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DA1



Estimating the effect of operational loading conditions from ultrasonic guided wave measurements using an iterated Unscented Kalman Filter

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Guided Waves for Structural Health Monitoring – gw4shm.eu



Context

- **Context:**
 - Modeling and Simulation for NDT, in particular SHM.
 - SHM: Environmental and Operational conditions (EOCs).
Mechanical loading conditions *in situ*. [Gorgin et al. 2020](#)
- **Colloque GdR MecaWave 2021 (direct model):**
Combining shell elements and transient spectral finite elements for guided wave propagation problems in prestressed thin structures.
- **Objective:**
Propose an inverse strategy to reconstruct loading conditions using ultrasonic guided waves measurements.
 - Direct model: Linearized elastodynamic model solved using 3D Spectral Finite Elements. [Dalmora et al. 2022](#)
 - Inversion: Iterated Unscented Kalman Filtering.

Slide 1

DA1 Notes:

- Pass sometime explaining (why, what, how) you are presenting.
- Always put references
- Do one repetition
- Be sure that the content will be explainable

DALMORA Andre; 05/07/2021



- The pipe is subjected to a force of 220kN which oscillates up to 2Hz.
- A weld with a defect is located between the transducer rings.
- The effect of prestresses significantly reduced the probability of detection of the defect.

Test stand of 4-point bending tests at IMA Dresden GmbH within the project QuantSHM (funding code 100207022) which was funded by the federal state of Saxony via the Sächsische Aufbaubank. (Image provided by Fraunhofer IKTS, a partner in the GW4SHM project.)

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Input



Mechanical loading
causing deformation
+ Ultrasonic Source



Measurements by sparse
ultrasonic transducers

Direct Model

Output



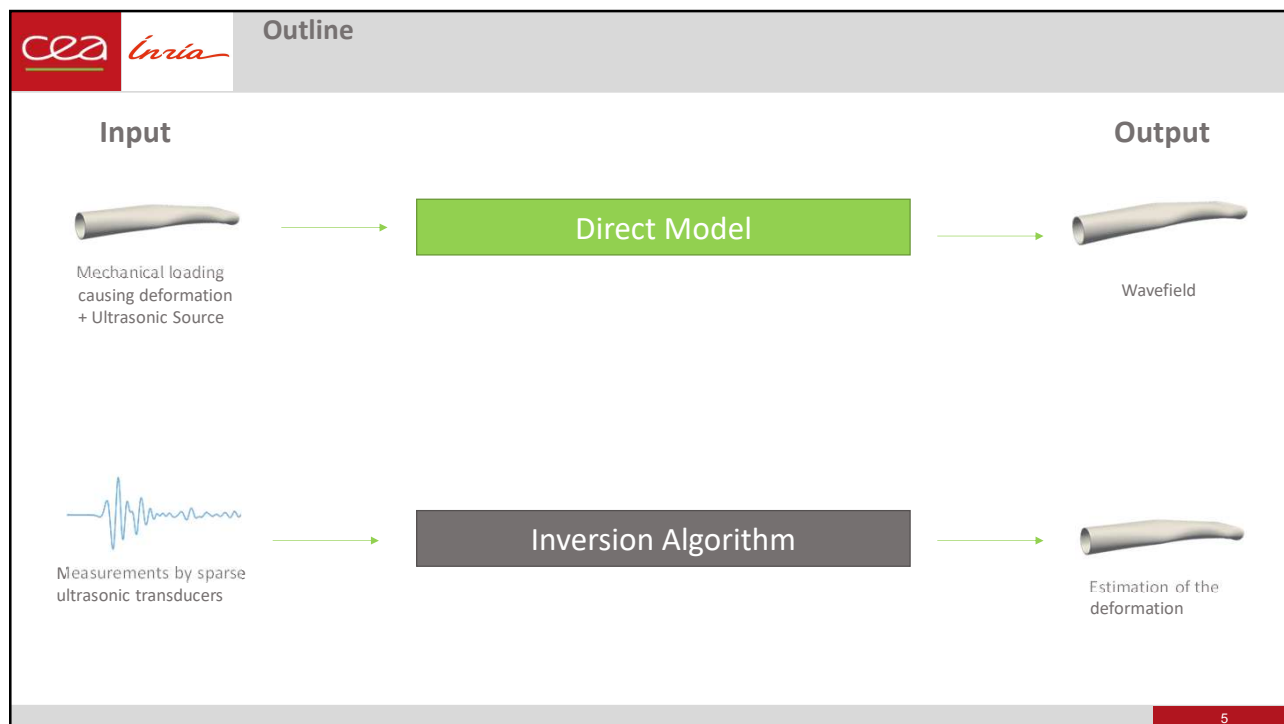
Wavefield

Inversion Algorithm



Estimation of the
deformation

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cea Inria **Direct model**
Mechanical and Numerical modeling

