Plasma-wall interaction: comparison between kinetic simulations and fluid theory

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Y. Munschy, E. Bourne, G. Dif-Pradalier, P. Donnel, Ph. Ghendrih, V. Grandgirard, Y. Sarazin

Ack. : GYSELA team, TSVV4

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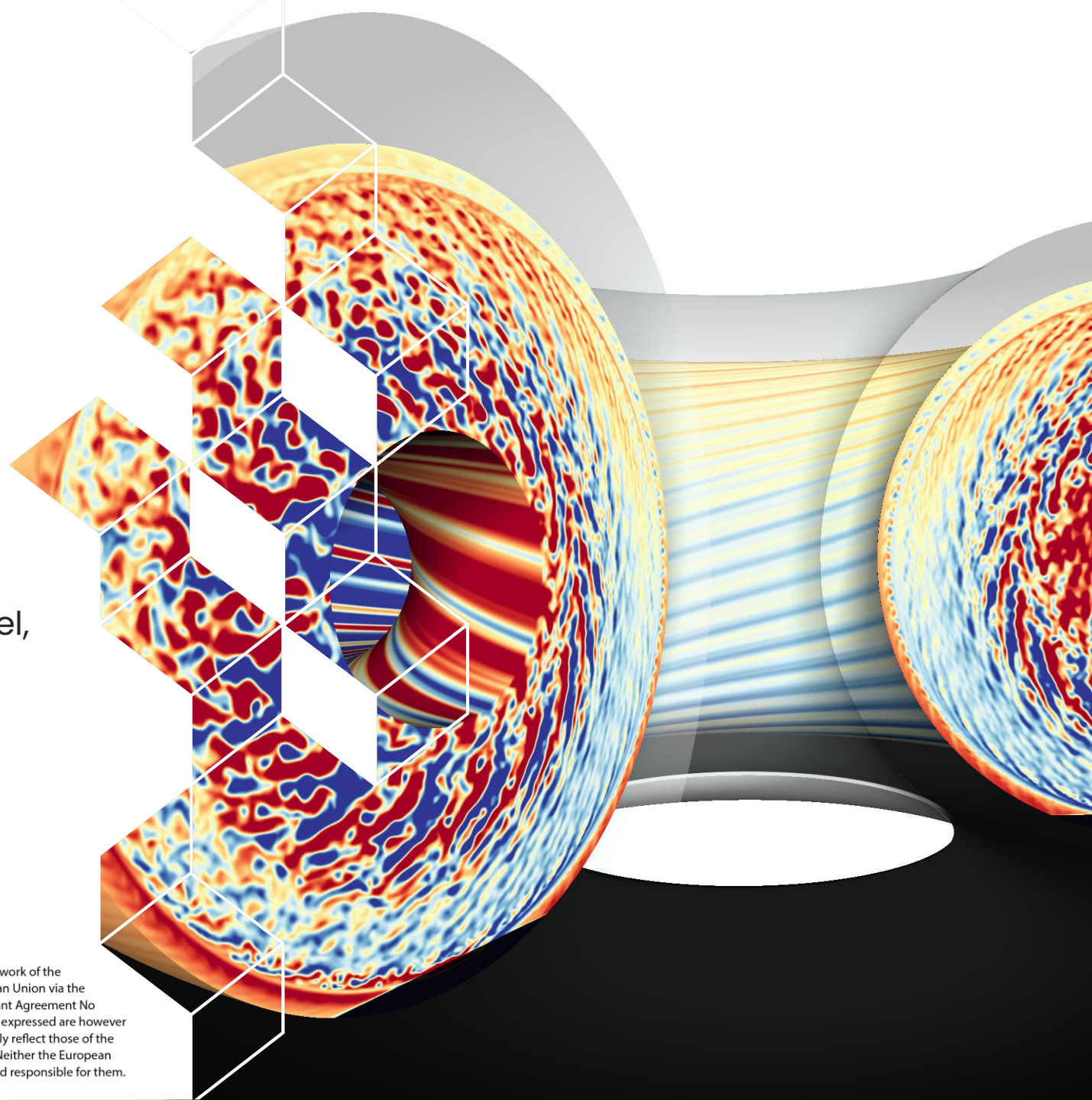

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Core-edge-wall interaction influences tokamak confinement quality

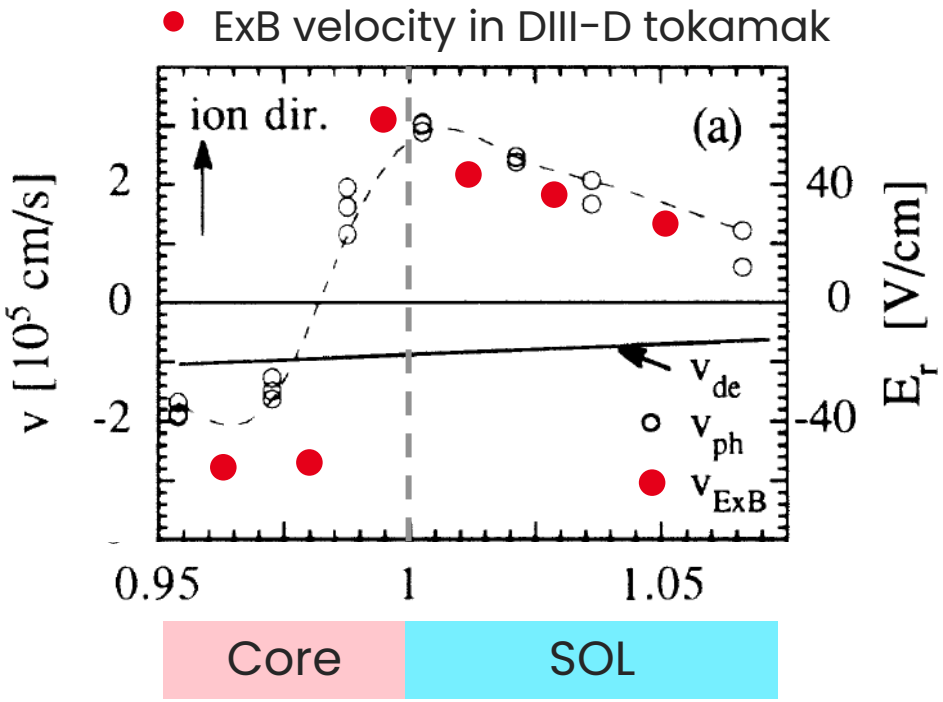
- Microscale turbulence ⇒ cross-field transport ⇒ degrades the confinement unless a transport barrier exists
  [Wagner1982]
- Turbulence regulated by ExB sheared flow at LCFS

$$\text{core}\quad E_r \propto \frac{\nabla P}{en} \qquad \text{SOL}\quad E_r \propto -\frac{\nabla T_e}{e}$$

E_r in SOL influenced by plasma-wall interaction
[Stangeby]

- Goals : 1) study of kinetic plasma wall interaction
  2) implementation in GYSELA gyrokinetic code (WIP)

[Ritz 1991]



Plasma wall interaction results from absorbing wall and inertia difference between ions and electrons

[Stangeby 2000, …Ghendrih 2011 and many more]

- Electrons: low inertia compared to ions
- Positively charged layer: Debye sheath (Poisson)

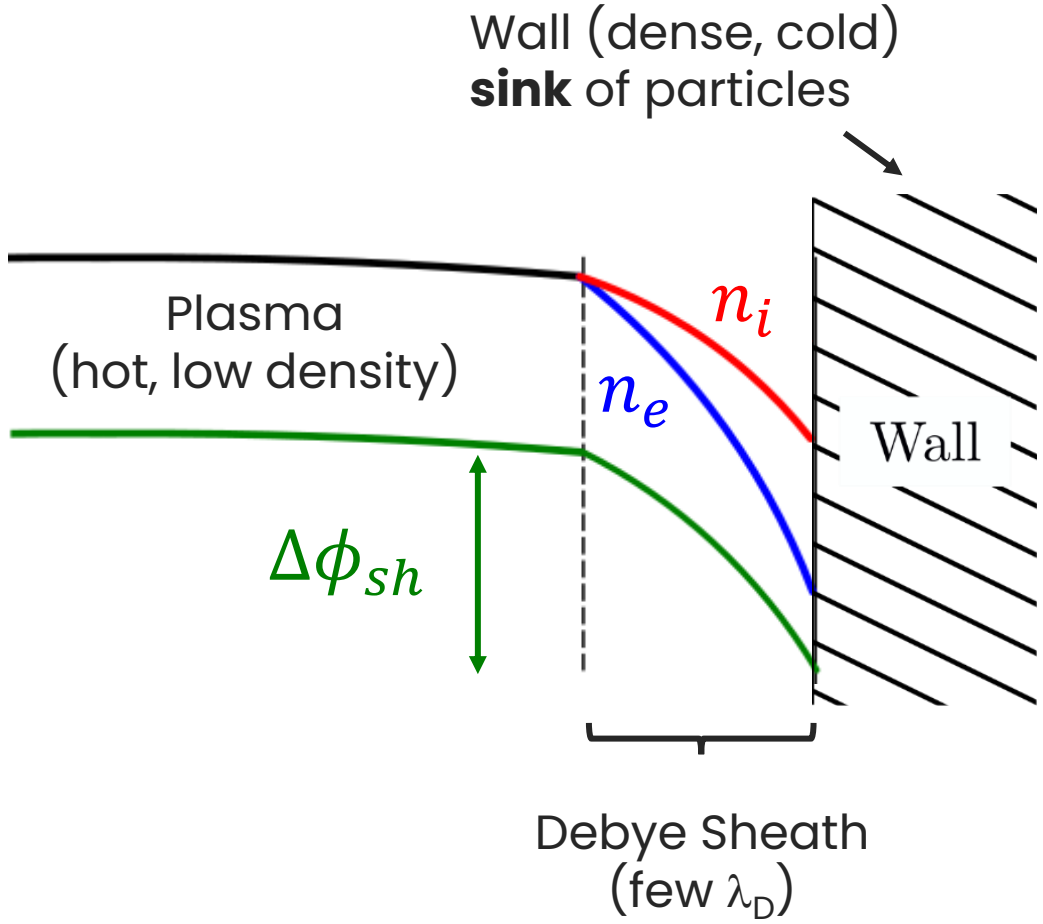
$$\lambda_D = \sqrt{\frac{\varepsilon_0 T_0}{n_0 e^2}} \approx 10^{-5} m$$

- Main properties of sheath:

Repels slow electrons	Accelerates ions

$\Rightarrow$ In avg. no net current at wall [Bohm1949]

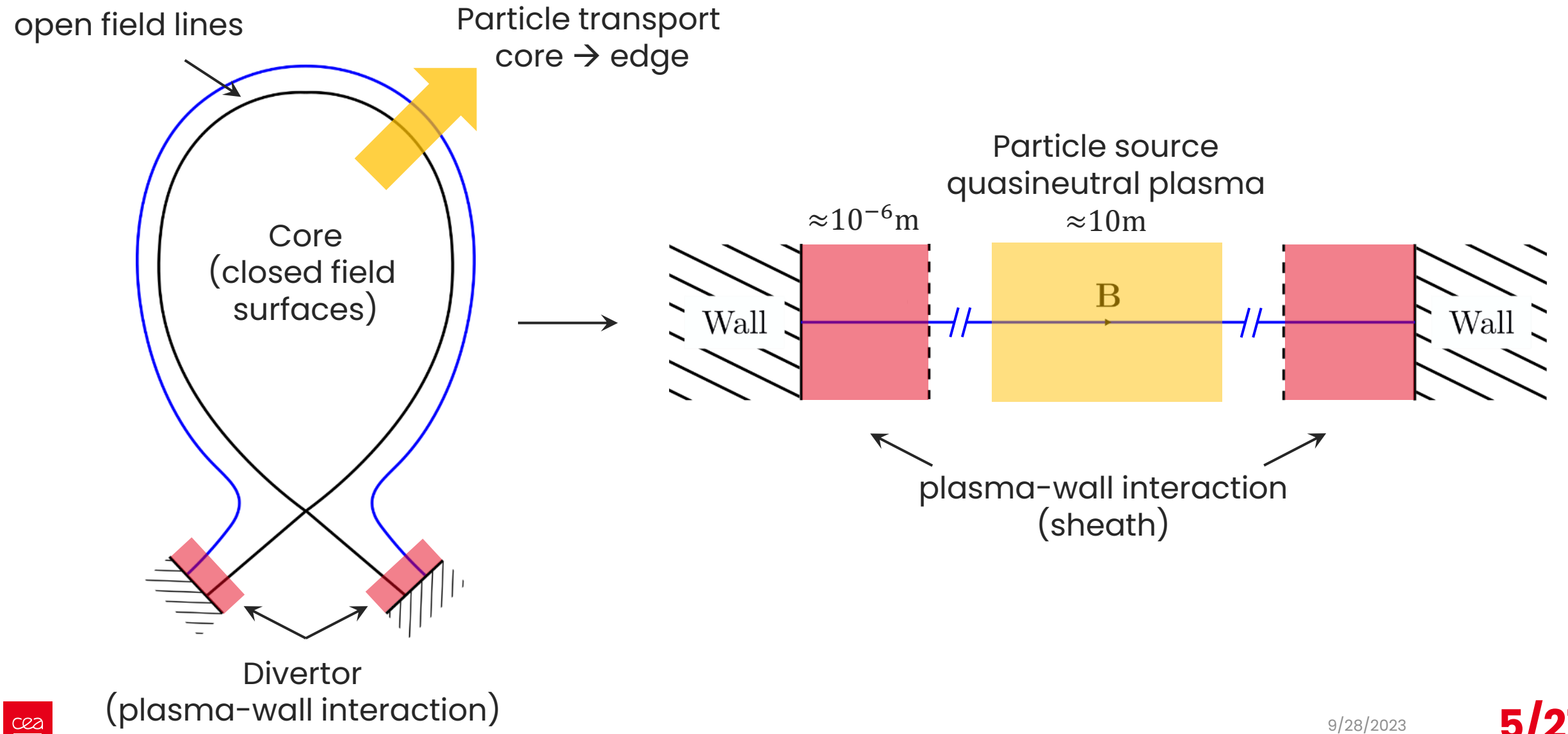
- $\lambda_D \ll \rho_c$ turbulent scale → sheath difficult to describe in turbulence code, GK quasineutrality ≠ Poisson eq. (no ∥ Laplacian)



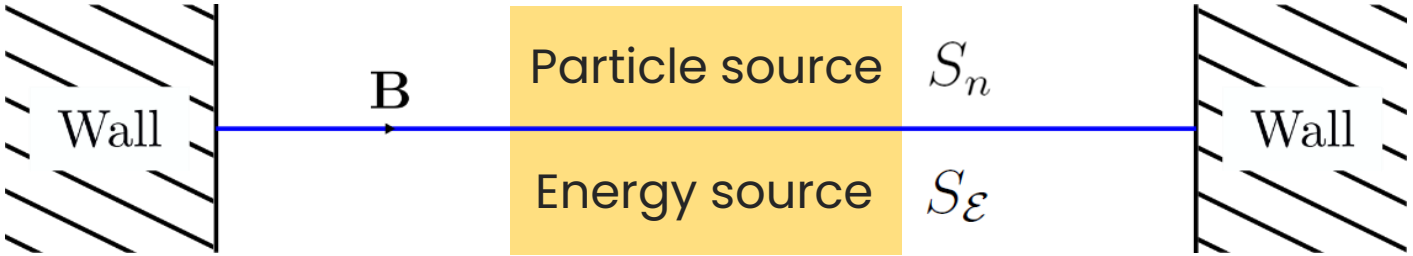
1. Fluid theory of plasma-wall interaction

2. Comparison with kinetic simulations

Simplified 1D geometry: single magnetic field line, no curvature



1D fluid model to describe plasma along a magnetic field line



x : position along magnetic field line

$a \in \{e, i\}$ ions and electrons

Density n_a particle flux Γ_a momentum flux $\Pi_a = n_a T_a + m_a n_a u_a^2$

Conservation of density

$$\partial_t n_a + \nabla_x \Gamma_a = S_n$$

Conservation of energy

$$m_a \partial_t \Pi_a / 2 + \nabla_x Q_a = -e_a \Gamma_a \nabla_x \phi + S_\mathcal{E}$$

Conservation of momentum

$$m_a \partial_t \Gamma_a + \nabla_x \Pi_a = -e_a n_a \nabla_x \phi$$

Energy flux $Q_a = u_a \left(\frac{3}{2} n_a T_a + \frac{1}{2} m_a n_a u_a^2 \right) + Q_a^{\text{heat}}$

Heat flux $Q_a^{\text{heat}} = \int_{-\infty}^{+\infty} dv \, \frac{1}{2} m_a \left(v - u_a \right)^3 f_a$

Closure required ! (zero for a maxwellian)

Particle flux governed by the source
Total pressure constant in quasineutral limit

Steady state eq.

