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# The spectral functions method for elastic plane wave diffraction by a wedge

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Non destructive examination of industrial structures requires the modeling of specimen geometry echoes generated by the surfaces (entry, back-wall, etc.) of inspected blocks. For that purpose, the study of plane elastic wave diffraction by a wedge is of great interest since surfaces of complex industrial specimen often include dihedral corners. These inspections often deal with high frequency ( $f = 2-5$  MHz) ultrasonic waves. Simulation of realistic inspections by finite elements and finite differences can therefore be time consuming because such methods require a small mesh step for a better description of the scattered wave. Semi-analytical methods are therefore preferred for high frequency problems. There exist various approaches for semi-analytical modeling of plane elastic wave diffraction by a wedge but the theoretical and numerical aspects of these methods have so far only been developed for wedge angles lower than  $\pi$ . Croisille and Lebeau [1] have introduced a resolution method called the Spectral Functions method in the different case of an immersed elastic wedge of angle less than  $\pi$ . Kamotski and Lebeau [2] have then proven existence and uniqueness of the solution derived from this method to the diffraction problem of stress-free wedges embedded in an elastic medium. The advantages of this method are its validity for wedge angles greater than  $\pi$  and its adaptability to more complex cases. The methodology of Croisille and Lebeau [1] has been first extended by the authors to the simpler case of an immersed soft wedge [3] (submitted). It has then been developed here for the 2D scattering problem of an elastic longitudinal wave and a numerical validation of the method for all wedge angles is proposed.

## 1 INTRODUCTION

Ultrasonic inspection of a specimen generates echoes from the entry and backwall surfaces. If these surfaces contain wedges, it is then necessary to provide a correct model of the interaction between the ultrasonic beam and these wedges.

For that purpose, based on the Physical Theory of Diffraction (PTD) introduced by Ufimtsev [4], an ultrasonic system model has been developed for embedded cracks and extended to mimic ultrasonics with some head waves by Zernov et al. [5,6]. Nevertheless, this ultrasonic PTD model can be time consuming for large specimen surfaces. A second solution to this problem, called the Uniform Theory of Diffraction (UTD) has been proposed in elastodynamics by Kamta Djakou et al. [7] and developed for a half-plane scatterer. To apply the aforementioned UTD method to the case of a wedge, a generic and trustworthy wedge diffraction model is necessary. Budaev [9,10] has developed a technique to reduce the wedge diffraction problem to a singular integral equation in various cases, but a numerical resolution method has only been presented in the case of a Rayleigh wave by Budaev and Bogoy and clarified by Kamotski et al. [11,12]. The results of Kamotski et al. have been applied by Gautesen and Fradkin [8] in order to present the Sommerfeld integral method. Gautesen and Fradkin have also developed a method called the Laplace transform method [8]. However, both models have only been developed for wedge-angles lower than  $\pi$  in 2D configurations (for which the incident wave vector is in the plane normal to the edge) and have only been developed for stress-free wedges.

The spectral functions method for diffraction by a wedge presents numerous advantages. First, it works for all wedge angles (and notably for angles higher than  $\pi$ ). Secondly, it requires a very short computation time. It is a generic method adaptable to much more complex cases such as 3D configurations or various types of media (impedance

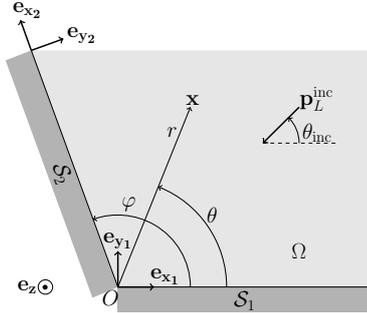


Figure 1: Plane wave incident on a stress-free wedge of angle  $\varphi$ .

wedges for instance). It has originally been developed by Croisille and Lebeau [1] and applied to an immersed elastic wedge. The methodology has been extended to the case of a stress-free wedge embedded in an elastic solid by Kamotski and Lebeau [2] to prove existence and uniqueness of the solution to the diffraction problem but the numerical aspects and the final solution have not been dealt with. The study of further configurations is to come and an article concerning the case of a stress-free wedge in acoustics has been submitted [3].