Starting from the nonlinear elastodynamics in its strong formulation

$$\begin{cases} \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} - \nabla_{\mathbf{x}} \cdot \boldsymbol{\sigma} = \rho \mathbf{f} & \text{on } \Omega(t), \\ \boldsymbol{\sigma} = \mathbf{G}(\mathbf{e}) & \text{(hyperelastic),} \\ \mathbf{u} = 0 & \text{on } \Gamma^D, \end{cases} \quad \text{with} \quad \mathbf{F} = \nabla_{\xi} \phi, \quad \mathbf{C} = \mathbf{F}^T \mathbf{F}, \quad \mathbf{e}(\mathbf{u}) = \frac{1}{2}(\mathbf{C} - \mathbf{I}).$$

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cea Inria Direct model

Method: Knowing that the associated forces differ in their natures, we decompose the problem in two sub-problems

For $\delta \ll 1$, we look for

$$\mathbf{u}(\xi, t) = \mathbf{u}_0(\xi) + \delta \tilde{\mathbf{u}}(\xi, t) + O(\delta^2),$$

neglecting the terms of second order.

scaled deformation (1e4)

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cea Inria Modelling loaded wave propagation
Decomposition and numerical solving

Weak formulation $\forall w \in \mathcal{V}(\hat{\Omega})$, • **Structural deformation: Nonlinear and static.**

$$\frac{d^2}{dt^2} m[u, w] + a(u)[w] = l[w]$$

$$\begin{cases} m[u, w] = \int_{\hat{\Omega}} \hat{\rho} \mathbf{u} \cdot \mathbf{w} d\hat{\Omega}, \\ a(u)[w] = \int_{\hat{\Omega}} \boldsymbol{\Sigma}(e) : \mathbf{D}_u \mathbf{e}(u)[w] d\hat{\Omega}, \\ l[w] = \int_{\hat{\Omega}} \hat{\rho} \mathbf{f} \cdot \mathbf{w} d\hat{\Omega}. \end{cases}$$

+

Linearization

$$\mathbf{f}(\xi, t) = \mathbf{f}_0(\xi) + \delta \tilde{\mathbf{f}}(\xi, t)$$

$$\mathbf{u}(\xi, t) = \mathbf{u}_0(\xi) + \delta \tilde{\mathbf{u}}(\xi, t) + O(\delta^2)$$

+ separating terms in δ

• **Wave propagation: Linearized and high frequency.**

$$\frac{d^2}{dt^2} m[\tilde{u}, w] + \mathbf{D}_u a(u_0)[\tilde{u}, w] = \tilde{l}[w]$$

• 3D Shell Finite Elements, avoiding numerical locking in thin structures. Chapelle and Bathe 2011

• MoReFEM code (MEDISIM - Inria team)

• Time domain (Explicit Leap Frog).
• SFEM kernel CIVA(CEA)

Maday and Patera. 1989 ; Cohen 2002

Operators

$$\begin{aligned} m[\tilde{u}, w] &= \int_{\hat{\Omega}} \hat{\rho} \tilde{\mathbf{u}} \cdot \mathbf{w} d\hat{\Omega}, \\ \mathbf{D}_u a(u_0)[\tilde{u}, w] &= \int_{\hat{\Omega}} \mathbf{D}_u \mathbf{e}(u_0)[\tilde{u}] : \mathbf{D}_e \boldsymbol{\Sigma}(e_0) : \mathbf{D}_u \mathbf{e}(u_0)[w] d\hat{\Omega} + \int_{\hat{\Omega}} \boldsymbol{\Sigma}(e_0) : \mathbf{D}_e^2 \mathbf{e}[\tilde{u}, w] d\hat{\Omega}, \\ l_0[w] &= \int_{\hat{\Omega}} \hat{\rho} \mathbf{f}_0 \cdot \mathbf{w} d\hat{\Omega}, \\ \tilde{l}[w] &= \int_{\hat{\Omega}} \hat{\rho} \tilde{\mathbf{f}} \cdot \mathbf{w} d\hat{\Omega}. \end{aligned}$$

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Remark on implementation $\forall w \in \mathcal{V}(\hat{\Omega}), \quad \frac{d^2}{dt^2} m[\tilde{u}, w] + D_u a(u_0)[\tilde{u}, w] = \tilde{l}[w]$

Discretizing by conform FE we have $\frac{d^2}{dt^2} \mathbb{M} \vec{u} + \mathbb{K}'(u_0) \vec{u} = \vec{f}$

