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Online Spatio-Temporal Fuzzy Relations

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Abstract

The democratization of localization sensors allows to get the coordinates of mobile entities at a cheap cost but with a variable precision. Mobile devices are almost all equipped with such sensors and it is now possible to locate humans, animals or vehicles in order to monitor their activities.

In this paper, we present fuzzy online relations which allow to describe some behaviors of a mobile entity from its coordinates and regarding a spatial region described as a crisp or a fuzzy geometry. Those operators are used within a fuzzy expert system in order to make decisions like yielding alerts, summarizing activities or assessing performance measures, regarding the behavior of localized entities.

1 Introduction

With the continuous fall of the prices of localization sensors, either indoor or outdoor (for instance GPS), and the proliferation of the Internet of Things, it is easy nowadays to locate entities in different systems: Cartesian, geodesic... Moreover, sensors are not the only way to locate entities: video processing based techniques also lead to accurate results [1]. When localization is streamed, it is then possible to characterize the behavior of those entities, like human beings, animals, vehicles (possibly unmanned).

Traditionally, activity recognition is achieved with several sensors [2] and videos [3], and has been applied to many domains: from the monitoring of elder people [4] to vehicle fleet management [5], or from assistance and security [6] to general scene understanding [7] ...

Since the position is not accurate and since the definition of the activity is itself vague, fuzzy logic is suitable [8] to process spatio-temporal information [1]: for instance, fuzzy C-means [9] and fuzzy ontologies [10] lead to successful applications in human body motion analysis.

Other papers put the stress on spatio-temporal relations. On the one hand, in [11], authors use a fuzzy dissimilarity measure which compares histograms of fuzzy Allen relations and apply their technique to a set of videos. On the other hand, [12] distinguishes different spatio-temporal objects like moving fuzzy points, lines and regions and studies the fuzzification of Allen relations between them. The authors apply their relations to fuzzy spatio-temporal databases. Both of these works use rich spatio-temporal information (for instance, the shape of the objects).

In this work, we consider that the input of our system is a stream of a point's locations from localization sensors which give no more information than the timestamped coordinates (i.e. no information about the nature, the shape or the condition of the entity). In our work, entities can be humans, vehicles or any kind of objects. Only few papers consider only this feature as input: for instance, [13] uses GPS positions to compute speed based features in order to determine the travel mode of the user. Taking into account only the coordinates ensures the usability of the system at a cheap cost since almost all devices, potentially carried by humans, are geolocalized (e.g. laptops, smartphones, etc.).

In this paper, we focus on the definition of online fuzzy spatio-temporal relations for a fuzzy inference system in order to trigger events regarding the trajectory of mobile entities and predefined spatial regions. We start from our previous work on online temporal relations [14] and fuzzy spatio-temporal relations [15]. We use the same compositional paradigm to build new relations. These relations determine simple behaviors and can thus be combined to detect more complex behaviors or scenarios.

The paper is organized as follows: we recall the previous work on online temporal fuzzy relations useful for this article (section 2) and we improve existing spatio-temporal relations (section 3). Then, we describe new spatio-temporal relations (section 4) like entrance and exit, compartmentalization and the crossing relation. Section 5 describes experiments conducted with geolocalized cars. Finally, section 6 points out the conclusions and gives perspectives to this work.

2 Previous work

In [14], we introduced a paradigm which consists in deriving specialized relations from base operators in the temporal domain. In this paper, we take advantage from these operators to build spatio-temporal relations for online characterization of behaviors.

The temporal operators use two concepts to deal with event streams [14]. On the one hand, expiration is the faculty for a temporal expression to yell that its value has expired and must be re-evaluated. On the other hand, they are applied on a scope. A scope is a fuzzy set defined on a temporal domain, anchored at the present moment, and whose membership function gives the importance of a moment in this temporal domain. For instance, figure 1 shows such a scope representing "the last 24 hours".