The following paper deals with the longitudinal wave diffracted by a wedge illuminated by an incident longitudinal wave in the plane normal to the wedge edge. It begins by presenting the problem and defining the wedge diffraction coefficient. Section 3 deals with finding an integral formulation of the solution. This integral formulation is expressed in terms of two unknown functions called the spectral functions. In section 4, a system of functional equations solved by the spectral functions is determined. This system can be solved semi-analytically. Section 5 presents some first validation results and section 6 gives the conclusions and perspectives of this work.

## 2 PROBLEM STATEMENT

Let us consider the diffraction problem of a plane longitudinal elastic wave  $\mathbf{u}^{\text{inc}}$  incident on a wedge delimited by the stress-free infinite plane faces  $\mathcal{S}_1$  and  $\mathcal{S}_2$ . The interior of the wedge, constituted of an isotropic solid medium, is defined by

$$\Omega = \{(r \cos \theta, r \sin \theta) \mid \theta \in ]0, \varphi[ \}.$$

And the incident plane wave is of the form

$$\mathbf{u}^{\text{inc}}(\mathbf{x}, t) = \mathbf{A}_L e^{i(\mathbf{p}_L^{\text{inc}} \cdot \mathbf{x} - \omega t)}.$$

$\mathbf{A}_L$  and  $\mathbf{p}_L^{\text{inc}}$  are respectively the amplitude vector and the incident wave vector. The Cartesian coordinate system  $(O; \mathbf{e}_{x_1}, \mathbf{e}_{y_1})$  is attributed to the face  $\mathcal{S}_1$  of the wedge and  $(O; \mathbf{e}_{x_2}, \mathbf{e}_{y_2})$  is attributed to the face  $\mathcal{S}_2$ , as shown in Fig. 1. These coordinate systems have the same origin located on the wedge edge which coincides with the  $z$ -axis. Let  $\mathbf{x} = (x_1, y_1)_{(\mathbf{e}_{x_1}, \mathbf{e}_{y_1})} = (x_2, y_2)_{(\mathbf{e}_{x_2}, \mathbf{e}_{y_2})}$  be a position vector  $\mathbf{x} = (r, \theta)$  in a local basis of polar coordinates associated to the coordinates  $(x_1, y_1)$ .

In the coordinate system  $(\mathbf{e}_{x_1}, \mathbf{e}_{y_1})$ , the incident wave vector is given by

$$\mathbf{p}_L^{\text{inc}} = -\frac{\omega}{c_L} \begin{pmatrix} \cos \theta_{\text{inc}} \\ \sin \theta_{\text{inc}} \end{pmatrix},$$

where  $c_L$  is the velocity of the longitudinal waves ( $c_T$  being the transversal wave velocity and  $c_R$  the Rayleigh wave velocity).

In the following, vectors are expressed in the coordinate system  $(O; \mathbf{e}_{x_1}, \mathbf{e}_{y_1})$ , except when explicitly stated otherwise. The displacement field  $\mathbf{u}$  is then solution to the linear elasticity problem for an isotropic homogeneous material and satisfies stress-free boundary conditions on  $\mathcal{S}_1$  and  $\mathcal{S}_2$ . Bold letters will hereafter be reserved for matrices in order to simplify notations and the harmonic time-factor  $e^{-i\omega t}$  is omitted.

Let us suppose that the total field is the sum of the incident field and of an edge diffracted field:

$$u = u^{\text{inc}} + u_0.$$

The dimensionless problem is obtained by applying the following change in variables:

$$u_0(x, y) = v \left( \frac{\omega}{c_L} x, \frac{\omega}{c_L} y \right).$$

The problem we wish to solve is now

$$(\mathcal{P}^L) \quad \begin{cases} (E + 1)v = 0 & (\Omega), \\ Bv = -Bv_L^{\text{inc}} & (\mathcal{S}_1 \cup \mathcal{S}_2), \end{cases} \quad (1)$$

where  $E$  is the dimensionless elasticity operator and  $B$  is the normal stress operator ( $n$  being in equation (3) the inward normal to each face of the wedge):

$$Ev = \mu \Delta v + (\lambda + \mu) \nabla \nabla v, \quad (2)$$

$$Bv = (\lambda \nabla v \cdot \mathbb{I}_2 + 2\mu \varepsilon(v)) \cdot n, \quad (3)$$

