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## Study of the effectiveness of Bayesian kriging for the decommissioning and dismantling of nuclear sites

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DE LA RECHERCHE À L'INDUSTRIE

**Study of the effectiveness of Bayesian kriging for the decommissioning and dismantling of nuclear sites.**

Martin Wieskotten

13-15 septembre 2021

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*Institut pour les sciences et technologies pour une économie circulaire des énergies bas carbone*

- **Context**
- **Geostatistical Model**
- **Bayesian Kriging**
- **Conclusion**

- **Context**
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- Wave of Dismantling and Decommissioning of nuclear sites.
- The ASN (Autorité de Sûreté Nucléaire) identified in 2019 35 sites in France:

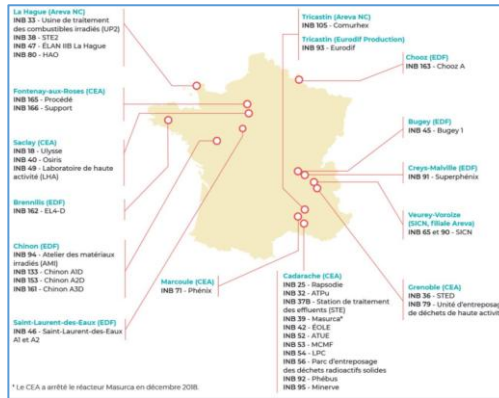


Figure 1 Map of stopped or in dismantling sites in January (ASN)

- Complex operational steps required, usually planned during the creation of the INB (Installation Nucléaire de Base) with the dismantling plan.

- Challenges due to radioactivity:
  - Radioprotection of operators.
  - Management of nuclear waste (sometimes substantial amount of low activity waste).
  - Contamination of the environment.
- Importance of a precise planning of the decommissioning steps for the optimisation of costs and logistics.

- Step of the radiological characterization, identify the presence of contamination.
- Measurements on site to obtain preliminary information (such as *in-situ* measurements).
- Recent use of geostatistics (or spatial statistics) (exemple (Desnoyers, 2010) ) : prediction on unobserved positions.

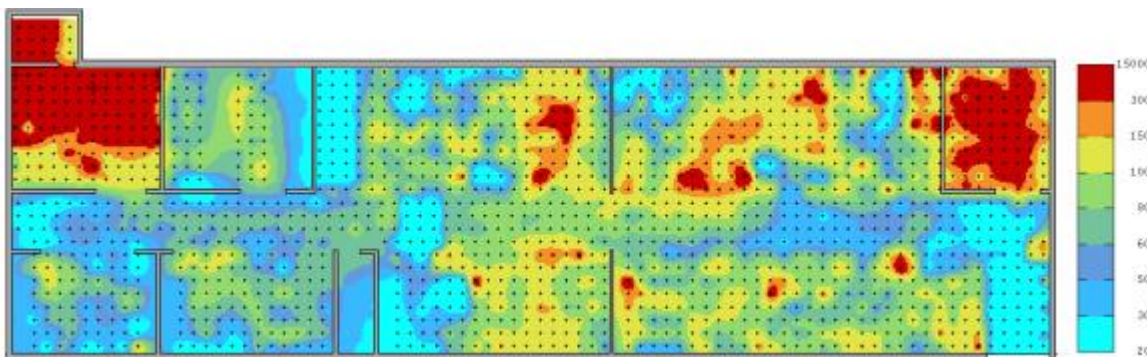


Figure 2 Exemple of radiological cartography (Desnoyers 2010)

- Context
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# MODEL AND HYPOTHESIS

■ Contamination modelled as a random field:

- $\{Z(\mathbf{x}), \mathbf{x} \in \mathbb{R}^d\}, Z(\mathbf{x}) \in \mathbb{R};$
- $Z(\mathbf{x}) = \mu(\mathbf{x}) + R(\mathbf{x})$
- $R(\mathbf{x})$  vérifie  $\mathbb{E}(R(\mathbf{x})) = \mathbf{0}.$

■ Stationnarity and Isotropy hypothesis:

- $\mathbb{E}[Z(\mathbf{x})] = \mu$
- $Cov(Z(\mathbf{x}), Z(\mathbf{x}')) = C(|\mathbf{x} - \mathbf{x}'|)$

■ Covariance :

