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# Propagation of elastic waves un buried waveguides: modelling of the forward problem and imaging with sampling methods

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## (1) Context

- Control of elongated structures (pipes, cables, plates...) partially or totally buried into an acoustic or elastic medium (water, cement, ...).
- Different techniques exist to identify defects in closed waveguides. Among them, a high-frequency **sampling-type method** called LSM gives good results.
- Few works on the modeling of wave propagation in these media and on the identification of defects in these open structures.



Left: suspended bridge (newcivilengineer.com), middle: underwater fiber-optics (acacia-inc.com), right: riser (ocean-energyresources.com).

### Difficulties:

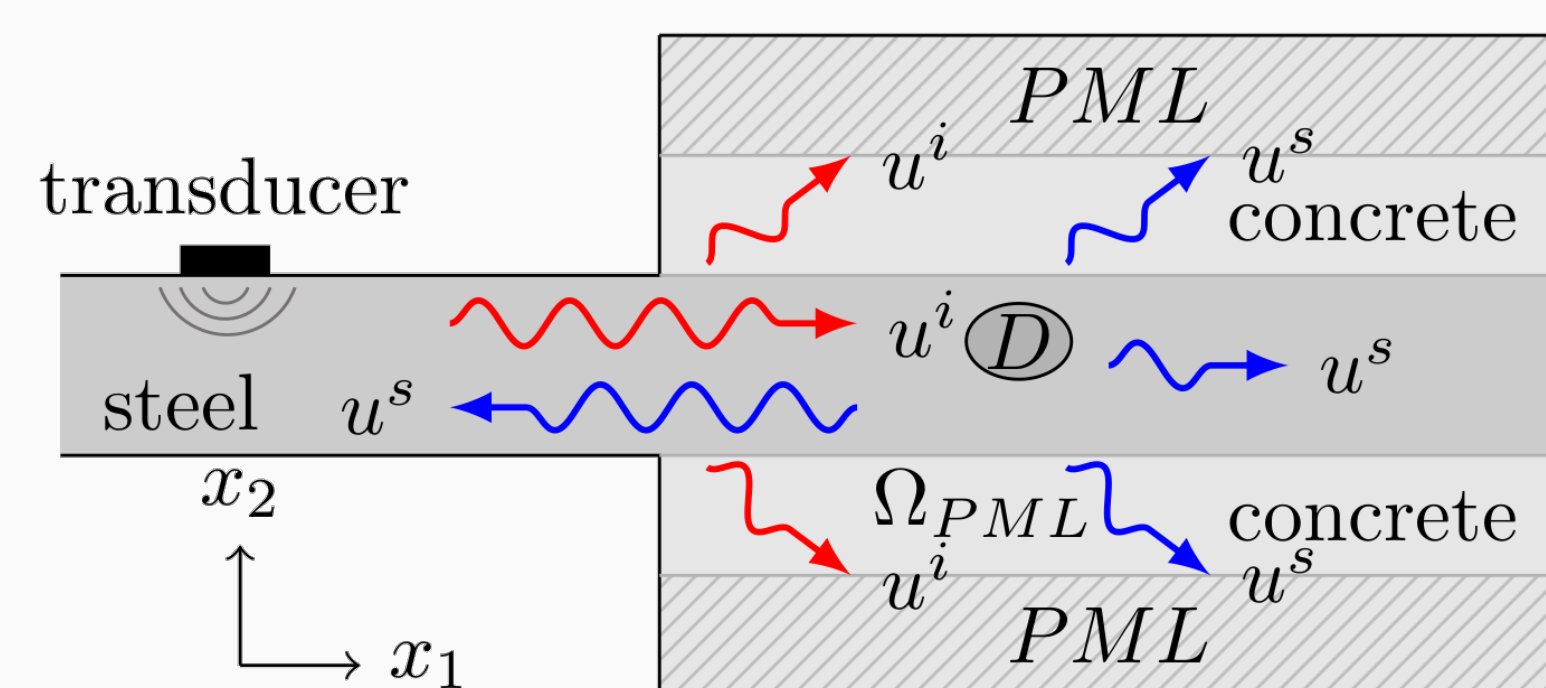
- only one side of the zone to control is accessible,
- waves propagating in the structure **leak** partially in the surrounding medium, the attenuation in the axial direction is more or less important depending on the mode,
- from a mathematical point of view: study of **non self-adjoint** problems.

## (2) SH waves in a 2D partially buried waveguide

We start by studying SH (Shear Horizontal) waves: acoustic case.

