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# Situational Assessment of Wildfires : a Fuzzy Spatial Approach

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Abstract—Decision support systems coupled with a Geographic Information System (GIS) are useful for assessing spatialized risks, which can be either natural, industrial or even anthropic. However, when the knowledge is not sufficiently precise or when the processes are complex, the methods of analysis of the GIS are not always sufficient. In this article, we propose to couple a GIS with a Fuzzy Expert System capable of evaluating fuzzy spatial rules, that is to say, fuzzy rules integrating spatial relations or properties. These are based on fuzzy mathematical morphology and model high-level metric or topological relationships such as "being close to", "being in the direction of" or "being adjacent to". The proposed approach has been applied to the identification of risky issues in the case of wildfires. We detail the expert knowledge modeled and show the results obtained in a graphic interface dedicated to the intervention of firefighters.

*Index Terms*—Fuzzy spatial relations, fuzzy mathematical morphology, vulnerability assessment, situational assessment, wildfire

#### I. INTRODUCTION

Decision support systems have often been used to assess risks, whether of natural, industrial or human origins. In particular, forest fires have been widely studied [1], [2], notably in the Mediterranean [3], [4] where they are frequent. Most of the proposed systems collect a large amount of information in order to jointly assess occurrence and intensity of fires : historical data from forest fires, meteorological data, vegetation type and topography of places. The difficulty is to combine these elements, all the more since there are interactions among them [1].

Different approaches have been proposed: for instance, they are based on machine learning [3], [5], dynamic Bayesian networks [6] and Fuzzy Expert Systems [2], [4]. To help the people involved in fire management, the first works proposed different fire propagation models [1]. However, the unpredictable nature of forest fires has led to an interest in very simplified models which allow calculations in real time [7].

The use of a Geographic Information System (GIS) quickly became obvious to aggregate this localized information and to allow a hierarchical and interactive visualization. State of the art also highlights the need to deal with the uncertainty linked to knowledge about the phenomenon [1], [2]. Moreover, we must also consider the input information is not always precise: for example, the delimitation of the burned area, whether provided manually or by an algorithm, cannot be precise by nature. Fuzzy sets and fuzzy logic [8] are an effective way to reason with uncertainty and to deal with ill-delimited objects. Schneider [9] defined three different fuzzy objects : fuzzy points, fuzzy lines and fuzzy regions. More recently, since vague objects are often extracted from a field (e.g., from a raster image), five categories of vague objects based on fuzzy logic are introduced in [10], like direct field-cutting objects or dynamic boundary objects. Furthermore, different formulations have been proposed to model spatial relations in a fuzzy manner [11]–[16].

In particular, ideas emerge for coupling fuzzy reasoning and a GIS [4], [17]–[19]. The majority of these approaches [4], [17], [18] consider as inputs some indicators of interest that must be assessed at different locations. They can also measure either non-spatial characteristics related to the location (e.g., population density) or spatial characteristics (e.g., elevation, distance to a road). Fuzzy reasoning with fuzzy spatial objects and a GIS is more integrated in [19], where fuzzy rules can be applied to them. However, the antecedent of fuzzy rules does not consider fuzzy relations between fuzzy spatial objects. This point is addressed by our approach that can handle both fuzzy spatial reasoning and reasoning on localized measurements.

This article is composed as follows. We briefly describe in section II the principles of fuzzy morpho-mathematics that allows to model vague spatial relations. Then, we present the proposed approach in section III which consists of interoperating a GIS like GéoPortail<sup>1</sup> and our fuzzy inference engine [20] that integrates high level operators to process directly spatial data. Then, we present in section IV an application to the assessment of risks related to the growth of a forest fire. Finally, we draw some conclusions and perspectives.

#### II. FUZZY MATHEMATICAL MORPHOLOGY

Bloch and Maître [11] proposed the fuzzy mathematical morphology approach. It evaluates if specific relations between

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<sup>1</sup>https://www.geoportail.gouv.fr/

an object of interest A and a reference object R occur. A and R are both localised in the space S. These relations are either topological ones, such as intersection [13], or metric relations such as proximity relations [14] and directional relative positions [13]. All these relations play a major role in spatial reasoning.

Several advantages of this approach should be emphasised, because spatial relations:

- may be computed on fuzzy or crisp objects,
- may be well defined or intrinsically vague,
- do not rely only on a point-to-point computation between fuzzy objects, but may also consider the spatial distance between them.

We give details on the metric relations computations that will be used in the next sections.