Using a leapfrog time scheme $\vec{u}^{n+1} = -\Delta t^2 \mathbb{M}^{-1} \mathbb{K}'(u_0) \vec{u}^n + \Delta t^2 \mathbb{M}^{-1} \vec{f} + 2\vec{u}^n - \vec{u}^{n-1},$

Lumped mass

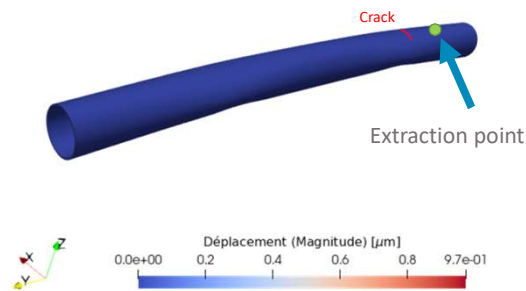
Parallelized non-assembled operation

Important aspects:

- Solving for **arbitrary hyperelastic constitutive law, geometry and mechanical loading**.
- High efficiency due to lumped-mass and parallelization.
- Non-assembled stiffness: low memory usage and inexpensive “parameter” update.
- Stability: operator non-negativeness must be assured.
- Strategy **validated** with experimental data.

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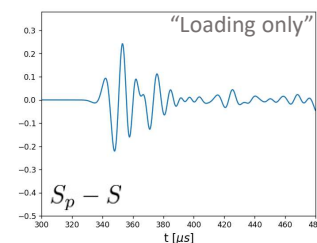
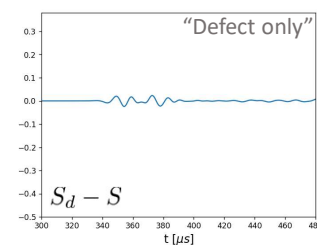
Wave propagation



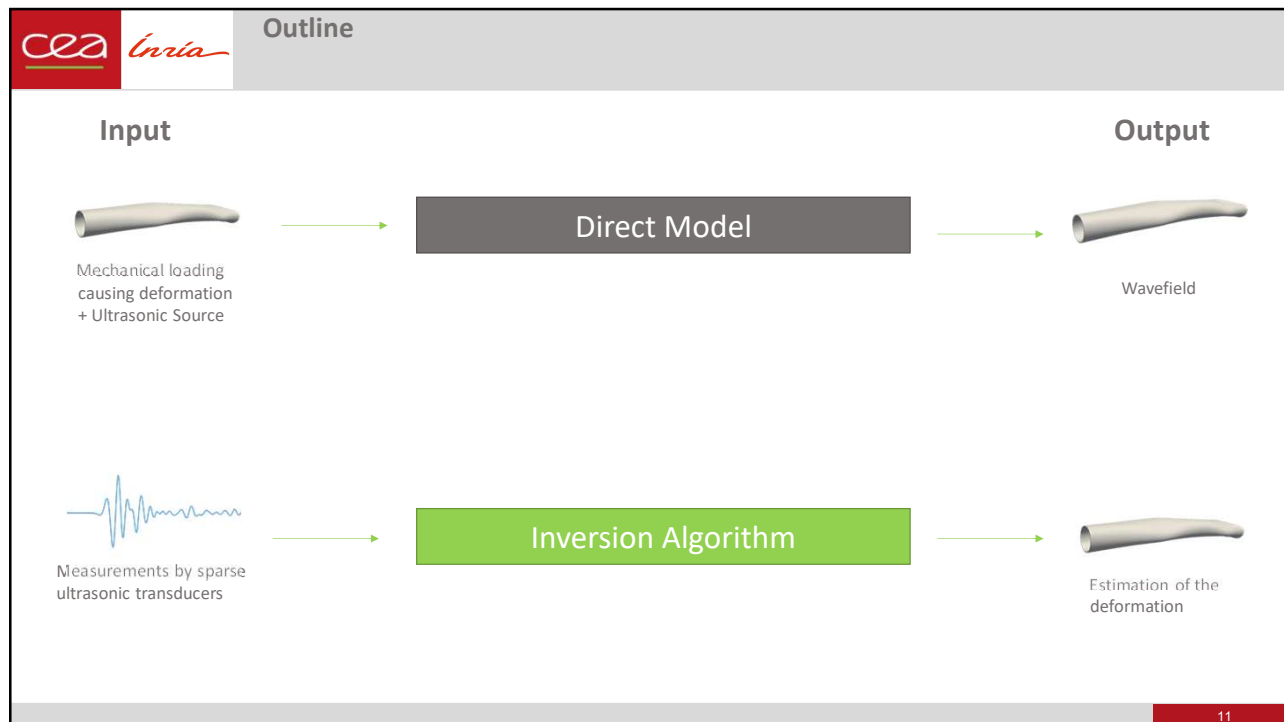
The effect of loading on wave propagation is evaluated numerically and compared with the effect of a defect. Considering the extracted simulated signals

- S : non deformed pipe (baseline; no defects).
- S_p : deformed pipe (without defect).
- S_d : pipe with defect (without deformation).

