Maximum gain of an Electrically Small Cosines-dipole-based end-fire array

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***Abstract* — Electrically Small Antennas (ESAs) are of high interest in applications where a compact size wants to be preserved. With the objective of the realization of a highly efficient and superdirective array within a very compact space, a new radiating element and array design is proposed, optimized for maximum attainable gain. A cosines-dipole printed antenna is designed by adjusting the amplitude and periodicity of the cosine microstrip, to resonate at the desired frequency of 916 MHz. Then, to achieve high gain two elements are closely spaced, mirrored to their excitation point, one fed and the other parasitic. An analytical method for gain optimization based on the Spherical Wave Expansion (SWE) theory is used to maximize the array gain. Two different array geometries are compared by looking at the power distribution of the spherical modes, and the configuration that returns the highest gain is chosen. A realized gain of 6.5 dBi is achieved in simulation, for a total size included in a radiansphere of *ka=1*, with the advantage of having a printed antenna, thus planar, easy to integrate, and low cost. The proposed antenna solution presents in theory an excellent trade-off between electrical size and realized gain.**

Keywords—antenna arrays, electrically small antenna, superdirectivity, high efficiency, antenna gain optimization.

I. Introduction

Electrically Small Antennas (ESAs) have proven to be of particular interest due to their small size (relative to the wavelength of interest) and yet good performance. However, the difficulty in their implementation has always been a matter of debate, given the fundamental limitations that bound gain and bandwidth to the antenna's physical size [1]. The milestone works of Chu [2] and Harrington [3] set the limits to antenna directivity as $D_{\max} = (kr)^2 + 2(kr)$, with k the wavenumber, r the radius of the smallest sphere enclosing the antenna, and kr the size of the "radiansphere". To agree with realizations of antennas presenting higher directivity values, Kildal and Best [4] proposed a heuristic formula that re-computes this limit as $D_{\max} = (kr)^2 + 3$. For a vanishing kr, this limit converges to 3 (or 4.77 dBi) agreeing with the directivity of an infinitesimal Huygens source. More recently, in [5] a new effective radiansphere of size $kr+1$ that allows the efficient propagation of the fundamental mode, even for a vanishing kr, is considered. This results in the re-calculation of the limit as $D_{\max} = [k(r+1/2\pi)]^2 + 2[k(r+1/2\pi)]$. On the other hand, already in 1946 Uzkov [6] observed for end-fire arrays of P isotropic sources an increasing directivity up to P^2 when tending to a near-zero inter-element distance. Hence, the size of a radiating source is not the only parameter determining its potential directivity. Any source whose directivity exceeds these limits is considered "superdirective". Superdirective sources that are also efficient are referred to as "supergain".

It is generally known that end-fire arrays present high values of gain, but when their inter-element spacing is reduced ($< 0.2\lambda$) [6]. The compact dimensions lead to strong levels of mutual-coupling and current magnitudes on the elements, and high power dissipation in the resistive part of the antennas. The synthesis of supergain arrays can be mathematically seen as a relaxation of the optimum problem when directivity is optimized. In practice, all the available modes, in the given volume and radiating in the desired direction, should be correctly excited with the additional constraint of keeping the radiation efficiency high.

While the literature on superdirectivity is quite large, only few works deal with the synthesis of supergain end-fire arrays. In [7], the array factor theory is used to optimize a three-element end-fire array presenting 8.6 dBi of gain with a kr=1.4; in [8] a convex optimization problem is developed and theoretical results are presented for two- and three-dipole end-fire arrays; and finally in [9] a method for gain optimization based on Spherical Wave Expansions (SWE) theory is presented. Recently, in [10] another approach to the design and optimization of high-gain antennas is presented based on the multipole excitations in a very compact space. An antenna within a radiansphere of $kr=0.98$ is realized and a realized gain of 6.3 dBi is measured.

This paper aims to present a new cosines-dipole antenna geometry, the design of a new two-element compact array geometry with a total kr<1, and the use of the SWE method [9] to investigate the best array geometry and maximize the gain.

