



Propagation of elastic waves un buried waveguides: modelling of the forward problem and imaging with sampling methods

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(1) Context

- **Control of elongated structures** (pipes, cables, plates...) partially or totally **buried** into an acoustic or elastic medium (water, cement,...).
- Different techniques exist to identify defects in closed waveguides. Among them, a high-frequency **sampling-type method** called LSM gives good results.
- Few works on the modeling of wave propagation in these media and on the identification of defects in these open structures.



Left: suspended bridge (newcivilengineer.com), middle: underwater fiber-optics (acacia-inc.com), right: riser (ocean-energyresources.com).

Difficulties:

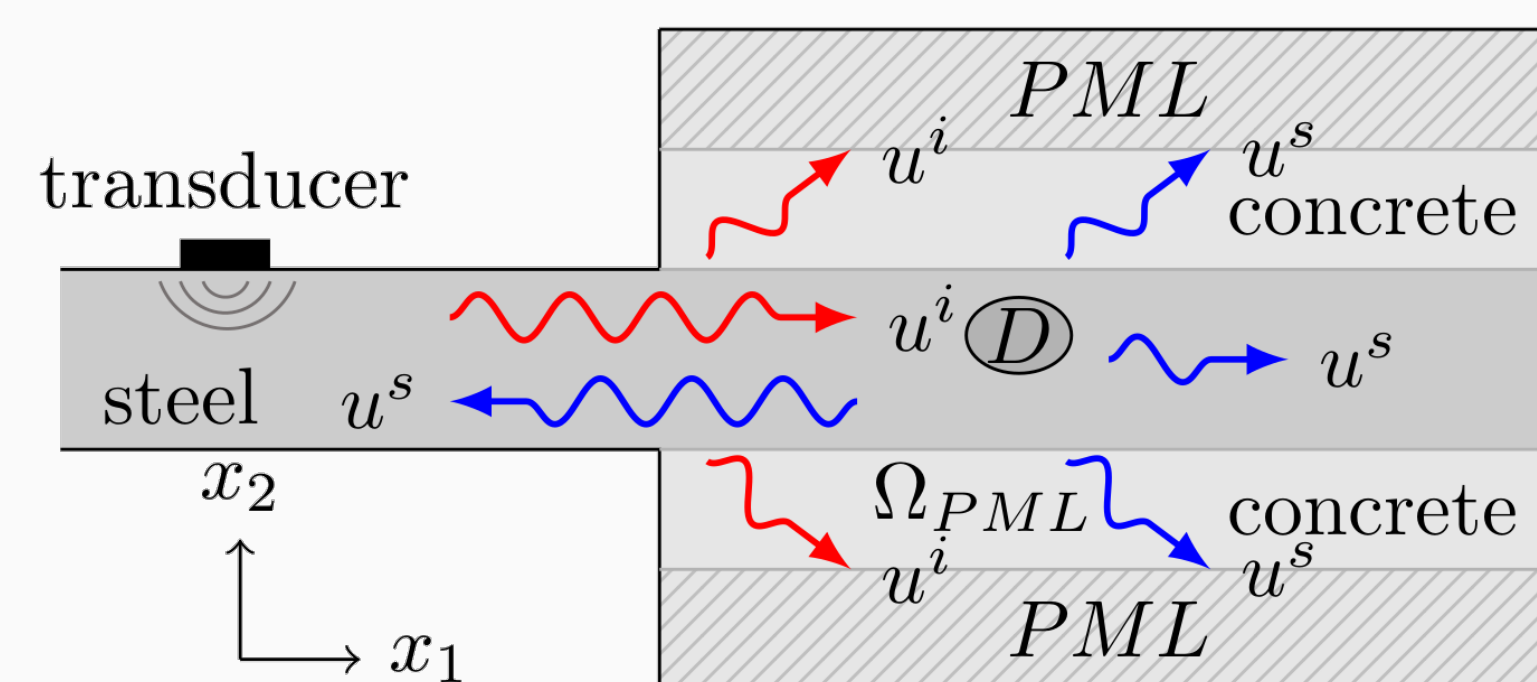
- only one side of the zone to control is accessible,
- waves propagating in the structure **leak** partially in the surrounding medium, the attenuation in the axial direction is more or less important depending on the mode,
- from a mathematical point of view: study of **non self-adjoint** problems.

(2) SH waves in a 2D partially buried waveguide

We start by studying SH (Shear Horizontal) waves: acoustic case.