$$\nabla_x \Gamma = S_n$$

$$\nabla_x \Pi = -e(n_i - n_e)\nabla_x \phi$$

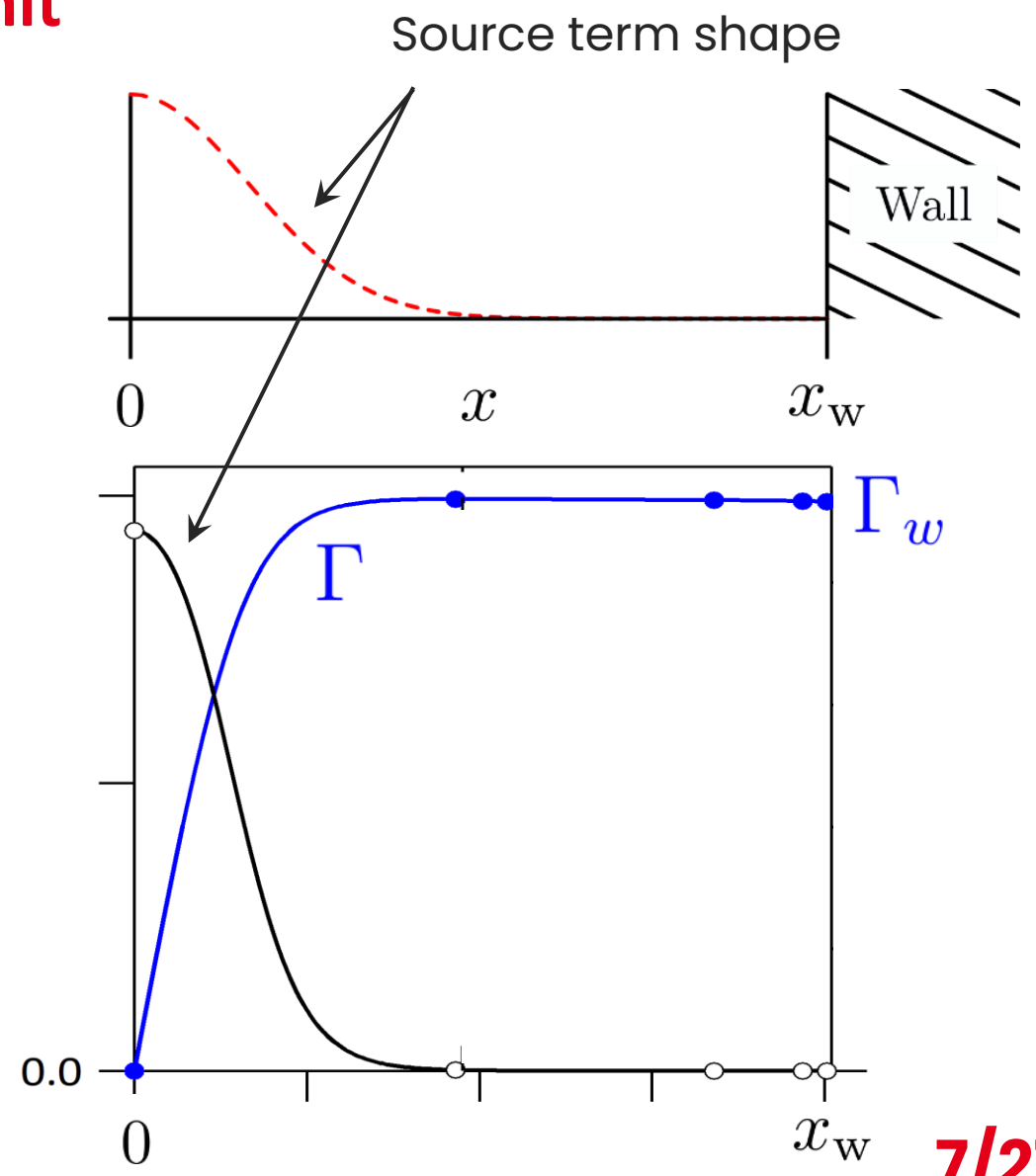
$$\Gamma = \Gamma_e = \Gamma_i \qquad \Pi = \Pi_e + \Pi_i$$

Total pressure

- Quasineutral limit: $n_i - n_e \approx 0$

$$\Pi = \text{constant} \qquad \Gamma = \int_0^x ds\ S_n$$

- Rq: $\Pi \approx \text{cte.}$ at sheath entrance



Quasineutral plasma flow is necessarily subsonic

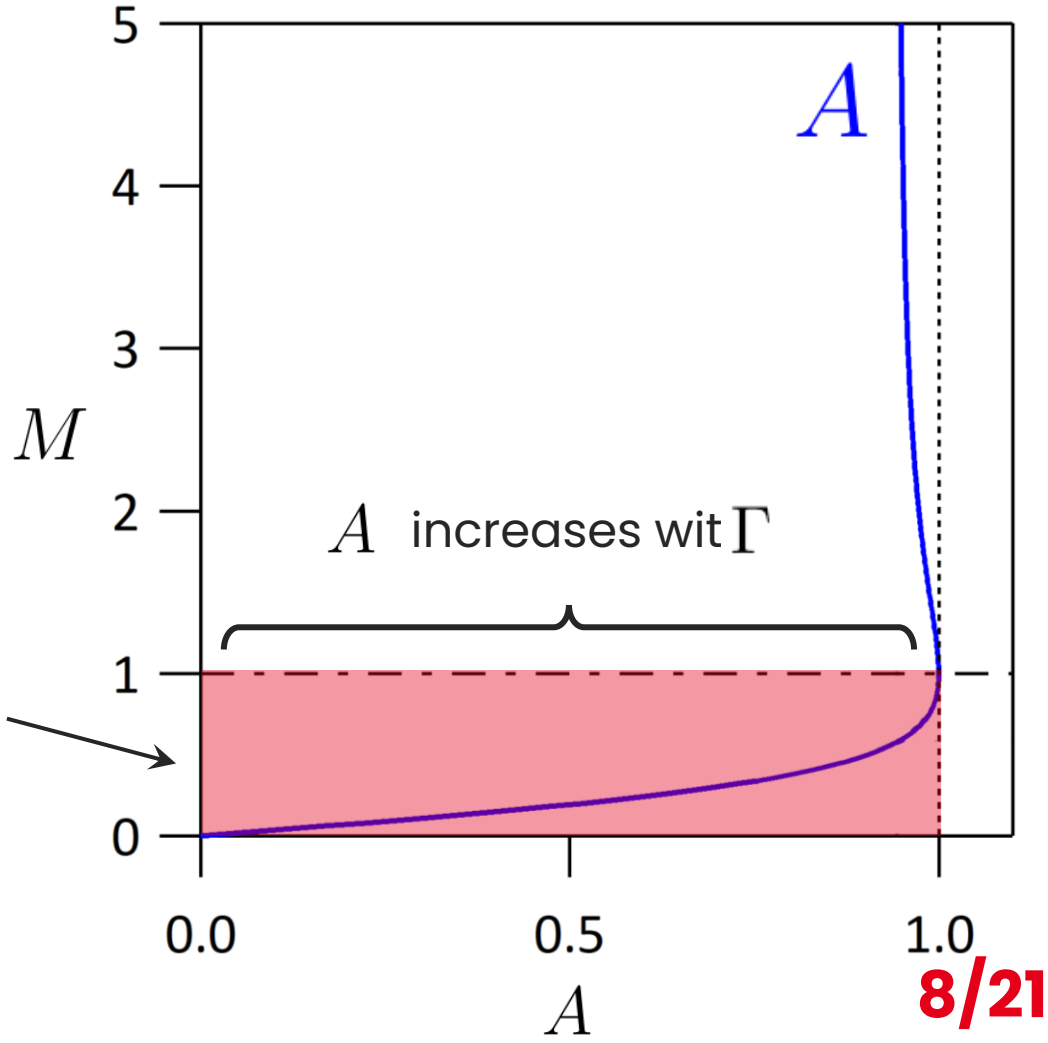
- Control parameter $A = \dfrac{4\sqrt{m_i T_s}\,\Gamma}{3\left(\Pi + (n_i - n_e)T_e\right)} = \dfrac{\sqrt{8\left(1+M^2\right)}M}{\left(1+3M^2\right)}$

Constant
(quasineutral plasma)

$$\Gamma = \int_0^x ds\; S_n \longrightarrow \begin{cases} A & \text{is maximum after source region} \\ A & \text{increases} \end{cases}$$

Mach number $M = u_i/c_s \leq 1$ in quasineutral plasma

$c_s = \sqrt{3T/m_i}$ from Maxwellian closure



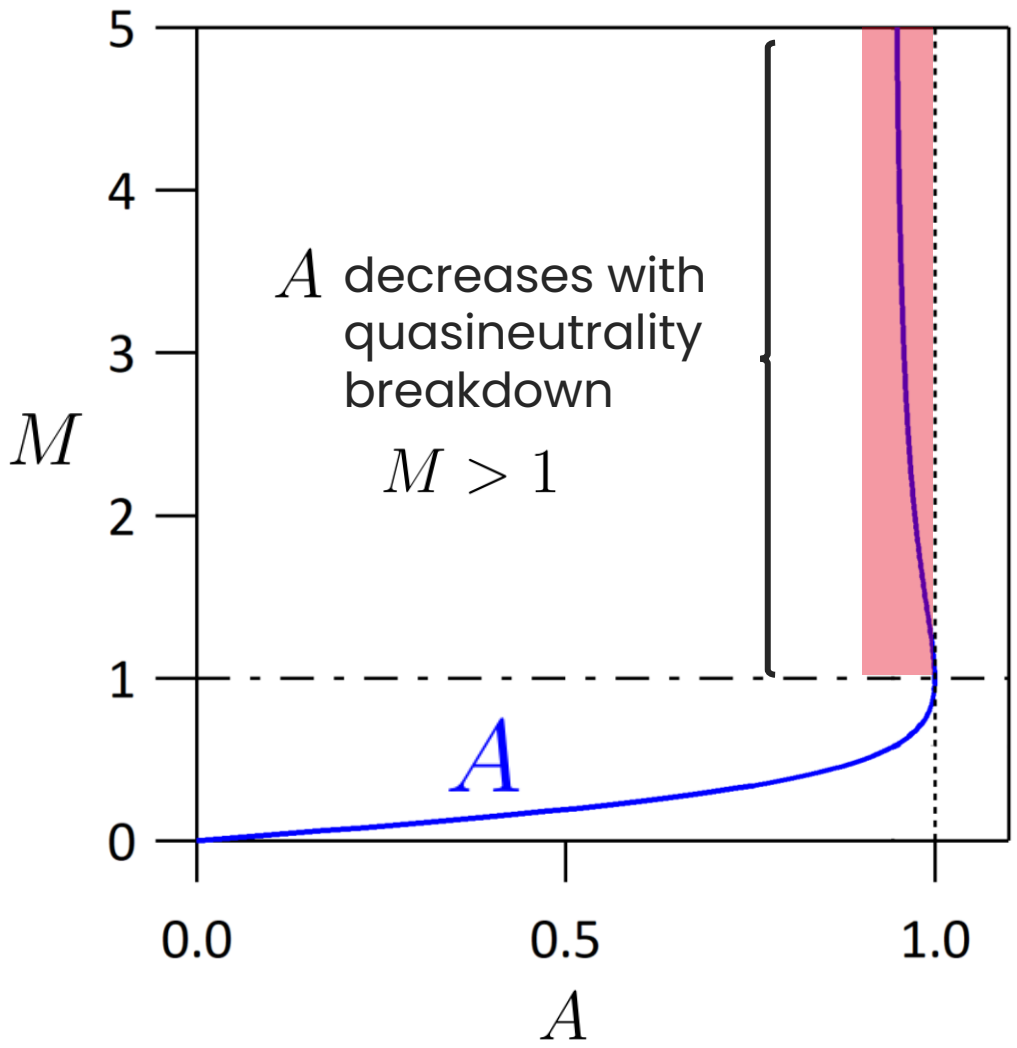
Supersonic transition driven by quasineutrality breakdown

$$A = \frac{4\sqrt{m_i T_s}\,\Gamma}{3\left(\Pi + (n_i - n_e)T_e\right)} = \frac{\sqrt{8\left(1+M^2\right)}M}{\left(1+3M^2\right)}$$

- Outside source region $\Gamma = \text{constant}$
- At the sheath entrance $n_i - n_e > 0$
- $\partial_x \Pi \ll \partial_x \left\{(n_e - n_i)\,T_e\right\}$

Decrease of A driven by quasineutrality breakdown

- Scale separation sheath size << plasma size → $M > 1$ in sheath



Sound speed critically depends on the chosen closure

- $M = 1$ At sheath entrance $M = u_i/c_s$ c_s **closure-dependent !**

- **Isothermal** closure $T_a = cst$ $c_s = \sqrt{T/m_i}$ ⟶ Similar expression for A_s

$$A = 2\sqrt{m_i T_0}\frac{\Gamma}{\Pi} = \frac{2M}{1+M^2}$$

[Ghendrih2011]

- **Maxwellian** closure $Q_a^{\text{heat}} = 0$ $c_s = \sqrt{3\,T/m_i}$

- **Polytropic** closure $\dfrac{dp}{p} - \gamma_p \dfrac{dn}{n} = 0$ $c_s = \sqrt{\gamma_p\, p/m_i n}$

- **High order** linear closure $N_\ell = 0$ $c_s = \left(\ell \dfrac{N_{\ell-1}}{n}\right)^{1/(\ell-1)}$ with $N_\ell = \int dv\ (v-u)^\ell f$

1. Fluid theory of plasma-wall interaction

2. Comparison with kinetic simulations

Boltzmann-Poisson system 1D in space and 1D in velocity to study kinetic plasma wall interaction

Normalized Boltzmann eq.

$$\partial_t f_a + \sqrt{A_a}\left(v_a \partial_x f_a - Z_a \partial_x \phi \, \partial_{v_a} f_a\right) = \mathcal{C}(f_a) + \mathcal{S}(f_a)$$

Electrons & ions distribution function $f_a(x, v_a, t)$

mass ratio $A_a = \dfrac{m_e}{m_a}$

Normalized Poisson eq.

$$\partial_x^2 \phi = -\rho \qquad \rho = \sum_{\text{species}} Z_a n_a$$

$$n_a = \int_{-\infty}^{+\infty} dv \, f_a$$

- Normalizations:

time $\omega_{pe_0} = \sqrt{\dfrac{\varepsilon_0 T_0}{n_0 e^2}}$ length $\lambda_{D_0} = \sqrt{\dfrac{\varepsilon_0 T_0}{n_0 e^2}}$ velocity $v_{T_{0a}} = \sqrt{\dfrac{T_0}{m_a}}$

Single species collisions → Fokker-Planck operator
Inter species collisions → fluid-like momentum/energy exchange

$$Df_a/Dt = \mathcal{S}(f_a) + \boxed{\mathcal{C}(f_a)} \longrightarrow \mathcal{C}_{\text{intra}}(f_a) + \mathcal{C}_{\text{inter}}(f_a)$$ [Dif-Pradalier2011, Estève 2015]

Intra species Fokker-Planck collision operator

$$\mathcal{C}_{\text{intra}}(f_a) = \partial_v \left[D_v f_M \partial_v \left(\frac{f_a}{f_M} \right) \right]$$

$D_v \propto 1/v^3$ Collisionality decreases with velocity

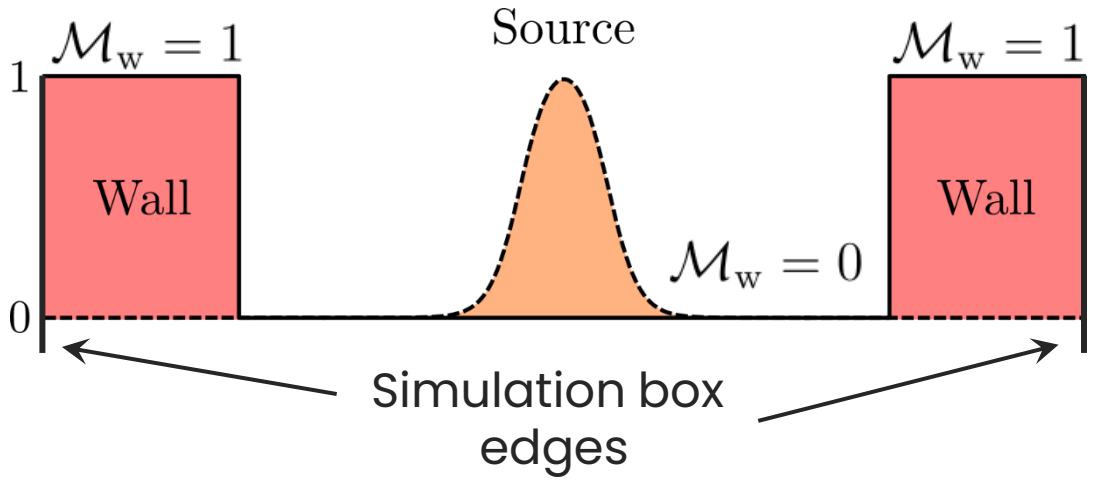
Inter species collision operator

$$\mathcal{C}_{\text{inter}}(f_a) = \frac{2\mathcal{Q}_{ab}}{n_a T_a} \left[\frac{m_a}{2T_a} (v_a - u_a)^2 - \frac{1}{2} \right] F_M + \frac{\mathcal{R}_{ab}}{n_a T_a} (v_a - u_a) F_M$$

$\mathcal{Q}_{ab}$ $\mathcal{R}_{ab}$ Momentum & energy exchange terms

Wall = particle, momentum, energy sink immersed in plasma

$$Df_a/Dt = \mathcal{C}(f_a) + \mathcal{S}(f_a) \longrightarrow \mathcal{S}_{\mathrm{src}}(f_a) + S_{\mathrm{sink}}(f_a)$$



- Relaxation of f_a towards g in the wall

$$\mathcal{S}_{\mathrm{sink}}(f_a) = -\nu \mathcal{M}_{\mathrm{w}}(x)\,(f_a - g)$$