Let E, E_1 and E_2 be fuzzy expressions, eval(E, t) be the value of the expression E at time t. Let S be a fuzzy scope, μ_S its membership function and S^* the same fuzzy scope but excluding the present moment t. Let supp(S) be



Figure 1: Fuzzy scope for the last 24 hours

the support of the membership function μ_S of S and $\neg E$ the negation of the fuzzy expression E which is often defined by the function f(x) = 1 - x. In the remainder of this paper, we will use the following temporal operators introduced in [14]:

• the ratio operator which aggregates the different activations of the operand expression *E* over a scope *S*:

$$cRatio(E, S, t) = \frac{\int_{t' \in supp(S)} eval(E, t') \land \mu_S(t')}{\int_{t' \in supp(S)} \mu_S(t')}$$
(1)

• the occurrence operator which indicates how much an expression E has been true in the scope S:

$$cOcc(E, S, t) = \bigvee_{t' \in supp(S)} eval(E, t') \land \mu_S(t')$$
(2)

• the precedence operator StartBef which indicates if an expression E_1 starts being true before E_2 :

$$lStartBef(E_1, E_2, S, t) =$$
$$Occ(E_2 \land Occ(E_1 \land \neg E_2, S, t'), S^*, t)$$
(3)

• the precedence operator *Before* which indicates if an expression E_1 becomes true before E_2 and which is compliant with Allen's semantics (see below):

$$lBefore(E_1, E_2, S, t) =$$

$$StartBef(E_1, E_2, S^*, t)$$

$$\land \neg eval(E_1, t)$$

$$\land Occ(E_2, S, t).$$
(4)

The deepest Occ operator of the StartBef operator (equ. 3) indicates whether in the scope there exists a moment when the phenomenon E_1 was observed while the phenomenon E_2 was not. The precedence is true if this Occ is true and if E_2 is observed. The other Occ operator in equation 3 looks for such a moment in the scope.

The other precedence operator Before defines a stronger precedence relation. Indeed, the semantic of the relation "before" has been defined in [16] and has been used in many papers: to Allen, " E_1 precedes E_2 " is true if E_1 has started and has stopped before E_2 began. The StartBef relation above does not match the semantics of Allen's precedence. To define the value of a " E_1 before E_2 " relation, E_1 must have started before E_2 , and E_1 must not be observed anymore while E_2 must have been observed. This gives the equation 4 to the *Before* operator.

3 Improvement of existing relations

In the spatio-temporal domain, we defined spatio-temporal operators under the same hypotheses, in order to describe human behavior like moving, going close to, going away and going along [15]. To better handle the temporal uncertainty of such relations, we changed their formulation, particularly thanks to the *Ratio* operator, replacing the average in the latter definitions. In the next section, we will use the IsMoving operator as a base operator, defined by:

$$lIsMoving(e, S, t) =$$

$$eval(P_{>0}, t) \lor Ratio(P_{>0}, S^*, t)$$
(5)

where $P_{>0}$ is the fuzzy proposition "the distance traveled by e since the previous position is not null". We voluntary omit $P_{>0}$ in the operator parameter list for the sake of readability. This nested proposition handles the spatial uncertainty, specifically because of the inaccuracy of GPS sensors. Figure 2 presents an example of a membership function suitable for the proposition $P_{>0}$. The *IsMoving* operator is thus describing the fact that the entity e is moving without any direction or speed considerations. The disjunction in the formula is important because it handles the temporal uncertainty of this operator: either e has traveled a certain distance between two last known positions at times t-1 and t, either it happened in the recent past. For instance, if e is a walking human, if he drops its keys and stops to pick them up, regarding the application we may want to consider the overall behavior, i.e. that he is walking. Another example with a car: we may want to characterize the overall trajectory without taking into account the stops at traffic lights.

To describe the other relations, let us introduce two notations:

• -o is the exterior of a region o;



Figure 2: Example of a membership function used for computing $P_{>0}$ ="the distance traveled by *e* since the previous position is not null". The distance is expressed in meters (m)

• $\in (e, o, t)$ is a geometrical operator which states if e belongs to a certain region o at time t, either crisp or fuzzy [17]: in the latter case, it supports *IsMoving* operator in the handling of the spatial uncertainty.

The IsGoingCloseTo relation indicates if an entity e is approaching a region o:

$$lIsGoingCloseTo(e, o, S, t) =$$

$$Ratio(\in (e, -o, t), S, t) \land IsMoving(e, S, t)$$

$$\land ((eval(E_{dir}, t) \land eval(E_{close}, t)))$$

$$\lor Ratio(E_{dir} \land E_{close}, S^*, t)).$$
(6)

where E_{dir} is measuring how much e is directing towards o and E_{close} indicates if e is close enough to o. Figure 3 shows some clues of the computation of E_{dir} . At time t, we consider that the optimal direction towards o is given by $\overline{P_e(t-1)M(t)}$ and that the acceptable directions are inside the cone $P_e(t-1)P_{\Delta 1}(t)P_{\Delta 2}(t)$. We thus use angles $\alpha(t)$ and $\beta(t)$ and a fuzzy set to quantify the adequacy of the direction. $E_{dir}(t)$ is defined by the expression " $D_{to}(t)$ is close to 1" where

$$D_{to}(t) = \min\left(\cos(\max(0, \alpha(t) - \beta(t))), 0\right). \tag{7}$$

Exploiting the same geometrical formalization of the direction, we can define the opposite relation IsGoingAway. Let $E_{op.dir}$ be the fuzzy proposition "the entity e is moving in the opposite direction of the region o", which can be expressed by $E_{op.dir}(t) = "D_{op}(t)$ is close to -1" where

$$D_{op}(t) = \min(\cos(\alpha(t)), 0).$$
(8)