Find $u^s \in H_{loc}^1(\Omega_{PML} \setminus \bar{D})$ such that :

$$\begin{cases} \alpha \partial_{x_2}(\alpha \mu \partial_{x_2} u^s) + \partial_{x_1}^2 u^s + k^2 u^s = 0 & \text{in } \Omega_{PML} \setminus \bar{D} \\ u^s = -u^i & \text{on } \partial D \\ \partial_n u^s = 0 & \text{on } \partial \Omega_{PML} \setminus \partial D \\ u^s \text{ is outgoing.} \end{cases}$$



Numerical testing configuration.

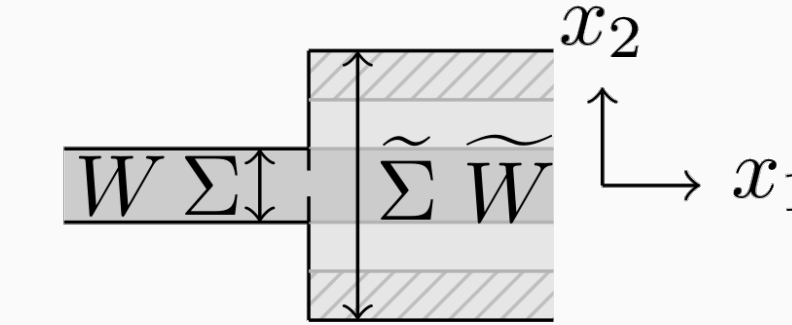
Difficulties:

- need to truncate the domain in the transverse direction for computation \Rightarrow use of PML (model the physical medium with absorption)
- ⊕ easy to implement, extension to elasticity ⊖ choice of its parameters not easy,
- need to truncate the domain in the axial direction.

(3) Modal analysis of both half-guides

Mode = solution of the homogeneous Helmholtz problem with separate variable:

$$g_n^\pm(x_1, x_2) = \varphi_n(x_2) e^{\pm i \beta_n x_1}$$



Left closed half-guide

No interaction with the outside medium.

$$\begin{cases} \Delta u + k^2 u = 0 & \text{in } W \\ \partial_n u = 0 & \text{on } \partial W \end{cases}$$

- known analytically,
- functions $(\varphi_n)_n$ form a complete basis of $L^2(\Sigma)$,
- orthogonality relationship.

Right open half-guide

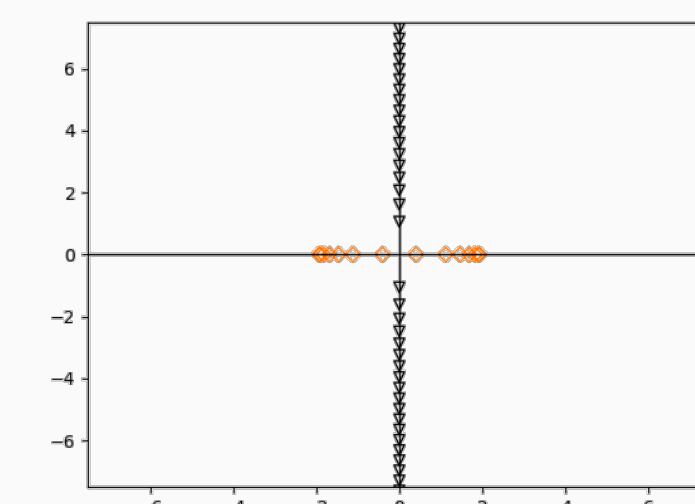
Interaction with the outside medium.

$$\begin{cases} \alpha \mu^{-1} \partial_{x_2}(\alpha \mu \partial_{x_2} u) + \partial_{x_1}^2 u + k^2 u = 0 & \text{in } W \\ \partial_n u = 0 & \text{on } \partial W \end{cases}$$

- not known analytically,
- functions $(\varphi_n)_n$ form a complete basis of $L^2(\Sigma)$ with the Riesz property?
- bi-orthogonality relationship.

(4) $\pm \beta_n$ in the complex plane

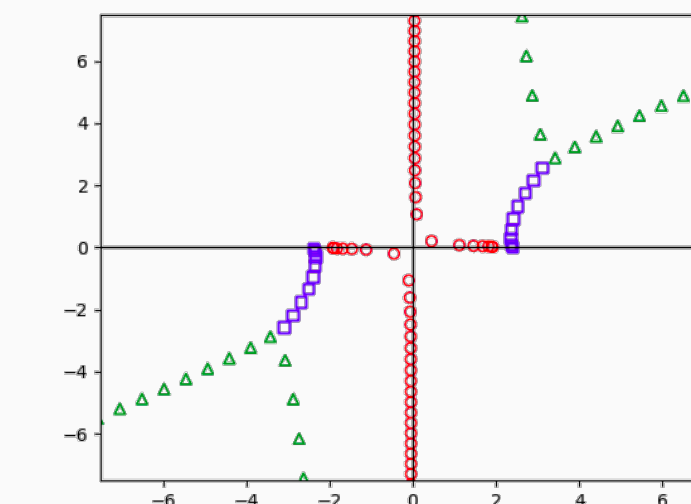
Left closed half-guide



Simple spectrum:

- ◇: propagating modes
 - oscillating without loss along the axial direction
 - finite number,
- ▽: evanescent modes
 - exponentially decreasing in the axial direction
 - infinite number.

Right open half-guide



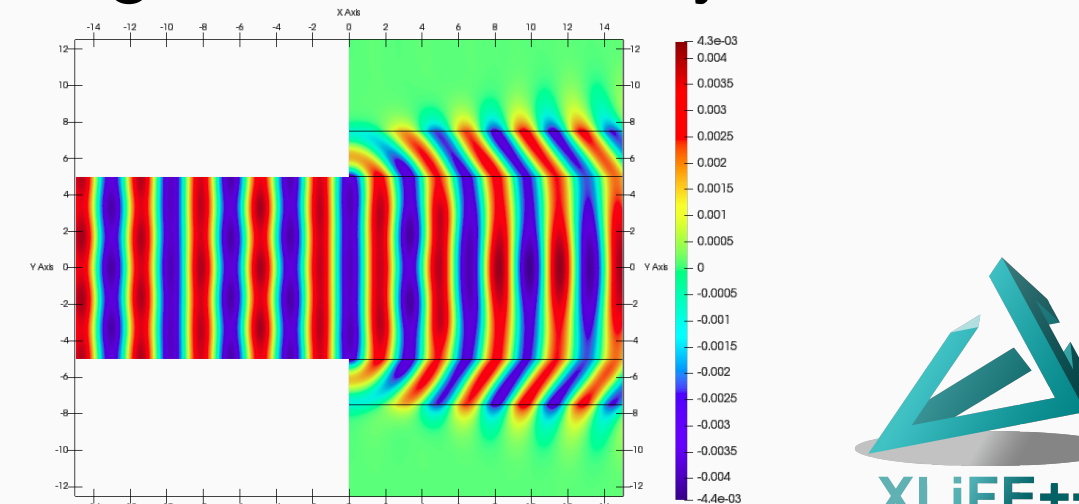
Complex spectrum \Rightarrow need of filtering:

- ○: leaky modes (physical objects)
 - oscillating with loss along the axial direction, exponentially decreasing in the transverse direction (in the PML)
 - infinite number,
- □: PML modes (not physical, in PML),
- ▲: polluting modes (due to discretization).

(5) Direct problem

Theoretical work: well-posedness of scattering problems in the junction with PML and asymptotic behaviour of scattered waves using Kondratiev theory.

Numerical work: truncation of the computational domain in the axial direction with Dirichlet-to-Neuman operator to synthesize data to test imaging method.



(6) Imaging with the Linear Sampling Method

Near-field operator \mathcal{N} : integral operator defined with data obtained on the accessible part of the structure (on the section Σ_{-R} of the left half-guide).

- For each point y of a sampling grid \mathcal{G} , solve the near-field equation:

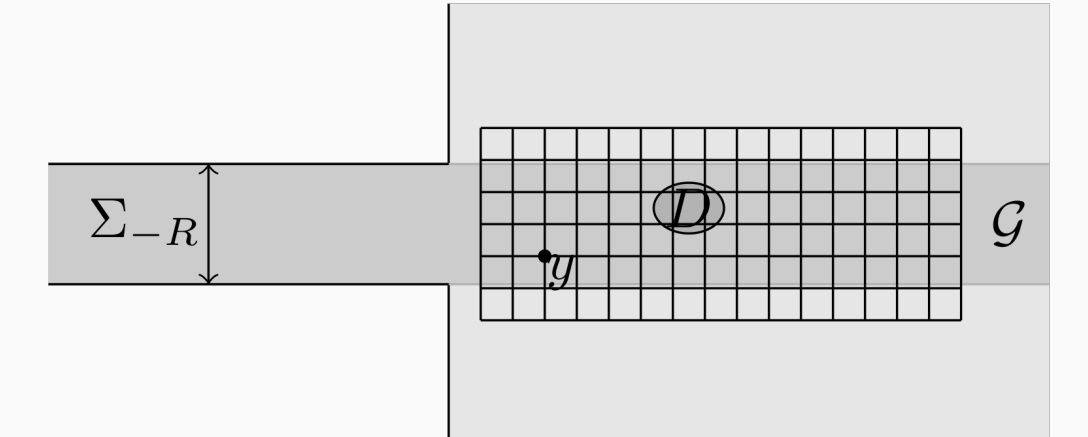
$$\mathcal{N}h(\cdot, y) = G(\cdot, y)|_{\Sigma_{-R}}$$

\mathcal{N} is compact \Rightarrow regularization.