$\lambda$  and  $\mu$  are the dimensionless Lamé coefficients,  $\mathbb{I}_2$  is the two-dimensional identity matrix and we have

$$\varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right).$$

The dimensionless incident longitudinal wave is given by

$$v_L^{\text{inc}}(r, \theta) = \begin{pmatrix} \cos \theta_{\text{inc}} \\ \sin \theta_{\text{inc}} \end{pmatrix} e^{ir\nu_L \cos(\theta - \theta_{\text{inc}})},$$

with

$$\nu_L = 1, \quad \nu_T = \frac{c_L}{c_T}, \quad \nu_R = \frac{c_L}{c_R}.$$

In the far field approximation ( $\frac{\omega r}{c_L} \gg 1$ ,  $r$  being the distance of propagation, see Fig. 1), it can be shown using the steepest descent method that the field  $v$  is the sum of two types of contributions:

$$v = v^{\text{diff}} + v^{\text{sing}},$$

where  $v^{\text{diff}}$  is the contribution of the saddle-point and describes the cylindrical waves diffracted by the edge. The contribution  $v^{\text{sing}}$  is due to the singularities of the solution. It will be shown in the following that these can only be simple poles (which describe the plane bulk waves reflected by the wedge faces and the surface Rayleigh waves) or branching points (which describe head waves).

The subject of this study is the edge-diffracted longitudinal wave. It can be expressed as a cylindrical wave, proportional to the incident wave and weighted by a coefficient  $D_L^L$  called the 2D diffraction coefficient which depends on the direction of observation  $\theta$  and on the parameters of the problem. The diffraction coefficient is therefore defined by

$$\begin{aligned} v_L^{\text{diff}}(r \cos \theta, r \sin \theta) \\ = D_L^L(\theta) \frac{e^{-ir}}{\sqrt{r}} v_L^{\text{inc}}(r \cos \theta, r \sin \theta). \end{aligned} \quad (4)$$

In order to obtain a far-field approximation of the longitudinal waves diffracted by a wedge, it is sufficient to compute the diffraction coefficient, defined by equation (4). The aim of the present work is to compute this coefficient.

### 3 INTEGRAL FORMULATION OF THE SOLUTIONS

Kamotski and Lebeau [2] have partially applied the spectral functions method to the case of an elastic wave incident on a stress-free wedge. They have used the method to prove the existence and uniqueness of the outgoing solution to this diffraction problem but have not computed such a solution. In the following, the main steps of their method and its application to the computation of the solution are presented.

Croisille and Lebeau [1] have shown that the problem satisfies the limiting absorption principle. The solution to the problem with absorption being the sum of two single layer potentials, the exact solution of the problem ( $\mathcal{P}^*$ ) is

$$v = v_1 + v_2, \quad (5)$$

with

$$\begin{aligned} v_j(x_j, y_j) = \frac{1}{4\pi^2} \lim_{\epsilon \rightarrow 0} \int_{-\infty}^{+\infty} e^{ix_j \xi} \Sigma_j(\xi) \\ \times \int_{-\infty}^{+\infty} e^{iy_j \eta} (\mathbf{M}(\xi, \eta) - e^{-2i\epsilon \mathbb{I}_2})^{-1} d\eta d\xi, \end{aligned} \quad (6)$$

where  $\Sigma_j$  are unknown functions called the spectral functions

$$\Sigma_j(\xi) = \begin{pmatrix} \hat{\alpha}_j(\xi) \\ \hat{\beta}_j(\xi) \end{pmatrix},$$

and the operator  $\mathbf{M}$  is the Fourier transform of the operator  $\mathbf{E}$  defined in (2). The unknown functions  $\alpha_j, \beta_j$  are sought for in a special class of distributions  $\mathcal{A}$  defined by Kamotski and Lebeau [2] in the following manner.

**Definition 1** We say that  $f \in \mathcal{A}$  if  $f \in \mathcal{S}'(\mathbb{R})$ ,  $\text{supp } f \subset \mathbb{R}^+$  and  $\exists C_0 > 0$  such that

$$\sup_{-\pi < \theta < 0} \int_{C_0}^{\infty} |\hat{f}(\rho e^{i\theta})|^2 d\rho < \infty, \quad (7)$$

$\hat{f}$  is holomorphic in neighbourhoods of  $\nu_L, \nu_T, \nu_R$ .

The computation of the unknown functions  $\hat{\alpha}_j, \hat{\beta}_j \in \hat{\mathcal{A}}$  ( $\hat{\mathcal{A}}$  being the Fourier transform of class  $\mathcal{A}$ ) will be treated in section 4. The inner integral in (6) can be computed using Gauss' residue theorem.