Effect of the defect vs prestresses



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cea Inria **Loading condition reconstruction**

Reconstructing loading deformation by means of limited ultrasonic measurements.

Direct model: $\frac{d^2}{dt^2} m[\tilde{u}, w] + D_u a(u_0)[\tilde{u}, w] = \tilde{l}(w)$

Objective: reconstruct u_0 using limited information over \tilde{u} .

We interpret the inverse problem as minimizing a least-squares misfit between measurements and data generated from the model.

Functional:

$$\mathcal{J}(u_0) = \min_{u_0} \frac{1}{2} \left\{ \|u_0\|_{\mathbb{P}_{\nu_0}^{-1}}^2 + \frac{1}{N_s} \sum_i \int_0^T \|y(t) - C_i(\tilde{u}|_{u_0}(t))\|_{\mathbb{S}}^2 dt \right\}.$$

number of measured signals N_s

measurements $y(t)$

Regularization norm $\|u_0\|_{\mathbb{P}_{\nu_0}^{-1}}^2$

observation operator $C_i(\tilde{u}|_{u_0}(t))$

wavefield generated for the deformation u_0 $\tilde{u}|_{u_0}(t)$

It represents a nonlinear minimization problem.

Diagram illustrating the reconstruction process:

- Initial state: $\hat{\Omega}$ (undeformed tube) with coordinate ξ .
- Deformation: u_0 (loading condition) leads to a deformed state Ω_0 with coordinate X .
- Wavefield: \tilde{u} (wavefield) is generated for the deformation u_0 , resulting in a wavefield $\Omega(t)$ with coordinate $x(t)$.

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Loading condition reconstruction

Remark on the dependency of direct problem

Direct model: $\frac{d^2}{dt^2} m[\tilde{u}, w] + D_u a(u_0)[\tilde{u}, w] = \tilde{l}[w]$

Dependency on u_0 (assuming know constitutive law and parameters):

$$D_u a(u_0)[\tilde{u}, w] = \int_{\Omega} \underbrace{D_u e(u_0)[\tilde{u}] : D_e \Sigma(e_0) : D_u e(u_0)[w]}_{\text{"}\varepsilon(\tilde{u}) : C : \varepsilon(w)\text{"}} d\Omega + \int_{\Omega} \underbrace{\Sigma(e_0) : D_u^2 e[\tilde{u}, w]}_{\text{geometrical nonlinearity}} d\Omega$$

Through the strain tensor and the stress tensor (for an arbitrary hyperelastic law):

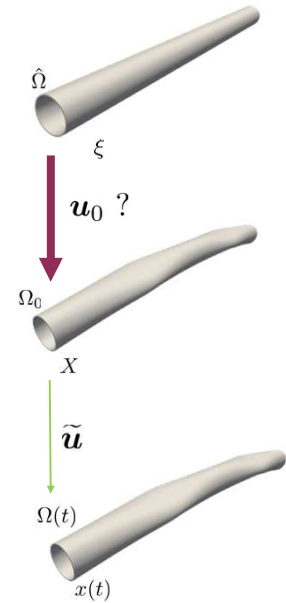
$$D_e \Sigma = D_e^2 W = 4 \sum_{i=1}^n \frac{\partial W}{\partial I_i} \frac{\partial^2 I_i}{\partial C \partial C} + 4 \sum_{i,j=1}^n \frac{\partial^2 W}{\partial I_i \partial I_j} \frac{\partial I_i}{\partial C} \otimes \frac{\partial I_j}{\partial C}$$

Descent methods such as Full Waveform Inversion requires the tangent dynamics for computing an adjoint model. [Virieux et al. 2014](#)

Further differentiation of our direct model would require unwieldy operators as

$$D_e^3 W = 8 \sum_{i,j,k=1}^n \left[\frac{\partial^3 W}{\partial I_i \partial I_j \partial I_k} \frac{\partial I_i}{\partial C} \otimes \frac{\partial I_j}{\partial C} \otimes \frac{\partial I_k}{\partial C} \right] + 8 \sum_{i,j,k=1}^n \frac{\partial^2 W}{\partial I_i \partial I_j} \left[\frac{\partial^2 I_i}{\partial C \partial C} \otimes \frac{\partial I_j}{\partial C} + \dots \right].$$

Derivative-free methods are preferred.