II. Directivity And Gain Optimization

Directivity limits for end-fire arrays of P elements, when their spacing vanishes, are known from the literature in the case of isotropic sources, according to Uzkov's observation [6] as P^2, and for electrical dipoles and Huygens sources as $P^2+P-1/2$ and P^2+2P respectively, according to [11]. In the latter, it is also pointed out that when directivity tends to its maximum, according to the efficiency, a drop in the gain is observed. The gap between directivity and gain depends on the efficiency values of the array elements.

When $P=2$ half-wave electrical dipoles are considered, their maximum directivity is 7.3 dBi, while for the gain not the same limit can be stated. However, despite losses severely impacting the array performance when compact sizes are targeted, gain optimization can be a solution. As reported in [9] by taking into account the radiation efficiency of the elements the optimization can be modified and target maximum gain rather than directivity. Based on the SWE theory, this optimization searches for a trade-off between the highest possible number of radiating modes and their efficiency. In the case of directivity, one may observe higher order modes excited and a consequent high directivity, but an actual very low gain. Such higher-order modes can be excited in the structure, but they are mainly dissipative and cannot radiate energy. The gain optimization cuts away that portion of modes that are not efficient. In practice, it reduces the directivity and increases the gain. By employing the method presented in [9], one can find the optimal excitation, i.e. the complex coefficient a_p, necessary to excite the best combination of spherical mode for maximum gain, as detailed in the following paragraph.

A. Synthesis of Supergain End-Fire Arrays

According to the SWE theory, outside of the radiansphere of radius *kr*, the electric field radiated by *P* radiating sources can be expressed with a combination of spherical far-field functions $\vec{K}_{smn}(\theta,\phi)$ as

$$\vec{E}_p(\theta,\phi) = \frac{k}{\sqrt{\eta_0}} \frac{1}{\sqrt{4\pi}} \frac{e^{-jkr}}{kr} \sum_{smn} Q_{smn,p} \vec{K}_{smn}(\theta,\phi) \qquad (1)$$

with η_0 being the complex impedance of the vacuum and $Q_{smn,p}$ the spherical wave coefficients for each of the $p=1,...,P$ elements. The term $\sum_{smn}$ denotes the triple sum over the indexes *s,m,n*. The degree *n* can be truncated to a N_{max} normally proportional to the size of the sphere, while the order *m* is limited by $|m| \le n$. The index $s=1,2$ indicates TE- or TM-modes, respectively. According to [9]

$$G(\theta,\varphi) = \frac{\left| \sum_p \sum_{smn} Q_{smn,p} \vec{K}_{smn}(\theta,\phi) \cdot \vec{i}_{co}^{\,*} \right|^2}{\sum_p \sum_{smn} \left| Q_{smn,p} \cdot \sqrt{1+\delta_{n,p}} \right|^2} \qquad (2)$$

is the total gain of the array. The $\vec{i}_{co}^{\,*}$ is the unit vector in the polarization direction, and the term $\delta_{n,p}$ is the dissipation factor, or the ratio $P_{loss,n}/P_{rad,n}$, expressing the losses of each spherical mode of *n-th* degree radiated by the *p-th* element. In [9], [12] it is demonstrated that the maximum of (2) is achieved whit the set of spherical wave coefficients

$$Q_{smn}^{\max} = c \cdot \left(\frac{\vec{K}_{smn}(\theta_0,\phi_0)}{\sqrt{1+\delta_n}} \cdot \vec{i}_{co}^{\,*} \right)^* \qquad (3)$$

with *c* an arbitrary constant. Then, the desired coefficients a_p are the solution of the matrix problem where the initial conditions are expressed by a matrix containing all the SWE coefficients $\mathbf{Q_{initial}} = \left[Q_{smn,1},...,Q_{smn,P} \right]$ of equation (2), calculated for the *P* elements. The coefficients of the optimal modal distribution are known by (3) as $\mathbf{Q_{optimal}} = \left[Q_{smn}^{\max} \right]$ calculated with the same number of spherical modes N_{max}. Then, a matrix inversion of these two quantities

$$\mathbf{a} = \mathbf{Q}_{\mathbf{initial}}^{-1} \times \mathbf{Q}_{\mathbf{optimal}} \qquad (4)$$

is the solution to the problem. The vector **a** in our case would contain two complex numbers a_1 and a_2, proportional to the optimal elements feed for gain maximization.

