Electromagnetic Analysis of Multilayer Structures with Arbitrarily Shaped Interfaces

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***Abstract*—A general formulation for electromagnetic analysis of multilayer structures with arbitrarily shaped interfaces is presented. The formulation belongs to modal methods. Its originality lies in the use of coordinates systems attached to the profile shape of the interfaces that separate the layers and to Maxwell's equations written under the covariant form. Each interface coincides with a coordinate surface which makes it elementary to enforce boundary conditions.**

***Index Terms*—Modal Methods, Curvilinear Coordinate Method, Computational Electromagnetics.**

I. Introduction

Many applications rely on the interaction of an electromagnetic field with multilayer structures. For instance, radar remote sensing of soils content requires an accurate model of soils which often involves multilayer rough surfaces and dielectric profiles. In an other frequency range, the performance of solar cells highly depend on light trapping which may be optimized by front and back gratings. At low frequencies, non-destructive testing of stratified conductors characterized by a set of arbitrary interfaces of complex shape may rely on the computation of the response of a 3D Eddy Current probe [1]. From the electromagnetic point of view, all three cited examples are similar. They represent problems of diffraction by layers with non-parallel faces, the case of parallel faces being a particular case. This paper is aimed at presenting a general method based on curvilinear coordinates for solving such problems.

II. Formulation

A. New Coordinates

Let us consider a stratified media characterized by a set of interfaces as is shown in Fig.1. Each interface separating two consecutive media with optical indexes ν_{p-1} and ν_p is described by an analytic function $a_p(x,y)$ depending on two directions x and y. The air domain, denoted by 0, contains the source. The last media is assumed to be infinite.

Each interface is associated with a non-orthogonal coordinate system such as:

$$x = x^1, \quad y = x^2, \quad z = x_p^3 + a_p(x^1, x^2). \tag{1}$$

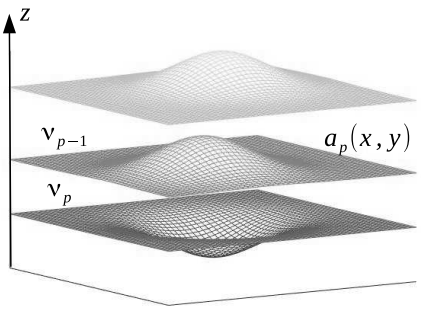

Fig. 1. Illustration of a multilayer structure. The pth layer is between a_{p-1} and a_p.

Therefore, boundary conditions at the interface of two adjacent layers becomes elementary since the latter is now a coordinate surface characterized by $x_p^3 = \text{constante}$.. Due to the change of coordinates, a new metric tensor must be introduced [2]. Denoting (x,y,z) by $(x^{1'}, x^{2'}, x^{3'})$, the components of the metric tensor g_{ij} are given by $\sum_{i'j'}(\partial x^{i'}\partial x^{j'}/(\partial x^i \partial x^j)\delta_{i'j'}$ where $\delta_{i'j'}$is the Kronecker symbol. The inverse of the matrix formed by the g_{ij} gives the components of the conjugate metric tensor g^{ij}:

$$[g^{ij}] = [g_{ij}]^{-1} = \begin{bmatrix} 1 & 0 & -\dot{a}_1 \\ 0 & 1 & -\dot{a}_2 \\ -\dot{a}_1 & -\dot{a}_2 & 1+\dot{a}_1^2+\dot{a}_2^2 \end{bmatrix}, \tag{2}$$

where $\dot{a}_i$, $i = 1,2$ are the partial derivative of $a_p(x^i)$ with respect to x^i. Here, the determinant g of $[g_{ij}]$ is equal to one. Since the coordinate system is non-orthogonal, it is convenient to write Maxwell's equations under the covariant form. Assuming a time dependence of the form $\exp(i\omega t)$, we have:

$$\begin{aligned} \xi^{lmn}\partial_m E_n &= -ik\sqrt{g}g^{lm}Z_0 H_m, \\ \xi^{lmn}\partial_m Z_0 H_n &= ik\nu^2\sqrt{g}g^{lm}E_m, \end{aligned} \tag{3}$$

where $l,m,n \in \{1,2,3\}$, k is the wave number, Z_0 is the vacuum wave impedance, ∂_j, $j \in \{1,2,3\}$ stands for $\partial/\partial x^j$, ξ^{lmn} is the Levi-Civita indicator.

B. Method of Solution

Since the components of the metric tensor do not depend on the x^3 variable, it is possible to express the four tangential components E_1, E_2, H_1 and H_2 in terms of the two longitudinal components E_3 and H_3 which satisfy the same scalar propagation equation [3]:

$$\left(\partial_m g^{mn} \partial_n + k^2 \nu^2\right) \psi = 0 \quad \psi = E_3 \text{ or } \psi = H_3. \tag{4}$$

Moreover any of the six components takes the form :

$$\Psi(x^1, x^2, x^3) = \psi(x^1, x^2) \exp(-i\gamma x^3). \tag{5}$$

Taking into account (5), (4) can be recast as a first order differential system of the form:

$$[\mathcal{L}_p^q] \begin{bmatrix} \psi_p^q \\ \psi_p'^q \end{bmatrix} = \gamma_p^q \begin{bmatrix} \psi_p^q \\ \psi_p'^q \end{bmatrix} \quad \psi_p'^q = -i\gamma_p^q \psi_p^q. \tag{6}$$