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Loading condition reconstruction

Remark on the parametric space to be reconstructed

Direct model: $\frac{d^2}{dt^2} m[\tilde{u}, w] + D_u a(u_0)[\tilde{u}, w] = \tilde{l}[w]$

The deformation sought u_0 is the result of a mechanical loading / solution of a quasi-static nonlinear problem

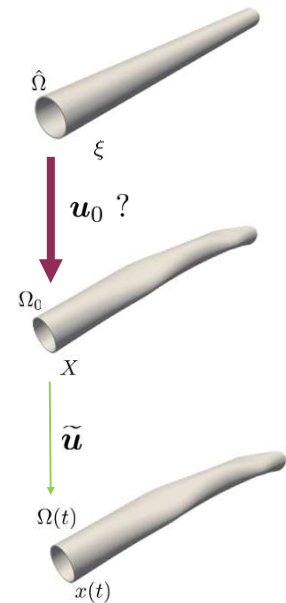
$$a(u_0)[w] = l_0(w).$$

Decomposing the deformation with the eigenmodes of $a(0)$

$$u_0(\xi) = \sum_{i=1}^{N_\theta} u_{i,0} \Psi_i(\xi)$$

we can significantly reduce the size of the parametric space to be reconstructed by reconstructing the coefficients $u_{i,0}^{(i)}$ with $N_\theta \ll \text{Number of DoFs}$.

The parametric space is reduced and a selection of eigenmodes can be made from a first guess or sensitivity analysis.



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Variational Method

$$\begin{cases} \frac{d}{dt}p(t) + D_u A^* p(t) = -D_u C^*(y(t) - C(\tilde{u}(t))), \\ p(T) = 0, \end{cases}$$

$$D\mathcal{J}(u_0) = \mathbb{P}_{u_0}^{-1} u_0 - p(0).$$

Sequential Method

$$\begin{cases} \frac{d}{dt}\hat{z}(t) = A(\hat{z}(t)) + G(t)(y(t) - C(\tilde{u}(t))), \\ \hat{z}(0) = z_0, \end{cases}$$

$$z(T)$$

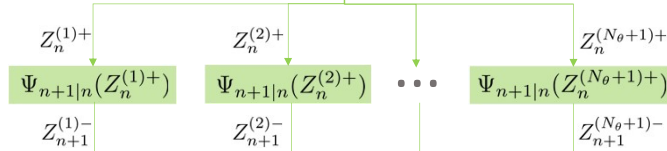
- With a reduced parametric space, the **Reduced Order Unscented Kalman Filter (ROUKF)** was chosen as the derivative-free method.
- Seem as a derivative-free version of the Extended Kalman Filter, where the propagation of the covariance is done empirically instead of applying the tangent dynamics.

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Sampling:

$$Z_n^{(i)+} = Z_n^+ + L_n \sqrt{U_n^{-1}} I^{(i)} \quad i \in [1, N_\theta + 1]$$

Prediction:



Correction:

$$\begin{aligned} Z_{n+1}^- &= E_\alpha(Z_{n+1}^{(i)-}) \\ L_{n+1} &= [Z_{n+1}^{*-}] D_\alpha [I^*]^\top \\ U_{n+1} &= \mathbb{1} + (CL)_{n+1}^* (CL)_{n+1} \\ Z_{n+1}^+ &= Z_{n+1}^- \\ &\quad + L_{n+1} U_{n+1}^{-1} (CL)_{n+1}^* (y_{n+1} - CE_\alpha(Z_{n+1}^{*+})) \end{aligned}$$

Extended state:

$$Z_n = \begin{pmatrix} \tilde{u}_n \\ \tilde{v}_n \\ \vec{u}_{0,n} \end{pmatrix}$$

Forward operator:

$$\Psi_{n+1|n}(Z_n)$$

Covariance:

$$U^{-1}$$

Particles matrix:

$$[Z_n^*] = \begin{pmatrix} Z_n^{(1)} & Z_n^{(2)} & \dots & Z_n^{(N_\theta+1)} \end{pmatrix}$$

Sigma points and weights matrix:

$$[I^*] \quad D_\alpha = \begin{pmatrix} \alpha_1 & & & \\ & \ddots & & \\ & & \ddots & \\ & & & \alpha_{N_\theta+1} \end{pmatrix}$$

Moireau and Chapelle 2011

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Reduced-Order Unscented Kalman Filter

Sampling:

$$Z_n^{(i)+} = Z_n^+ + L_n \sqrt{U_n^{-1}} I^{(i)} \quad i \in [1, N_\theta + 1]$$

Prediction:

$Z_n^{(1)+}$
 $\Psi_{n+1|n}(Z_n^{(1)+})$
 $Z_{n+1}^{(1)-}$

$Z_n^{(2)+}$
 $\Psi_{n+1|n}(Z_n^{(2)+})$
 $Z_{n+1}^{(2)-}$

\dots

$Z_n^{(N_\theta+1)+}$
 $\Psi_{n+1|n}(Z_n^{(N_\theta+1)+})$
 $Z_{n+1}^{(N_\theta+1)-}$

