

DA1



## Numerical solver for the modelling of guided waves propagation under stress condition in the context of SHM applications.

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Guided Waves for Structural Health Monitoring – [gw4shm.eu](http://gw4shm.eu)

### Introduction

Context

- **Context:**
  - Modeling and Simulation for NDT, in particular SHM.
  - SHM: Environmental and Operational conditions (EOCs). Gorgin et al. 2020
- **Proposed model and computation sequence:**
  - Propagation of ultrasonic waves on thin structures subjected to deformation.
  - Compute prestresses and deformations caused by quasi-static external forces.
  - Simulate wave propagation in the deformed structure.
- **Arbitrary hyperelastic laws and structural loading forces (under non-buckling conditions).**
- **The aerospace and oil industry are potential areas of application.**

## Slide 1

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### DA1 Notes:

- Pass sometime explaining (why, what, how) you are presenting.
- Always put references
- Do one repetition
- Be sure that the content will be explainable

DALMORA Andre; 05/07/2021



Test stand of 4-point bending tests at IMA Dresden GmbH within the project QuantSHM (funding code 100207022) which was funded by the federal state of Saxony via the Sächsische Aufbaubank. (Image provided by Fraunhofer IKTS, a partner in the GW4SHM project.)

Tschöke et al. 2017

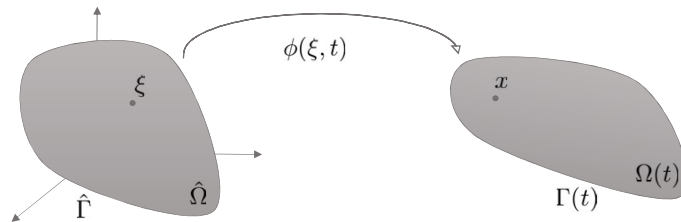
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- The pipe is subjected to a force of 220kN which oscillates up to 2Hz.
- A weld with a defect is located between the transducer rings.
- The effect of prestresses significantly reduces the sensitivity of defects detection.

#### Equation of Motion

$$\begin{cases} \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} - \nabla_{\mathbf{x}} \cdot \boldsymbol{\sigma} = \rho \mathbf{f} & \text{on } \Omega(t), \\ \boldsymbol{\sigma} \cdot \mathbf{n} = 0 & \text{on } \Gamma^N, \\ \mathbf{u} = 0 & \text{on } \Gamma^D, \end{cases}$$

$$\mathbf{u}(\boldsymbol{\xi}, t) = \mathbf{x}(\boldsymbol{\xi}, t) - \boldsymbol{\xi}$$



#### Weak formulation

$$\forall w \in \mathcal{V}(\hat{\Omega}), \quad \mathcal{V}(\hat{\Omega}) = \{v \in H^1(\hat{\Omega})^3 \mid v = 0 \text{ on } \Gamma^D\},$$

Find  $u \in \mathcal{V}(\hat{\Omega})$ , such that,  $\forall w \in \mathcal{V}(\hat{\Omega})$ ,

$$\frac{d^2}{dt^2} m[u, w] + a(u)[w] = l[w]$$

$$\begin{cases} m[u, w] = \int_{\hat{\Omega}} \hat{\rho} \mathbf{u} \cdot \mathbf{w} \, d\hat{\Omega}, \\ a(u)[w] = \int_{\hat{\Omega}} \boldsymbol{\Sigma}(\mathbf{e}) : D_{\mathbf{u}} \mathbf{e}(\mathbf{u})[w] \, d\hat{\Omega}, \\ l[w] = \int_{\hat{\Omega}} \hat{\rho} \mathbf{f} \cdot \mathbf{w} \, d\hat{\Omega}. \end{cases}$$

$$\begin{cases} J = \det \mathbf{F}, \\ \mathbf{F} = \nabla_{\boldsymbol{\xi}} \phi, \\ \boldsymbol{\Sigma}(\mathbf{e}) = J \mathbf{F}^{-1} \boldsymbol{\sigma} \mathbf{F}^{-T}. \end{cases}$$

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**cea Inria** Modelling prestressed wave propagation  
Decomposition of the problem

For  $\delta \ll 1$ , we look for

$$\mathbf{u}(\xi, t) = \mathbf{u}_0(\xi) + \delta \tilde{\mathbf{u}}(\xi, t) + O(\delta^2),$$

neglecting the terms of second order.

