}}), \quad i = 1, 3, 4,$$

$$\mathbb{M}_2 = \alpha_2 \mathbb{P}_{\mathbf{S}} + \beta_2 (\mathbb{I} - \mathbb{P}_{\mathbf{S}}).$$

Proposed scaling correspond to the low frequency but *not so high* conductive case:  $\kappa_0 L_c = \eta$ ,  $\sigma = O(1)$  ( $\kappa_1^2 = O(\eta)$ ).

$$M_1 = (\omega \varepsilon_0)^{1/2} \Pi_{\mathbf{S}} + (\omega \mu_0)^{-1/2} (\mathbf{I} - \Pi_{\mathbf{S}}) \quad ,$$

$$M_3 = (\omega \varepsilon_0)^{1/2} \Pi_{\mathbf{S}} + (\omega \mu_0)^{-1/2} (\mathbf{I} - \Pi_{\mathbf{S}}) \quad ,$$

$$M_4 = (\omega \mu_0)^{1/2} \Pi_{\mathbf{S}} + (\omega / \sigma)^{1/2} (\mathbf{I} - \Pi_{\mathbf{S}}) \quad ,$$

$$\mathbb{M}_2 = (\omega \sigma)^{-1/2} \mathbb{P}_{\mathbf{S}} + (\omega \mu_0)^{1/2} (\mathbb{I} - \mathbb{P}_{\mathbf{S}}) \quad .$$

[Chhim, Merlini, Rahmouni, Guzman, Andriulli, IEEE 2020]

# Quasi-Helmholtz (QH) BIE

Standard PMCHWT:

$$\widehat{\mathbf{Z}} \widehat{\mathbf{X}} = \widehat{\mathbf{Y}}$$

$$\mathbf{Z} := \langle \widetilde{\mathbf{X}}, \mathbf{Z} \mathbf{X} \rangle_{\times}, \quad \mathbf{Y} := \langle \widetilde{\mathbf{X}}, \mathbf{Y} \rangle_{\times}, \quad \langle \mathbf{u}, \mathbf{v} \rangle_{\times} := \int_{\Gamma} (\mathbf{n} \times \mathbf{v}) \cdot \mathbf{u} \, d\mathbf{S}.$$

QH projectors - PMCHWT:

$$\mathbf{Z}^{\text{QH}} \mathbf{X}^{\text{QH}} = \mathbf{Y}^{\text{QH}}$$

$$\mathbf{Z}^{\text{QH}} = \mathbf{L}^{\text{QH}} \widehat{\mathbf{Z}} \mathbf{R}^{\text{QH}},$$

$$\mathbf{Y}^{\text{QH}} = \mathbf{L}^{\text{QH}} \widehat{\mathbf{Y}},$$

$$\widehat{\mathbf{X}} = \mathbf{R}^{\text{QH}} \mathbf{X}^{\text{QH}},$$

$$\mathbf{L}^{\text{QH}} = \begin{bmatrix} M_1 & 0 \\ 0 & \mathbb{M}_2 \mathbb{G}^{-1} \end{bmatrix} \quad \mathbf{R}^{\text{QH}} = \begin{bmatrix} M_3 & 0 \\ 0 & M_4 \end{bmatrix}$$

$$M_i = \alpha_i \Pi_{\text{S}} + \beta_i (\mathbf{I} - \Pi_{\text{S}}), \quad i = 1, 3, 4,$$

$$\mathbb{M}_2 = \alpha_2 \mathbb{P}_{\text{S}} + \beta_2 (\mathbb{I} - \mathbb{P}_{\text{S}}).$$

We suggest a rescaling based on the previous asymptotic analysis with  $\kappa_0 L_{\text{c}} = \eta$ ,  $\sigma = O(\eta^{-1})$ .