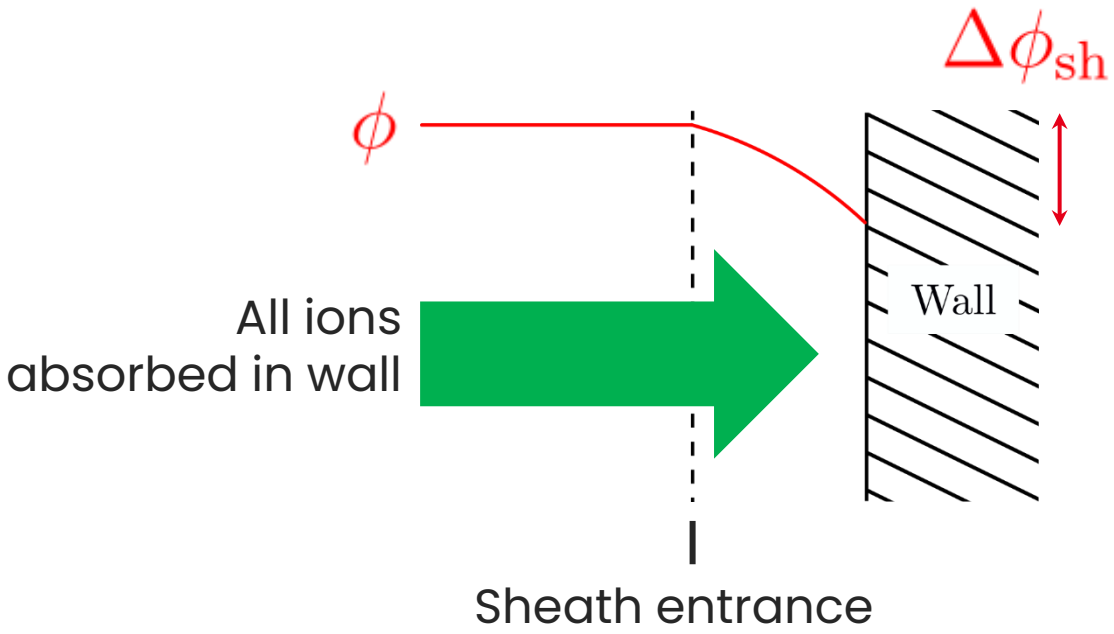
$$g(v) = \frac{n_{\mathrm{w}}}{\sqrt{2\pi T_{\mathrm{w}}}}\, e^{-\frac{v^2}{2T_{\mathrm{w}}}}$$

$$n_{\mathrm{w}} \approx 10^{-9} \ll 1$$
$$T_{\mathrm{w}} < T_{\mathrm{src}}$$

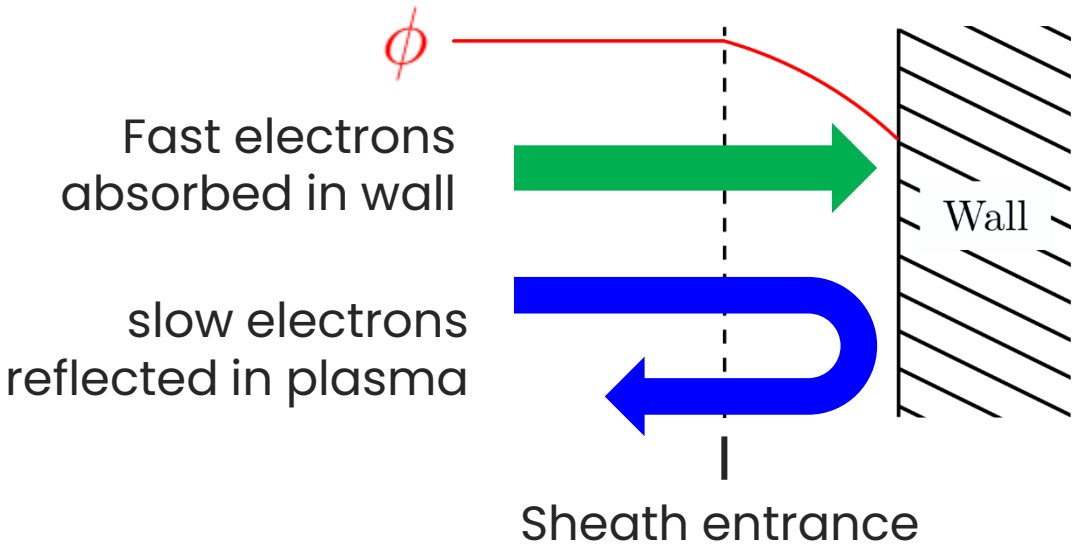
- Particles, momentum energy absorption in wall
- Wall **immersed** inside simulation domain
- No boundary condition @ plasma-wall transition
- Decorrelate sheath physics from numerical BC

Expected kinetic physics: ion accelerated towards the wall slow electrons reflected back to plasma

- **Acceleration of ions**



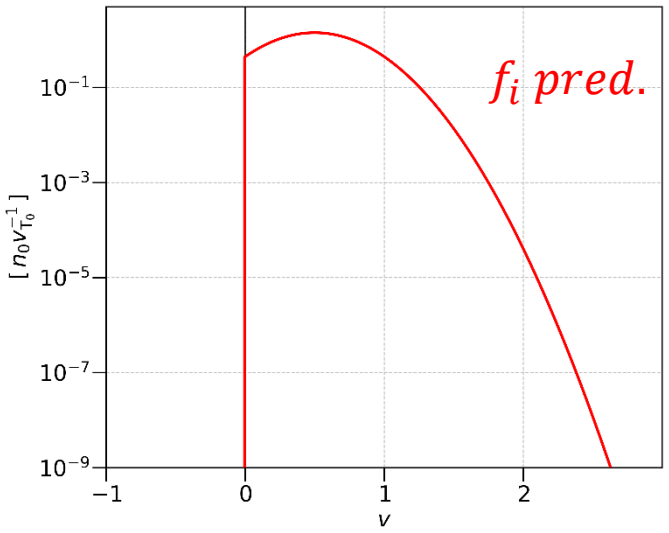
- **Reflection of slow electrons**



- Cut-off velocity between fast and slow electrons $v_{\mathrm{c}} = \sqrt{\dfrac{2e\,|\Delta\phi_{\mathrm{sh}}|}{m_e}}$
- Steady state at wall: $\Gamma_i = \Gamma_e$

Ion distribution function at sheath entrance: acceleration & no ions coming back from the wall

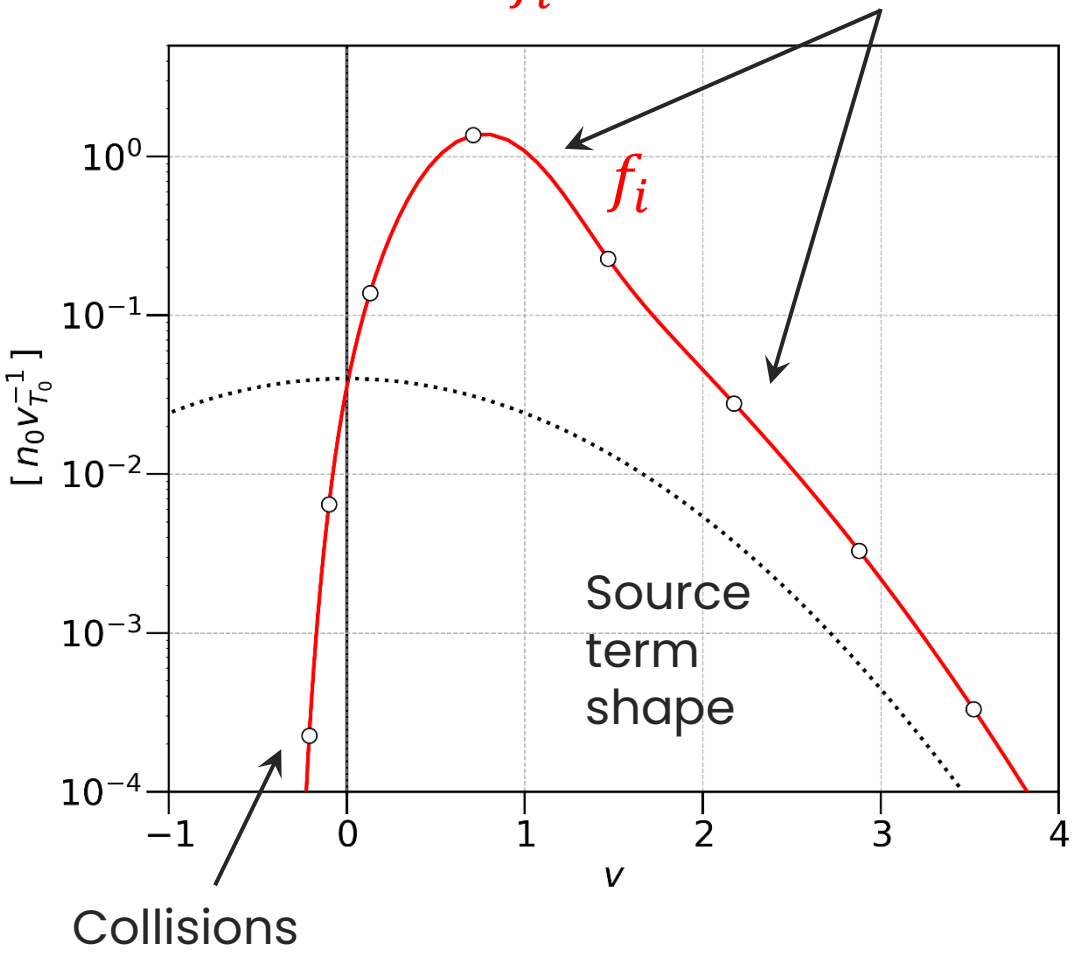
- **Prediction** for f_i



- Positive mean velocity (Bohm criterion ?)
- No ions coming back from wall

- **Simulation** f_i

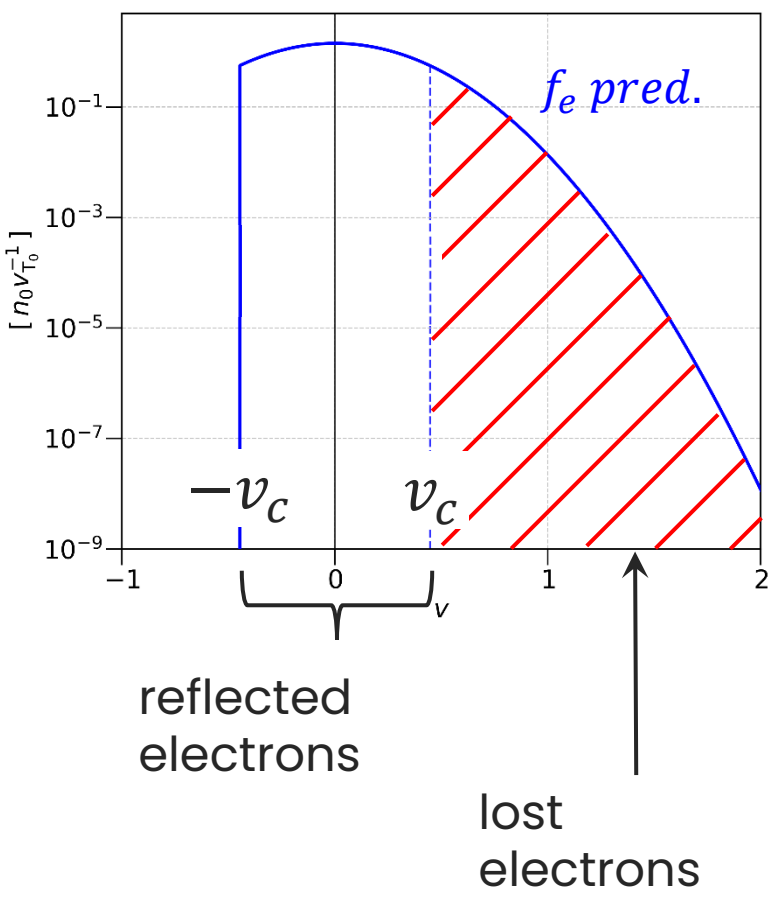
Competition between particle source & collisions



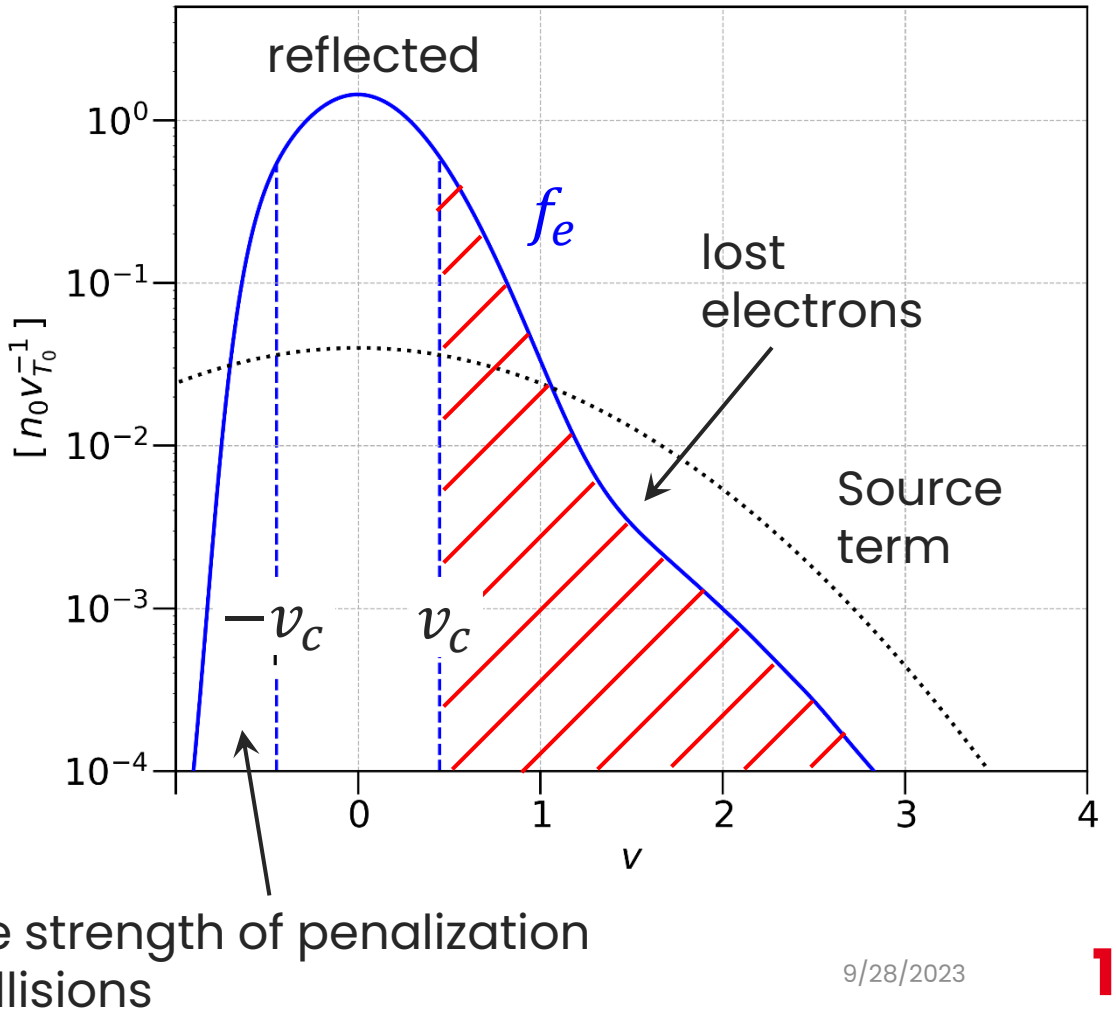
Electron distribution function at sheath entrance: slow electrons reflected into plasma, fast ones absorbed

$$v_c = \sqrt{\frac{2e\,|\Delta\phi_{\mathrm{sh}}|}{m_e}}$$

- **Prediction** for f_e



- **Simulation** f_e

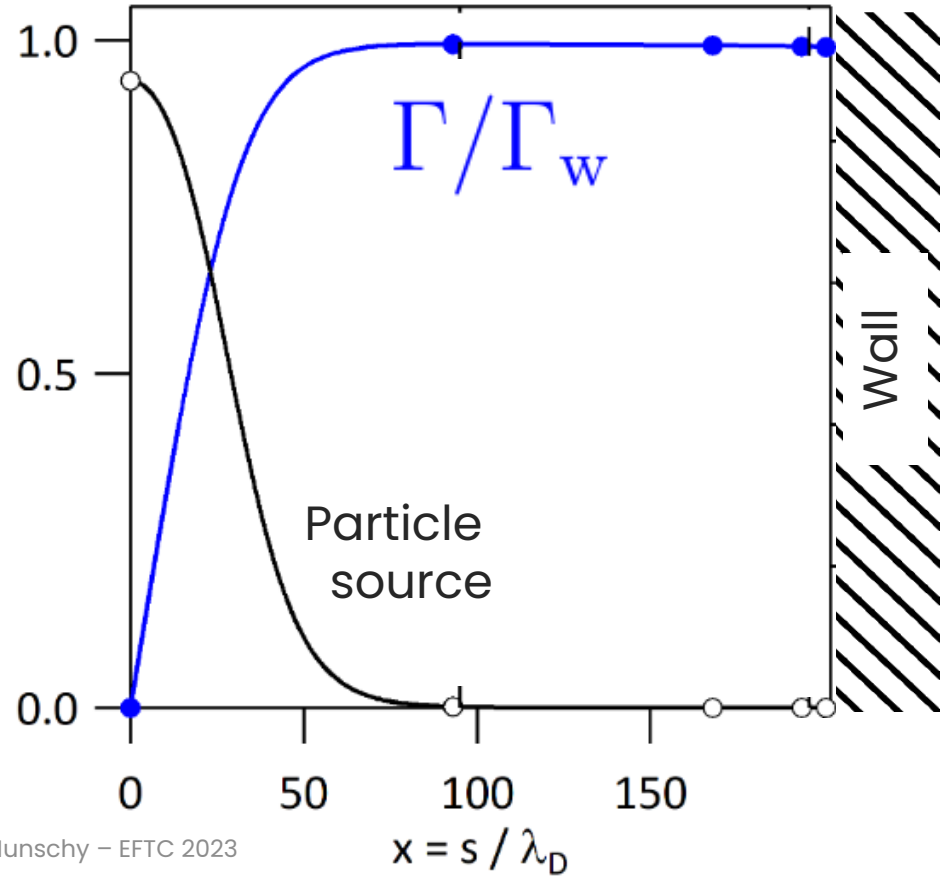


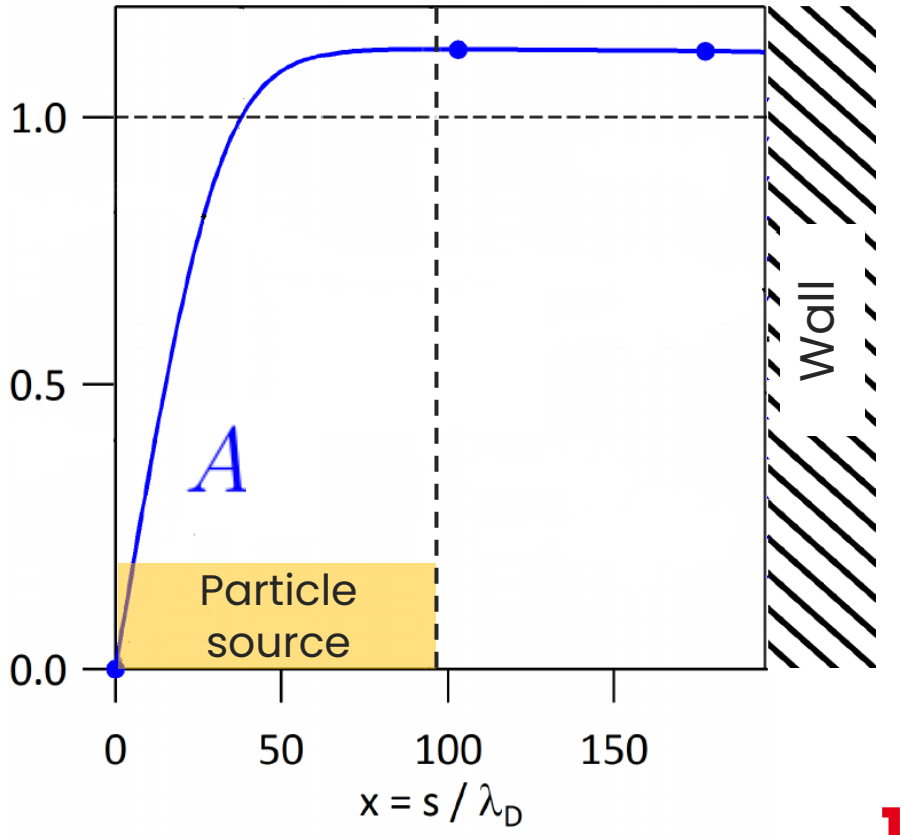
A parameter constant after source region → fluid theory suggests subsonic regime