This integral can be approached in the far-field approximation by using the stationary phase method. By identifying the result of this approximation with equation (4), we can express the longitudinal diffraction coefficient in terms of the spectral functions:

$$\begin{aligned} D_L^L(\theta) = \frac{e^{-i\pi/4}}{2\sqrt{2\pi}} (\hat{\alpha}_1(-\cos \theta) \cos \theta \\ + \hat{\beta}_1(-\cos \theta) \sin \theta \\ + \hat{\alpha}_2(-\cos(\varphi - \theta)) \cos(\varphi - \theta) \\ + \hat{\beta}_2(-\cos(\varphi - \theta)) \sin(\varphi - \theta)). \end{aligned} \quad (8)$$

In the following section, we will explain how the spectral functions can be computed semi-analytically.

#### 4 SEMI-ANALYTICAL RESOLUTION

The first step to computing the spectral functions is to determine a system of functional equations of which the spectral functions are solution. The second step is to use this system to prove that the spectral functions can be decomposed into a singular part and a regular part and to compute them.

##### 4.1 Functional equations

To determine a system of functional equations of which the spectral functions are solution, we begin with the substitution of  $v$  by the decomposition (5) in the wedge boundary conditions (1). The boundary condition can be taken separately on each face and expressed using the corresponding coordinate system:

$$\begin{cases} B(v_1(x_1, 0) + v_2(x_2 \cos \varphi, x_2 \sin \varphi)) = -Bv_L^{\text{inc}}|_{\mathcal{S}_1}, \\ B(v_2(x_2, 0) + v_1(x_1 \cos \varphi, x_1 \sin \varphi)) = -Bv_L^{\text{inc}}|_{\mathcal{S}_2}. \end{cases}$$

Thanks to this decomposition, two new operators can be defined.  $B_1$  is obtained by expressing the normal stress operator  $B$  defined in (3) on  $\mathcal{S}_j$  in terms of  $(x_j, y_j)$ .

The expression (3) of the normal stress operator can also be projected onto the coordinate system  $(O; e_{x_{3-j}}, e_{y_{3-j}})$  and expressed in terms of  $(x_{3-j}, y_{3-j})$ , yielding the operator  $B_2$ .

The system of boundary conditions can then be written as follows:

$$\begin{cases} B_1(v_1) + B_2(v_2) = -Bv_L^{\text{inc}}|_{\mathcal{S}_1}, \\ B_1(v_2) + B_2(v_1) = -Bv_L^{\text{inc}}|_{\mathcal{S}_2}. \end{cases} \quad (9)$$

Let us take the Fourier transform of (9). First, the integral formulation of  $v_1$  and  $v_2$  given by (6) is taken at points  $y = 0, x \geq 0$  and substituted into  $B_1$ . The Fourier transform is then applied to the result:

$$\begin{aligned} \int_0^{+\infty} e^{-ix\xi} B_1(v_1)(x) dx &= \frac{1}{2} \mathbf{DM}(\Sigma_1)(\xi) \\ &= \frac{1}{2} \int_{\Gamma_0} \mathbf{DM}(\xi, \zeta) \Sigma_1(\zeta) d\zeta, \end{aligned}$$

with

$$\mathbf{DM}(\xi, \zeta) = \frac{1}{2i\pi} \frac{1}{\xi - \zeta} \mathbf{dm}(\zeta).$$

The contour  $\Gamma_0$  is represented in Fig. 2. The matrices  $\mathbf{dm}$  and  $\mathbf{tm}_L$  and  $\mathbf{tm}_T$  defined in the following (10) are known explicitly. However, their

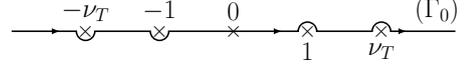


Figure 2: Integration contour  $\Gamma_0$ .

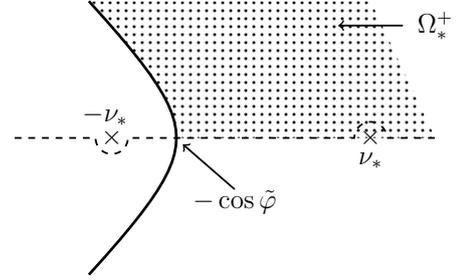


Figure 3: Domain  $\Omega_*^+$ .

expression, being cumbersome [1], are not reproduced here.