#### A. Metric Relation Evaluation

Let  $\mu_A$  be the membership function representing the fuzzy set A in S, such as  $\mu_A : S \to [0, 1]$ . We can interpret  $\mu_A(x)$ as the degree to which we consider that A really spreads over the location x.

Metric relations are assessed by a two-step process. First, we compute the fuzzy landscape  $\mu_{\alpha}(R)$  on S which gives the set of locations verifying the desired relation  $\alpha$  with the reference object R. Then, the adequacy of A with the fuzzy landscape  $\mu_{\alpha}(R)$  is assessed for all points x in S. Different functions can be used to assess this adequacy. The two extreme functions are given by the degree of possibility  $\Pi_{\alpha}^{R}$  (optimistic evaluation) and the degree of necessity  $N_{\alpha}^{R}$  (pessimistic evaluation):

$$\Pi^{R}_{\alpha}(A) = \sup_{x \in \mathcal{S}} \quad \top \left[ \mu_{\alpha}(R)(x), \mu_{A}(x) \right] \tag{1}$$

$$N^R_{\alpha}(A) = \inf_{x \in \mathcal{S}} \quad \perp \left[\mu_{\alpha}(R)(x), 1 - \mu_A(x)\right] \tag{2}$$

where  $\top$  is a t-norm (fuzzy conjunction),  $\perp$  is its associated t-conorm.

Fig. 1 shows the example of a crisp reference object R with a triangular shape and two fuzzy landscapes computed from it. We can see on the fuzzy landscape the locations that are considered to be close to R and to be at the right of R respectively. We can see that the values decrease when one moves away from R or from the desired direction.

#### B. Fuzzy Landscapes Computation

A fuzzy landscape  $\mu_{\alpha}(R)$  is obtained by the fuzzy dilation  $D_{\nu}(\mu)$  of the reference object R by a structuring element  $\nu$  from spatial domain  $\mathcal{V}$  in [0, 1] according to [11]:

$$D_{\nu}(\mu)(x) = \sup_{y \in \mathcal{S}} \quad \top \left[\mu(y), \nu(x-y)\right] \tag{3}$$

where x and y are two points in S,  $\mu$  is the membership function of the spatial fuzzy set representing R, and  $\nu$  is a structuring element that characterises the spatial relation. For simplify, we suppose  $\nu$  is centered in the center O of the image.



Fig. 1: Reference object R (left) and two examples of fuzzy landscapes "close to R" (in the middle) and "to the right of R" (right). The boundary of the reference object has been subtracted from the fuzzy landscapes in order to visualize its location (shown as gray pixels).

In the case of the proximity relation assessment, if we want to assess the same distance relation whatever the direction,  $\nu$ must be isotropic by rotation around the center O. We may use a generating function f from  $\mathbb{R}^+$  in [0,1] that gives the membership value for each x according to its distance to O. In the case of the "close to" relation, that is to say, "to be at a distance lower than  $d_1$ ", f should be decreasing, such that f(d(x, O)) is close to 0 when the distance between x and O is approximately equal to  $d_1$ . Considering computational costs, the generation of  $\nu$  could be restricted to its support.

In the case of directional relative positions between two objects like "to the right of", the structuring element  $\nu$  is symmetrical according to desired direction  $\overrightarrow{u_{\alpha}}$ . Obviously, the closer the direction  $\overrightarrow{Ox}$  to  $\overrightarrow{u_{\alpha}}$ , the higher the value of  $\nu(x)$ . We generalise the generating function  $\nu$  given in [13] :

$$\nu_{\alpha,\gamma}(x) = max \left[ 0, 1 - \gamma^{-1} \arccos\left(\frac{\overrightarrow{Ox} \cdot \overrightarrow{u_{\alpha}}}{\|\overrightarrow{Ox}\|}\right) \right] \quad (4)$$

where the new parameter  $\gamma$ , called opening angle, sets the maximal acceptable angular deviation for a minimal satisfaction of the relation. Actually,  $\nu_{\alpha,\gamma}$  is a simple linear function of the deviation angle  $\beta$  formed between  $\overrightarrow{Ox}$  and  $\overrightarrow{u_{\alpha}}$ . The size of  $\nu_{\alpha,\gamma}$  should be at least two times the size of  $A \cup R$  if it is centered around O.

#### **III. PROPOSED APPROACH**

We introduce in this section the proposed approach and then show the integration of fuzzy spatial operators into fuzzy rules.

