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COMBINING SHELL ELEMENTS AND TRANSIENT SPECTRAL FINITE ELEMENTS FOR GUIDED WAVE PROPAGATION PROBLEMS IN DEFORMED THIN STRUCTURES

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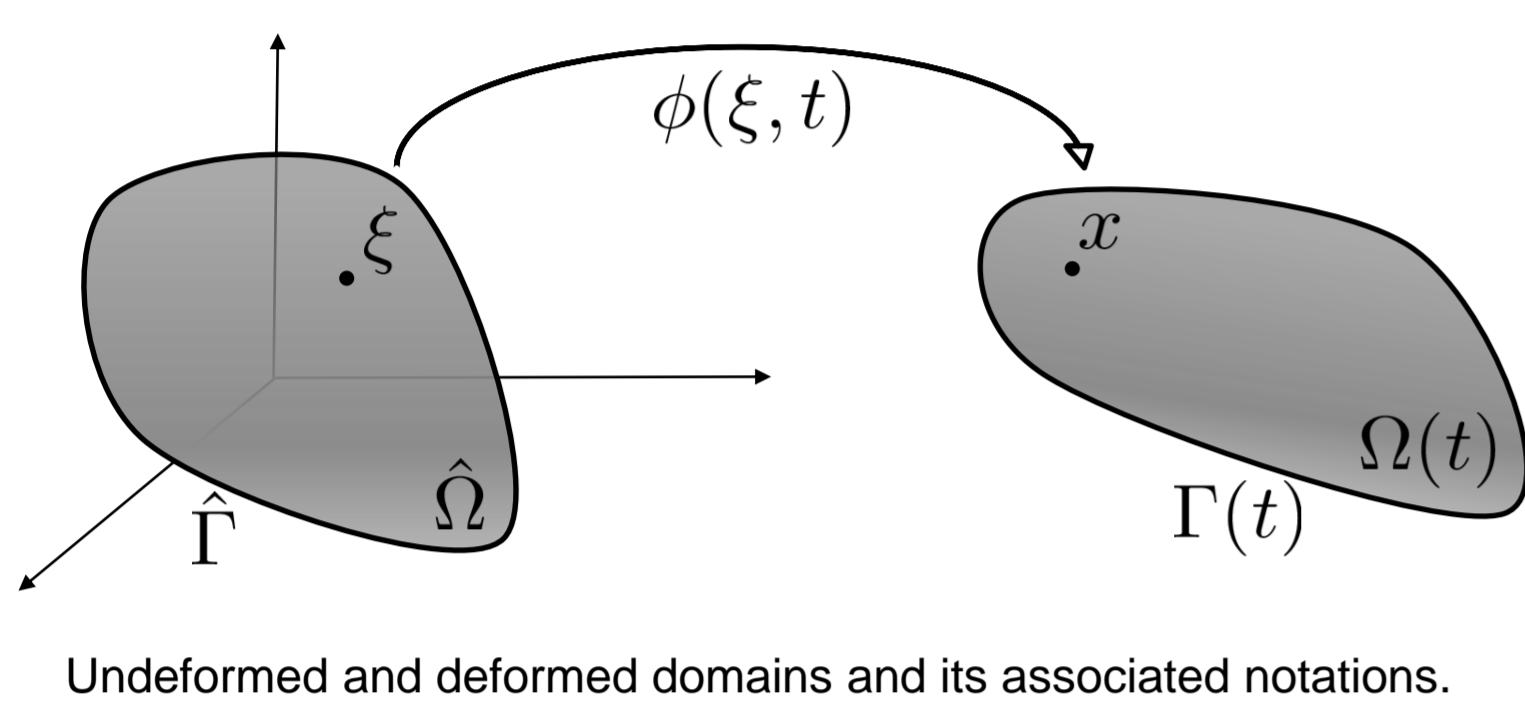
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In leading edge industrial applications, assessing structure integrity is an important aspect of safety requirements. **Structural Health Monitoring (SHM)** proposes to use sensors and signal processing units *in situ* for such purpose. Ultrasonic SHM systems are already implemented in industry but their usage are limited due to environmental and operational conditions [1], such as internal stresses due to structural loading/deformation. We present a model, and corresponding numerical methods, for **elastic wave propagation in a predeformed medium**. We use a **shell formulation** [2] for nonlinear mechanics to compute the effects of structure loading. The computed displacement is fed into a spectral elements method [3] (SFEM) kernel to solve the **time-domain linearized elastodynamics problem representing the wave propagation**.