B. Parasitic array

The complex coefficients a_p solution for maximum gain represents the excitations of the array elements. The amplitude and phase of the a_p are proportional to the magnitude and phase of the currents that must be excited. Setting such a specific current on each element with a fully driven configuration leads to the optimal result. However, it is often preferred, for a matter of complexity, or realization feasibility, to seek an alternative solution. Parasitic loading demonstrated an excellent agreement with full-driven excitation for superdirective arrays [13]. The amplitude and phase shift needed among the array elements are synthesized by loading with equivalent complex loads each element. In this way, the real and imaginary parts of the element's impedance can be tuned to build the desired current. The initial a_p complex coefficients is transformed then in a set of impedances Z_p.

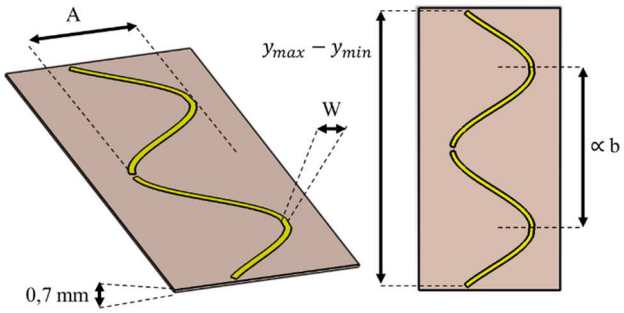

Fig. 1. Geometry illustration of the cosines-dipole printed antenna.

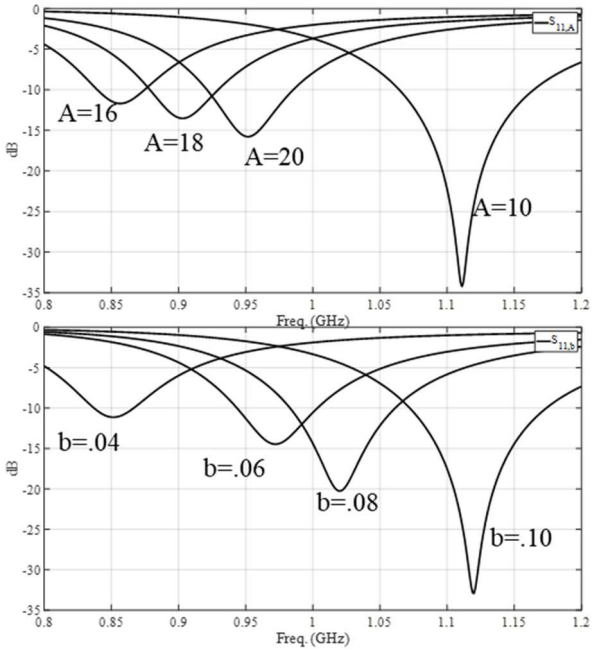

Fig. 2. Parametric results of the S_{11} modifying the parameter *A* (top figure) and *b* (bottom figure) of the cosine microstrip line.

In our case of a two-element array, parasitic loading is quite straightforward, as only one load must be calculated. First, the two complex values a_1 and a_2 are normalized to the value a_1 because represent the fed element. We are interested in the amplitude and phase shift of the second element compared with the first. Then the complex value $a_{2,norm}$ and the scattering matrix *S* are used to calculate the load Z_2. This impedance is connected in series with the second element and provides the same effect of the full-drive excitation of a_1 and a_2 as separate excitations.

III. Single Source and Array Design Method

The objective is the design of the highest possible gain antenna in a limited space of $kr<1$. End-fire arrays present compact dimensions and high gain. As revealed in the SotA of superdirectivity, dipolar-type radiators are preferred considering that they can be flexibly miniaturized without compromising radiation efficiency. Moreover, the results must be cheap and easy to fabricate, and for these reasons, a printed type of antenna is investigated. The design of a miniaturized electrical dipole, based on a co-sinusoidal bend of the two arms, and printed on a dielectric substrate is reported. Then, two configurations of two-element end-fire arrays of this new

proposed element are investigated. By using the SWE analysis on the radiated field and the array optimization, the best configuration for maximum gain is chosen.