- $C(|\mathbf{x} - \mathbf{x}'|) = \sigma^2 f(|\mathbf{x} - \mathbf{x}'|, \phi) + \tau^2$
- Exemple :  $f(|\mathbf{x} - \mathbf{x}'|, \phi) = \exp\left(\frac{-|\mathbf{x} - \mathbf{x}'|}{\phi}\right)$

■ Parametric model, estimation of  $(\mu, \sigma^2, \phi, \tau^2).$

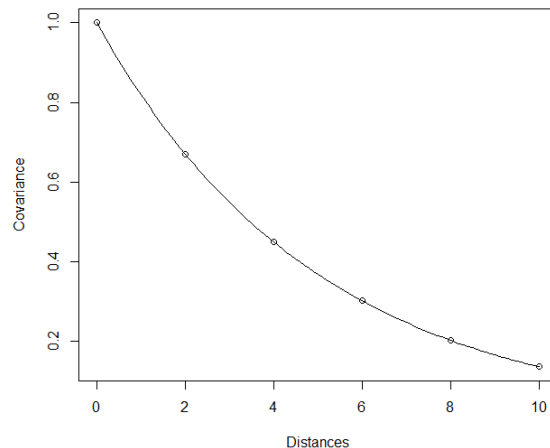


Figure 3 Exemple of exponential covariance function with  $\phi = 5$

- 
- Parameters estimation  $\theta = (\mu, \sigma^2, \phi, \tau^2)$ :

- Variographic analysis;

- Likelihood maximisation:

$$\hat{\theta} = \operatorname{argmax}_{\theta} (L(z(\mathbf{x}_1), \dots, z(\mathbf{x}_n), \theta))$$

For a random variable  $Z(x_i)$  with density  $p_{Z(x_i)}$ :

$$L(z(\mathbf{x}_1), \dots, z(\mathbf{x}_n), \theta) = \prod_{i=1}^n p_{Z(x_i)}(z(\mathbf{x}_i) | \theta)$$

- Kriging Predictions:

- $\hat{Z}(\mathbf{x}_0) = \sum_{i=1}^n \lambda_i Z(\mathbf{x}_i)$ ;

- Estimation of  $\lambda_i$  by minimizing  $\operatorname{Var}(\hat{Z}(\mathbf{x}_0) - Z(\mathbf{x}_0))$ .

■ Choix of a random gaussian process:

□ Trajectory  $(z(\mathbf{x}_1), \dots, z(\mathbf{x}_n))$

□  $\mathbf{Z} \sim \mathcal{N}(\boldsymbol{\mu}, \sigma^2 \boldsymbol{\Sigma})$  with  $\boldsymbol{\Sigma} = \left( f(|\mathbf{x}_i - \mathbf{x}_j|, \phi) \right)_{i,j \in [1,n]}$

■ Logarithm of the likelihood:

$$\log(L(\mathbf{z}, \boldsymbol{\theta})) = -\frac{1}{2} \left( n \log(2\pi) + \log(|\sigma^2 \boldsymbol{\Sigma}|) + (\mathbf{z} - \boldsymbol{\mu})' (\sigma^2 \boldsymbol{\Sigma})^{-1} (\mathbf{z} - \boldsymbol{\mu}) \right)$$

with  $\mathbf{z} = (z(\mathbf{x}_1), \dots, z(\mathbf{x}_n))'$ .

■ Prediction of  $Z(\mathbf{x}_0)$  :

$$E[Z(\mathbf{x}_0) | \mathbf{z}, \mu, \sigma^2, \phi] = \mu + \mathbf{k}' \boldsymbol{\Sigma}^{-1} (\mathbf{z} - \mu)$$

$$\text{Var}[Z(\mathbf{x}_0) | \mathbf{z}, \mu, \sigma^2, \phi] = \sigma^2 (1 - \mathbf{k}' \boldsymbol{\Sigma}^{-1} \mathbf{k})$$

with  $\mathbf{k} = \left( f(|\mathbf{x}_0 - \mathbf{x}_j|, \phi) \right)_{j \in [1,n]}$