#### A. Architecture Overview

To allow the specialization of our inference engine [20] to process geographical data, while maintaining its genericity, we add an adapter that the engine calls whenever needed. The goal of the adapter is to query the GIS and to gather information in models that can be processed by the inference engine. Fig. 2 represents an overview of the system. The end-user, who is most of the time on-field, consults the indications on his tablet or his laptop. The calculations are either deported or local (if no connection is available) and the graphical interface uses



Fig. 2: System overview

our inference engine as a web service as often as needed. The engine receives the geographic area that must be processed and uses a GIS, itself in the form of a web service, to gather the geographic data necessary for the calculations. Once the calculations are completed, the fuzzy expert system returns all the results to be displayed on a cartographic background of the graphic interface.

Access to geographic data or cartographic backgrounds of a GIS can be done using the WFS (*Web Feature Service*) and WMS (*Web Map Service*) data exchange protocols. An inference engine request can result in a set of GIS requests to obtain all the information in the area (different layers, different types of data, etc.). This data is then transformed into an internal data model using the adapter, then into fuzzy raster, that is to say fuzzy pixel arrays, on which the spatial rules can be applied.

#### B. Fuzzy Spatial Operators in Fuzzy Rules

A set of fuzzy spatial relations and properties presented in the literature is based on fuzzy mathematical morphology [13], [14], [21]. It can be usefully exploited in fuzzy rules as new operators. In our previous work, we proposed to represent a rule base by a graph [20]. Its nodes represent, for example, simple or compound propositions, the input data, the output data, or even the rules themselves. Such a representation is seamlessly extensible and allows eliminating common subexpressions and preventing from redundant computations, in particular when expressions are expensive to evaluate. Here, we consider new types of possible nodes: spatial inputs and expressions representing either spatial properties or relations. Spatial relations are constructed according to the compositional paradigm introduced in [22]: new operators are built upon basic ones. Thus, the new operators take benefits from the material acceleration of the base operators by the SIMD (Single Instruction Multiple Data) instructions [23].

We use the following notations :  $\top$  is a t-norm,  $\mu_A$  and  $\mu$  are the membership functions of the spatial fuzzy set representing the two objects (fuzzy or not) A and R respectively, x and y are points in S. Let Hg and  $\neg$  be the height and the complement of a membership function defined on the spatial domain, defined by :

$$Hg(A) = \sup_{x \in \mathcal{S}} (\mu_A(x)) \tag{5}$$

$$\neg A(x) = 1 - \mu_A(x).$$
 (6)

The overlap Rec(A, R) between A and R is a point-to-point operator, and is defined as follows for all x in S:

$$Rec(A, R)(x) = \top(\mu_A(x), \mu(x)).$$
(7)

We define the occurrence degree Occ between A and R and the dilatation of R by the structuring element  $\nu$  which gives fuzzy landscapes (3) based only on  $H_g$  and Rec:

$$Occ(A, R) = Hg(Rec(A, R))$$
(8)

$$D_{\nu}(R)(x) = Hg(Rec(R,\nu^{x})) \tag{9}$$

where  $\nu^x$  is defined from S in [0,1] as  $\nu^x(y) = \nu(x-y)$ . *Occ* can be written as the degree of possibility of A and R. It can be interpreted as the degree of intersection between Aand R. Thus, it is the first topological relation. Note that some operators, like *Rec* and  $D_{\nu}(R)$ , are defined from the spatial domain S to [0,1], while others, like *Hg* and *Occ*, directly give fuzzy values.

Let  $\nu_{\alpha,\gamma}$  be the structuring element representing the direction  $\alpha$  with an opening angle  $\gamma$  (4) and  $\nu_1$  and  $\nu_2$  those representing the distances less than  $d_1$  and  $d_2$  respectively, with the condition that  $d_2$  is higher than  $d_1$ . Then, we evaluate the two first metric relations "A is in the direction  $\alpha$  of R" (*Dir*), and "A is close to R" (*Close*) by the following definitions :

$$Dir(A, R, \nu_{\alpha, \gamma}) = Occ(A, D_{\nu_{\alpha, \gamma}}(R))$$
(10)

$$Close(A, R, \nu_1) = Occ(A, D_{\nu_1}(R)).$$
 (11)

Note that we use the degree of possibility (1) to assess if a metric relation is at least partially verified between two objects.

Now, we derive two other metric relations "A is far away from R" (Far) and "A is between the distances  $d_1$  and  $d_2$ from R" (Between) form previous operators :

$$Far(A, R, \nu_2) = \neg Close(A, R, \nu_2)$$
(12)

$$Between(A, R, \nu_1, \nu_2) = \top (Close(A, R, \nu_2),$$

$$Far(A, R, \nu_1)).$$
 (13)

Finally, based on these definitions and [14], an adjacency relationship Adj can be written as :

$$Adj(A, R, \nu_1) = \top (\neg Occ(A, R), Close(A, R, \nu_1), \\Close(R, A, \nu_1)).$$
(14)

From this definition, one can conclude that two objects are considered as adjacent if they do not intersect ( $\neg Occ$  part) and if each one is close to the second one considering a distance tolerance. With the same *modus operandi*, more relations can be defined.