A. The cosine dipole

Dipole miniaturization can be done by bending the dipole's arms. This is equivalent as an increase in the electrical path of the current which allows to match the desired frequency. The employee of a cosine line geometry presents a smoother current path, and even for a 40% of total length reduction a radiation efficiency higher than 99% is observed. The cosine dipole is defined by three parameters, corresponding to the dimensions of the cosine function, assuming it written as $A\cos(b \cdot y)$. One has that

- the larger the value of b, the smaller the period and the distance between two maxima or minima are;
- the larger the value of A, the greater the amplitude of the cosine line;
- y is the direction where the line is shaped, and it varies from $y_{\min}$ to y_{max}, defining the vertical dipole length.

To match the interested frequency of 916 MHz the parameters A and b must be tuned. At first, for a fixed amplitude $A = \lambda/16$ the effect of the parameter b is investigated. One can observe that the lower is b, the closer is the cosine to a straight line, and the return loss improve as the resonance frequency increase (Fig. 2). The choice of b affects the resonance and the length of the line, but not the overall length of the dipole, which is defined by $y_{max} - y_{min}$. The amplitude A scale up or down the line according to its value, and has also an effect on the resonance of the antenna (Fig. 2). Finally, the parameters are set as $y_{max} - y_{min} = 0.3\,\lambda$ (10 cm), $A=\lambda/22$ (14.9 cm), and b=0.107. The substrate chosen is the Roger 5880 ($\varepsilon_r = 2.2$) with thickness 0.7 mm. The printed line is made of copper ($\sigma = 5.7e^8$) with a width of 2 mm and thickness 0.07 mm. The final geometry is presented in Fig. 1.

B. Two-elements array

The single element is none other than a dipole, as described in the previous section. The advantageous low profile permits the design of a two elements array within the radiansphere of an electrically small antenna ($kr<1$). The curved shape takes advantage of the available surface of the sphere, even if just on its central section (circle of radius $1/2\pi$). It has been highlighted in Sections I and II that high gain is possible only when radiation efficiency is high, and element spacing is critical when lower than a certain threshold [9]. Therefore, two different array configurations, as illustrated in Fig. 3, have been investigated and compared. The first case favors the length of cosines-bended dipole's arm, but the two elements are very close to fit into the fixed sphere. The second case considers more distant elements, and a further length reduction of the arms to fit in the same sphere.

C. Gain optimization and simulated results

In the first case, the two elements have a central axe-to-central axe distance of 0.06λ. The overall length of the dipoles is reduced by 38%. The configuration is illustrated in Fig. 3 with green lines. The structure is symmetric and the spherical modes can be more evenly excited. When the gain optimization method of Section II is applied, the SWE of the total array field is depicted in Fig. 3. One can distinguish the

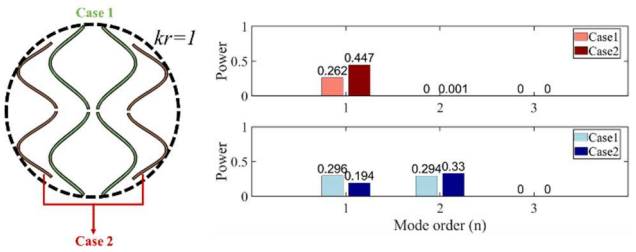

Fig. 3. Two possible element geometry and disposition fitting in the radiansphere of $kr=1$. On the right the power distribution of the SWE for Cases 1 and 2 when the gain is optimized. TE modes are shown in red bars (first line) and TM modes in blue bars (second line).

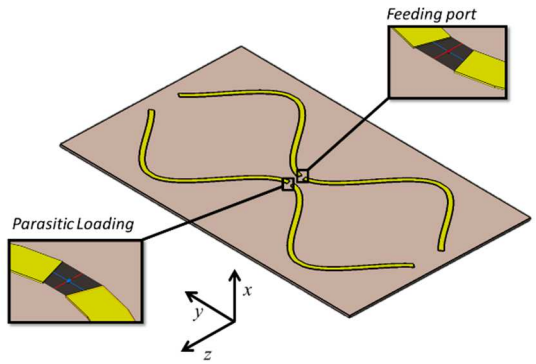

Fig. 4. Geometry of the two elements cosine-dipoles mirrored to the feeding point. The zooms highlight the feeding port of the active elements and the parasitic loading of the second one.

