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# Multi-trace boundary integral formulations with eddy current models

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**Abstract**—We are interested in boundary integral formulations adapted to the solution of low frequency inductive electromagnetics in the case where the geometry is partitioned in (potentially irregular) subdomains. In the context of electromagnetics in piecewise homogeneous media, the multi-trace formalism (MTF) provides boundary integral formulations for Maxwell’s equations posed at the interfaces between different media, with the unknowns associated to one medium a priori decoupled from the unknowns associated to other media. This makes MTF a comfortable paradigm for integral equation based domain decomposition.

This formalism is recent and, so far, has been studied only for electromagnetics in time harmonic regime. In this contribution, we adapt the multi-trace approach to the case where effective permittivity vanishes in certain subdomains. We study the theoretical properties of these new formulations, and discuss its numerical stability.

## I. MOTIVATIONS

Eddy current testing (ECT) is widely used by industrials for the inspection of conductive components, and simulation techniques are needed for the design of new testing procedures as well as for the evaluation of existing ones. Most often based on a finite element (FEM) calculation of the quasi-static fields, the modelling of a realistic inspection requires the introduction of specific discretization tools or the coupling to the boundary element method (FEM-BEM) for finely handling the air gap between the inspected part and the moving probe. As soon as the inspected workpiece is considered as an isotropic and homogeneous medium, it is possible to use the BEM on both sides of the interface between the air and the conductive component. For few years, the CEA LIST has been developing a BEM solution dedicated to low frequency electromagnetic testing [1], especially to model the ECT of steam generator tubes in nuclear pressurized water reactors [2]. This new contribution is the result of a recent collaboration with the JLL laboratory, and aims to extend this BEM solution to the modelling of piecewise homogeneous media with several components in contact, as illustrated in Figure 1.

## II. A PROBLEM OF LOW FREQUENCY ELECTROMAGNETICS

Consider a partition of free space  $\mathbb{R}^3 = \cup_{j=0}^n \bar{\Omega}_j$  where  $\Omega_j \cap \Omega_k = \emptyset$ , each  $\Omega_j$  being a potentially irregular subdomain that

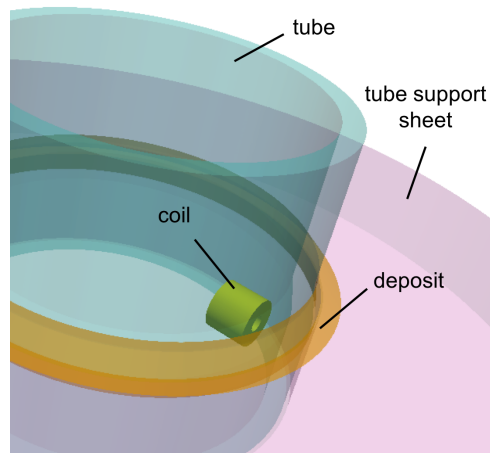


Fig. 1. Example of ECT involving several components in contact.

is bounded for  $j \neq 0$ . In this contribution, we are interested in solving the equations of time harmonic electromagnetics

$$\begin{aligned} \mathbf{curl}(\mathbf{E}) - \omega\mu\mathbf{H} &= 0 && \text{in } \mathbb{R}^3 \\ \mathbf{curl}(\mathbf{H}) + \omega\epsilon\mathbf{E} &= \mathbf{j} && \text{in } \mathbb{R}^3 \end{aligned} \quad (1)$$

+ Silver-Müller’s radiation condition

where  $\text{div}(\mathbf{j}) = 0$ , and where  $\epsilon$  (resp.  $\mu$ ) is the piecewise constant electric permittivity (resp. the magnetic permeability):  $\mu(\mathbf{x}) = \mu_j > 0$  and  $\epsilon(\mathbf{x}) = \epsilon_j$  with  $\Re\{\epsilon_j\}, \Im\{m\{\epsilon_j\}\} \geq 0$  for  $\mathbf{x} \in \Omega_j$ . Denoting

$$\kappa_j := \omega\sqrt{\mu_j\epsilon_j}$$

the effective wave number in the subdomain  $\Omega_j$ , a particular feature of the present contribution is that  $\kappa_j = 0$  in certain subdomains.