Find  $u^s \in H_{loc}^1(\Omega_{PML} \setminus \bar{D})$  such that :

$$\begin{cases} \alpha \partial_{x_2} (\alpha \mu \partial_{x_2} u^s) + \partial_{x_1}^2 u^s + k^2 u^s = 0 & \text{in } \Omega_{PML} \setminus \bar{D} \\ u^s = -u^i & \text{on } \partial D \\ \partial_n u^s = 0 & \text{on } \partial \Omega_{PML} \setminus \partial D \\ u^s \text{ is outgoing.} \end{cases}$$



Numerical testing configuration.

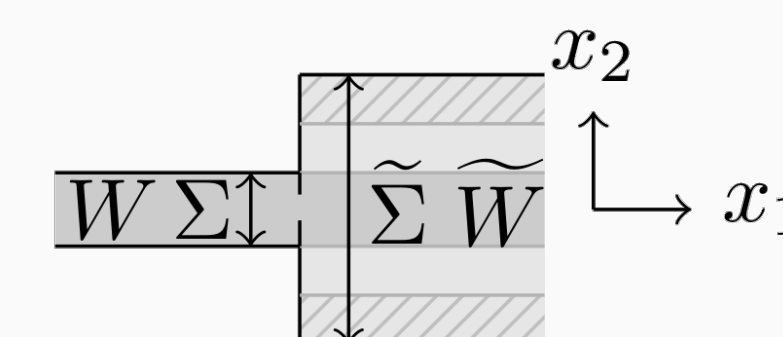
### Difficulties:

- need to truncate the domain in the transverse direction for computation  $\Rightarrow$  use of PML (model the physical medium with absorption)
- $\oplus$  easy to implement, extension to elasticity  $\ominus$  choice of its parameters not easy,
- need to truncate the domain in the axial direction.

## (3) Modal analysis of both half-guides

**Mode** = solution of the homogeneous Helmholtz problem with separate variable:

$$g_n^\pm(x_1, x_2) = \varphi_n(x_2) e^{\pm i \beta_n x_1}$$



### Left closed half-guide

No interaction with the outside medium.

$$\begin{cases} \Delta u + k^2 u = 0 & \text{in } W \\ \partial_n u = 0 & \text{on } \partial W \end{cases}$$

- known analytically,
- functions  $(\varphi_n)_n$  form a complete basis of  $L^2(\Sigma)$ ,
- orthogonality relationship.

### Right open half-guide

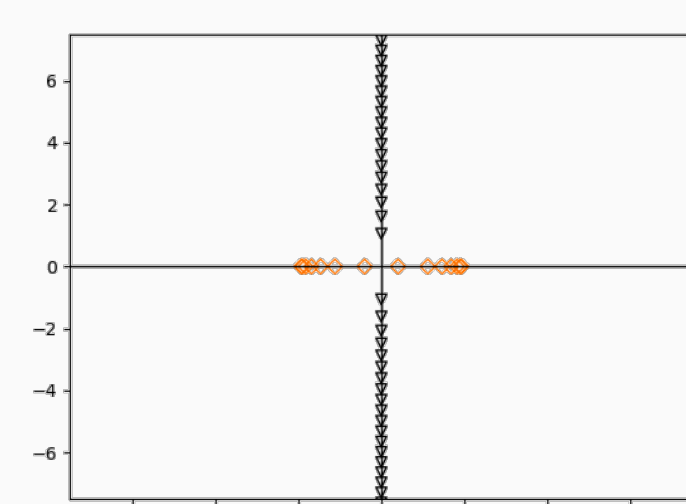
Interaction with the outside medium.

$$\begin{cases} \alpha \mu^{-1} \partial_{x_2} (\alpha \mu \partial_{x_2} u) + \partial_{x_1}^2 u + k^2 u = 0 & \text{in } W \\ \partial_n u = 0 & \text{on } \partial W \end{cases}$$

- not known analytically,
- functions  $(\varphi_n)_n$  form a complete basis of  $L^2(\Sigma)$  with the Riesz property?
- bi-orthogonality relationship.