In equation (6), the subscript p refers to the interface whereas the superscript q refers to the media. According to the notations illustrated on Fig.1, we see that $q = p$ or $q = p - 1$. Equation (6) is transformed into an algebraic eigenvalue system by using the method of moments after a suitable basis is chosen. The numerical solution is then obtained thanks to standard linear algebra libraries. It can be shown that there are two sets of modes which correspond to forward waves and backward waves respectively. Finally the solution is a linear combination of eigen-modes depending on both the media and the coordinate system. Hence, inside a layer whose faces are not identical, the field is represented by two different sets of eigenvectors:

$$\psi_p^{\pm q} = \sum_{l=1}^{L} \left(A_{e\,l,p}^{\pm\, q} \psi_{e\,l\,p}^{\pm\, q} + A_{h\,l\,p}^{\pm\, q} \psi_{h\,l\,p}^{\pm\, q} \right) \exp(-i\gamma_p^{\pm q}(x_p^3)), \tag{7}$$

with $q = p - 1$ or $q = p$. Vector ψ is the concatenation of the four tangential components. The subscript e, (respectively h) designates a vector generated by E_3, (respectively H_3). L is the number of numerically computed eigen-modes.

C. Boundary Conditions

The unknown constant coefficients depend on two physical phenomena of different nature. On the one hand, crossing an interface, the total tangential field components are continuous which implies that forward and backward waves are coupled. On the other hand, the propagation from one interface to the next one in the same media, creates only a phase shift or an attenuation without coupling the two kind of waves. Boundary conditions are enforced thanks to the the S-matrix formalism. At the pth interface which separates media $p - 1$ and media p, that is at $x_p^3 = 0$, we have an interface $S_p^{p-1,p}$ matrix such that:

$$\begin{bmatrix} A_{e\,l\,p}^{+\,(p-1)} \\ A_{h\,l\,p}^{+\,(p-1)} \\ A_{e\,l,p}^{-p} \\ A_{h\,l\,p}^{-p} \end{bmatrix} = S_p^{p-1,p} \begin{bmatrix} [A_{e\,l\,p}^{-\,(p-1)} \\ A_{h\,l\,p}^{-\,(p-1)} \\ A_{e\,l\,p}^{+p} \\ A_{h\,l\,p}^{+p} \end{bmatrix}. \tag{8}$$

Between interface $p-1$ and interface p, we have derived two modal expansions for the field: one is attached to a_{p-1} and the other one to a_p. The corresponding coordinate systems are linked by

$$x_{p-1}^3 = x_p^3 + a_{p-1}(x^1, x^2) - a_p(x^1, x^2) = x_p^3 + t_p(x^1, x^2). \tag{9}$$

In the case of layers with parallel faces, function $t_p(x^1, x^2)$ reduces to a constant. We consider the pth layer as a generalized transmission line which connects input field $\psi_{p-1}^{-\,p-1}$ and $\psi_p^{+\,p-1}$ to output fields $\psi_{p-1}^{+\,p-1}$ and $\psi_p^{-\,p-1}$. As already mentioned, no coupling occurs between forward waves and backward waves. Hence the definition of the bloc anti-diagonal layer $S_{p-1\,p}^{p-1}$ matrix:

$$\begin{bmatrix} A_{e\,l\,(p-1)}^{+\,(p-1)} \\ A_{h\,l\,(p-1)}^{+\,(p-1)} \\ [A_{e\,l,p}^{-,p} \\ A_{h\,l\,p}^{-,p} \end{bmatrix} = S_{p-1\,p}^{p-1} \begin{bmatrix} [A_{e\,l\,p-1}^{-\,(p-1)} \\ A_{h\,l\,p-1}^{-\,(p-1)} \\ [A_{e\,l\,p}^{+p} \\ A_{h\,l\,p}^{+p} \end{bmatrix}. \tag{10}$$

At the input of the pth layer that is at $z_{p-1}^3 = 0$, the outgoing waves correspond to the incoming waves of the output plane. Similarly, at the output of the same layer, that is at $z_p^3 = 0$, the outgoing waves correspond to the incoming waves of the input. Hence the two relations from which the coefficients of the layer S matrix are computed:

$$\begin{array}{l} \psi_{p-1}^{+\,(p-1)}(x_{p-1}^3 = 0) = \psi_p^{+\,(p-1)}(x_p^3 = t_p), \\ \psi_p^{-\,(p-1)}(x_p^3 = 0) = \psi_{p-1}^{-\,(p-1)}(x_p^3 = -t_p). \end{array} \tag{11}$$

D. Excitation

The structure is exited from air with a known field which is expanded onto the eigenvectors associated to the first interface which determine input amplitude coefficients $A_{e\,l,1}^{-0}$,$A_{h\,l,1}^{-0}$. All other coefficients are obtained from the usual recursion formulas of the S matrix algorithm.

III. Conclusion

We have described an efficient and versatile algorithm for modeling arbitrarily shaped faces multilayer structures. Its advantage over pure numerical methods like Finite Elements Methods is that no mesh is needed in the z direction which makes it competitive in terms of computational speed especially for layers with rather large thickness. In the presentation, we will focus on the numerical implementation and give some examples of applications.

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