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**cea Inria** Modelling prestressed wave propagation  
Decomposition of the problem

**Weak formulation**

$$\frac{d^2}{dt^2} m[u, w] + a(u)[w] = l[w]$$

$$\left\{ \begin{array}{l} m[u, w] = \int_{\hat{\Omega}} \hat{\rho} \mathbf{u} \cdot \mathbf{w} \, d\hat{\Omega}, \\ a(u)[w] = \int_{\hat{\Omega}} \boldsymbol{\Sigma}(\mathbf{e}) : \mathbf{D}_u \mathbf{e}(\mathbf{u})[w] \, d\hat{\Omega}, \\ l[w] = \int_{\hat{\Omega}} \hat{\rho} \mathbf{f} \cdot \mathbf{w} \, d\hat{\Omega}. \end{array} \right.$$

**Structural deformation: Nonlinear and static.**

Find  $u_0 \in \mathcal{V}(\hat{\Omega})$ , such that,  $\forall w \in \mathcal{V}(\hat{\Omega})$ ,

$$a(u_0)[w] = l_0(w),$$

$$\left\{ \begin{array}{l} a(u_0)[w] = \int_{\hat{\Omega}} \boldsymbol{\Sigma}(\mathbf{e}_0) : \mathbf{D}_u \mathbf{e}(\mathbf{u})[w] \, d\hat{\Omega} \\ l_0[w] = \int_{\hat{\Omega}} \hat{\rho} \mathbf{f}_0 \cdot \mathbf{w} \, d\hat{\Omega}. \end{array} \right.$$

**Linearization**

$$\mathbf{f}(\xi, t) = \mathbf{f}_0(\xi) + \delta \tilde{\mathbf{f}}(\xi, t)$$

$$\mathbf{u}(\xi, t) = \mathbf{u}_0(\xi) + \delta \tilde{\mathbf{u}}(\xi, t) + O(\delta^2)$$

+ separating terms in  $\delta$ .

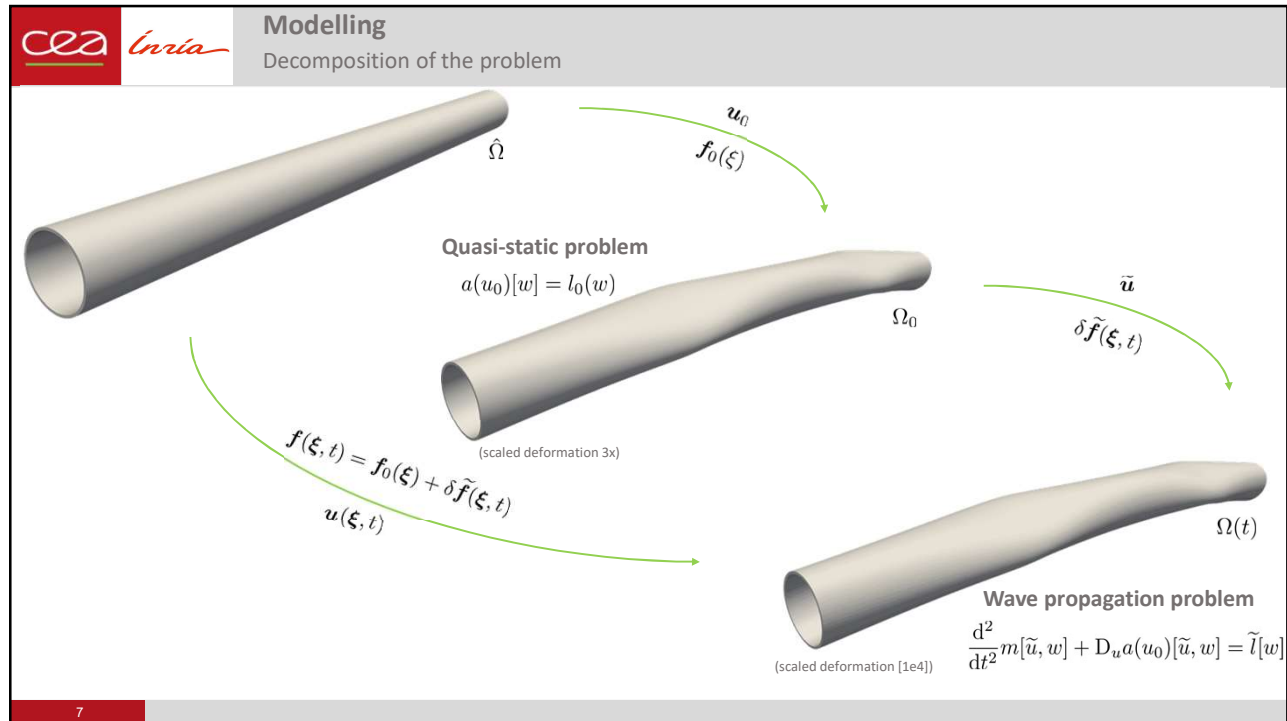
**Wave propagation: Linearized and high frequency.**