$$M_1^{\text{HC}} = (\Pi_{\text{S}} + (\mathbf{I} - \Pi_{\text{S}})) (i\omega\mu_0)^{-1},$$

$$M_3^{\text{HC}} = \eta^2 \Pi_{\text{S}} + (\mathbf{I} - \Pi_{\text{S}}),$$

$$M_4^{\text{HC}} = (\Pi_{\text{S}} + (\mathbf{I} - \Pi_{\text{S}})) (i\omega\mu_0 L_{\text{c}}),$$

$$\mathbb{M}_2^{\text{HC}} = (\mathbb{P}_{\text{S}} + (\mathbb{I} - \mathbb{P}_{\text{S}})) L_{\text{c}}.$$

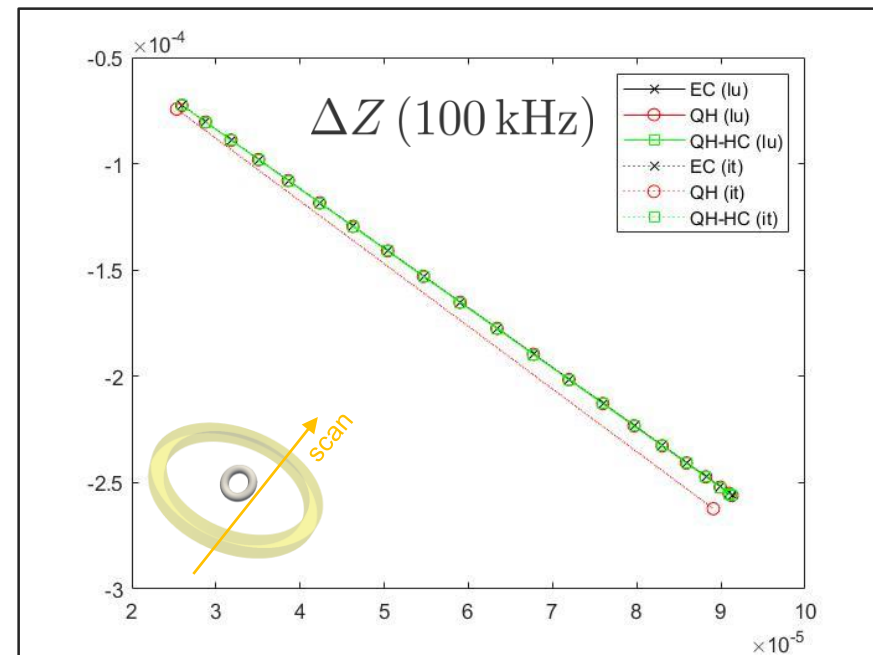
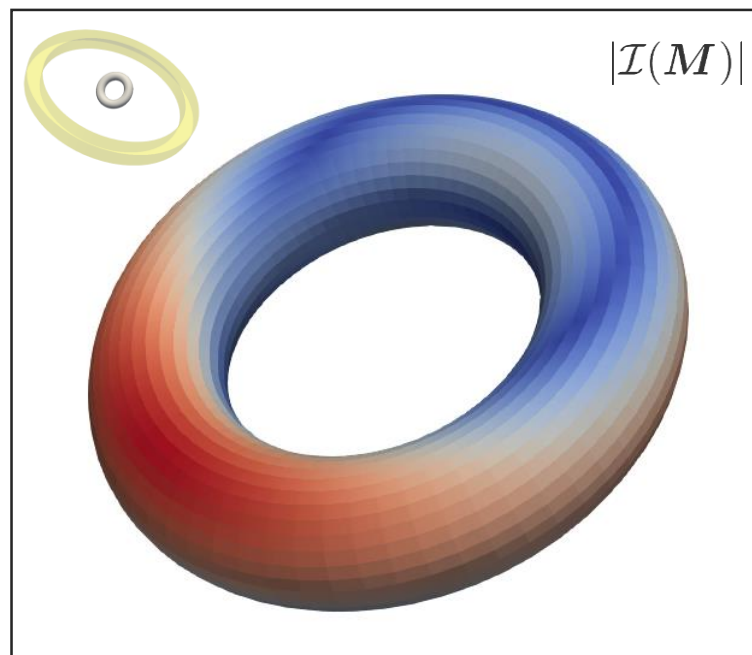
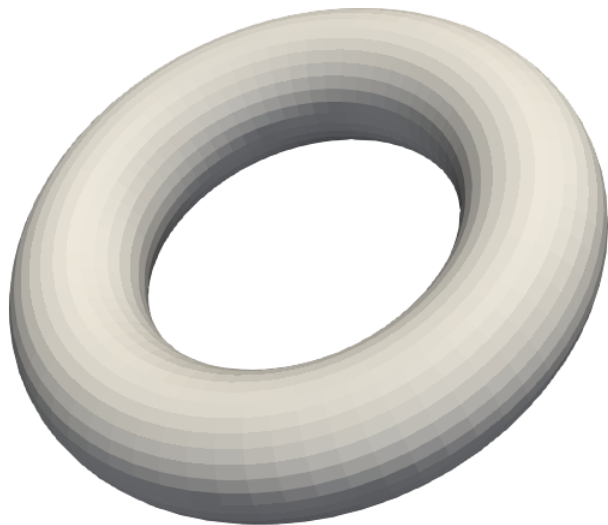