To compute the Fourier transform of the operator  $B_2$ , a new translation operator must be defined,

$$\begin{aligned} T_*(\xi = \nu_* \cos z) &= \zeta \cos \varphi + \zeta_*(\zeta) \sin \tilde{\varphi} \\ &= \nu_* \cos(z + \tilde{\varphi}), \end{aligned}$$

where the following notation is used:

$$\tilde{\varphi} = \begin{cases} \varphi & \text{if } \varphi < \pi \\ 2\pi - \varphi & \text{if } \varphi \geq \pi \end{cases}$$

This translation operator is well defined on the following domain  $\Omega_*^+$ , represented in Fig. 3,

$$\Omega_*^+ = \{\xi = \nu_* \cos z, 0 \leq \text{Re } z < \pi - \tilde{\varphi}\}.$$

The Fourier transform of  $B_2$  is obtained in a similar manner as for  $B_1$ . The integral formulation of  $v_1$  and  $v_2$  given by (6) is taken at points  $x' = x \cos \varphi, y' = x \sin \varphi$  and substituted into  $B_2$ . The Fourier transform is then applied to the result:

$$\begin{aligned} \int_0^{+\infty} e^{-ix\xi} B_2(v_2)(x) dx &= \frac{1}{2} \mathbf{TM}(\Sigma_2)(\xi) \\ &= \frac{1}{2} \int_{\Gamma_0} \mathbf{TM}(\xi, \zeta) \Sigma_2(\zeta) d\zeta, \end{aligned}$$

with

$$\begin{aligned} \mathbf{TM}(\xi, \zeta) &= \frac{1}{2\pi i} \sum_{*=L,T} D_*(\xi, \zeta) \mathbf{tm}_*(\zeta, \text{sgn } \sin \varphi), \\ D_*(\xi, \zeta) &= \frac{1}{\xi - T_*(\zeta)}. \end{aligned} \quad (10)$$

By taking the sum of these two operators **DM** and **TM**, the Fourier transform of the boundary conditions on each face of the wedge is finally obtained. This is a system of functional equations solved by the spectral functions:

$$\begin{cases} \mathbf{DM}(\Sigma_1) + \mathbf{TM}(\Sigma_2) = \frac{W_1^L}{\xi - \cos \theta_{\text{inc}}}, \\ \mathbf{TM}(\Sigma_1) + \mathbf{DM}(\Sigma_2) = \frac{W_2^L}{\xi - \cos(\varphi - \theta_{\text{inc}})}. \end{cases} \quad (11)$$

The explicit expression of the residues in the right-hand side can be obtained by taking the Fourier transform of the right-hand side of (9).

This system of functional equations will be resolved in  $\hat{\mathcal{A}}^2 \oplus \hat{\mathcal{A}}^2$ , defined by (7) to obtain an evaluation of the spectral functions.

#### 4.2 Method of resolution

The resolution of system (11) is done semi-analytically. The first step is to compute  $y_j$ , which is called the ‘‘singular part’’. First, the subsets of  $\mathbb{C}$ ,  $\{z_1^k\}_{k \geq 0}$  and  $\{z_2^k\}_{k \geq 0}$  are defined recursively by Kamotski and Lebeau [2].

These sets are the poles of the spectral functions and Croisille and Lebeau [1] have shown that they are finite, meaning that any incident ray on the wedge will eventually become an outgoing ray after a certain number of reflections.

Using these sets, Croisille and Lebeau [1] have also proven the following result.

**Lemma 1** *There exist two functions  $Y_1, Y_2$  being the finite sum of principal parts corresponding to simple poles*

$$Y_j(\xi) = \sum_k \frac{a_j^k}{\xi - z_j^k}, \quad a_j^k \in \mathbb{C}^2, \quad z_j^k \in \mathbb{C},$$

such that  $u_1$  and  $u_2$  defined by

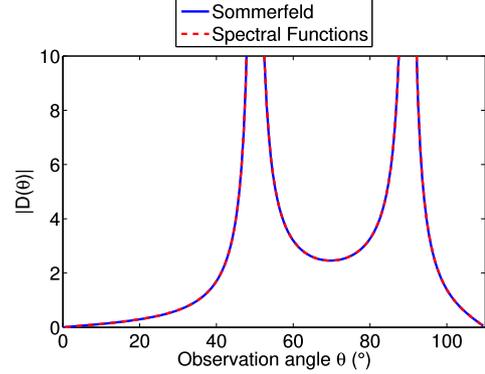
$$u_j(\xi) = \frac{W_j^L}{\xi - z_j} - \mathbf{DM}(Y_j)(\xi) - \mathbf{TM}(Y_{3-j})(\xi) \quad (12)$$

are analytical on  $\mathbb{C} \setminus ]-\infty, -1]$ .