We propose a novel formulation of the above problem as a system of coupled boundary integral equations, by means of the multi-trace approach [3], [4]. To deal with subdomains where  $\epsilon$  vanishes we rely on the Hiptmair-Ostrowski eddy-current coupling scheme [5]. In what follows, we describe the bilinear form associated to our eddy-current multi-trace formulation.

### III. VARIATIONNAL SPACE AND TRANSMISSION OPERATOR

Our low-frequency multi-trace formulation is posed on the skeleton  $\Sigma := \cup_{j=0}^n \Gamma_j$  where  $\Gamma_j := \partial\Omega_j$  of the subdomain partition. Let  $\mathcal{I}_* = \{j = 0 \dots n \mid \kappa_j = 0\}$ . The variational space takes the form

$$\begin{aligned} \mathbb{H}(\Sigma) := \{ & \mathbf{u} = (\mathbf{u}_j)_{j=0}^n, \mathbf{u}_j = (\mathbf{u}_j^E, \mathbf{u}_j^H) \\ & \mathbf{u}_j^E, \mathbf{u}_j^H : \text{vector fields tangent to } \Gamma_j \\ & \text{div}_{\Gamma_j} \mathbf{u}_j^H = 0 \text{ for } j \in \mathcal{I}_*\}. \end{aligned}$$

In this variationnal setting,  $\mathbf{u}_j^E$  (resp.  $\mathbf{u}_j^H$ ) is related to the tangential trace of the electric (resp. magnetic) field at  $\Gamma_j$ . Next we introduce an operator that allows an enforcement of transmission conditions across interfaces between adjacent media. This is a local operator acting on pairs of tangent traces at the skeleton. For  $\mathbf{u}, \mathbf{v} \in \mathbb{H}(\Sigma)$  it is given by

$$\begin{aligned} \Pi(\mathbf{u}, \mathbf{v}) = & \sum_{0 \leq j < k \leq n} \int_{\Gamma_j \cap \Gamma_k} \mathbf{u}_j^E \times \mathbf{v}_k^H + \mathbf{u}_k^H \times \mathbf{v}_j^E d\sigma \\ & + \int_{\Gamma_j \cap \Gamma_k} \mathbf{u}_k^E \times \mathbf{v}_j^H + \mathbf{u}_j^H \times \mathbf{v}_k^E d\sigma \end{aligned} \quad (2)$$

### IV. INTEGRAL OPERATORS

We also need to describe the integral operators coming into play in our formulation. Consider the outgoing Green kernel associated to Helmholtz equation  $\mathcal{G}_\kappa(\mathbf{z}) := \exp(i\kappa|\mathbf{z}|)/(4\pi|\mathbf{z}|)$ , as well as  $\mathcal{B}(\mathbf{z}) := (i\kappa|\mathbf{z}| - 1)\mathbf{z}/|\mathbf{z}|^2$ , and introduce the classical bilinear forms for EFIE and MFIE at each boundary

$$\begin{aligned} \mathbf{a}_\kappa^\Gamma(\mathbf{p}, \mathbf{q}) &:= \int_{\Gamma \times \Gamma} \mathcal{G}_\kappa(\mathbf{x} - \mathbf{y}) [\mathbf{p}(\mathbf{x}) \cdot \mathbf{q}(\mathbf{y}) \\ & \quad - \kappa^{-2} \text{div}_\Gamma \mathbf{p}(\mathbf{x}) \text{div}_\Gamma \mathbf{q}(\mathbf{y})] d\sigma(\mathbf{x}, \mathbf{y}) \\ \mathbf{b}_\kappa^\Gamma(\mathbf{p}, \mathbf{q}) &:= \int_{\Gamma \times \Gamma} \mathcal{G}_\kappa(\mathbf{x} - \mathbf{y}) \\ & \quad \mathcal{B}(\mathbf{x} - \mathbf{y}) \cdot [\mathbf{p}(\mathbf{x}) \times \mathbf{q}(\mathbf{y})] d\sigma(\mathbf{x}, \mathbf{y}) \end{aligned}$$