Figure 3: Elements of comprehension for the *IsGoingCloseTo* relation

$$lIsGoingAway(e, o, S, t) =$$

Ratio($\in (e, {}^{-}o, t), S, t$) \land IsMoving(e, S, t)
 $\land (eval(E_{op.dir}, t) \lor Ratio(E_{op.dir}, S^*, t)).$ (9)

This definition of the *IsGoingAway* relation is here intended to a local use. Actually, it did not use a proximity predicate: in this case, an object moving in the opposite direction of the region, but located very far from it, will be considered going away. For a general use, a fuzzy proposition stating the proximity of the entity to the region must be added to the equation 9.

The last relation, IsGoingAlong, indicates if the entity e is moving while following the boundaries of the region o:

$$lIsGoingAlong(e, o, S, t) =$$

Ratio($\in (e, -o, t), S, t$) \land IsMoving(e, S, t)
 \land Ratio(E_{close}, S, t). (10)

The *IsGoingAlong* is very tolerant because the direction is not taken into account. If needed, we will work on a new version which takes direction in consideration for this kind of behavior.

From those previous operators, we describe in the next section new online spatio-temporal operators.

4 Definition of new spatio-temporal relations

4.1 Entrance and exit operators

Let's consider the fact that an entity e is entering a closed geometrical surface o with membership function μ_o indicating how much a point belongs to o. o can be crisp or fuzzy. The formulation of such an operator is quite straightforward given the existing operators: entering a closed geometrical surface means being outside this area before being inside this area. Thus,

$$lIsEntering(e, o, S, t) =$$

$$IsMoving(e, S, t)$$

$$\land Before(\in (e, -o, t), \in (e, o, t), S, t). \tag{11}$$

In practice, the scope S must be a little greater than the sampling rate of the position of e: if S is too big, IsEntering will trigger only once during the scope S because of *Before* operator: so cases of a reentrance will not be detected.

We can derive from *IsEntering* the *IsExiting* operator:

$$lIsExiting(e, o, S, t) =$$

$$IsMoving(e, S, t)$$

$$\land Before(\in (e, o, t), \in (e, {}^{-}o, t), S, t).$$
(12)

4.2 Compartmentalization operators

This type of operators monitors the entity e and states about its behavior in two ways:

- either it keeps moving in a closed area,
- either it is following a path that it has been asked to follow.

On the one hand, the first operator is straightforward to define. It states if e keeps moving while staying in a defined closed area o. We can define the operator IsMovingInside with the following equation :

$$lIsMovingInside(e, o, S, t) =$$

$$eval(E_{MovingIn}, t) \lor Ratio(E_{MovingIn}, S^*, t)$$
(13)

where the fuzzy expression $E_{MovingIn}$ evaluated at time t states if e is moving inside o at that moment. It can be defined by :

$$eval(E_{MovingIn}, t) = eval(P_{>0}, t) \land \in (e, o, t).$$

$$(14)$$

The disjunction in equation 13 handles the spatial uncertainty: for instance, if e is out of o for a brief moment, the ratio will smooth this fact. As for

Is Exiting and Is Entering operators, the inclusion relationship $\in (e, o, t)$ between e and o can be either crisp or fuzzy.

On the other hand, to monitor if e is following a path, we define a path Π , also called an itinerary, as an oriented polyline. Thus, to follow a path, the entity e must move in the same direction of Π and must be close enough to Π while moving (figure 4).

Let $P_e(t)$ be the position of e at time t. Let $\overline{QQ'}$ the line segment of Π which $P_e(t)$ has been projected onto. Thus, the distance $d(e, \overline{QQ'})$ between e and $\overline{QQ'}$ must be kept low. Let E_{close} be the fuzzy proposition " $d(e, \overline{QQ'})$ is low".