- Maxwellian closure

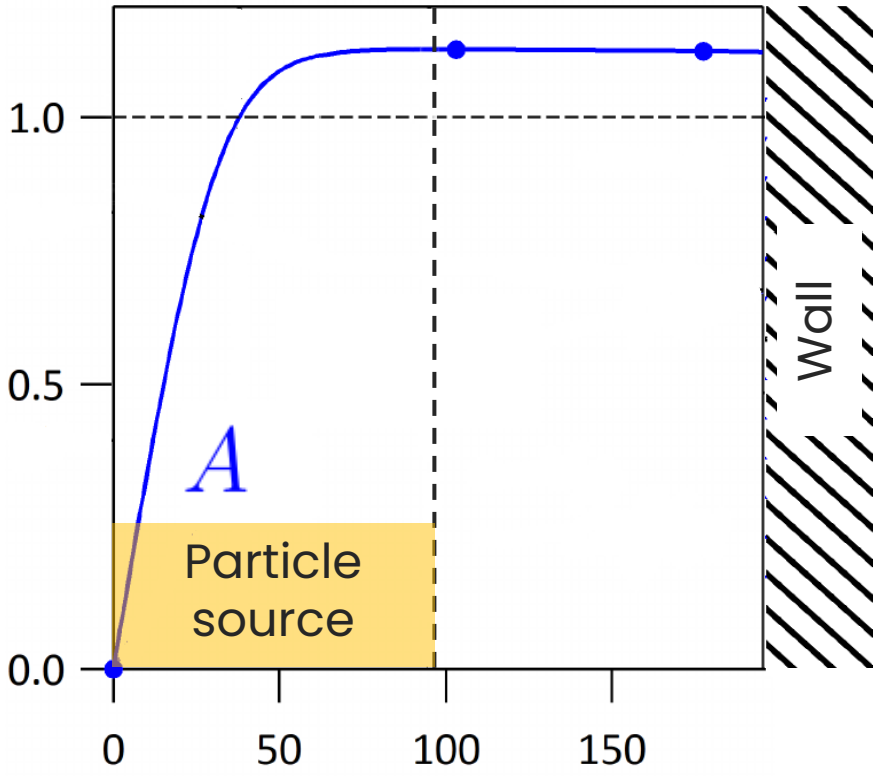
$$\begin{cases} \Pi = \text{constant} \\ \Gamma = \int_0^x ds\ S_n \end{cases}$$

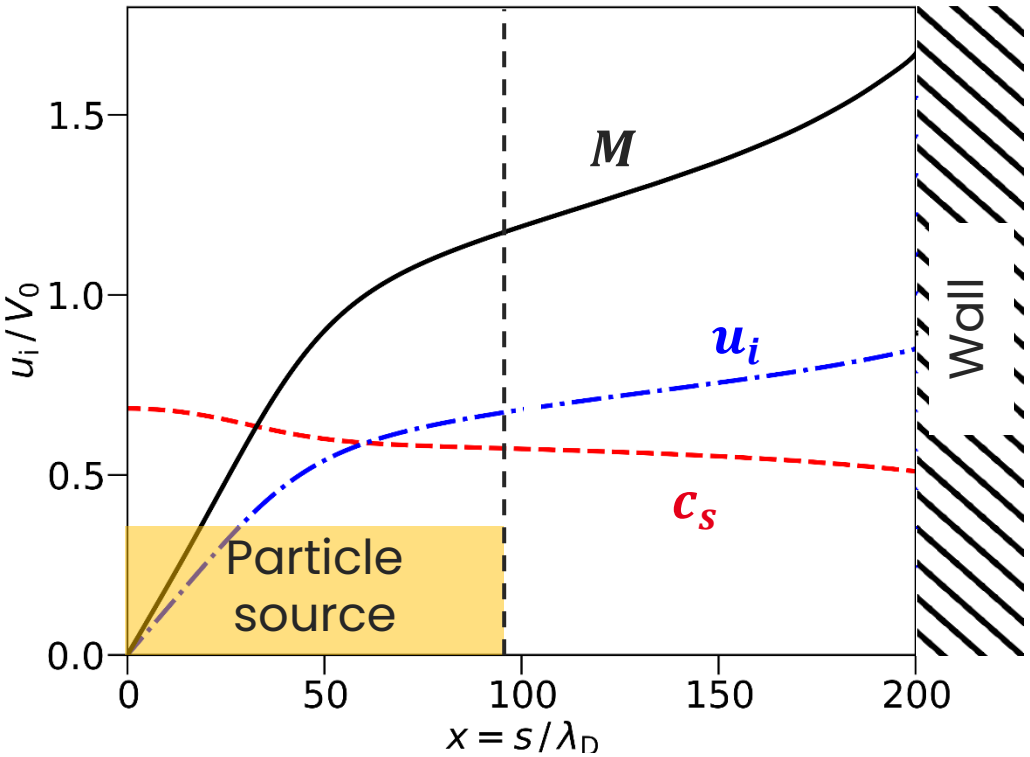
$$A^2 = \frac{16 m_i T_s \Gamma^2}{9\left(\Pi + (n_i - n_e)T_e\right)^2}$$





Limit of fluid theory : Mach number cannot only depend on *A*





$$A = \frac{4\sqrt{m_i T_s}\,\Gamma}{3\left(\Pi + (n_i - n_e)T_e\right)} = \frac{\sqrt{8\left(1+M^2\right)}M}{\left(1+3M^2\right)}$$

From closure

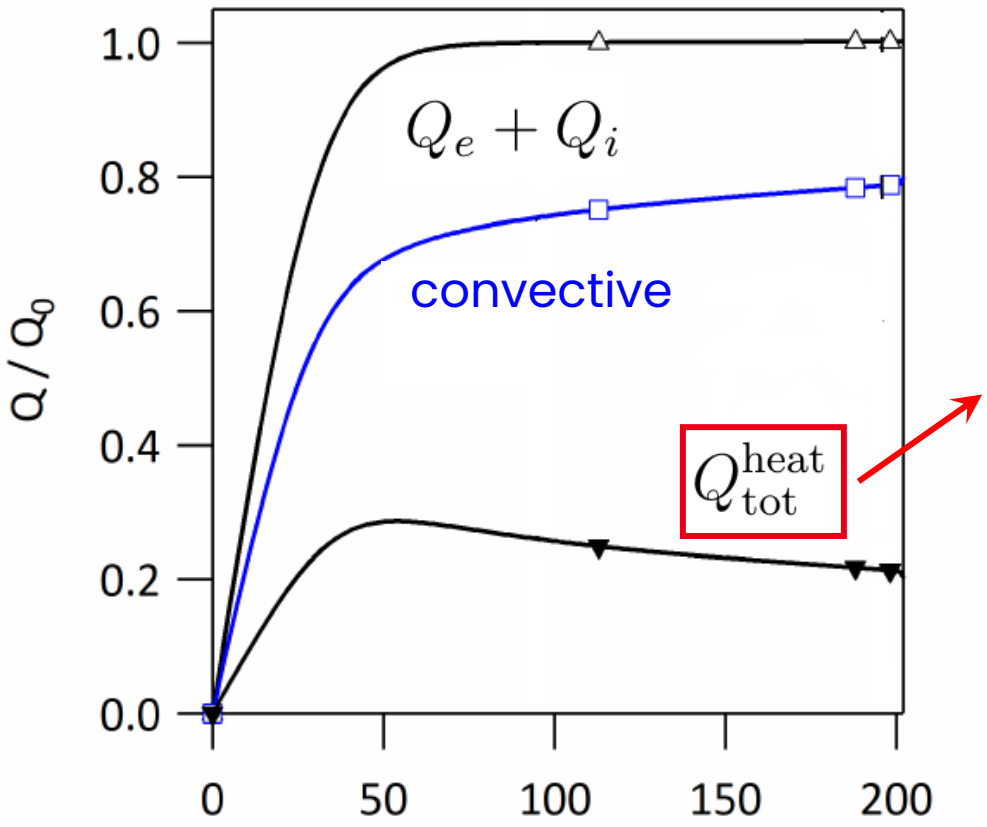
M increases with constant A

$$M = u_i/c_s \qquad c_s = \sqrt{3\,T/m_i}$$

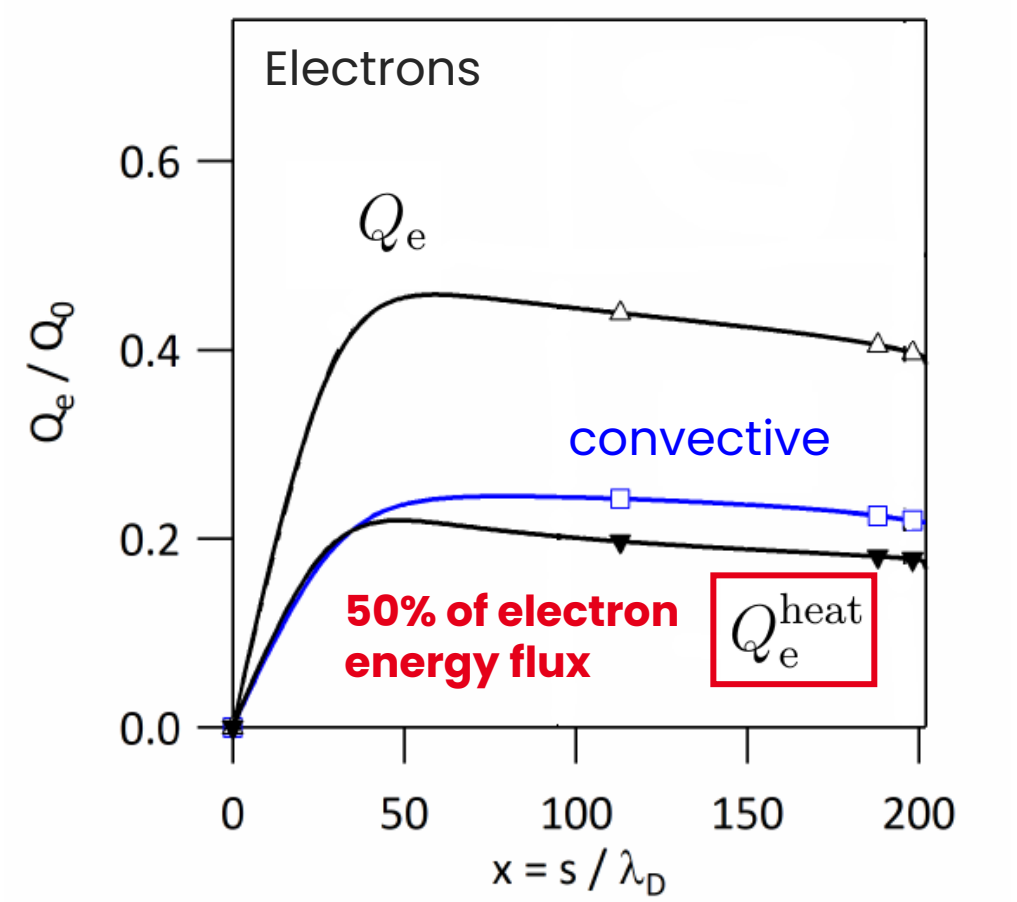
Incompatible with fluid theory

Non vanishing electron heat flux: $Q_e^{heat} \approx Q_e/2$

Energy flux = convective + heat



30% of total energy flux



$Q^{heat} \neq 0$ Maxwellian closure irrelevant

No known closure leads to correct prediction of heat flux

Conclusion

- Comparison of kinetic simulation of plasma-wall interaction with fluid theory

- Trajectories of particles in plasma highlight kinetic sheath physics at play :
  - Acceleration of ions, reflection of slow electrons
  - Collisions

- Reduced fluid model fails to predict plasma behavior :
  - Sound speed critically depends on closure : **Bohm's criterion is unoperational**
  - Vanishing heat flux closure not relevant
  - Validity of **other closures can be questioned** : isothermal, zero collisional flux, high order moments…
  - Sheath heat transmission factor in good agreement with predictions (6% difference)

[Munschy et al. 2023] *Kinetic plasma-sheath self organization*

[Munschy et al. 2023] *Kinetic plasma-wall interaction using immersed boundary conditions*

Sheath heat transmission factor: good agreement with predictions

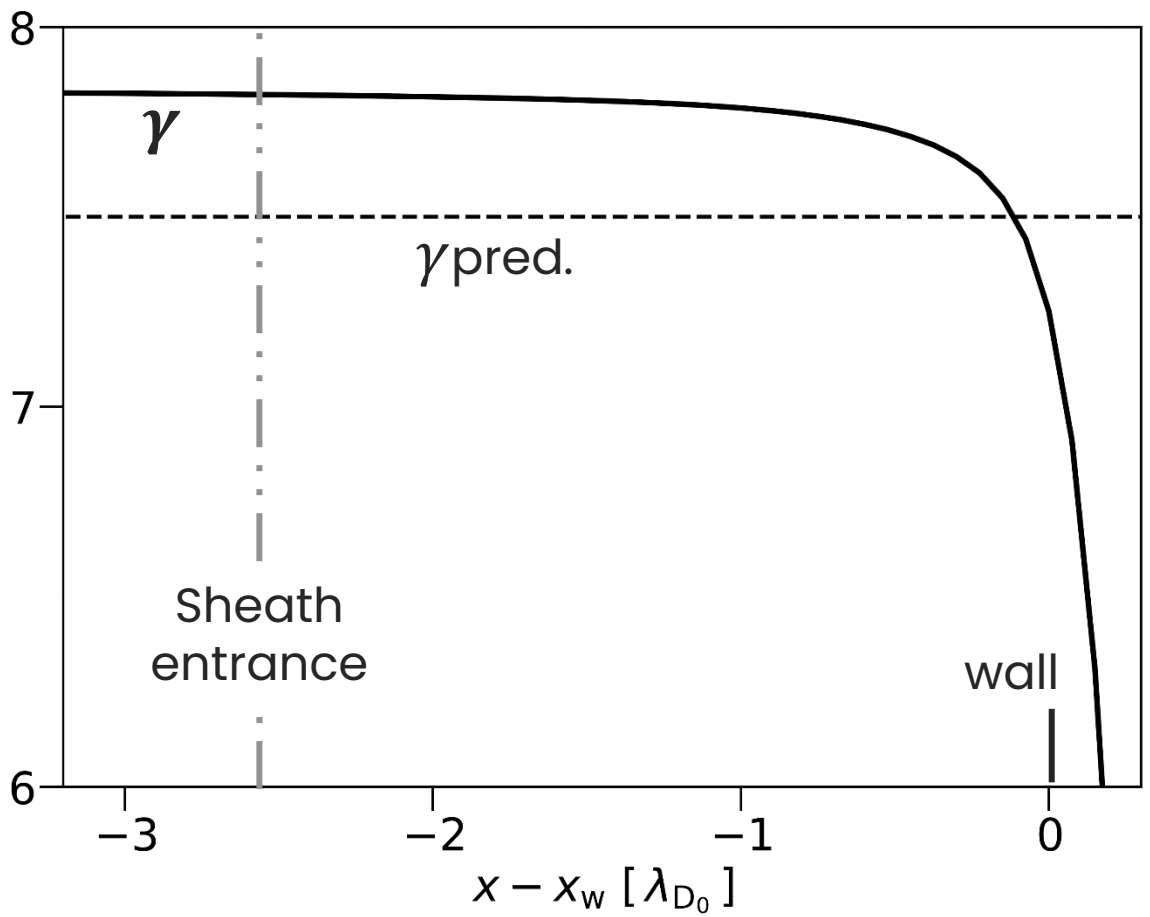
- Γ and T_e measured experimentally (Langmuir probes)