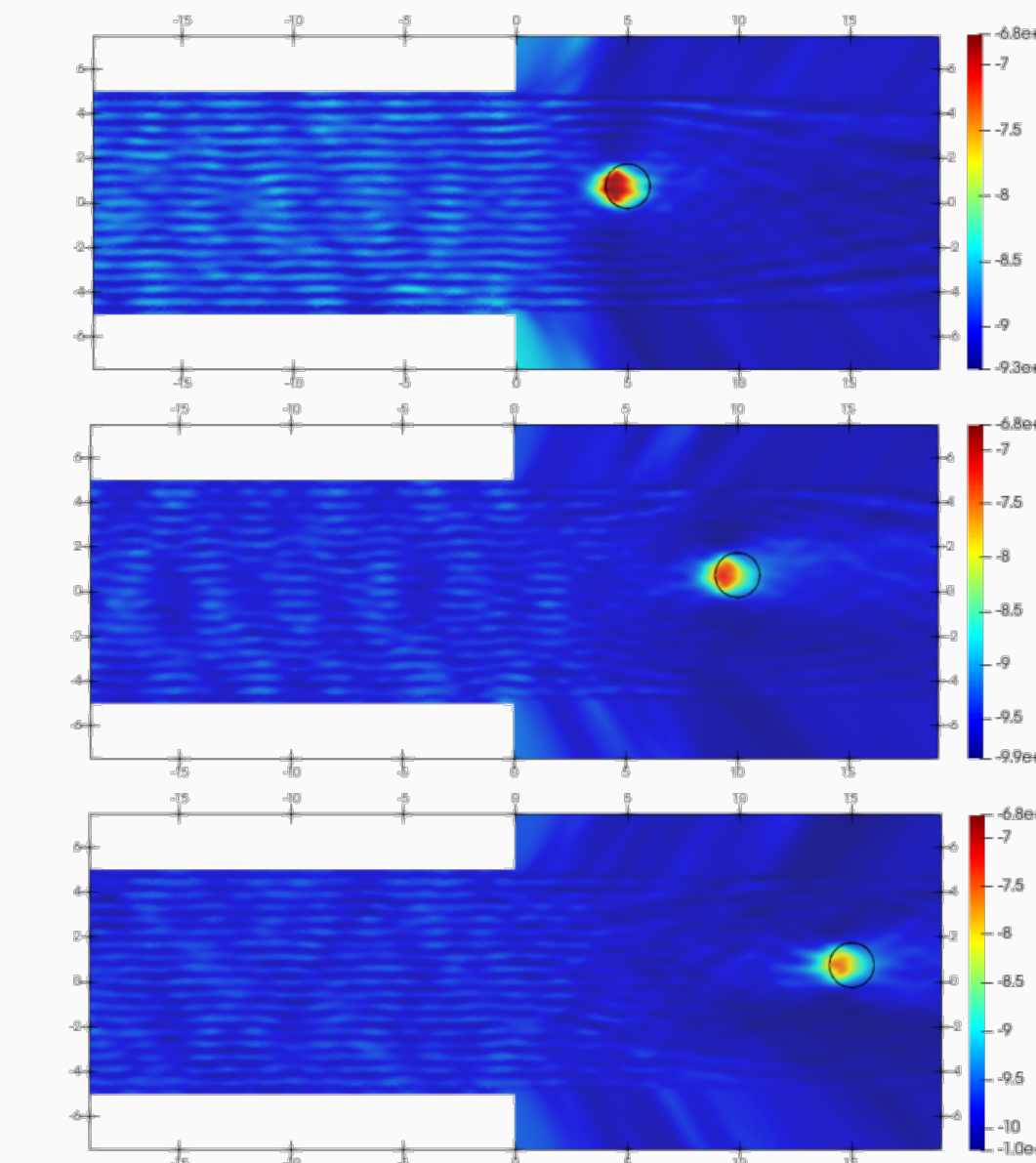
- Plot the level curves of the function

$$H : y \mapsto \log \left(1 / \|h(\cdot, y)\|_{L^2(\Sigma_{-R})} \right)$$

- Property: the near-field equation has a solution " \iff " $y \in D$.
- H is a "characteristic" function of the obstacle \Rightarrow we image the defect.