Fig. 3: Hierarchical fuzzy rule base system for each issue and each time horizon h. The different variables (inputs, intermediate and outputs) are shown upon arrows.

#### IV. SITUATIONAL ASSESSMENT OF WILDFIRES

In the case of intervention on wildfires, firefighters we met expressed the need to keep their own decisions during an intervention. These decisions may concern the number of engaged workforce, the type of intervention, as well as terrestrial and aerial resources. These decisions derive from the chosen strategy. This one depends on numerous factors, constraints, situational assessment, possible hazards anticipation and past experience. In the case of wildfires, some influencing factors are weather conditions, local topography and vegetation type. The considered constraints can be the available resources, the response time, and the access pathways.

In this context, we proposed to help to accelerate the situational assessment by highlighting potential risky issues according to the evolution of the wildfire. The designed fuzzy rule base system (FRBS) is depicted in Fig. 3 as a hierarchical fuzzy system [24]. The latter advocates that a hierarchical fuzzy system helps to reduce system complexity and it is more interpretable than a flat FRBS when the intermediate variables are themselves meaningful. In that case, each block of fuzzy rules can be analysed and interpreted independently.

The hierarchical fuzzy system consists of tree blocks:

- fuzzy spatial rules are defined as a first block in order to evaluate a vulnerability score for each recorded issue according to its location and the potential trajectory of the fire,
- a severity score for each issue is evaluated in a second block according to its typology and its nature,
- finally, a risk score for each issue and for each time horizon *h* is estimated by aggregating both vulnerability and severity scores by fuzzy rules.

On the graphical interface, the end-user can set the time horizons h to be considered for each request to the inference engine. The considered issues in this application are the geographical items contained in nearly ten layers of BDTOPO<sup>®</sup> of French National Geographical Institute located in the area of the fire. The requested layers are for instance layers with buildings (undifferentiated, industrial and remarkable buildings, lightweight and linear constructions) and Points of Activity or Interest (PAI). The current adapter can be easily replaced by another one that accesses to another GIS.

The next three subsections describe the vulnerability, the severity and the risks assessments. We then detail the spatial and the rule base settings. We finally show the results.

#### A. Vulnerability Assessment

An issue is considered vulnerable according to the fire if it is on its trajectory or nearby at some time horizon. The human knowledge about fire evolution can be expressed as follows. The fire spreads in the direction of the wind in an elliptical shape [7] on a flat land with homogeneous vegetation. In the South-East of France, the wildfire propagates with a speed around 3 to 8 % of wind speed according to the land topography, the type and the density of vegetation. These parameters are considered during the estimation of the shape of fire perimeter (fire shape for short), which consists of the location of the fire and of the burned area at a given time horizon. The vulnerability of an issue reaches its highest values when it is located in the spread direction of the fire and close to the estimated fire shape.

As we can see, the knowledge about fire evolution is rough and uncertain. This knowledge is translated by the two following fuzzy spatial rules:

- "If the issue A is close to the fire shape at the time horizon h and A is in the wind direction according to the fire shape at the time horizon h Then the vulnerability of the issue A at the time horizon h is high",
- "If the issue A is not close to the fire shape at the time horizon h or A is not in the wind direction according to the fire shape at the time horizon h **Then** the vulnerability of the issue A at the time horizon h is low".

To take into account the estimated weather parameters for each time horizon h, the structuring elements defining the tolerable distances and angular deviations are computed on the fly at the creation of each set of rules. They are represented in Fig. 3 as parameters in the block of the fuzzy spatial rules.

#### B. Severity Assessment

The assessment of severity depends only on the typology of the issue considered. To do this, we consider the database layer from which the issue comes and its nature. The maximum level of severity is reached for all buildings that are open to the public (such as campsites, schools and hospitals) and for all constructions which have an interest for the population (e.g., water tanks) or a significant risk (e.g., industrial buildings). An intermediate level is assigned to remarkable buildings, natural spaces and specific constructions. Finally, a low level of severity is given to lightweight, linear and undifferentiated constructions.