$$Q = \gamma \Gamma T_e$$

γ gives an estimation of deposited power on divertor plates

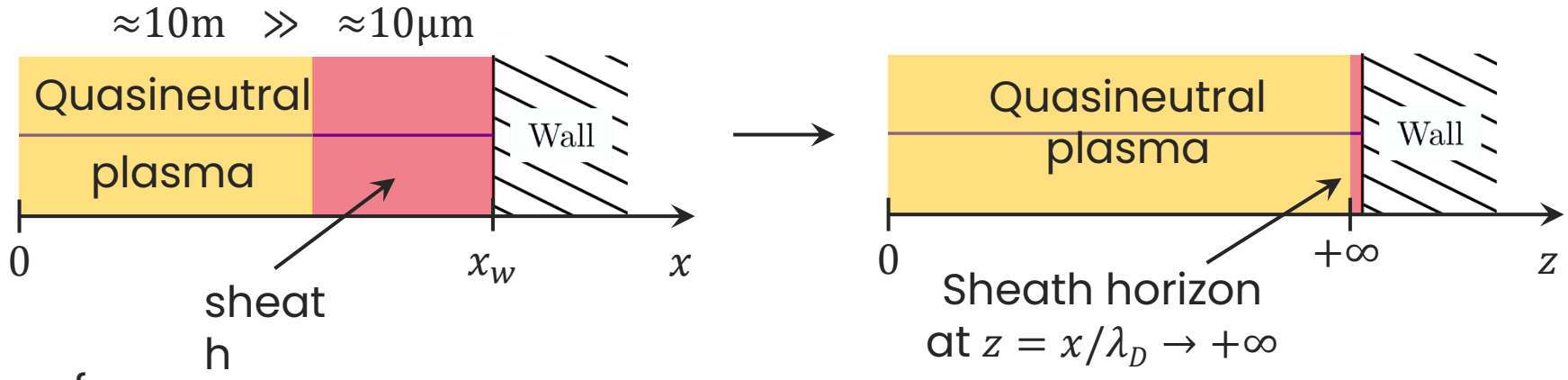
- Usual value $\gamma = \gamma_{\mathrm{i}}^{\mathrm{isoth.}} + \gamma_{\mathrm{e}} \approx 7.5.$
- Good agreement with kinetic code

$$\gamma_{\mathrm{code}} \approx 7.9$$



Scale separation ⇒ supersonic transition at sheath entrance

- Scale separation



- Expansion in powers of $1/z$
  - Adiabatic electrons $n_e = n_\infty e^{\phi-\phi_\infty}$ ϕ_∞ n_∞ : taken at sheath horizon $z \to +\infty$
  - Poisson equation $n_i = n_\infty \left(1 + F(\phi-\phi_\infty) - \Delta\phi\right)$

$$F(\phi-\phi_\infty) = e^{\phi-\phi_\infty} - 1 = \frac{1}{z^K} A_K^F + \dots \qquad K > 0$$

- $\forall z \geq 0 \quad \Pi(z) = \Pi_\infty$ gives $M_\infty = 1$

  and : taken at sheath horizon $z \to +\infty$ $c_s = \sqrt{3T/m_i}$

Supersonic transition at sheath horizon

Bohm's criterion

[Ghendrih2011, Munschy2023]

Robust physics with respect to the choice of the penalization term

1. Constant coefficients : conducting wall

Charge conservation from Boltzmann eq.

$$\partial_t \rho + \partial_x j_{\mathrm{tot}} = -\mathcal{M}_{\mathrm{w}} \{\nu_{\mathrm{iw}}(n_{\mathrm{i}} - n_{\mathrm{w}}) - \nu_{\mathrm{ew}}(n_{\mathrm{e}} - n_{\mathrm{w}})\}$$

$j_{\mathrm{e}} + j_{\mathrm{i}}$

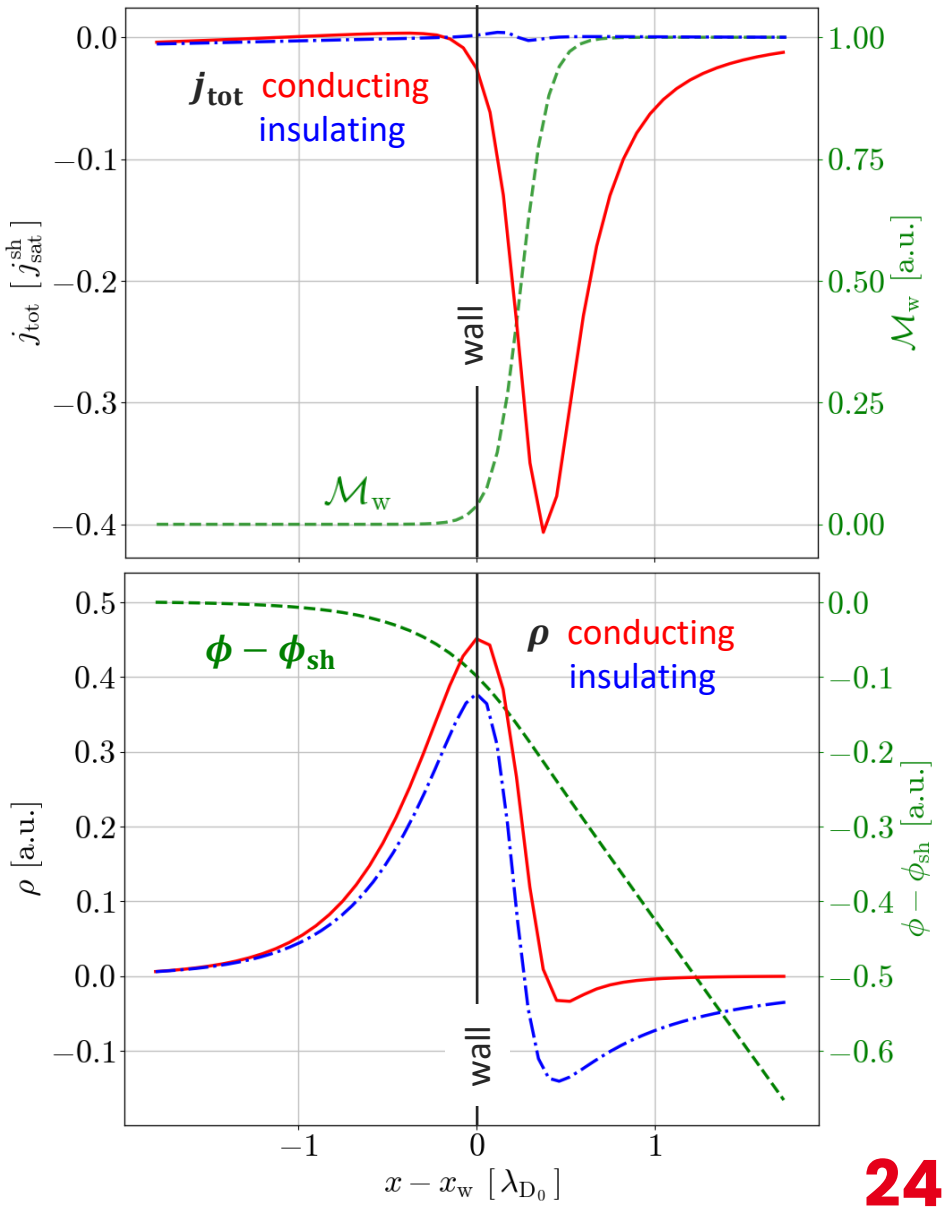
→ currents in wall

2. Adaptive coefficients : insulating wall

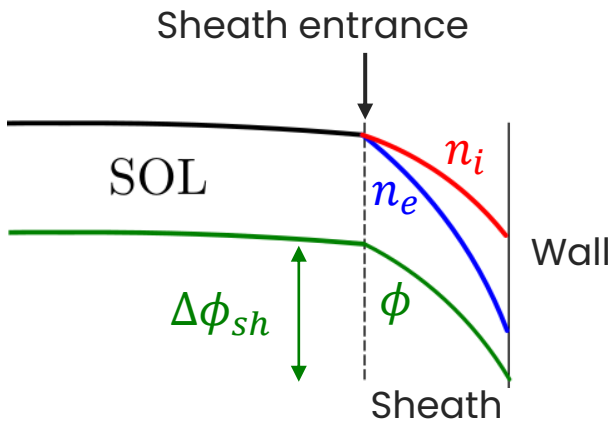
$$\nu_{\mathrm{iw}} = constant \qquad \nu_{\mathrm{ew}}(x,t) = \nu_{\mathrm{iw}} \frac{n_{\mathrm{i}} - n_{\mathrm{w}}}{n_{\mathrm{e}} - n_{\mathrm{w}}}$$

Rhs. vanishing → no net current in wall

- Robust physics : almost identical potential drop in the sheath in both cases : focus on the conducting case in the following



Distribution functions at sheath entrance

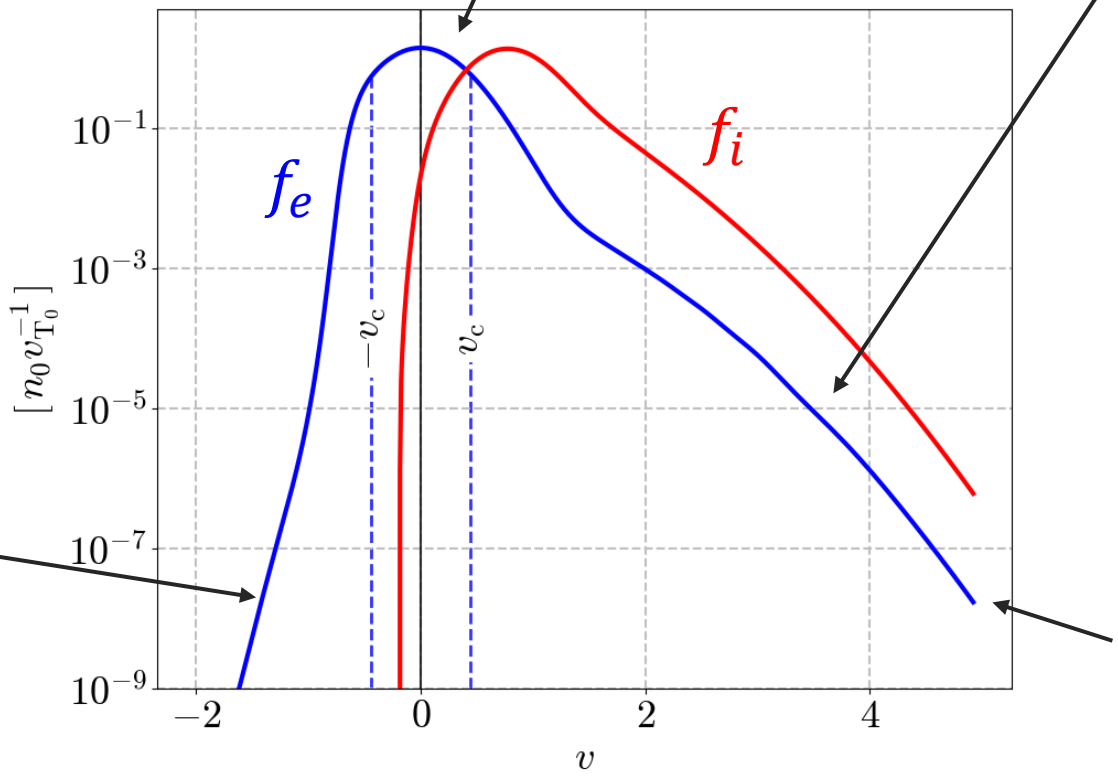


Escaping velocity:

$$v_c = \sqrt{\frac{2e|\Delta\phi_{sh}|}{m_e}}$$

Bulk electrons **reflected** into the plasma

Fast electrons **absorbed** in the wall



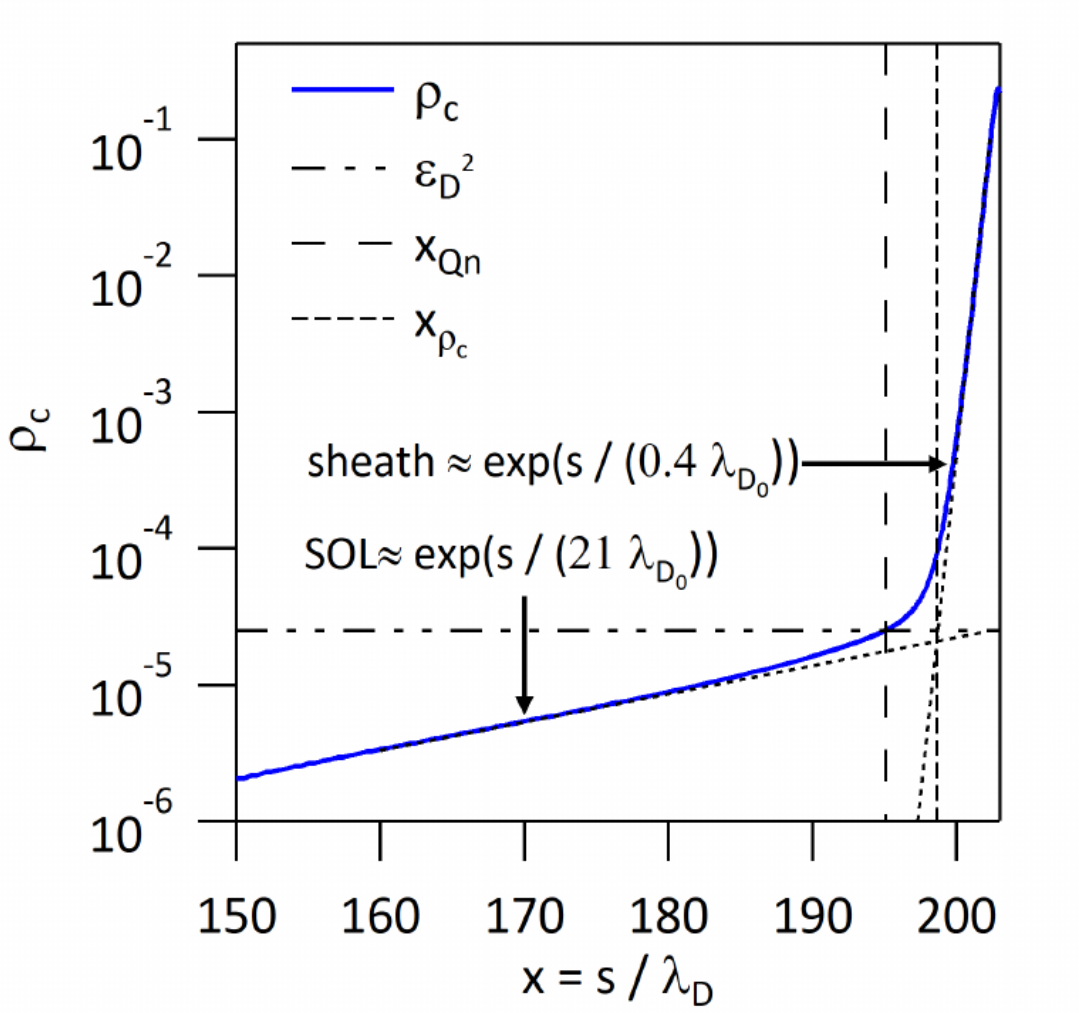
Electron **reemission** from the wall

- finite strength of penalization & collisions

$$sink = -\mathcal{M}_w \nu_{wa} (f_a - g_w)$$

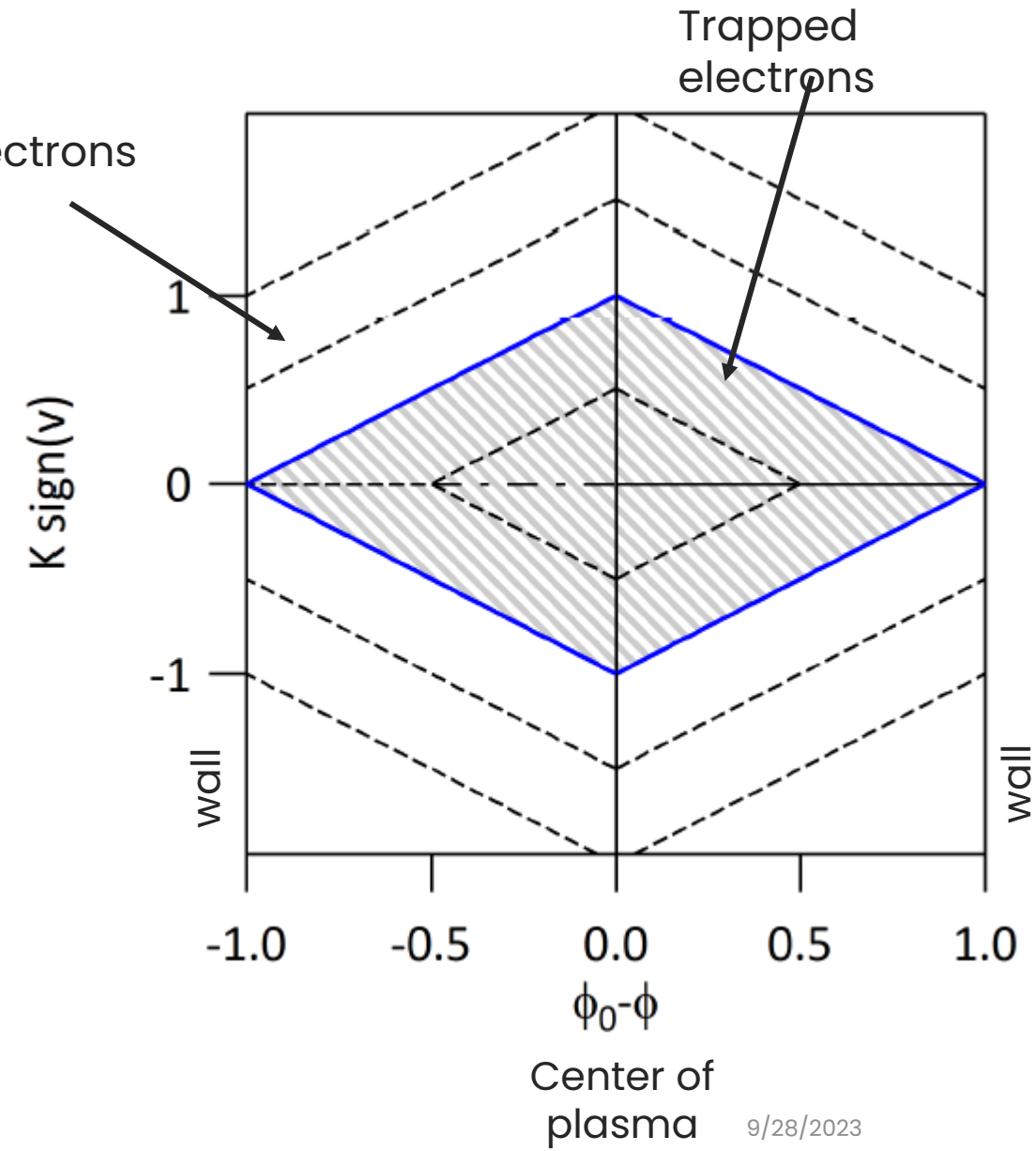
Shape resembles sum of two Maxwellian with different temperatures : collisions vs. injection of particles by the source