(7) Numerical results



Top: $d = 5$, middle: $d = 10$, bottom: $d = 15$.

Parameters:

- $N_p = 19$ propagating modes in the left half-guide,
- $\sigma = 10\%$ of noise in the data,
- Dirichlet obstacle.

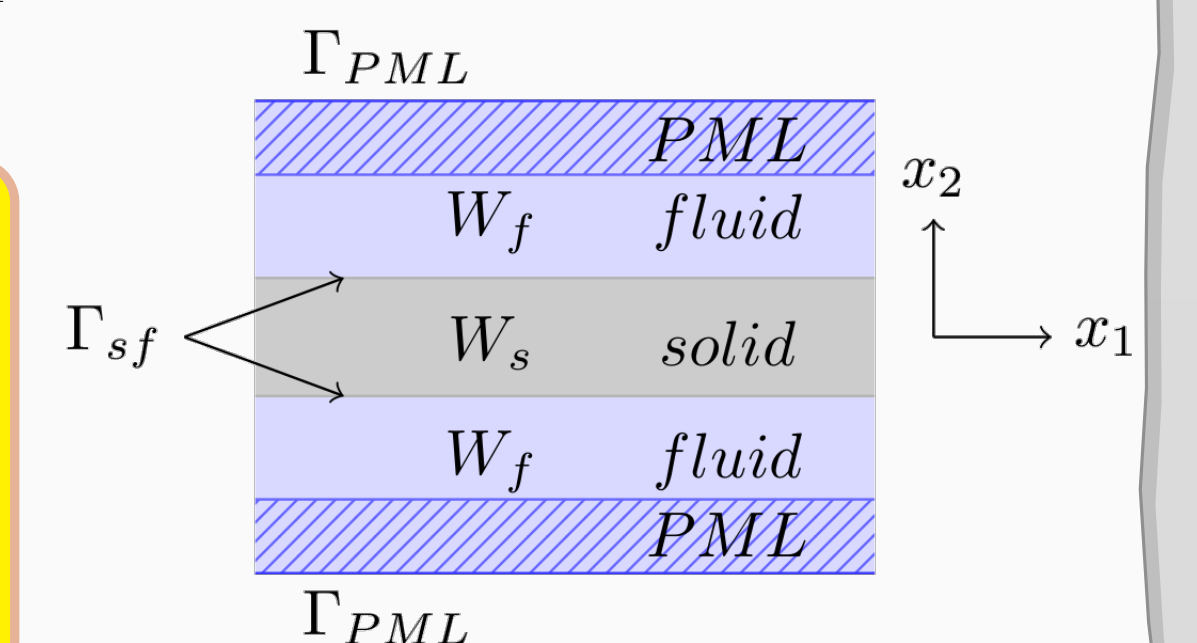
Interpretation:

The quality of the reconstruction is deteriorated when the distance d of the obstacle to the junction increases: data is lost through the partial leakage of waves in the surrounding medium.

(8) Elastic plate immersed in a fluid

Mode=solution $g_n^\pm(x_1, x_2) = (\mathbf{u}_n, \varphi_n)^t(x_2) e^{\pm i \beta_n x_1}$ of the homogeneous fluid-structure problem:

$$\begin{cases} \operatorname{div}(\sigma(\mathbf{u})) + \rho_s \omega^2 \mathbf{u} = 0 & \text{in } W_s, \\ \alpha \partial_{x_2}(\alpha \mu \partial_{x_2} \varphi) + \partial_{x_1}^2 \varphi + k_f^2 \varphi = 0 & \text{in } W_f, \\ \sigma(\mathbf{u}) \mathbf{n}_s = -i \omega \rho_f \varphi \mathbf{n}_s & \text{on } \Gamma_{sf}, \\ -i \omega \mathbf{u} \cdot \mathbf{n}_s = \nabla \varphi \cdot \mathbf{n}_s & \text{on } \Gamma_{sf}, \\ \nabla \varphi \cdot \mathbf{n}_f = 0 & \text{on } \Gamma_{PML}. \end{cases}$$



- Different families of modes are computed: leaky Lamb modes, Scholte modes (interface modes) requiring a specific treatment, PML modes.
- XY formalism is derived: bi-orthogonality, modal decomposition of the Green tensor.
- **Perspectives:** direct problem, adaptation of the LSM and experimental validation.