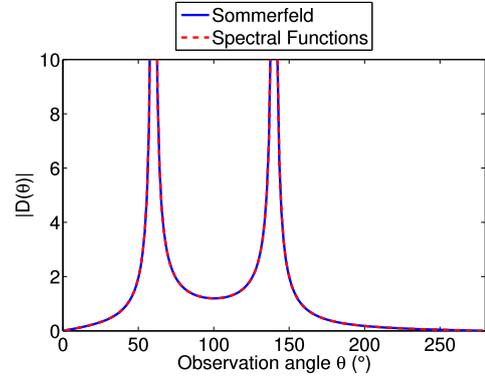
The functions  $Y_1$  and  $Y_2$  are the singular parts of the spectral functions  $\Sigma_1$  and  $\Sigma_2$ . Their poles and residues are computed explicitly using a recursive procedure given by Croisille and Lebeau [1].

Let us now compute the ‘‘regular parts’’  $X_1$  and  $X_2$  of the spectral functions, defined by

$$X_j(\xi) = \Sigma_j(\xi) - Y_j(\xi).$$



(a)  $\varphi = 110^\circ$ ,  $\theta_{\text{inc}} = 90^\circ$



(b)  $\varphi = 280^\circ$ ,  $\theta_{\text{inc}} = 240^\circ$

Figure 4: Diffraction coefficient computed with the spectral functions method and with the Sommerfeld method.

According to (12), these functions are solutions of the following system:

$$\begin{cases} \mathbf{DM}(X_1)(\xi) + \mathbf{TM}(X_2)(\xi) = u_1(\xi), \\ \mathbf{TM}(X_1)(\xi) + \mathbf{DM}(X_2)(\xi) = u_2(\xi). \end{cases}$$

Solvability of this system is proven by Kamotski and Lebeau [2]. The functions  $X_1$  and  $X_2$  are analytical on  $\mathbb{C} \setminus ]-\infty, -1]$  and are approached numerically by a Galerkin collocation method, yielding a linear system of equations where the coefficients are integrals that can be computed exactly. However, the details of this computation are quite long and are not presented here. They will hopefully be the subject of a future publication.

The semi-analytical computation of the spectral functions leads to the numerical evaluation of the diffraction coefficients, presented in the following section.

## 5 NUMERICAL RESULTS AND VALIDATION

In this section, the “acoustic limit” of the spectral functions code is taken by setting the longitudinal wave velocities to satisfy  $c_L \gg c_T$  in order to simulate the diffraction of an acoustic wave.

We consider two examples: first a longitudinal wave normally incident on a wedge of angle  $\varphi = 110^\circ$  (Fig. 4a) and secondly a longitudinal wave incident with an angle  $\theta_{\text{inc}} = 240^\circ$  on a wedge of angle  $\varphi = 280^\circ$  (Fig. 4b).

The longitudinal diffraction coefficient is computed for 200 observation angles using the spectral functions method (8) and compared to the expression of the far-field Geometrical Theory of Diffraction (GTD) coefficient of the scattering of a plane acoustic wave with a soft wedge, expressed by Sommerfeld [13].

There is an excellent agreement between the results produced by the spectral functions method and by Sommerfeld’s GTD.

## 6 CONCLUSION

This communication deals with the modeling of the longitudinal waves scattered by a wedge illuminated by a longitudinal plane elastic wave. A far-field approximation of the elastic waves diffracted by a stress-free wedge can be obtained by computing a function called the diffraction coefficient. This coefficient is expressed in terms of two unknown functions  $\Sigma_1$  and  $\Sigma_2$  called the spectral functions.

A first numerical test of the developed elastodynamic code has shown that this method is valid in the limit case of an acoustic wave.

Future work on this method will deal with shear waves and include a validation of the two-dimensional diffraction of an elastic wave by a wedge for angles both lower and higher than  $\pi$ .

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