Correction:

$$\begin{aligned} Z_{n+1}^- &= E_\alpha(Z_{n+1}^{(i)-}) \\ L_{n+1} &= [Z_{n+1}^{*-}] D_\alpha [I^*]^\top \\ U_{n+1} &= \mathbb{1} + (CL)_{n+1}^* (CL)_{n+1} \\ Z_{n+1}^+ &= Z_{n+1}^- \\ &\quad + L_{n+1} U_{n+1}^{-1} (CL)_{n+1}^* (y_{n+1} - CE_\alpha(Z_{n+1}^+)) \end{aligned}$$

- Uncertainty only in u_0 .
- Iteration by time-step.
- Particles are evaluated in parallel.
- Propagates the covariance.

Moireau and Chapelle 2011

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Reduced-Order Unscented Kalman Filter

Sampling:

$$Z_n^{(i)+} = Z_n^+ + L_n \sqrt{U_n^{-1}} I^{(i)} \quad i \in [1, N_\theta + 1]$$

Prediction:

$Z_n^{(1)+}$
 $\Psi_{n+1|n}(Z_n^{(1)+})$
 $Z_{n+1}^{(1)-}$

$Z_n^{(2)+}$
 $\Psi_{n+1|n}(Z_n^{(2)+})$
 $Z_{n+1}^{(2)-}$

\dots

$Z_n^{(N_\theta+1)+}$
 $\Psi_{n+1|n}(Z_n^{(N_\theta+1)+})$
 $Z_{n+1}^{(N_\theta+1)-}$

Correction:

$$\begin{aligned} Z_{n+1}^- &= E_\alpha(Z_{n+1}^{(i)-}) \\ L_{n+1} &= [Z_{n+1}^{*-}] D_\alpha [I^*]^\top \\ U_{n+1} &= \mathbb{1} + (CL)_{n+1}^* (CL)_{n+1} \\ Z_{n+1}^+ &= Z_{n+1}^- \\ &\quad + L_{n+1} U_{n+1}^{-1} (CL)_{n+1}^* (y_{n+1} - CE_\alpha(Z_{n+1}^+)) \end{aligned}$$

- Eigenmodes are stored and the non-assembled aspect allows inexpensive change of parameter.
- Attention must be given to the range explored by the particles to assure stability. This is done when choosing the initial covariance.

$$U_0 = \mathbb{P}_{u_0}$$


The initial components covariance of the set of eigenmodes are chosen from a u_0 *a priori*.

Moireau and Chapelle 2011

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Loading condition reconstruction

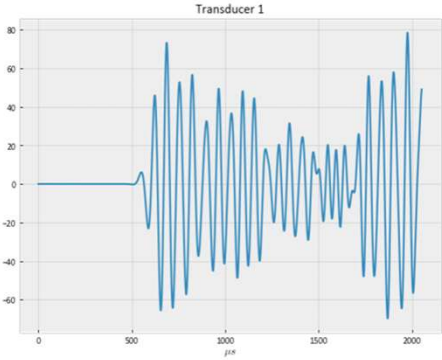
Study Cases



Transducers, natural state and target deformed state.
(300x deformation scale)

- Dimensions: 600 x 300 x 6.35 mm
- Measurements: 5 integration patches (radial).
- Excitation: 200kHz 5-cycles.
- Noise: 0.1%
- 180μs simulation time.
- 120 Smallest eigenvalues pairs computed.

Example of measured signal:




Obs. operator: $C_i(\tilde{u}(t)) = \int_{\Gamma_i} \tilde{u}(t) \cdot \mathbf{s}_i \, d\Gamma, \quad i \in [1, 5]$