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- The parameters are considered random variables (Kitanidis, 1986):

$$E[Z(\mathbf{x}_0)|\mathbf{z}] = E_{\theta}[E[Z(\mathbf{x}_0)|\mathbf{z}, \theta]] = \int_{\Theta} (\mu + \mathbf{k}'\Sigma^{-1}(\mathbf{z} - \mu)) p_{\theta}(\theta|\mathbf{z}) d\theta$$

$$\begin{aligned} \text{Var}[Z(\mathbf{x}_0)|\mathbf{z}] &= E_{\theta}[\text{Var}[Z(\mathbf{x}_0)|\mathbf{z}, \theta]] + \text{Var}_{\theta}[E[Z(\mathbf{x}_0)|\mathbf{z}, \theta]] \\ &= \int_{\Theta} \sigma^2(1 - \mathbf{k}'\Sigma^{-1}\mathbf{k}) p_{\theta}(\theta|\mathbf{z}) d\theta \\ &\quad + \int_{\Theta} ((\mu + \mathbf{k}'\Sigma^{-1}(\mathbf{z} - \mu)) - E[Z(\mathbf{x}_0)|\mathbf{z}])^2 p_{\theta}(\theta|\mathbf{z}) d\theta \end{aligned}$$

- For the expectation: weighted average
- For the variance: weighted average + uncertainty in parameters estimation
- Estimation of  $p_{\theta}(\theta|\mathbf{z})$ , the posterior:

- Bayes theorem:  $p_{\theta}(\theta|\mathbf{z}) = \frac{L(\mathbf{z}, \theta) p_{\theta}(\theta)}{\int_{\Theta} L(\mathbf{z}, \theta) p_{\theta}(\theta) d\theta}$ .
- $p_{\theta}(\theta)$  joint law to specify.

- Monte-Carlo approximation of the prediction posterior (Diggle et Ribeiro, 2007) :
  - ❑ Discretization of  $p_\phi(\phi|\mathbf{z})$ .
  - ❑ Sampling  $\phi$  in  $p_\phi(\phi|\mathbf{z})$ , then sampling values for  $(\mu, \sigma^2)$  in  $p_{\mu, \sigma^2}(\mu, \sigma^2|\mathbf{z}, \phi)$  by plugging in the sampled  $\phi$ .
  - ❑ Plugging in the values of  $\theta$  to estimate  $p_{Z(\mathbf{x}_0)}(z(\mathbf{x}_0)|\mathbf{z}, \theta)$  then sampling  $Z(\mathbf{x}_0)$  to approximate  $p_{Z(\mathbf{x}_0)}(z(\mathbf{x}_0)|\mathbf{z})$ .

■ Prior's choice:

$$\begin{aligned} \pi(\phi) &= U(0, D_{max}) \\ \pi(\sigma^2) &\propto \frac{1}{\sigma^2} \\ \pi(\mu) &\propto 1 \end{aligned}$$

- **Objective** : Comparing Bayesian kriging to Simple Kriging on simulated data sets.
- 100 simulations of a random gaussian process on a square grid of 25x25 size and 0,4 with the following parameters :

$$\mu = 0,5 \quad \sigma^2 = 0,1 \quad \phi = 3,5$$

- From these 625 points,  $n$  are sampled to constitute the data set.
- Cross-validation is then applied and models are compared with the following criterion (Demay et al., 2021):

$$Q^2 = 1 - \frac{\sum_{i=1}^n (z(\mathbf{x}_i) - \widehat{z(\mathbf{x}_i)})^2}{\sum_{i=1}^n (z(\mathbf{x}_i) - \widehat{\mu})^2}$$

$$PVA = \left| \log \left( \frac{1}{n} \sum_{i=1}^n \frac{(z(\mathbf{x}_i) - \widehat{z(\mathbf{x}_i)})^2}{s_{-i}^2} \right) \right|$$

$$\Delta_\alpha = \frac{1}{n} \sum_{i=1}^n \phi_i, \quad \phi_i \begin{cases} 1 & \text{if } z(\mathbf{x}_i) \in IC \\ 0 & \text{else} \end{cases}$$