Find  $\tilde{u}(t) \in \mathcal{V}(\hat{\Omega})$ ,  $\forall t > 0$ , such that,  $\forall w \in \mathcal{V}(\hat{\Omega})$ ,

$$\frac{d^2}{dt^2} m[\tilde{u}, w] + D_u a(u_0)[\tilde{u}, w] = \tilde{l}[w]$$

$$\left\{ \begin{array}{l} m[\tilde{u}, w] = \int_{\hat{\Omega}} \hat{\rho} \tilde{\mathbf{u}} \cdot \mathbf{w} \, d\hat{\Omega}, \\ D_u a(u_0)[\tilde{u}, w] = \int_{\hat{\Omega}} \mathbf{D}_u \mathbf{e}(\mathbf{u}_0)[\tilde{u}] : \mathbf{D}_e \boldsymbol{\Sigma}(\mathbf{e}_0) : \mathbf{D}_u \mathbf{e}(\mathbf{u}_0)[w] \, d\hat{\Omega} + \int_{\hat{\Omega}} \boldsymbol{\Sigma}(\mathbf{e}_0) : \mathbf{D}_u^2 \mathbf{e}[\tilde{u}, w] \, d\hat{\Omega}, \\ \tilde{l}[w] = \int_{\hat{\Omega}} \hat{\rho} \tilde{\mathbf{f}} \cdot \mathbf{w} \, d\hat{\Omega}. \end{array} \right.$$

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**cea Inria** Modelling prestressed wave propagation  
Choice of constitutive law

**Weak formulation**

$$\frac{d^2}{dt^2} m[u, w] + a(u)[w] = l[w]$$

$$\begin{cases} m[u, w] = \int_{\hat{\Omega}} \hat{\rho} \mathbf{u} \cdot \mathbf{w} \, d\hat{\Omega}, \\ a(u)[w] = \int_{\hat{\Omega}} \boldsymbol{\Sigma}(\mathbf{e}) : \mathbf{D}_u \mathbf{e}(\mathbf{u})[w] \, d\hat{\Omega}, \\ l[w] = \int_{\hat{\Omega}} \hat{\rho} \mathbf{f} \cdot \mathbf{w} \, d\hat{\Omega}. \end{cases}$$

**Hyperelastic law**

Current stress state only depends on the current deformation and the work done is independent of the deformation path. It exists an energy density

$$W = W(\mathbf{e}) \quad \boldsymbol{\Sigma}(\mathbf{e}) = \frac{\partial W(\mathbf{e})}{\partial \mathbf{e}}$$

that in isotropic cases is dependent only on three invariants of E:

$$W = W(I_1, I_2, I_3)$$

The Murnaghan's constitutive law is often used for acoustoelasticity.

Murnaghan. 1937

Hughes and Kelly. 1953


Toupin and Bernstein. 1961

Pao and Gamer. 1985

Strain-energy potential:

$$W^{MUR}(\lambda, \mu, l, m, n) = \frac{\lambda + 2\mu}{8} (I_1 - 3)^2 - \frac{\mu}{2} (3 - 2I_1 + I_2) + \frac{l + 2m}{24} (I_1 - 3)^3 - \frac{m}{4} (9I_1 - 3I_2 - 2I_1^2 + I_1 I_2 - 9) - \frac{n}{8} (I_1 - I_2 + I_3 - 1).$$

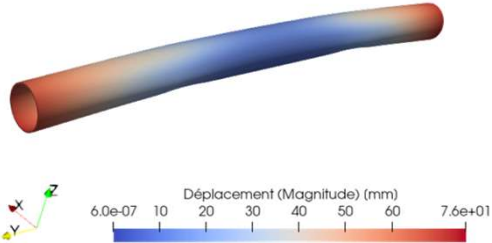
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## Numerical Illustration