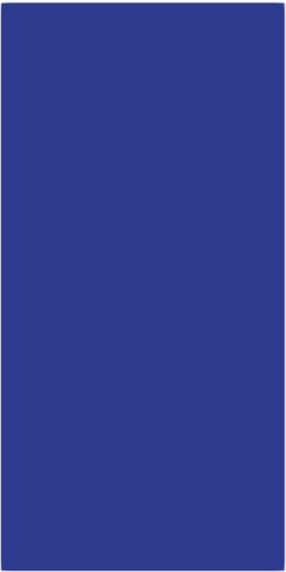
19

Loading condition reconstruction

Study Cases

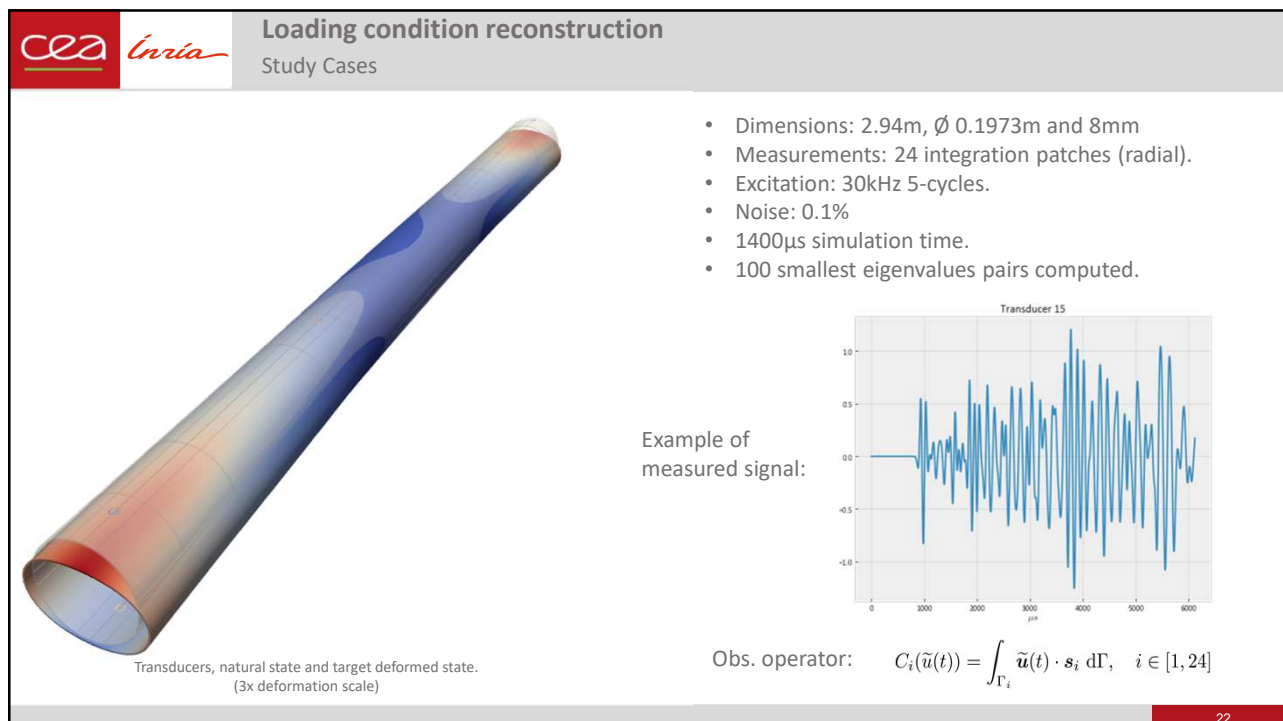
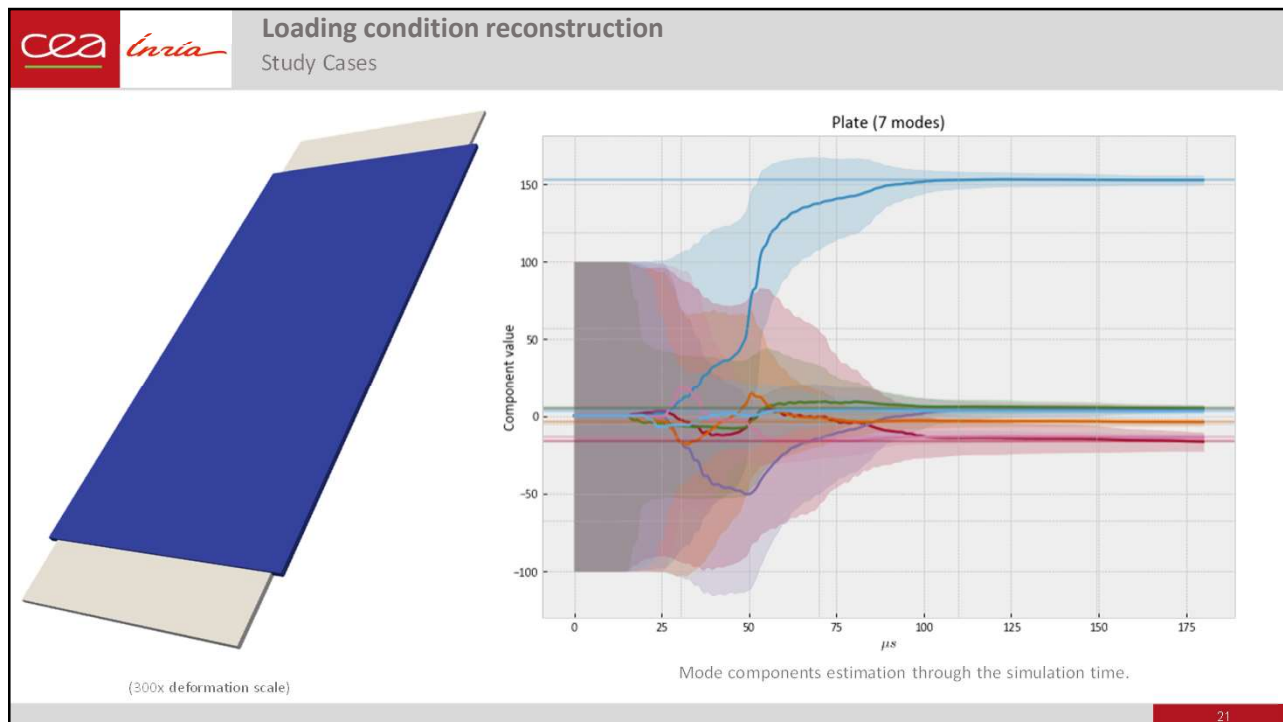