where  $\Gamma = \Gamma_j := \partial\Omega_j, j = 0 \dots n$ . Since we are interested in the low frequency regime, we shall also resort on bilinear forms related the behaviour of  $\mathbf{a}_\kappa^\Gamma$  as  $\kappa \rightarrow 0$ . Define

$$\begin{aligned} \mathbf{a}_*^\Gamma(\mathbf{p}, \mathbf{q}) &:= \int_{\Gamma \times \Gamma} \mathcal{G}_\kappa(\mathbf{x} - \mathbf{y}) \mathbf{p}(\mathbf{x}) \cdot \mathbf{q}(\mathbf{y}) d\sigma(\mathbf{x}, \mathbf{y}) \\ \mathbf{a}_\sharp^\Gamma(\mathbf{p}, \mathbf{q}) &:= - \int_{\Gamma \times \Gamma} \mathcal{G}_\kappa(\mathbf{x} - \mathbf{y}) \text{div}_\Gamma \mathbf{p}(\mathbf{x}) \text{div}_\Gamma \mathbf{q}(\mathbf{y}) d\sigma(\mathbf{x}, \mathbf{y}) \end{aligned}$$

Then, to each subdomain we associated a bilinear form  $A_{\Gamma_j}$  involving an integral operator. We have to distinguish two cases.

**Case 1:  $\kappa_j \neq 0$ .** In this case,  $A_{\Gamma_j}$  is the bilinear form associated to the classical Rumsey formulation. For  $\mathbf{u}, \mathbf{v}$  two pairs of traces tangent to  $\Gamma_j$ , it is defined by

$$\begin{aligned} A_{\Gamma_j}(\mathbf{u}, \mathbf{v}) := & \kappa_j^2 \mathbf{a}_{\kappa_j}^{\Gamma_j}(\mathbf{u}^E, \mathbf{v}^E) + \mathbf{b}_{\kappa_j}^{\Gamma_j}(\mathbf{u}^E, \mathbf{v}^H) \\ & + \mathbf{b}_{\kappa_j}^{\Gamma_j}(\mathbf{u}^H, \mathbf{v}^E) + \mathbf{a}_{\kappa_j}^{\Gamma_j}(\mathbf{u}^H, \mathbf{v}^H). \end{aligned} \quad (3)$$

**Case 2:  $\kappa_j = 0$ .** In this case,  $A_{\Gamma_j}$  is the bilinear form coming

into play in Hiptmair-Ostrowski's eddy-current coupling. It is given by

$$\begin{aligned} A_{\Gamma_j}(\mathbf{u}, \mathbf{v}) := & \mathbf{a}_*^{\Gamma_j}(\mathbf{u}^E, \mathbf{v}^E) + \mathbf{b}_0^{\Gamma_j}(\mathbf{u}^E, \mathbf{v}^H) \\ & + \mathbf{b}_0^{\Gamma_j}(\mathbf{u}^H, \mathbf{v}^E) + \mathbf{a}_\sharp^{\Gamma_j}(\mathbf{u}^H, \mathbf{v}^H). \end{aligned} \quad (4)$$

### V. MULTI-TRACE FORMULATION

The bilinear form associated to our multi-trace formulation can then be obtained simply by adding the transmission bilinear form with the bilinear form (3)-(4) associated to each subdomain. It takes the form

$$\begin{cases} \text{Find } \mathbf{u} = (\mathbf{u}_j) \in \mathbb{H}(\Sigma), \\ \Pi(\mathbf{u}, \mathbf{v}) + \sum_{j=0}^n A_{\Gamma_j}(\mathbf{u}_j, \mathbf{v}_j) = \mathbf{f}(\mathbf{v}) \\ \forall \mathbf{v} = (\mathbf{v}_j) \in \mathbb{H}(\Sigma). \end{cases} \quad (5)$$

where  $\mathbf{f}$  is a source term whose precise expression is related to the current  $\mathbf{j}$  appearing in the right-hand side of (1).

### VI. CONCLUSION

In this talk, we shall provide further details on the derivation of (5) and discuss its numerical stability in the low frequency regime. This will be illustrated by numerical experiments related to concrete applications in non-destructive testing considered at CEA.

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