Sheath entrance definition

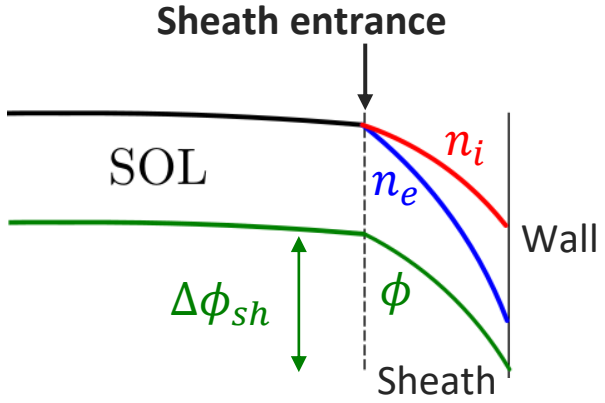


Collisions = detrapping mechanism trapped and passing

- Phase space characteristics for electrons in the whole plasma
- Particle source fraction injected in passing region : 1% for a typical value of sheath potential drop
- Without collisions trapped particles stay trapped while passing are absorbed in the wall
- Collisions : necessary transfer mechanism between trapped and passing, and necessary condition to reach steady state (the particle source must be compensated by the sink in the wall region)

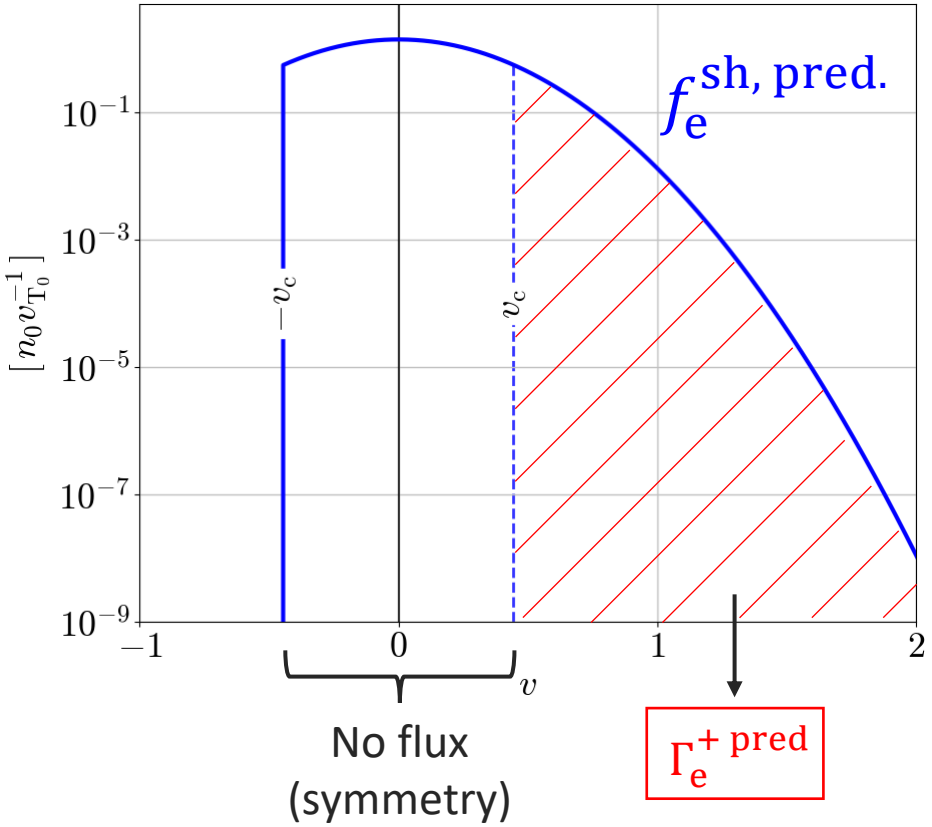


Using Stangeby's prediction of the sheath potential drop to explain parametric dependencies



Escaping velocity:

$$v_c = \sqrt{\frac{2e|\Delta\phi_{\mathrm{sh}}|}{m_{\mathrm{e}}}}$$



Potential drop prediction [Stangeby]

$$\Delta\phi_{\mathrm{sh}}^{\mathrm{pred.}} = \frac{T_{\mathrm{e}}^{\mathrm{sh}}}{e}\log\left(\sqrt{2\pi\frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}M^2\left(1+\frac{T_{\mathrm{i}}^{\mathrm{sh}}}{T_{\mathrm{e}}^{\mathrm{sh}}}\right)}\right)$$

-> Comes from $\Gamma_{\mathrm{i}}^{\mathrm{sh}} = \boxed{\Gamma_{\mathrm{e}}^{+\,\mathrm{pred}}}$

$$\boxed{\Gamma_{\mathrm{e}}^{+\,\mathrm{pred}}} = \int_{v_c}^{+\infty} dv\, v f_{\mathrm{e}}^{\mathrm{sh,\,pred.}}$$

With $f_{\mathrm{e}}^{\mathrm{sh,\,pred.}}$ truncated maxwellian of temperature $T_{\mathrm{e}}^{\mathrm{sh}}$

Correction terms for the potential drop prediction

Potential drop prediction [Stangeby]

$$\Delta\phi_{\mathrm{sh}}^{\mathrm{pred.}} = \frac{T_{\mathrm{e}}^{\mathrm{sh}}}{e}\log\left(\sqrt{2\pi\frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}M^2\left(1+\frac{T_{\mathrm{i}}^{\mathrm{sh}}}{T_{\mathrm{e}}^{\mathrm{sh}}}\right)}\right)$$

Correction term 1:

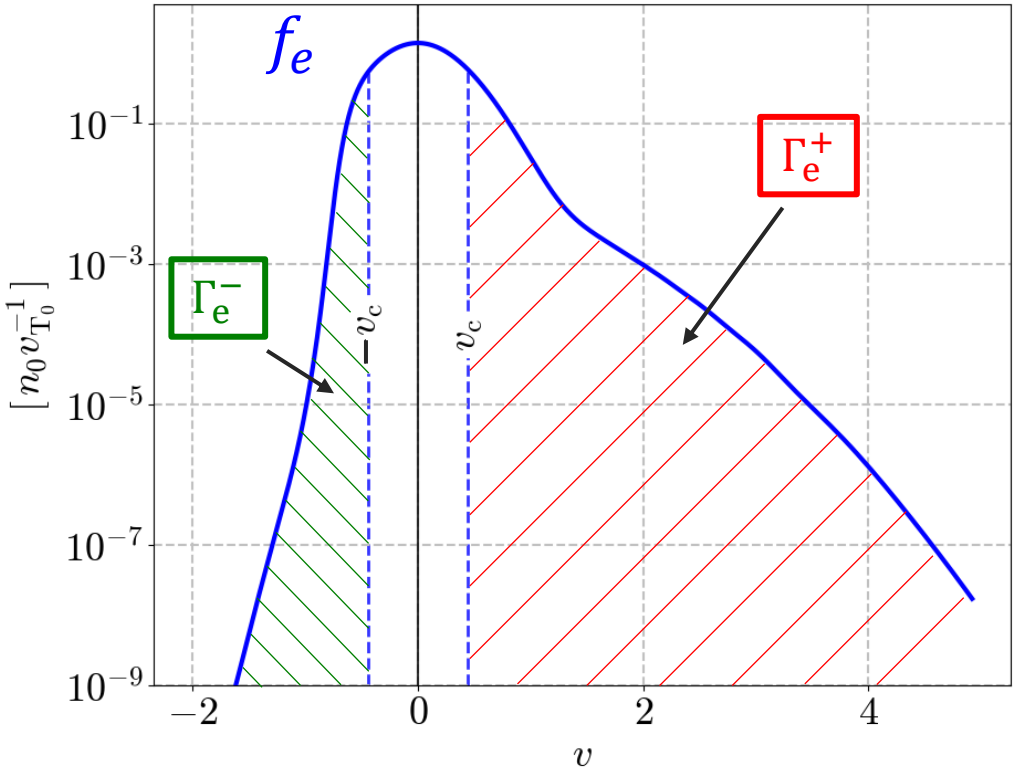
- electron reemission from the wall due to finite penalization strength

$$\Gamma_{\mathrm{i}}^{\mathrm{sh}} = \Gamma_{\mathrm{e}}^{+} + \boxed{\Gamma_{\mathrm{e}}^{-}} \leq \Gamma_{\mathrm{e}}^{+} \qquad \boxed{\Gamma_{\mathrm{e}}^{-}} = \int_{-\infty}^{-v_{\mathrm{c}}} dv\, v f_{\mathrm{e}}^{\mathrm{sh}}$$

Correction term 2:

- Effective temperature to match forward flux to predicted Maxwellian flux

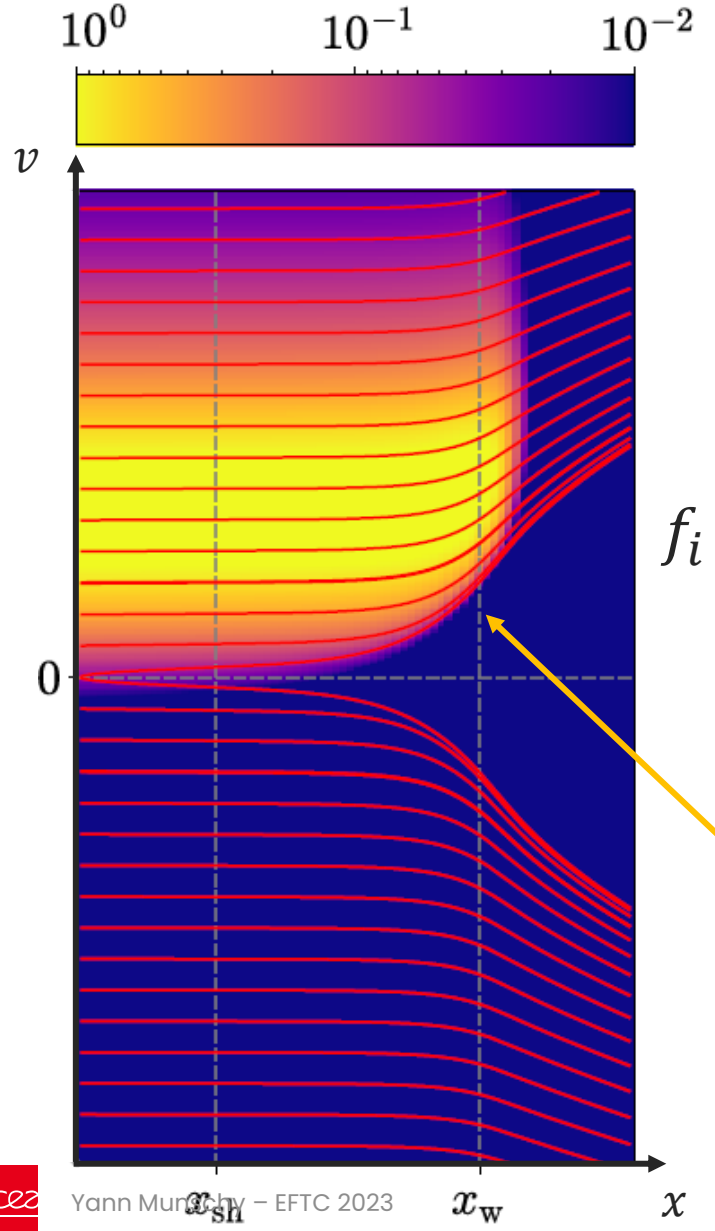
$$\Gamma_{\mathrm{e}}^{+\,\mathrm{pred}} = \int_{v_c}^{+\infty} dv\, v\ \mathrm{maxw.}\,(T_{\mathrm{e}}^{\mathrm{sh}}) \neq \boxed{\Gamma_{\mathrm{e}}^{+}} \longrightarrow \int_{v_c}^{+\infty} dv\, v\ \mathrm{maxw.}\,(\boldsymbol{\tilde{T}_e^{sh}}) = \boxed{\Gamma_{\mathrm{e}}^{+}}$$



Corrected potential drop prediction

$$\Delta\phi_{sh}^{pred.} = \frac{\boldsymbol{\tilde{T}_e^{sh}}}{e}\log\left(\sqrt{2\pi\frac{m_e}{m_i}M^2\left(1+\frac{T_i^{sh}}{\boldsymbol{\tilde{T}_e^{sh}}}\right)\left(1+\frac{\boxed{\Gamma_e^-}}{\Gamma_i^{sh}}\right)}\right)$$

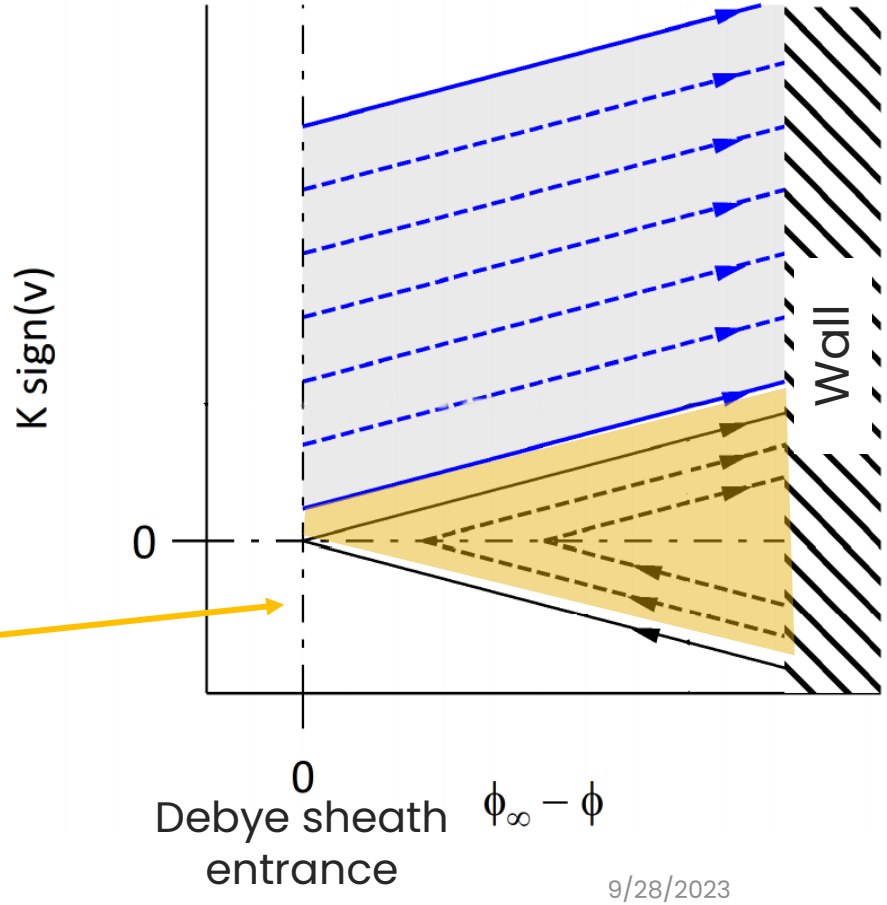
Ions trajectories in phase space: acceleration & gap