**1** Eddy current testing & Motivations

**2** Boundary integral formalisms

**3** **Preliminary results**

# Preliminary results (in progress)



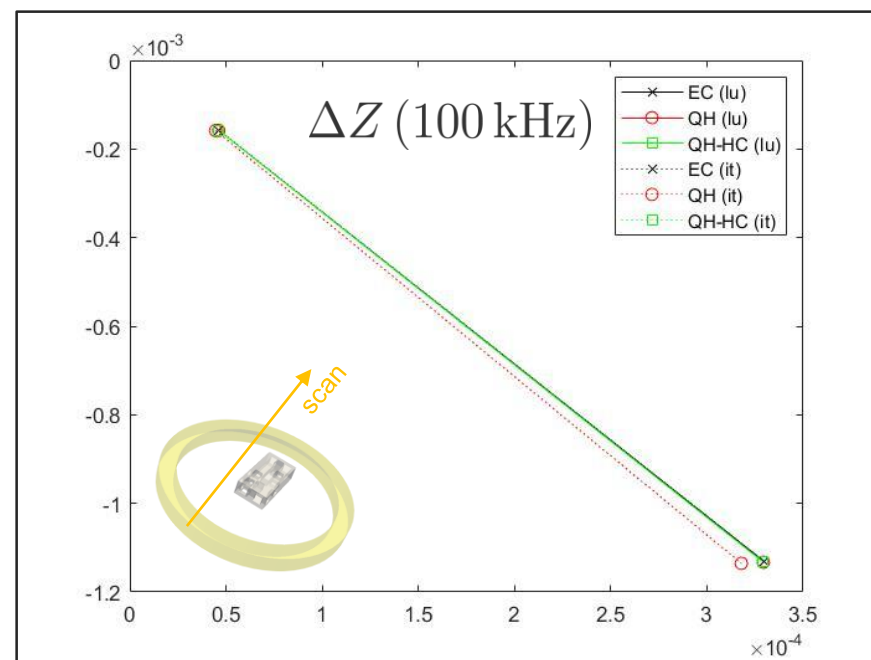
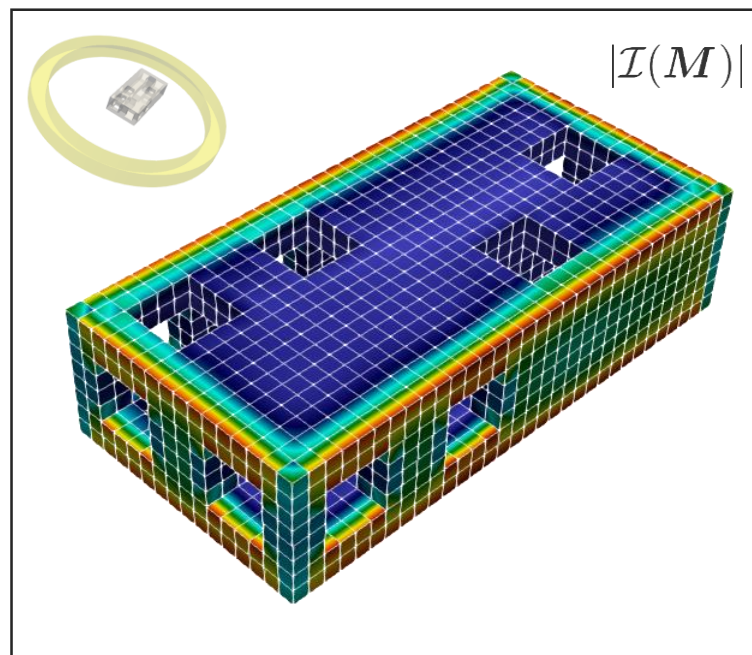
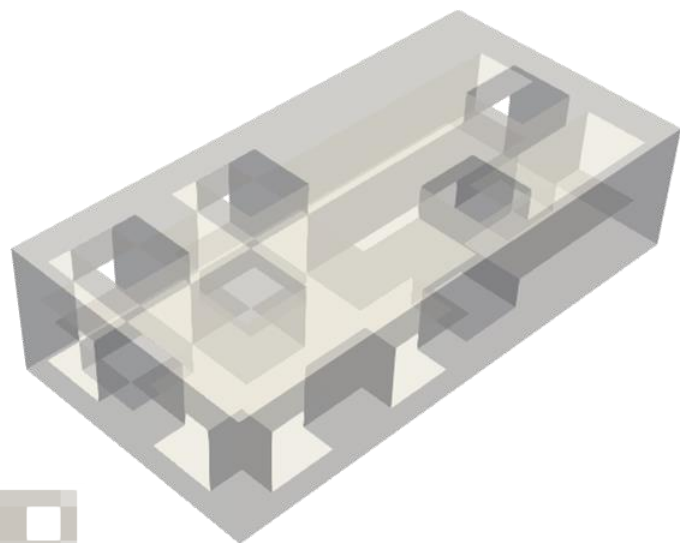
$R = 10 \text{ mm}$ ,  
 $r = 3 \text{ mm}$ ,  
 $\sigma = 1 \text{ MS/m}$   
 $n_{\text{el}} = 2048$