Some geographical items coming from different layers are overlapping : indeed, there can be one item for the commercial area from the PAI layer and one item for each building of this area coming from another layer. Nevertheless, their provenance and their nature are different, so their severity could be different. For instance, an undifferentiated building will not trigger an alert considering its severity. If it is on a campsite, the item representing the campsite area will trigger



Fig. 4: Combination of fuzzy landscapes associated to the extension of the fire with the Zadeh t-norm (left) and probabilistic t-norm (center, right) and the opening angle  $\gamma = \pi/2$  (left, center) and  $\gamma = \pi/4$  (right). A threshold at a minimum value of 0.5 is applied in order to compare the different maps.

a high alert. The use of different data layers is very useful in this case.

#### C. Risks Assessment

Finally, additional fuzzy rules evaluate the risk scores of each issue at the different time horizons h according to their estimated vulnerabilities and severity. Therefore, it is possible to highlight issues that are risky and very vulnerable but also issues that are less vulnerable but that would present significant risks if they were affected. These are issues that are not located exactly in the spread direction of the fire or in the immediate vicinity of the estimated fire shape. For instance, we have the following rules:

- "If the vulnerability of issue A at the time horizon h is high and the severity of issue A is high **Then** the risk of issue A at the time horizon h is high",
- "If the vulnerability of issue A at the time horizon h is low and the severity of issue A is high **Then** the risk of issue A at the time horizon h is medium".

#### D. Parameters Setting

Spatial parameter setting consists in choosing an opening angle  $\gamma$  (4) and a t-norm for the vulnerability assessment block. At the beginning of the intervention, the fire may be approximated by a triangular shape, as the reference object shown on Fig. 1. Fig. 4 shows the influence of these two parameters when combining conjunctively the two fuzzy land-scapes representing the expressions "close to the fire" and "in the spread direction of the fire". By using the probabilistic t-norm and by thresholding at a value of 0.5, the shapes obtained approach an ellipsoidal propagation of a forest fire, more or less open, as shown in several works [6], [7]. In the following, we will show the results obtained with  $\gamma = \pi/2$  triggering intermediate alerts in directions that are not totally in the spread direction of the fire.

The chosen FRBS is a Mamdani-type fuzzy system with the center of gravity defuzzification method. Following the analysis in [25], the output variables are composed of triangular membership functions, even on the extremities of their domain (Fig. 5). Their parameters are adjusted in order to get



Fig. 5: Definition of the output linguistic variables "vulnerability" with two fuzzy sets (left), "severity" and "risk" with four fuzzy sets (right).

the defuzzified values of the output variables in [0, 5]. This homogeneous scale helps the understanding of the results.

#### E. Results

We show in Fig. 6 the results obtained in a human-machine interface dedicated to the intervention of firefighters. The inference engine evaluates the fuzzy spatial rules and fuzzy rules to infer the vulnerability, severity and risk scores of each identified issue. Different layers of the web interface show the computed indicators: each geographic item is represented on the respective layer. Some items are well highlighted even if they are not the closest to the forecast fire shape or not directly in the wind direction considering the fire location. Indeed, the fuzzy rules computing the risk scores aggregate the vulnerability and severity scores.

An advantage of this approach is that we can easily change the fuzzy rules in order to refine the knowledge domain. We can also easily configure the parameters of the structuring elements if the distances or the angular deviations for risky issues are not satisfying. New rules and indicators can be introduced as well in order to take into account more data and information. Moreover, fuzzy logic captures the vagueness of spatial relations and allows the end-users to easily author the rules. In particular, the mathematical complexity of spatial operators are hidden behind words that are manipulated by the end-users.

#### V. CONCLUSION AND PERSPECTIVES

In this article, we have presented an extension of fuzzy expert systems to the processing of spatial data from GIS. We have exploited spatial relationships proposed in the literature and shown that they fit into the compositional paradigm introduced in our previous work. These relations make it possible to construct fuzzy spatial rules. Compared to previous works of the literature, our inference engine can handle both fuzzy spatial reasoning between fuzzy spatial objects and fuzzy reasoning on spatial measurements or objects. We have applied the proposed approach to the assessment of the risky issues in the case of wildfires.

We will continue our work to integrate more spatial relationships to increase the available vocabulary to be used in rules. This will allow us to improve the knowledge modeling, such as a more direct use of topography. This approach can be extended to other types of issues (people, vehicles) as well as to the management of other types of crises (floods, industrial incidents, etc.).



(a) Vulnerability assessment

(b) Severity assessment

(c) Risk assessment

Fig. 6: Situational assessment of constructions and points of activities and interest (PAI) at the  $4^{th}$  hour of the fire represented as colored points and filled polygons. The color scale goes from green (very low value) to red (very high value). The red polygons represent the forecast of the fire shape for each hour from the ignition point (red star).

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