Moreover, we have to take into account the orientation of e regarding Π . We could use exogenous orientation information (e.g. a magnetometer), but it is not a good characterization of the trajectory orientation. For instance, imagine a human moving sideways: his own orientation is thus perpendicular to the trajectory. We use the orientation of $\overrightarrow{P_e(t-1)P_e(t)}$ instead, which is thus deduced from the last move; as a consequence, we need two positions to start characterizing the behavior. If e follows exactly Π , $\overrightarrow{QQ'}$ and $\overrightarrow{P_e(t)P'(t)}$ are collinear and have the same direction. So if we note $\alpha'(t)$ the angle between the two vectors at time t, $\cos(\alpha'(t)) = 1$. Let E'_{dir} be the fuzzy proposition "eis moving in the good direction". For instance, the value of this proposition can be evaluated by $\max(\cos(\alpha'(t)), 0)$: the more $\max(\cos(\alpha'(t)), 0)$ tends to 1, the more e is moving in the right direction. Obviously, $\max(\cos(\alpha'(t)), 0)$ equals 0 for an opposite direction.

Thus, the *IsFollowingAPath* can be formulated as:

$$lIsFollowingAPath(e, \Pi, S, t) = IsMoving(e, S, t)$$

$$\land (eval(E_{close}, t) \lor Ratio(E_{close}, S^*, t))$$

$$\land (eval(E'_{dir}, t) \lor Ratio(E'_{dir}, S^*, t))$$
(15)

in which we voluntarily omit E_{close} and E'_{dir} in the parameter list for the sake of readability. Both expressions are important parameters because they allow to customize both the temporal and the spatial uncertainty of the operator regarding the application.

4.3 Going through operator

In this section, we define an operator to assess if e is crossing a closed area o. This operator is not easy to define and several attempts have been done in the past. In [18], the author studied the different meanings of the term "crossing" and asked a pool of 32 persons to choose among 8 figures which ones define the best "crossing". The conclusion is that "crossing" the points on which the entity enters and goes out of the region are located on "opposite" sides. Figure 5 shows different itineraries which cross the region. The examples are given in



Figure 4: Illustration of the different elements of the IsFollowingAPath operator equation



Figure 5: Different examples of trajectories which cross the region, adapted from [18]



Figure 6: Illustration of the different elements of the IsCrossing operator equation

increasing order of value for an intuitive definition of "crossing". The example (A) is not a case of crossing because the entity enters and leaves on the same side of the region. In (B), the entity has crossed the region but the sides are not opposed. The case (C) and (D) are really "crossing" examples but the path inside the region is shorter in (C) than in (D), so we expect a lower value in (C) than in (D).

We thus chose to introduce a criterion to assess how much e is crossing o. During the crossing of o, the trajectory of e is splitting o into two closed geometries $\mathcal{O}_1(t)$ and $\mathcal{O}_2(t)$, whose areas are respectively $\mathscr{A}_1(t)$ and $\mathscr{A}_2(t)$ (figure 6). Our criterion establishes that the value of "e is crossing o" takes its maximum value when it splits o into $\mathcal{O}_1(t)$ and $\mathcal{O}_2(t)$ such as $\mathscr{A}_1(t) = \mathscr{A}_2(t)$. Of course, it is not sufficient: e has to be moving inside o. Thus, we define:

$$lIsCrossing(e, o, S, t) =$$

$$IsMoving(e, S, t) \land \in (e, o, t)$$

$$\land (Criterion(t) \lor Ratio(Criterion(t), S^*, t))$$
(16)

where Criterion(t) is a predicate whose values are in [0,1] indicating if the two areas $\mathscr{A}_1(t)$ and $\mathscr{A}_2(t)$ are equal. As usual, we handle the uncertainty about the general trajectory with the *Ratio* operator.

After several tests, we chose the entropy as Criterion(t):

$$Criterion(t) = -\frac{\mathscr{A}_1(t)}{\mathscr{A}}\log_2\frac{\mathscr{A}_1(t)}{\mathscr{A}} - \frac{\mathscr{A}_2(t)}{\mathscr{A}}\log_2\frac{\mathscr{A}_2(t)}{\mathscr{A}}$$
(17)

where \mathscr{A} is the area of o.

To compute $\mathscr{A}_1(t)$ and $\mathscr{A}_2(t)$ when e is inside o but not on the boundary, we have to extrapolate its trajectory (see the dotted line in figure 6): although



Figure 7: Subsumption lattice for the spatio-temporal relations

there are a lot of extrapolation methods, we chose to simply extend the last line segment of the trajectory until it intersects the boundary of o. At each new position, the frontier between $\mathscr{O}_1(t)$ and $\mathscr{O}_2(t)$ is re-evaluated with the past positions and the extrapolation of the itinerary.