## (4) $\pm \beta_n$ in the complex plane

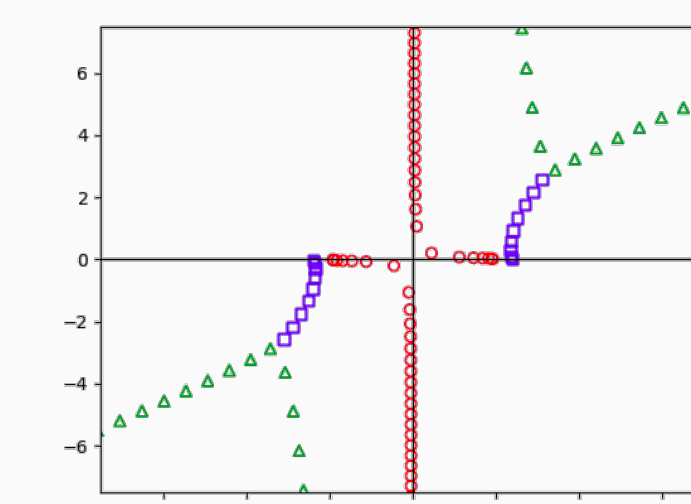
### Left closed half-guide



Simple spectrum:

- $\diamond$ : propagating modes
  - oscillating without loss along the axial direction
  - finite number,
- $\nabla$ : evanescent modes
  - exponentially decreasing in the axial direction
  - infinite number.

### Right open half-guide



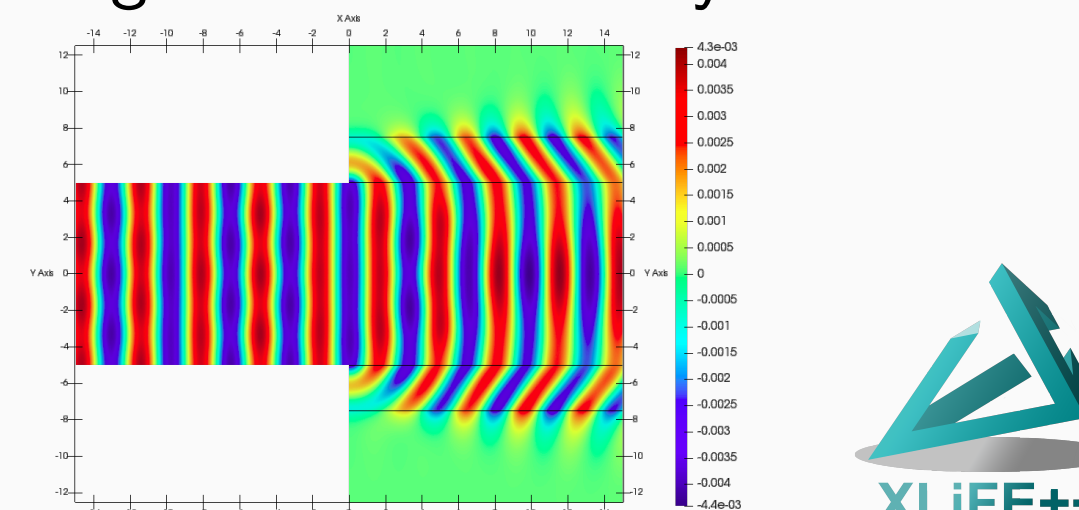
Complex spectrum  $\Rightarrow$  need of filtering:

- $\circ$ : leaky modes (physical objects)
  - oscillating with loss along the axial direction, exponentially decreasing in the transverse direction (in the PML)
  - infinite number,
- $\square$ : PML modes (not physical, in PML),
- $\triangle$ : polluting modes (due to discretization).

## (5) Direct problem

**Theoretical work:** well-posedness of scattering problems in the junction with PML and asymptotic behaviour of scattered waves using Kondratiev theory.

**Numerical work:** truncation of the computational domain in the axial direction with Dirichlet-to-Neuman operator to synthesize data to test imaging method.



## (6) Imaging with the Linear Sampling Method

**Near-field operator**  $\mathcal{N}$ : integral operator defined with data obtained on the accessible part of the structure (on the section  $\Sigma_{-R}$  of the left half-guide).

- For each point  $y$  of a sampling grid  $\mathcal{G}$ , solve the near-field equation:

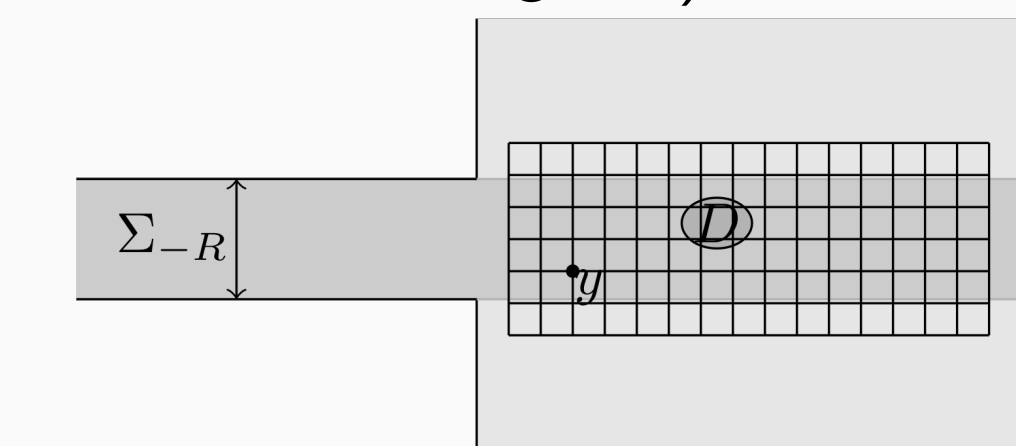
$$\mathcal{N}h(\cdot, y) = G(\cdot, y)|_{\Sigma_{-R}}$$