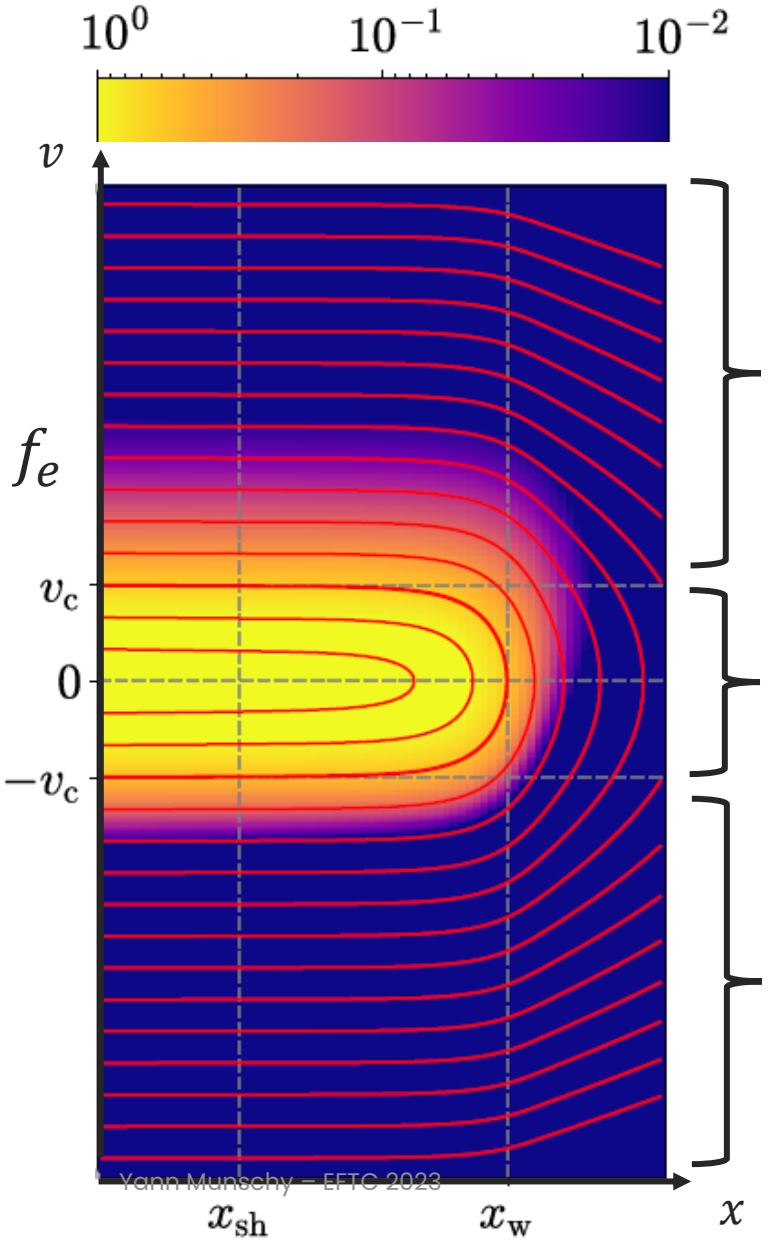
- kinetic energy $K = H_a - e_a\phi$
- No collisions → Hamiltonian invariant

Acceleration creates gap

(Filled by collisions in 1D, Electric field as well in higher dimensions)



Fast electrons overcome the sheath potential barrier

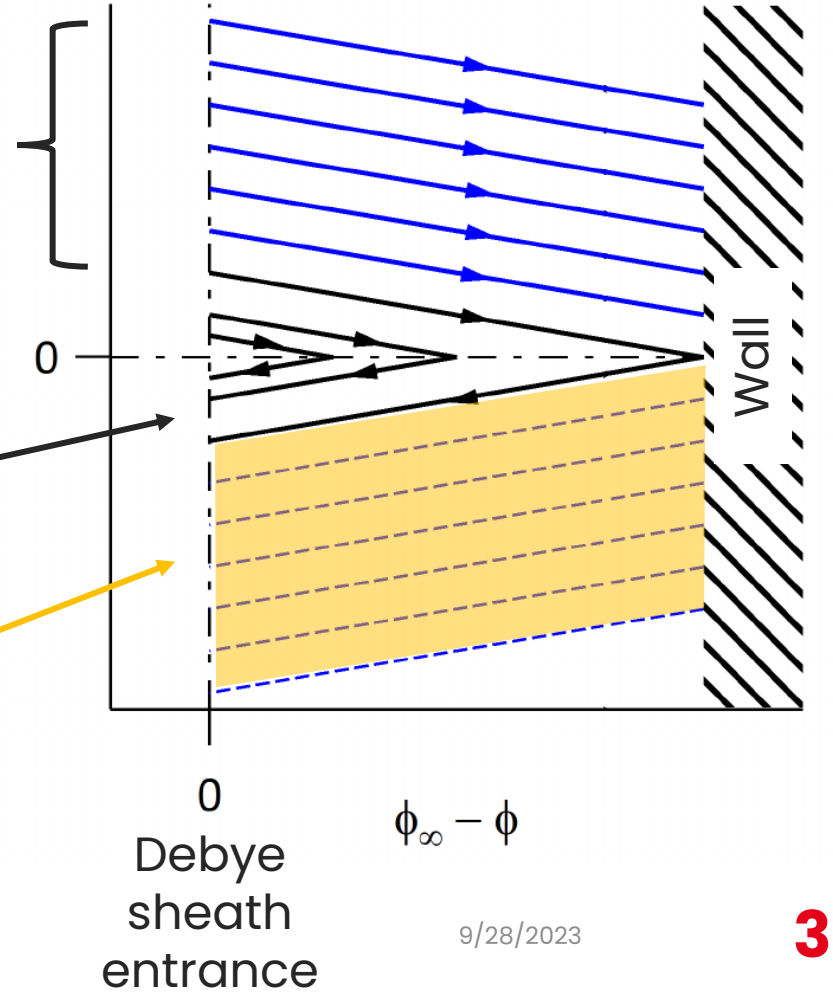


- kinetic energy $K = H_a + e_a\phi$
- No collisions → Hamiltonian invariant

fast electrons (absorbed in wall)

Trapped electrons (reflected in the plasma)

Empty phase space region



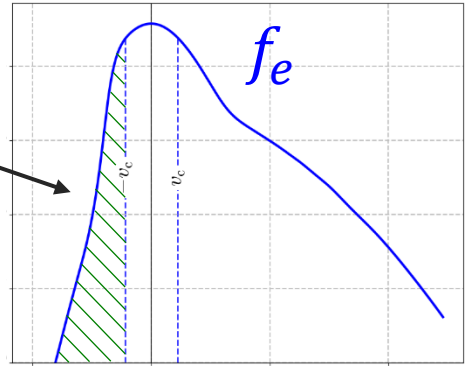
Ion-to-electron mass ratio dependency recovered

$$\Delta\phi_{\mathrm{sh}}^{\mathrm{pred.}} = \frac{T_{\mathrm{e}}^{\mathrm{sh}}}{e}\log\Bigg(\underbrace{\sqrt{2\pi M^2\left(1+\frac{T_{\mathrm{i}}^{\mathrm{sh}}}{T_{\mathrm{e}}^{\mathrm{sh}}}\right)}\left(1+\frac{\Gamma_{\mathrm{e}}^{-}}{\Gamma_{\mathrm{i}}^{\mathrm{sh}}}\right)}_{\Theta} \times \sqrt{\frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}}\Bigg) \longrightarrow \boxed{\Delta\bar{\phi}_{\mathrm{sh}}^{\mathrm{pred.}} = \frac{e\Delta\phi_{\mathrm{sh}}^{\mathrm{pred.}}}{T_{\mathrm{e}}^{\mathrm{sh}}} - \log(\Theta) = \frac{1}{2}\log\left(\frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}\right)}$$

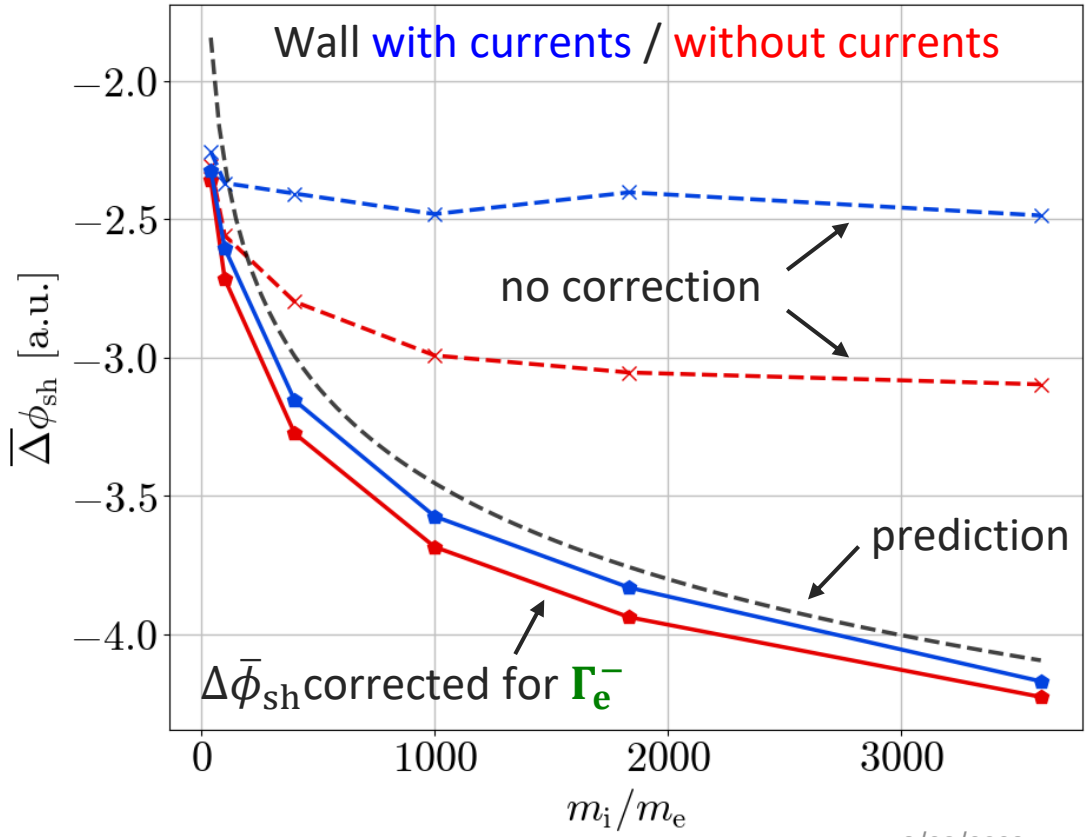
Relation $\Delta\bar{\phi}_{sh} \propto \log\left(\frac{m_e}{m_i}\right)$ verified

- Electron reemission from the wall explains the difference observed between theory and simulation

$$\Gamma_{\mathrm{e}}^{-} = \int_{-\infty}^{-v_{\mathrm{c}}} dv v f_{\mathrm{e}}^{\mathrm{sh}}(v)$$



- Electron reemission : finite steepness of penalization mask & absorption of electrons in wall not perfect

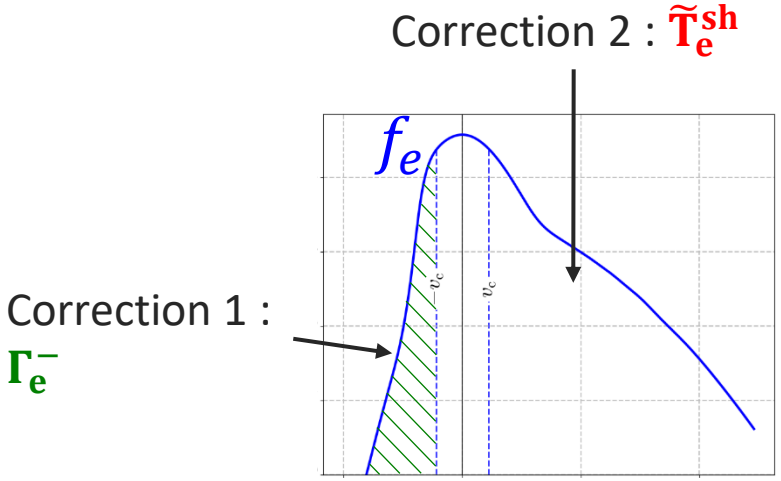


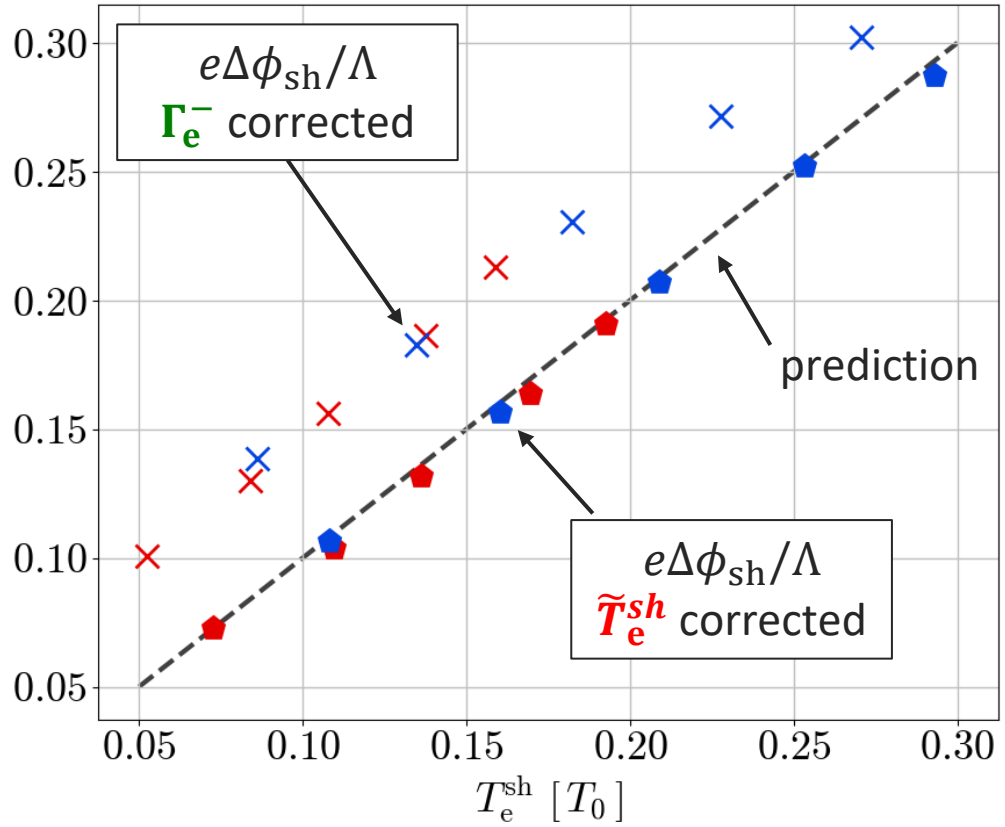
Electron temperature dependency $\Delta\phi_{sh} \propto T_e^{sh}$ recovered

$$\Delta\phi_{\mathrm{sh}}^{\mathrm{pred.}} = \frac{T_{\mathrm{e}}^{\mathrm{sh}}}{e}\underbrace{\log\left(\sqrt{2\pi\frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}M^2\left(1+\frac{T_{\mathrm{i}}^{\mathrm{sh}}}{T_{\mathrm{e}}^{\mathrm{sh}}}\right)\left(1+\frac{\Gamma_{\mathrm{e}}^{-}}{\Gamma_{\mathrm{i}}^{\mathrm{sh}}}\right)}\right)}_{\Lambda} \longrightarrow \boxed{\frac{e\Delta\phi_{\mathrm{sh}}^{\mathrm{pred.}}}{\Lambda} = T_{\mathrm{e}}^{\mathrm{sh}}}$$

Linear relation $\Delta\phi_{sh} \propto T_e^{sh}$ recovered

- The faster the electrons, the greater the potential drop



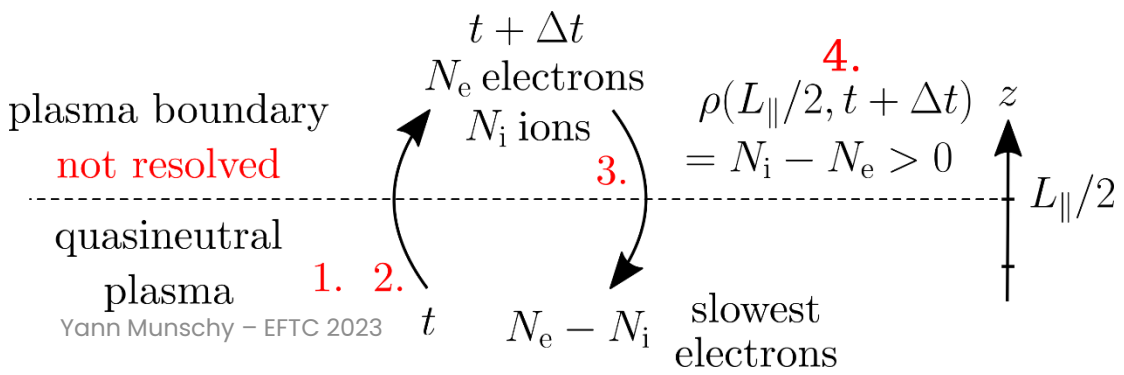


Existing solution for including sheath physics in GK codes

- **Logical sheath**
  - At a point of the simulation / plasma boundary
  - Count the number of ions going out of the plasma
  - Absorb an equal number of the fastest electrons
  - Reflect the other electrons back into the plasma

→ Equality of ion and electron fluxes at plasma boundary
→ Equality of fluxes defines the electron threshold velocity

$$\int_0^\infty dv\, v f_{\mathrm{i}}(x, y, L_\parallel/2, v, \mu) = \int_{v_c}^\infty dv\, v f_{\mathrm{e}}(x, y, L_\parallel/2, v, \mu).$$



- **Conducting sheath**
  - define electron threshold velocity is defined with the electric potential

$$\frac{1}{2} m_e v_c^2 = e\phi_{boundary}$$

  - Absorb all outcoming ions, and electrons faster than v_c
  - No direct constraint on electron and ion fluxes
  - Idea : the potential at plasma boundary reacts to the current

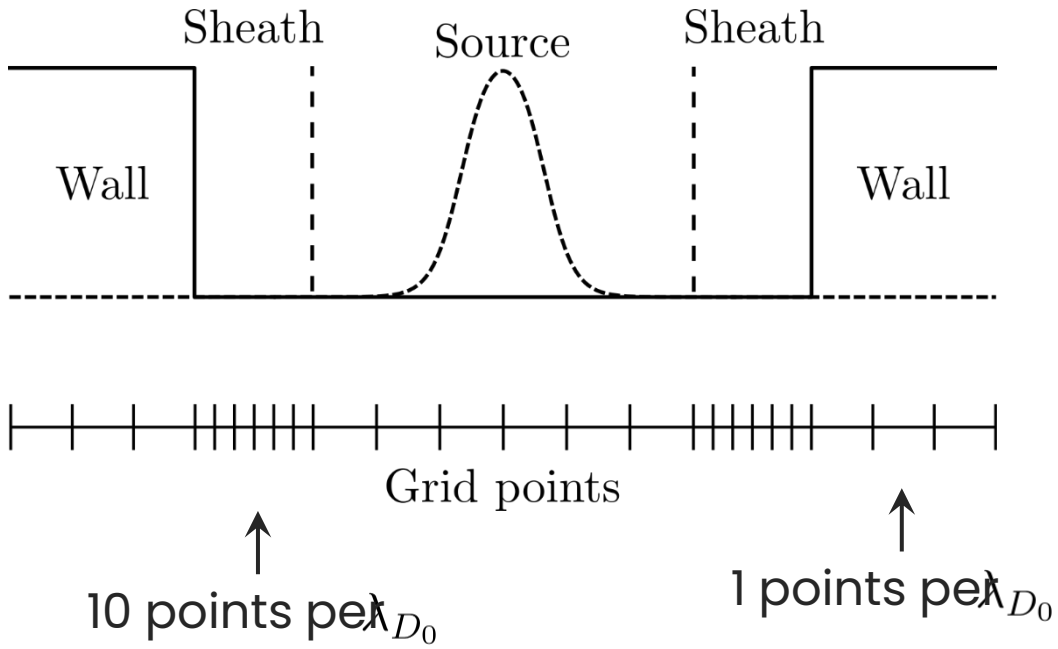
1D1V code parallelized on GPU with non equidistant grid points to mesh Debye length

- Backward semi Lagrangian scheme for Boltzmann eq.