4.4 Subsumption lattice of online spatio-temporal relations

To allow more research on these spatio-temporal relations, for instance relation learning, we start maintaining a subsumption lattice with the different spatiotemporal relations (figure 7) implemented in the system. Figure 7 only shows the relations between a mobile entity and a region (i.e. it does not include the IsFollowingAPath). For instance, as we characterize moves, all the relations are more specific than IsMoving, and IsGoingThrough is more specific than a scenario consisting in the succession of IsEntering, IsMovingInside and IsExiting.

5 Experiments

As we discussed before, there are many applications to this spatio-temporal monitoring. In this paper, we illustrate the work by a company's vehicle fleet. The goal is to display the different vehicles on a map and to summarize the different trajectories online. In this experiment, drivers were saving their activities.

In this case, each car is considered as a mobile entity and is equipped with a GPS sensor and a 3G connection to send their position every 3 seconds. Different map regions have been predefined, as the research center (in white in figure 8),

a golf field (in green in figure 8) and the different roads and car parks inside and around the research center.

Figures 9, 10 and 11 show the different parameters which will be used for the configuration of the relations, i.e. respectively the temporal, spatial and orientation tolerances. For instance, *IsMoving* will be parametrized with the fuzzy sets "very short time" and "distance tolerance".

Experiments show that the behaviors are correctly detected. We show in figure 8 one vehicle journey using Microsoft Bing Maps API. Figure 12 shows the output of the operator *IsFollowingAPath* applied on the road D306 (first part of the journey) and the car itself. It computes high values when the car is moving on road D306, and pits when the car stops at traffic lights. The last brutal pit is due to the change of direction (the car is turning right at the intersection). The slope is reflecting the temporal uncertainty. As the Microsoft API gives crisp geometrical regions, the spatial uncertainty cannot appear.

Figure 13 shows the output of the operator IsCrossing applied to the entity e and the spatial region corresponding to the golf field. After turning right, the vehicle was driven on a road which crosses the golf field. We can see the precedence of the two trajectories on both figures 12 and 13. The latter operator barely reaches its maximum value 1 because the region of the golf field is not split into two regions of equal areas, but it is high enough to early detect the intention of crossing the golf field. This operator is more intentional than factual, since at one moment, the direction can lead to the decision the entity is going through the region whereas the next moment it does not.

Figure 14 shows the results of IsEntering operator applied to e and the golf field region, with two different scopes to emphasize the effect of the temporal uncertainty. The golf field has also been defined as a crisp spatial region. Regarding the scope parameter S, the operators can have a high value during a long time. The longer the scope, the longer the output will have the maximum value. The figure shows that the relations which rely on geometrical criterion are noisier than the others. It emphasizes also the need for a subsumption lattice since we can detect the subsumption pattern between IsGoingThrough applied to the car and the golf field and the scenario IsEntering, IsMovingInside and IsExiting.

Finally, we represent the online evaluation of a subset of the relations into a "trajectogram" in figure 15. Each relation has a signal which represents its activation along the trajectory shown in figure 8.

6 Conclusion

In this paper, we have formalized spatio-temporal relations between a mobile entity, which is represented as a point, and predefined geometrical regions. We start from base operators and we build from them, by combination, new operators.

The compositional paradigm allows intuitive definition and formalization of the spatio-temporal relations. It takes also advantage of the property of



Figure 8: View of a specific itinerary



Figure 9: Temporal scopes



Figure 10: Spatial scopes



Figure 11: Membership functions used for assessing the direction of e in case of the computation of the operator IsGoingCloseTo



Figure 12: Output of the expression "Car is following road D306" during the example journey



Figure 13: Output of the expression "Car is crossing the golf field" during the example journey



Figure 14: IsEntering evaluation with two temporal scopes



Figure 15: Spatio-temporal relations activations during the journey

each relation which composes the new operators: for instance, the temporal uncertainty is handled by the base temporal operators, specifically by the *Ratio* operator, and the spatial uncertainty is handled by the *IsMoving* operator.

The intuitive definitions are very important for justification, especially in human centered applications. Indeed, it is possible to justify, for instance, why the system considers an entity is entering a region: because it was outside before being inside.

Since the operators are not specific to an application, we can use them in other applications and we are planing to complete the spatio-temporal operator collection with new general purpose relations and more specific ones. For instance, we aim to use these operators to characterize the behavior of unmanned vehicles to assess their ability to drive themselves, to change the lighting of a room regarding the behavior of its visitors, or to monitor the activity of security agents or elder people at home.

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