$\mathcal{N}$  is compact  $\Rightarrow$  regularization.

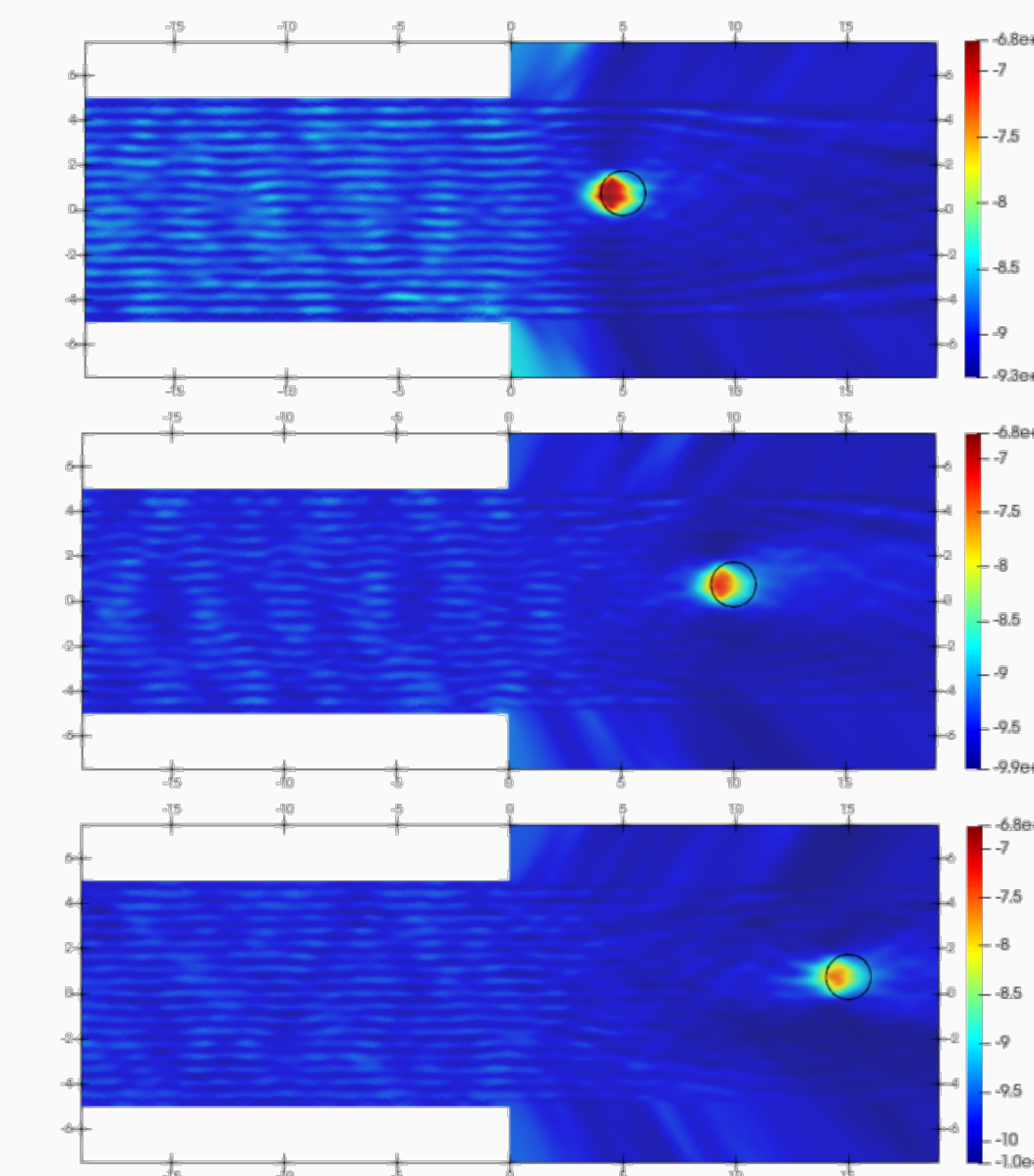
- Plot the level curves of the function

$$H : y \mapsto \log \left( 1 / \|h(\cdot, y)\|_{L^2(\Sigma_{-R})} \right)$$

- Property: the near-field equation has a solution "  $\iff$  "  $y \in D$ .
- $H$  is a "characteristic" function of the obstacle  $\Rightarrow$  we image the defect.