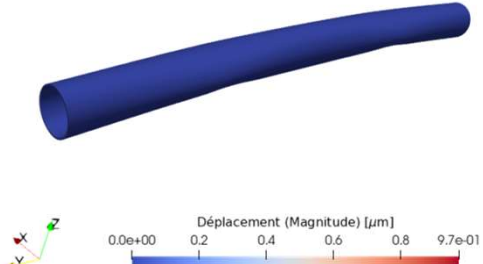
Application

### Prestresses computation




- Dimensions: 2.94m,  $\varnothing$  0.1973m and 8mm thick.
- 3D Shell Finite Elements. Chapelle and Bathe 2011
  - Static; 0.3 MDoFs; 4.7GB
- Max deformation: 76mm.
- MoReFEM code (MEDISIM - Inria team)
  - <https://gitlab.inria.fr/MoReFEM>

### Wave propagation



- Time domain (Explicit Leap Frog).
- Excitation at 80kHz.
- 3D High Order Spectral FE Cohen, Gary C. 2002
  - 12 MDoFs; 5100 timesteps; 20 minutes; 2.8GB
- SFEM code CIVA SHM (CEA)
  - Imperiale and Demaldent 2019 Carrascal-Manzanares et al. 2017

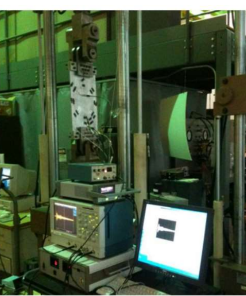
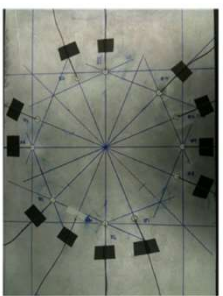
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## Validation

Experimental data from the literature

### Experimental setup

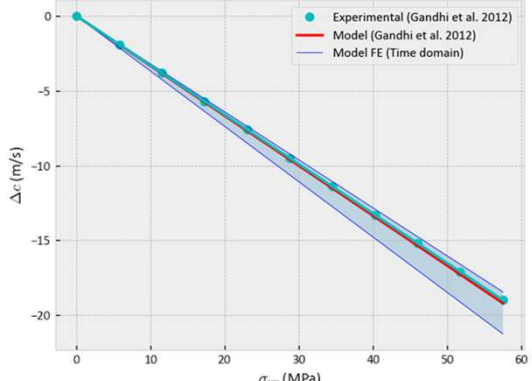
Experimental configuration for measuring velocity changes due to prestresses.

- Frequency: 250kHz (S0 mode)
- 610 x 305 x 6.35 mm aluminum plate
- Axial loading from 0MPa to 57.5MPa

Gandhi et al. 2012

### Comparing Models

Differences on the loading axis (fitted)



Velocity change (S0 mode) due to prestresses with fitted parameters.

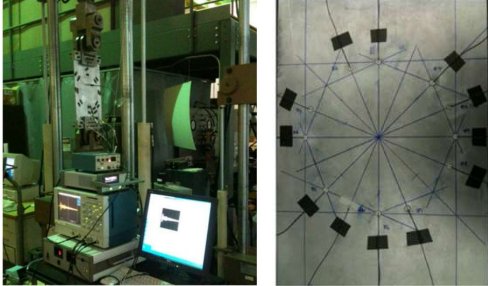
- Isotropic material: Murnaghan ( $\lambda$ ,  $\mu$ ,  $l$ ,  $m$  and  $n$ )

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## Validation

Experimental data from the literature

## Experimental setup

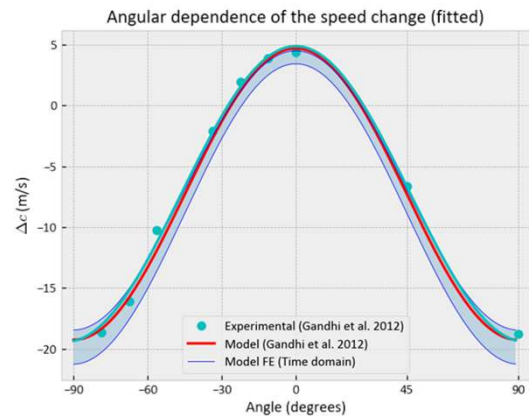


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## Comparing Models



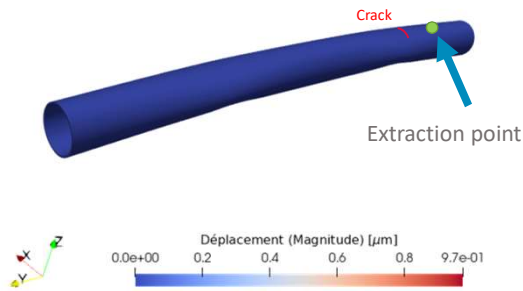
Velocity change (S0 mode) due to prestresses (57.5MPa) relative to the angle of propagation with fitted material parameters.