TE and TM components of $n=1$ fully excited, and the excitation of the TM mode of $n=2$. The power distribution is the one of two dipoles closely spaced, as demonstrated in [11]. All the modes are evenly excited, which means that the energy shared between the excited modes and it results in stable radiation. The simulated patterns of the realized gain the E- and H-planes are illustrated in Fig. 5. In this case, a directivity of 7.1 dBi and a gain of 6.5 dBi are observed. The second case considers, in the same radiansphere, a more distant element disposition. The central axe-to-central axe distance is of 0.17λ. To make the dipoles fit in the sphere, the cosines-line miniaturization is increased by up to a 46% reduction of the total dipole length, as illustrated in Fig. 3 (red lines). The efficiency observed is still close to the unity, but this configuration presents a different modal distribution when the gain is optimized (Fig. 3). The excited modes are the same, but the power is not evenly distributed compared to the first case. In particular, for $n=1$ the TM mode has the 19% of power, and the TE mode the 45% of power. This unbalance is the result of the increased element-spacing. Considering a dipole centered in the origin of the coordinate system and oriented along the y direction, it is known from its SWE that by shifting along the z direction TE modes components appear. Then, the resulting modal distribution differs from the optimal one which sees the TE and TM modes of $n=1$ equally excited (Huygens source). The dipole miniaturization is also responsible, since the total length is decreased, and the cosine-line miniaturization squeezes the arms over each other and makes the line closer to a loop, further enhancing the TE mode excitation. The radiation patterns in E- and H-planes of the realized gain is illustrated in Fig. 5. The directivity observed is 5.4 dBi and the gain of 5.3 dBi. In the latter case, directivity and gain are very close, telling us that the problem is not related to efficiency, but to the way it possible to excite the modes in the radiating structure.

IV. Discussion and Conclusions

In this paper, the process of synthesis of a supergain end-fire arrays is discussed, highlighting the difference with the optimization of directivity. Then, the design process of a compact and high-gain array is described. Firstly, a new printed cosine-dipole antenna is shown. Then, two different configurations of two-element arrays are analyzed, representing the compromise between single-element size and inter-element spacing, at a fixed radiansphere of kr=1. With the help of SWE, it was possible to maximize the gain and show the different power distribution of the radiated modes in the two cases, determining the causes of the difference in performances. It is noted that a homogeneous power distribution returns a higher gain. Moreover, the result obtained for the directivity is close to the limits found in [11] for dipole-based end-fire arrays, but with a surprisingly high level of realized gain. One can then again verify Uzkov's assertion, with directivity being more important the closer the elements are together. What is remarkable here is that this can be equally valid for gain, if certain design conditions occur. In particular, the first case with a reduced element spacing and 36% of dipole miniaturization shows a realized gain in simulations of 6.5 dBi, making this compact array of kr=1 a supergain planar antenna. This value surpasses the Chu-Harrington limit that this size is set to 4.77 dBi, and the Kildal-Best limit set to 6 dBi.

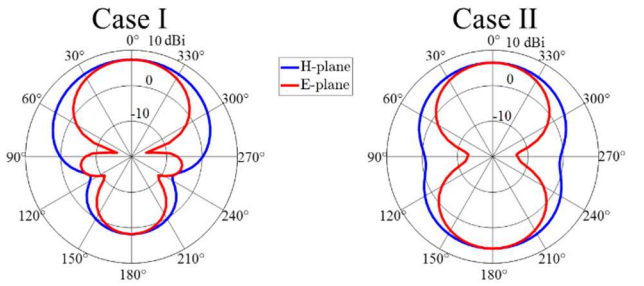

Fig. 5. Simulated (CST Microwave Studio) radiation patterns in E- and H-planes at 916 MHz for the two cases. Cosine-dipoles are closely spaced in Case 1 and farther apart in Case 2.