## (7) Numerical results



Top:  $d = 5$ , middle:  $d = 10$ , bottom:  $d = 15$ .

### Parameters:

- $N_p = 19$  propagating modes in the left half-guide,
- $\sigma = 10\%$  of noise in the data,
- Dirichlet obstacle.

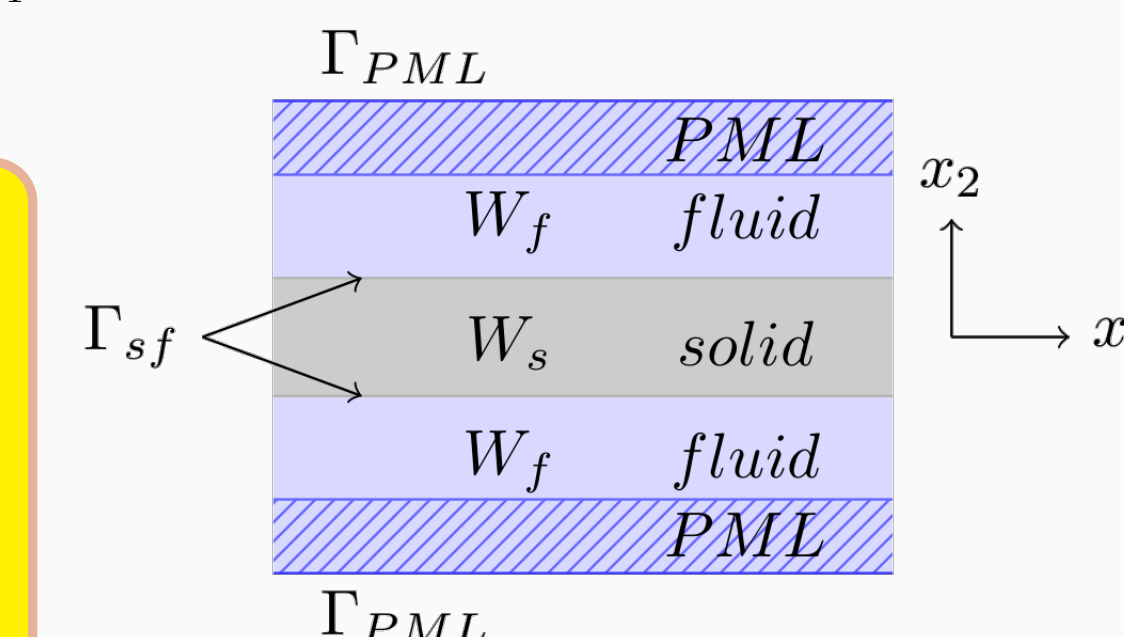
### Interpretation:

The quality of the reconstruction is deteriorated when the distance  $d$  of the obstacle to the junction increases: data is lost through the partial leakage of waves in the surrounding medium.

## (8) Elastic plate immersed in a fluid

**Mode**=solution  $g_n^\pm(x_1, x_2) = (\mathbf{u}_n, \varphi_n)^t(x_2) e^{\pm i \beta_n x_1}$  of the homogeneous fluid-structure problem:

$$\begin{cases} \operatorname{div}(\sigma(\mathbf{u})) + \rho_s \omega^2 \mathbf{u} = 0 & \text{in } W_s, \\ \alpha \partial_{x_2} (\alpha \partial_{x_2} \varphi) + \partial_{x_1}^2 \varphi + k_f^2 \varphi = 0 & \text{in } W_f, \\ \sigma(\mathbf{u}) \mathbf{n}_s = -i \omega \rho_f \varphi \mathbf{n}_s & \text{on } \Gamma_{sf}, \\ -i \omega \mathbf{u} \cdot \mathbf{n}_s = \nabla \varphi \cdot \mathbf{n}_s & \text{on } \Gamma_{sf}, \\ \nabla \varphi \cdot \mathbf{n}_f = 0 & \text{on } \Gamma_{PML}. \end{cases}$$



- Different families of modes are computed: leaky Lamb modes, Scholte modes (interface modes) requiring a specific treatment, PML modes.
- $XY$  formalism is derived: bi-orthogonality, modal decomposition of the Green tensor.
- Perspectives:** direct problem, adaptation of the LSM and experimental validation.