$$R = \frac{1}{m} \sum_{\alpha} (\Delta_\alpha - \alpha)^2$$

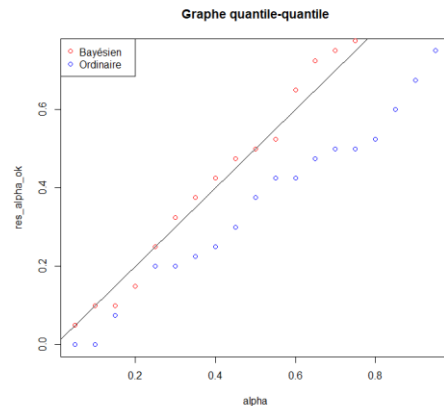


Figure 4 Illustration of the  $\Delta_\alpha$  graph

# RESULTS FOR SIMULATED DATA

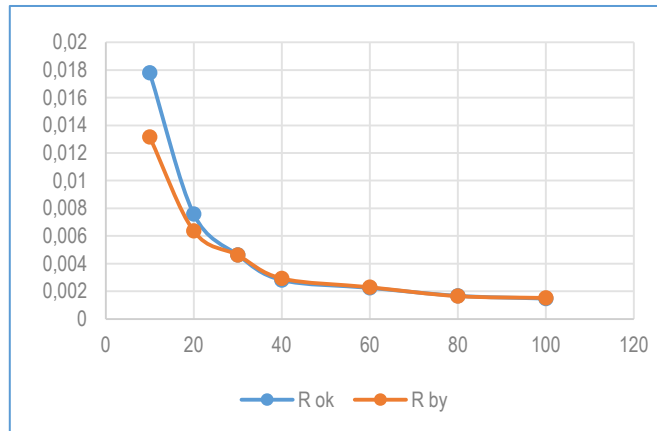
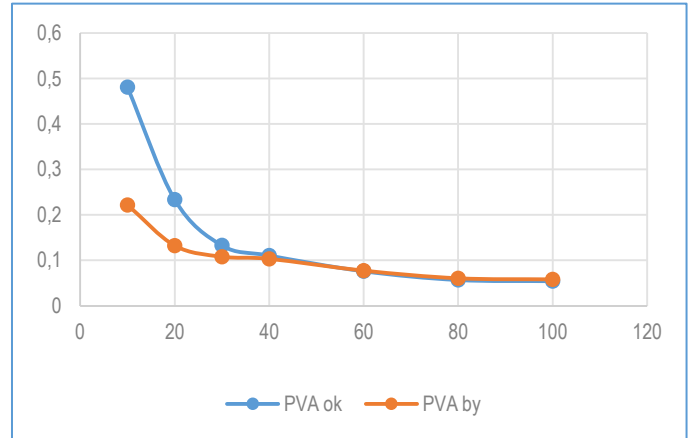
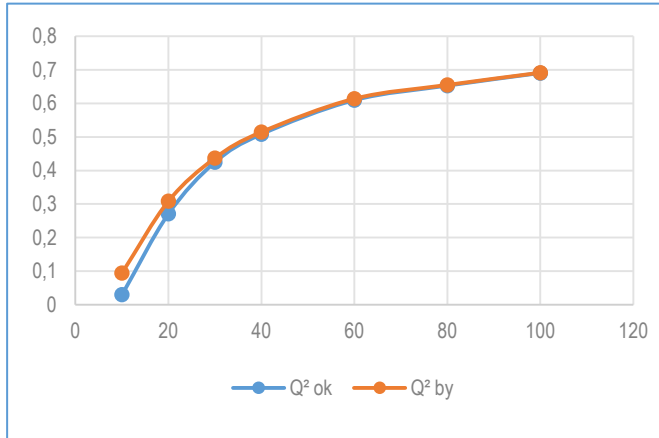


Figure 5 Average results for the Q², PVA and R criterion



- 70 measurement results from the G3 reactor in CEA Marcoule.
- Box-Cox Transformation applied, the cross-validation.