[Bourne2022]

- Cubic spline interpolation on **non equidistant mesh**
  - Test bench for C++ version of GYSELA (5D gyrokinetic code)
- GPU parallelization

- 1 simulation: ~20h on NVIDIA Tesla V100 (5120 CUDA cores)



Collisions

Intra species: $C_{aa}(f_a) = \partial_{v_a}\left[D_v f_{Ma}\partial_{v_a}\left(\frac{f_a}{f_{Ma}}\right)\right]$; $f_{Ma} = \frac{n_a}{\sqrt{2\pi T_{Ma}}}\exp\left\{\frac{1}{2T_{Ma}}(v_a - u_{Ma})^2\right\}$

[Dif-Pradalier2011]

Conservation of density: $\int dv\ C_{aa}(f_a) = 0$

Conservation of momentum: $\int dv\ v_a C_{aa}(f_a) = 0$

Conservation of energy: $\int dv\ v_a C_{aa}(f_a) = 0$

} Gives $T_{Ma}(x,t)$ and $u_{Ma}(x,t)$

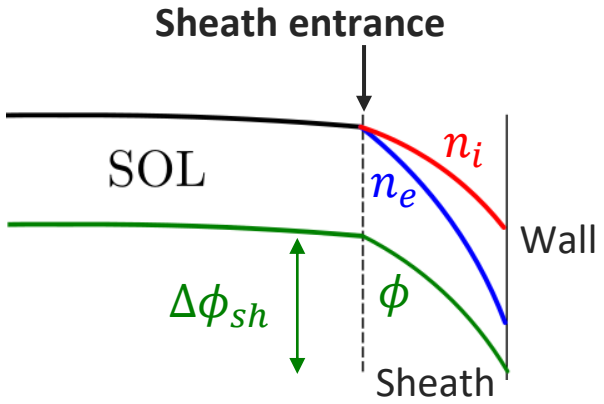
Inter species: $C_{ab}(f_a) = \frac{2Q_{ab}^M}{n_a T_a}\left[\frac{m_a}{2T_a}(v_a - V_a)^2 - \frac{1}{2}\right]F_{Ma} + \frac{R_{ab}^M}{n_a T_a}(v_a - V_a)F_{Ma}$

[Esteve2015]

$R_{ab}^M = -n_a \nu_{ab}\left(u_a - \sqrt{\frac{A_b}{A_a}}u_b\right)$ friction force (momentum transfer)

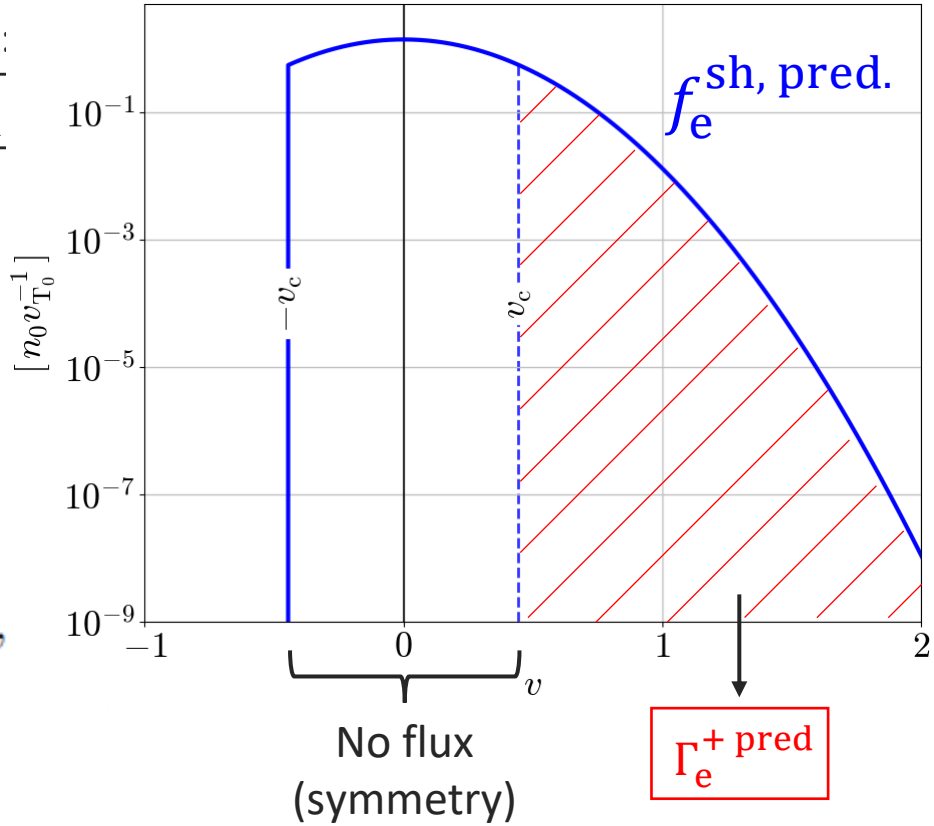
$Q_{ab}^M = -3n_a \frac{A_b}{A_b + A_a}\nu_{ab}(T_a - T_b) - u_a R_{ab}^M$ energy transfer

Prediction for potential drop in the sheath region



Escaping velocity:

$$v_c = \sqrt{\frac{2e|\Delta\phi_{sh}}{m_e}}$$



$$\Gamma_i^{sh} = n_i^{sh} M c_s.$$

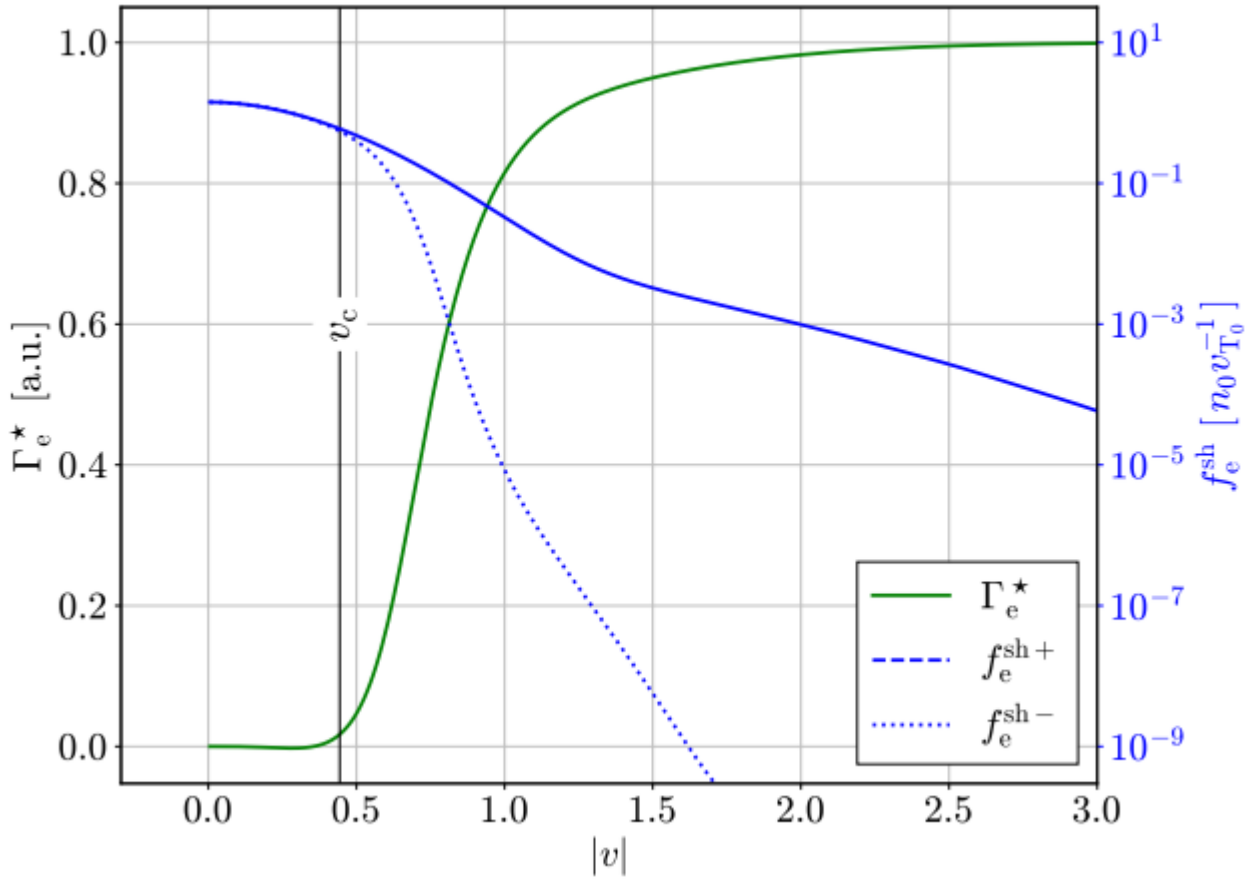
$$f_e(x_{sh}, v) = \overline{n}_e \sqrt{\frac{m_e}{2\pi T_e^{sh}}}\, e^{-\frac{m_e v^2}{2T_e^{sh}}}\, \mathbb{1}_{v \geq -v_c}(v),$$

$$\Gamma_e^{sh} = \int_{-v_c}^{+\infty} dv\, v n_e^{sh} \sqrt{\frac{m_e}{2\pi T_e^{sh}}} e^{-\frac{m_e v^2}{2T_e^{sh}}}$$

$$= n_e^{sh} e^{\frac{e\Delta\phi_{sh}}{T_e^{sh}}} \sqrt{\frac{T_e^{sh}}{2\pi m_e}}.$$

$$v(x_w)^2 = v(x_{sh})^2 + \frac{2e\Delta\phi_{sh}}{m_e}.$$

Breakdown of electron flux at sheath entrance

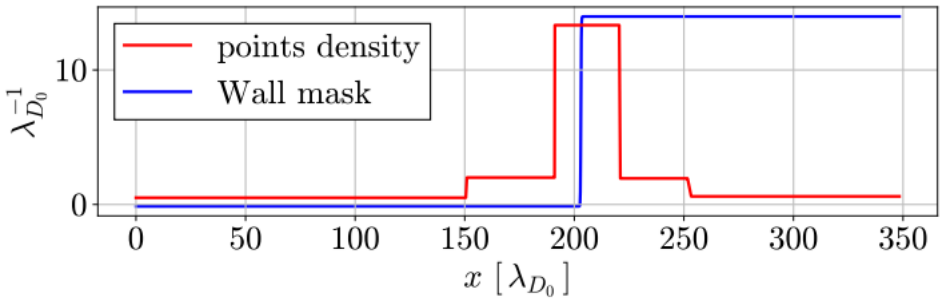


$$\Gamma_e^\star(v) = \frac{1}{\Gamma_e^{sh}} \int_{-v}^{v} dv'\, v' f_e^{sh}(v'),$$

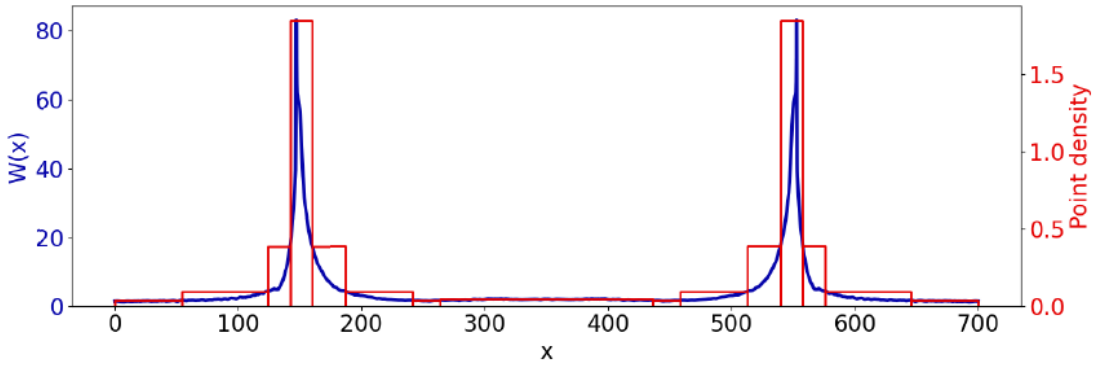
Numerical parameters for reference simulation

TABLE I. Reference simulation parameters.

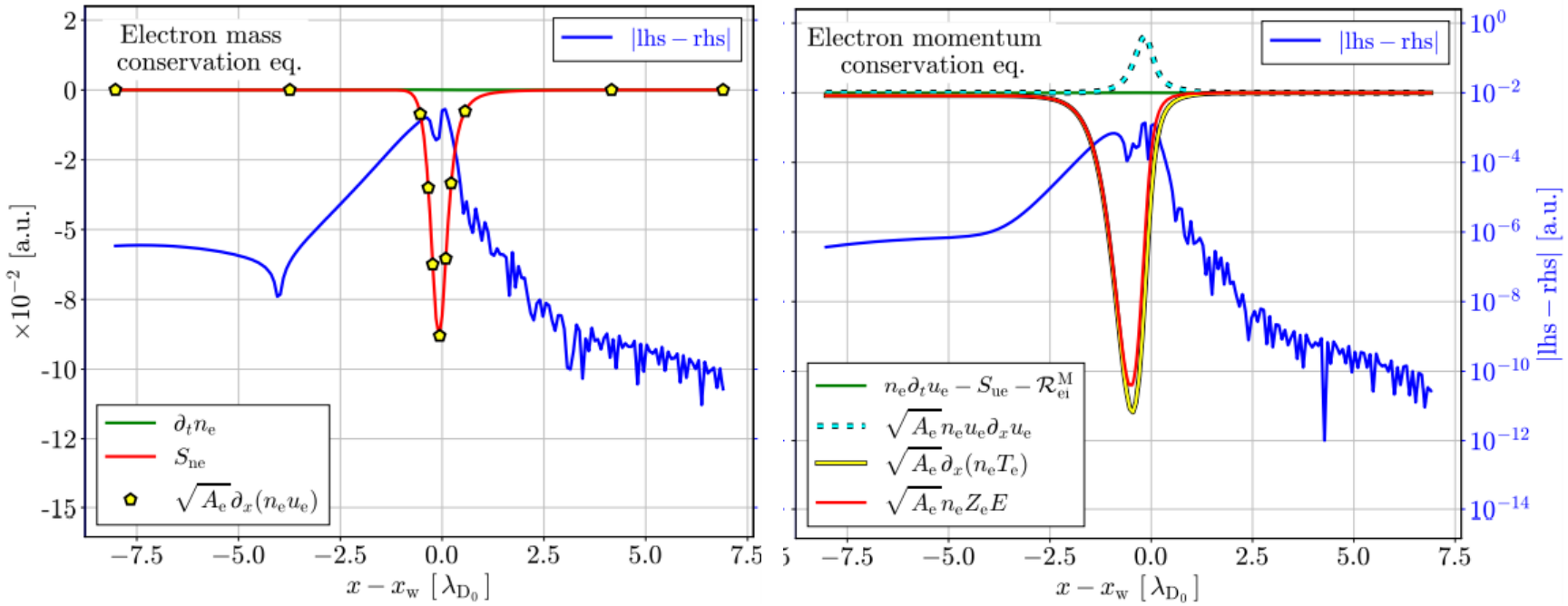
L_x	$L_\parallel$	d_w	n_w	$\nu_0^\star$
700	406	0.2	10^{-9}	0.1



[Bourne 2022]



Code verification : fluid conservation equations



$$\partial_t n_a + \sqrt{A_a}\partial_x(n_a u_a) = S_{na}, \quad \text{(B1)}$$

$$n_a(\partial_t u_a + \sqrt{A_a} u_a \partial_x u_a) + \sqrt{A_a}\partial_x(n_a T_a) = n_a Z_a E + S_{ua} + \mathcal{R}^M_{ab}, \quad \text{(B2)}$$

$$\partial_t(n_a T_a) + \sqrt{A_a} u_a \partial_x(n_a T_a) + \sqrt{A_a}\,(\partial_x Q_a + 3 n_a T_a \partial_x u_a) = S_{ha} + 2\mathcal{Q}^M_{ab}, \quad \text{(B3)}$$

Effect of collisions on the distribution function shape