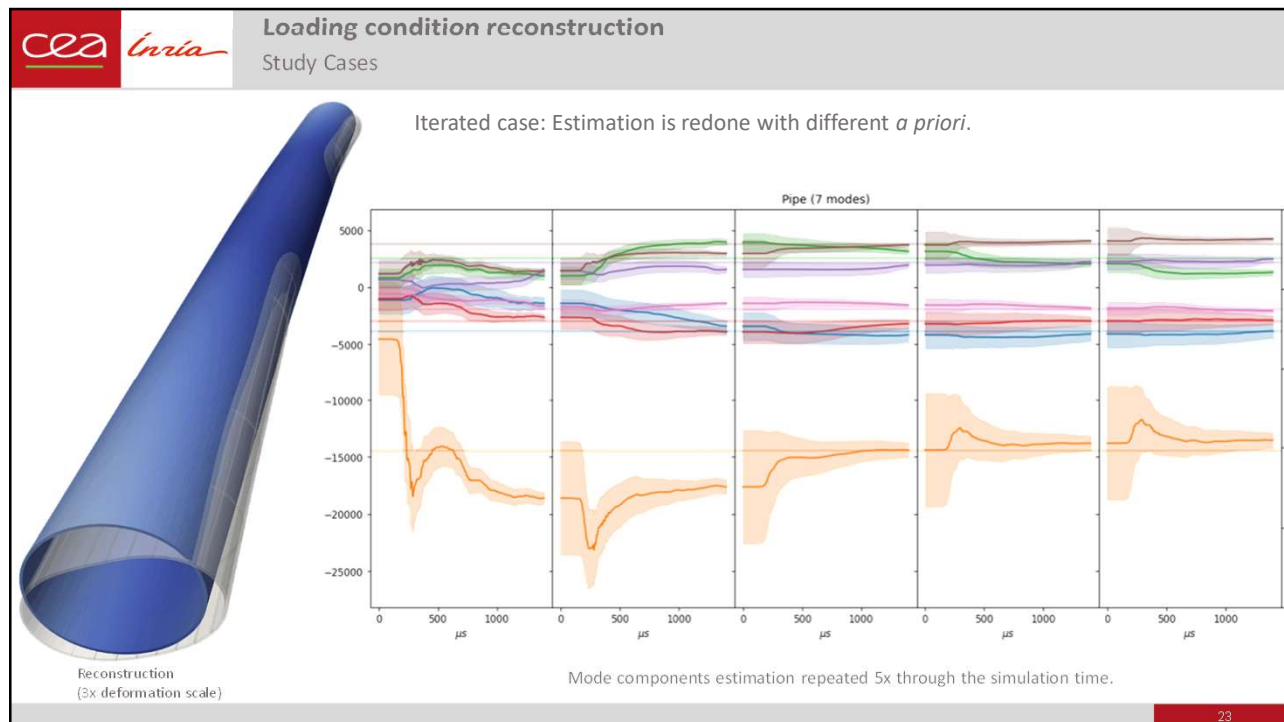
Transducers, natural state and target deformed state.
(300x deformation scale)





Target wavefield

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Conclusions and Perspectives

Conclusions

- A **mechanical and numerical model** was presented for **wave propagation on loaded structures**. It comprises arbitrary hyperelastic law and non-buckling load.
- The simulation model was **validated using experimental data**.
- Reconstruction of loading deformation** of the structure is done using the presented direct model and **Reduced order Kalman Filter (ROUKF)**.
- Reconstruction of loading deformation on realistic cases were presented.

Perspectives

- Further studies on the stability of the model and limitation of search range of estimator to **mitigate instability** during inversion.
- Mode selection: **sensitivity analysis** from estimation using observed data; different base with the *a priori*.

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Literature and Acknowledgements

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Thank you for your attention