limit of the leading order reduction  
  $|\Delta Z(\text{GMRES}) - \Delta Z(\text{LU})| > \text{tol}$

GMRES (1e-04)		EC		QH		QH-HC	
freq	$\delta$	Cond	iter	cond	iter	cond	iter
1000 kHz	0.5 mm	1.4e4	159	1.4e9	68	5.4e4	92
100 kHz	1.6 mm	3.1e4	276	4.6e8	83	6.8e4	151
10 kHz	5 mm	3.8e4	209	2.6e8	109	6.8e4	169
1 kHz	16 mm	1.2e5	207	2.7e8	117	6.8e4	148

$$\text{cond}(\mathbf{A}) = \|\tilde{\mathbf{A}}\| \cdot \|\tilde{\mathbf{A}}^{-1}\|, \quad \tilde{\mathbf{A}} = \mathbf{D}\mathbf{A}\mathbf{D}, \quad D_{ij} = \delta_{ij} \mathbf{A}_{ii}^{-1/2}.$$

# Preliminary results (in progress)



$L = 40 \times 20 \times 10 \text{ mm}^3$ ,  
 $\sigma = 1 \text{ MS/m}$   
 $n_{\text{el}} = 2848$   
 $\# \mathcal{V}_H = 34$



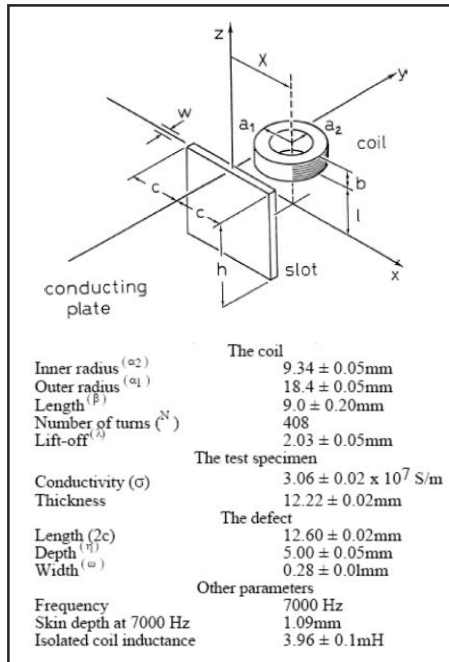