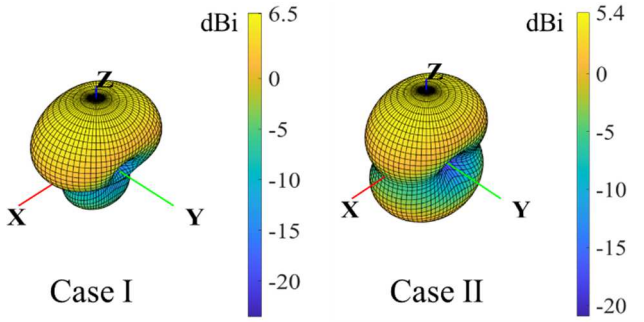

Fig. 6. Analytical reconstruction in MATLAB of the 3D Gain radiation pattern from the SWE coefficients for the two cases.

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TABLE I. Optimization Results

Element disposition	Optimization results			
	a_{opt}	Z_{opt}^{*}	*Dir [dBi]*	*Gain [dBi]*
Case 1	$0.45^{\angle -152.46}$	j28.85	7.1	6.5
Case 2	$0.48^{\angle -93.29}$	j67.25	5.4	5.3

*Optimal loads include a real part forced to zero for gain optimization.

References

[1] M. Koziol, "A four-year program to tackle a fundamental antenna challenge - IEEE Spectrum," *IEEE Spectrum*, Sep. 06, 2022. https://spectrum.ieee.org/electrically-small-antenna (accessed Dec. 06, 2022).

[2] L. J. Chu, "Physical Limitations of Omni-Directional Antennas," *Journal of Applied Physics*, vol. 19, p. 14, 1948.

[3] R. F. Harrington, "Effect of antenna size on gain, bandwidth, and efficiency," *J. Res. Natl. Bur. Stan. Sect. D. Rad. Prop.*, vol. 64D, no. 1, p. 1, Jan. 1960,

[4] P.-S. Kildal and S. R. Best, "Further investigations of fundamental directivity limitations of small antennas with and without ground planes," in *Proc. IEEE Antennas Propag. Soc. Int. Symp. (APS/URSI)*, San Diego, CA, Jul. 2008, pp. 1–4.

[5] M. Pigeon, A. Clemente, C. Delaveaud, and L. Rudant, "Analysis of Harrington limit for electrically small antenna directivity," in *Proc. Eu. Conf. Antennas Propag. (EuCAP 2014)*, The Hague, Netherlands, Apr. 2014, pp. 2921–2925.

[6] A. I. Uzkov, "An approach to the problem of optimum directive antenna design," 1946.

[7] A. Debard, A. Clemente, and C. Delaveaud, "Three-element end-fire linear arrays (super) directivity and gain optimization," in *Proc. Eu. Conf. Antennas Propag. (EuCAP 2020)*, 2020, Copenhagen, Denmark, pp. 1–4.

[8] A. Georgiadis and N. B. Carvalho, "A convex optimization approach for the design of supergain electrically small antenna and rectenna arrays comprising parasitic reactively loaded elements," *IEEE Trans. Antennas Propag.*, vol. 70, no. 6, pp. 4674-4682, Jun. 2022,

[9] A. Tornese, A. Clemente, and C. Delaveaud, "A new method for end-fire array gain optimization using spherical wave expansion," in *Proc. IEEE Antennas Propag. Soc. Int. Symp. (APS/URSI),* Denver, Colorado, pp.1232-1233.

[10] T. Shi, M.-C. Tang, R. Chai, and R. W. Ziolkowski, "Multipole-based electrically small unidirectional antenna with exceptionally high realized gain," *IEEE Trans. Antennas Propag.*, vol. 70, no. 7, pp. 5288–5301, Jul. 2022,

[11] A. Debard, A. Clemente, A. Tornese, and C. Delaveaud, "On the maximum end-fire directivity of compact antenna arrays based on electrical dipoles and Huygens sources," *IEEE Trans. Antennas Propagat.*, vol. 71, no. 1, pp. 299–308, Jan. 2023.

[12] J. E. Hansen, *Spherical Near- Field Antenna Measurements*, vol. 502. Iet, 2008.

[13] T. H. O'Donnell and A. D. Yaghjian, "Electrically small superdirective arrays using parasitic elements," in *Proc. IEEE Antennas Propag. Soc. Int. Symp. (APS/URSI)*, Albuquerque, NM, USA, 2006, pp. 3111–3114.