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## Numerical Illustration

Evaluating the effect of prestresses

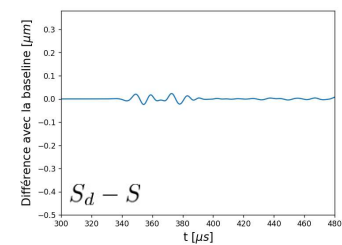
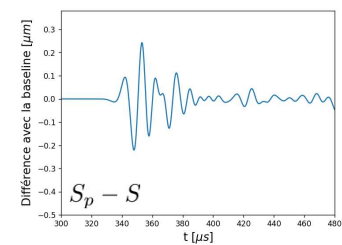
## Wave propagation



The effect of prestresses on wave propagation is evaluated numerically in comparison with the effect of a defect. Considering the extracted simulated signals

- $S$ : non deformed pipe (baseline; no defects).
- $S_p$ : deformed pipe (without defect).
- $S_d$ : pipe with defect (without deformation).

## Effect of the defect vs prestresses



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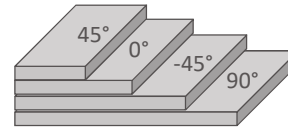
**Model**

Using a transversely isotropic hyperelastic law for each layer it is possible to model stratified composite materials such as CFRP.

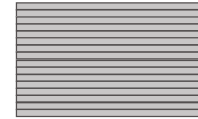
Bonet, J, and AJ Burton. 1998.

**16-ply quasi-isotropic CFRP plate**

Dimensions:  
900 x 300 x 2 [mm]



[45°, 0°, -45°, 90°]



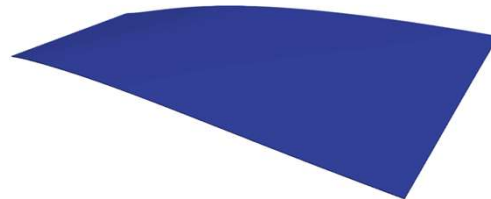
[45°, 0°, -45°, 90°]<sub>25</sub>

**Hyperelastic potential:**

$$W(\{I_i\}_{i=1}^5) = \frac{\mu}{2}(I_1 - 3) - \mu \log \sqrt{I_3} + \frac{\lambda}{2}(\sqrt{I_3} - 2)^2 \\ + [\alpha + \beta \log \sqrt{I_3} + \gamma(I_4 - 1)](I_4 - 1) \\ - \frac{\alpha}{2}(I_5 - 1)$$

$$I_4 = \mathbf{a} \cdot \mathbf{C} \cdot \mathbf{a} \quad I_5 = \mathbf{a} \cdot \mathbf{C}^T \mathbf{C} \cdot \mathbf{a} \quad (\mathbf{a} \text{ is the fiber direction for each layer})$$

Guided wave propagation in CFRP plate under torsion.



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**Conclusions**

- A tool for the propagation of waves in a prestressed thin structure is proposed for arbitrary hyperelastic constitutive laws and for arbitrary deformations (under conditions of non-buckling).
- Consistent results in comparison with experimental results and model for aluminium. [Gandhi et al. 2012](#)
- Transversely isotropic laws can be used to model composites and stratified materials such as CFRP.
- Evaluation of the effect of prestresses can be done with the proposed tool.

**Perspectives**

- Continue the validation (for isotropic and transversely isotropic cases).  
[Abderahmane et al. 2021](#) [Shi et al. 2013](#) [Muir et al. 2009](#)
- Prestresses reconstruction by using this direct model with Kalman filtering.
- Potential integration into the simulation CIVA platform for the NDT.

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Abderahmane, Abdellahi, Alain Lhémy, and Laurent Daniel. 2021. "Effects of Multiaxial Pre-Stress on Lamb and Shear Horizontal Guided Waves." *The Journal of the Acoustical Society of America* 149 (3): 1724–36.

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Murnaghan, F. D. 1937. "Finite Deformations of an Elastic Solid." *American Journal of Mathematics* 59 (2): 235–60.

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Toupin, R. A., and B. Bernstein. 1961. "Sound Waves in Deformed Perfectly Elastic Materials. Acoustoelastic Effect." *The Journal of the Acoustical Society of America* 33 (2): 216–25.

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