limit of the leading order reduction  
  $|\Delta Z(\text{GMRES}) - \Delta Z(\text{LU})| > \text{tol}$

GMRES (1e-04)		EC		QH		QH-HC	
freq	$\delta$	Cond	iter	cond	iter	cond	iter
1000 kHz	0.5 mm	3.3e4	158	2.3e9	119	1.7e4	149
100 kHz	1.6 mm	5.3e4	245	6.4e8	184	2.7e4	287
10 kHz	5 mm	1.0e6	625	1.9e8	496	2.8e4	572
1 kHz	16 mm	2.2e5	451	1.9e8	763	2.8e4	783

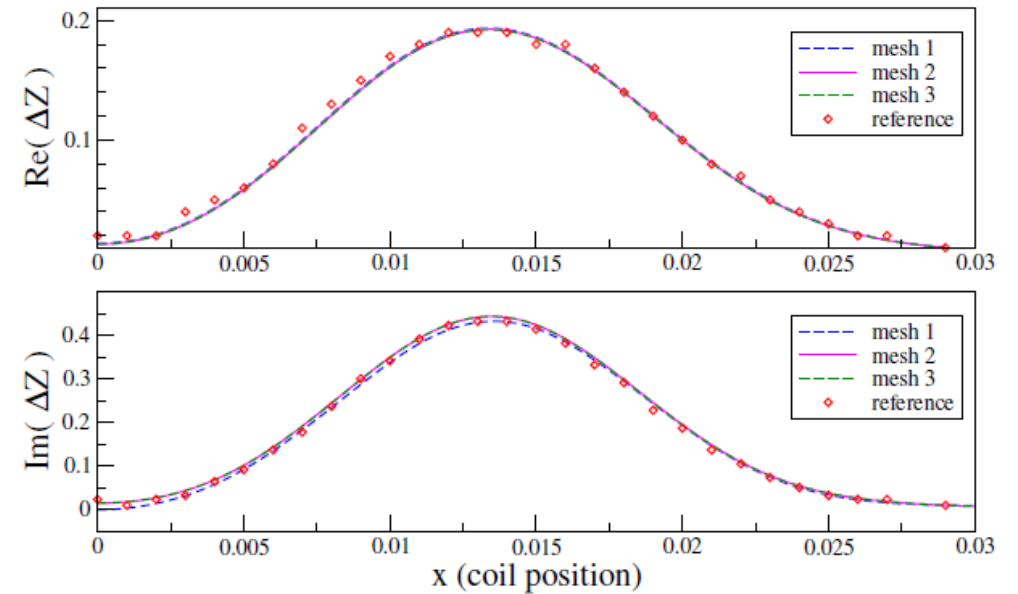
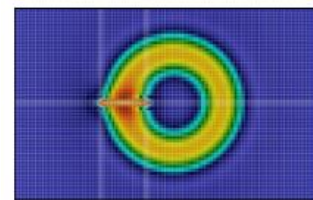
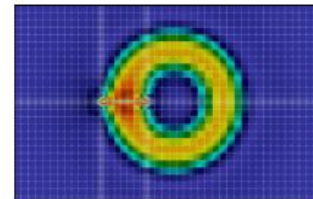
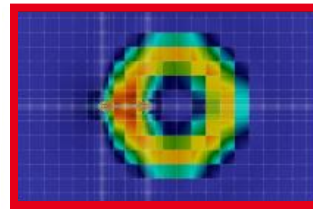
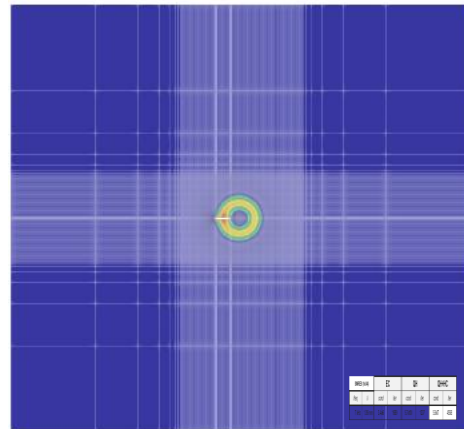
$$\text{cond}(\mathbf{A}) = \|\tilde{\mathbf{A}}\| \cdot \|\tilde{\mathbf{A}}^{-1}\|, \quad \tilde{\mathbf{A}} = \mathbf{D}\mathbf{A}\mathbf{D}, \quad D_{ij} = \delta_{ij} \mathbf{A}_{ii}^{-1/2}.$$