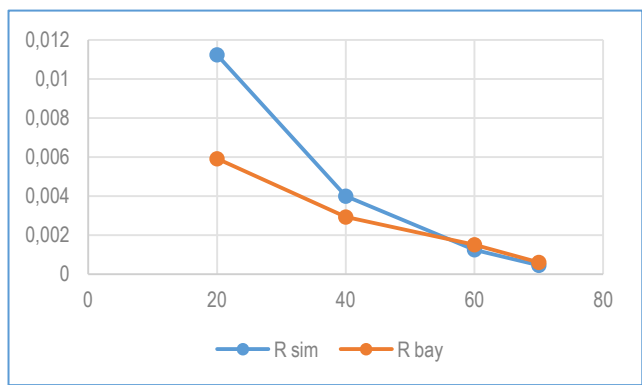
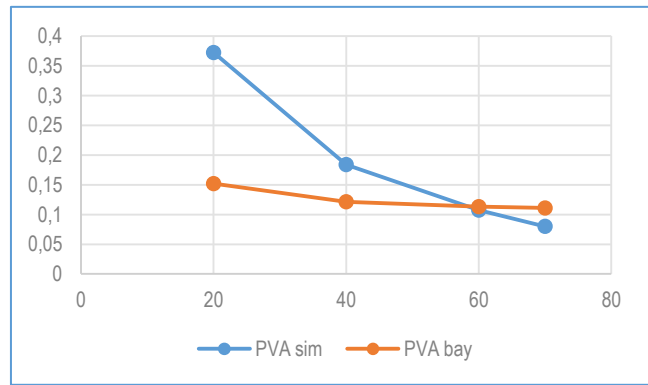
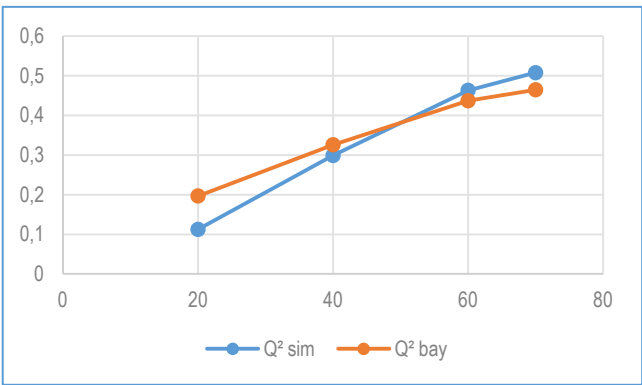
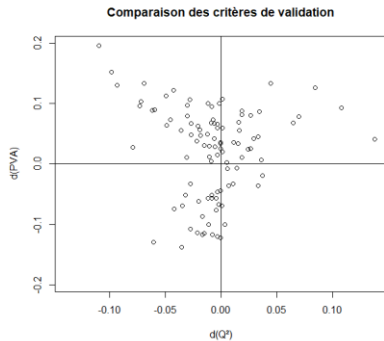
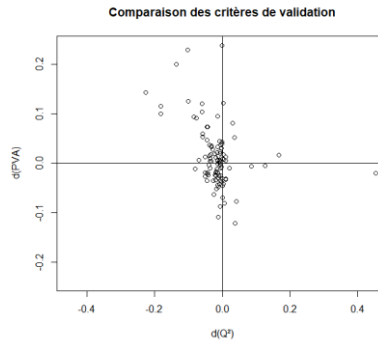


Figure 6 Average results for the Q², PVA and R criterion

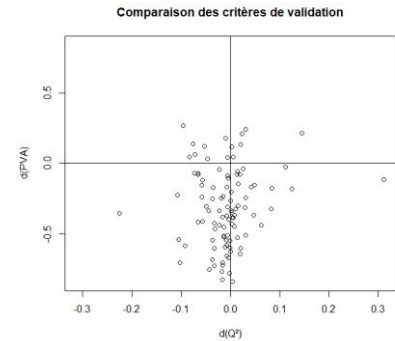
- Difficult choice (independent from the data).
- Sensibility analysis for prior specification, especially in the case of wrong specifications, for  $n = 30$ .



$$\begin{aligned}\pi(\phi) &= U(0, D_{max}) \\ \pi(\sigma^2) &\propto \frac{1}{\sigma^2} \\ \pi(\beta) &\propto 1\end{aligned}$$



$$\begin{aligned}\pi(\phi) &= U(0, D_{max}) \\ \sigma^2 &\sim \chi^{-2}(0.1, 40) \\ \beta &\sim \mathcal{N}\left(0.5, \frac{\sigma^2}{40}\right)\end{aligned}$$



$$\begin{aligned}\pi(\phi) &= U(0, D_{max}) \\ \sigma^2 &\sim \chi^{-2}(0.6, 40) \\ \beta &\sim \mathcal{N}\left(1, \frac{\sigma^2}{40}\right)\end{aligned}$$

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## CONCLUSION

- Bayesian Kriging gives better estimates (especially for the scale parameter) for small data sets
- Not efficient for larger data sets (computational costs)
- Allows for prior information specifications : future works for specifying priors for the nugget effect.

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# THANK YOU FOR YOUR ATTENTION

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