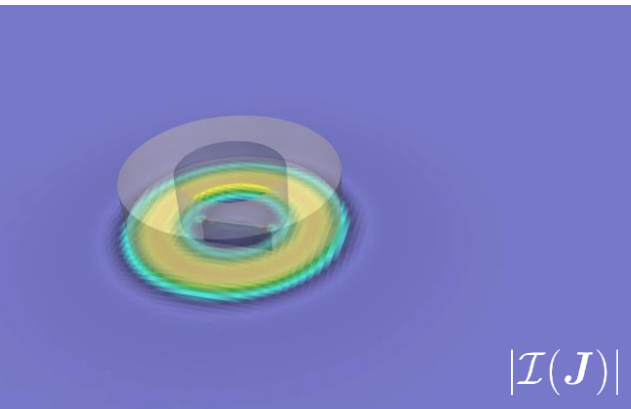
# Preliminary results - TEAM 15 benchmark



	$N$	Assembly (s)	Solution (s)
Mesh 1	7645	51	6
Mesh 2	15 343	177	37
Mesh 3	39 107	1264	586



[Sabbagh & Burke, 1992]



GMRES (1e-04)		EC		QH		QH+HC	
freq	$\delta$	cond	iter	cond	iter	cond	iter
7 kHz	1.09 mm	3.4e6	1999	5.7e10	1307	5.9e7	4558

$|\Delta Z(\text{GMRES}) - \Delta Z(\text{LU})| > \text{tol}$

$$\text{cond}(\mathbf{A}) = \|\tilde{\mathbf{A}}\| \cdot \|\tilde{\mathbf{A}}^{-1}\|, \quad \tilde{\mathbf{A}} = \mathbf{D}\mathbf{A}\mathbf{D}, \quad D_{ij} = \delta_{ij} \mathbf{A}_{ii}^{-1/2}.$$

# Conclusion

BEM solutions dedicated to eddy current inspection & heating

- EC: Eddy Current testing regime (low freq & high conductivity)
- QH: Quasi-Helmholtz projectors (low freq & medium conductivity)

Comparison on toy cases (naïve implementation at this stage)

- EC: fairly well conditioned
- QH: arbitrarily complex multi-connected geometry – warning lu vs gmres (further analysis required)
- QH-HC: High conductivity improvement – so far so good (further analysis required)

What's next ?

- Improved implementation (QH projectors *on-the-fly*)
- Evaluation on realistic use cases (degraded meshes)



# Thank you for your attention

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