Modelling context

We base our modeling on classical nonlinear elastodynamics.



$$\begin{cases} \mathbf{u}(\xi, t) = \mathbf{x}(\xi, t) - \xi \\ \mathbf{x}(\xi, t) = \phi(\xi, t) \\ \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} - \nabla_{\mathbf{x}} \cdot \boldsymbol{\sigma} = \mathbf{f} & \text{in } \Omega(t) \\ \boldsymbol{\sigma} = \mathbf{G}(\mathbf{e}) \text{ (Constitutive law)} \\ \boldsymbol{\sigma} \cdot \mathbf{n} = 0 & \text{in } \Gamma^N(t), \\ \mathbf{u} = 0 & \text{in } \Gamma^D(t), \\ \mathbf{u}(\cdot, 0) = \frac{\partial \mathbf{u}}{\partial t}(\cdot, 0) = 0 \end{cases}$$

We recall the definition of some relevant tensors

$$\mathbf{F} = \nabla_{\xi} \phi, \quad \mathbf{C} = \mathbf{F}^T \mathbf{F}, \quad \mathbf{e}(\mathbf{u}) = \frac{1}{2}(\mathbf{C} - \mathbf{I}), \quad \boldsymbol{\Sigma}(\mathbf{e}) = \mathbf{J} \mathbf{F}^{-1} \boldsymbol{\sigma} \mathbf{F}^{-T}, \quad \mathbf{J} = \det \mathbf{F}$$

and use them to write the formulation in the Total Lagrangian Weak Formulation [4]

$$\forall w \in \mathcal{V}(\hat{\Omega}), \quad m\left(\frac{\partial^2 \mathbf{u}}{\partial t^2}; w\right) + a(\mathbf{u}; w) = l(w; w), \quad \text{where} \quad \begin{cases} m\left(\frac{\partial^2 \mathbf{u}}{\partial t^2}; w\right) = \int_{\hat{\Omega}} \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} \cdot w \, J d\hat{\Omega}, \\ a(\mathbf{u}; w) = \int_{\hat{\Omega}} \boldsymbol{\Sigma}(\mathbf{e}) : \mathbf{D}_{\mathbf{u}} \mathbf{e}(\mathbf{u}) \cdot w \, d\hat{\Omega}, \\ l(w; w) = \int_{\hat{\Omega}} \mathbf{f} \cdot w \, J d\hat{\Omega} \end{cases} \quad (1)$$

Modelling of predeformed wave propagation

We assume external forces that leads to quasi-static displacements that are distinguished from ultrasonic wave propagation in time and amplitude. From Eq. (1):

Assumptions $\delta \ll 1$

$$\begin{aligned} \mathbf{f}(\xi, t) &= \mathbf{f}_0(\xi) + \delta \tilde{\mathbf{f}}(\xi, t) \\ \mathbf{u}(\xi, t) &= \mathbf{u}_0(\xi) + \delta \tilde{\mathbf{u}}(\xi, t). \end{aligned}$$

$$(1) \rightarrow \int_{\hat{\Omega}} \boldsymbol{\Sigma}(\mathbf{u}_0) : \mathbf{D}_{\mathbf{u}} \mathbf{e}(\mathbf{u}_0) \cdot w \, d\hat{\Omega} - \int_{\hat{\Omega}} \mathbf{f}_0 \cdot w \, J_0 \, d\hat{\Omega} + \delta \left(\int_{\hat{\Omega}} \rho \frac{\partial^2 \tilde{\mathbf{u}}}{\partial t^2} \cdot w \, J_0 \, d\hat{\Omega} + \int_{\hat{\Omega}} \boldsymbol{\Sigma}(\mathbf{u}_0) : \mathbf{D}_{\mathbf{u}} \mathbf{e}(\mathbf{u}_0; \tilde{\mathbf{u}}, w) \, d\hat{\Omega} + \int_{\hat{\Omega}} \mathbf{D}_{\mathbf{u}} \mathbf{e}(\mathbf{u}_0) \cdot \tilde{\mathbf{u}} : \mathbf{D}_{\mathbf{e}} \boldsymbol{\Sigma}(\mathbf{u}_0) : \mathbf{D}_{\mathbf{u}} \mathbf{e}(\mathbf{u}_0) \cdot w \, d\hat{\Omega} - \int_{\hat{\Omega}} \tilde{\mathbf{f}} \cdot w \, J_0 \, d\hat{\Omega} \right) = 0.$$

Quadratic terms were neglected. The resulting equation can be solved in two parts:

Quasi-static problem (canceling δ^0 terms)

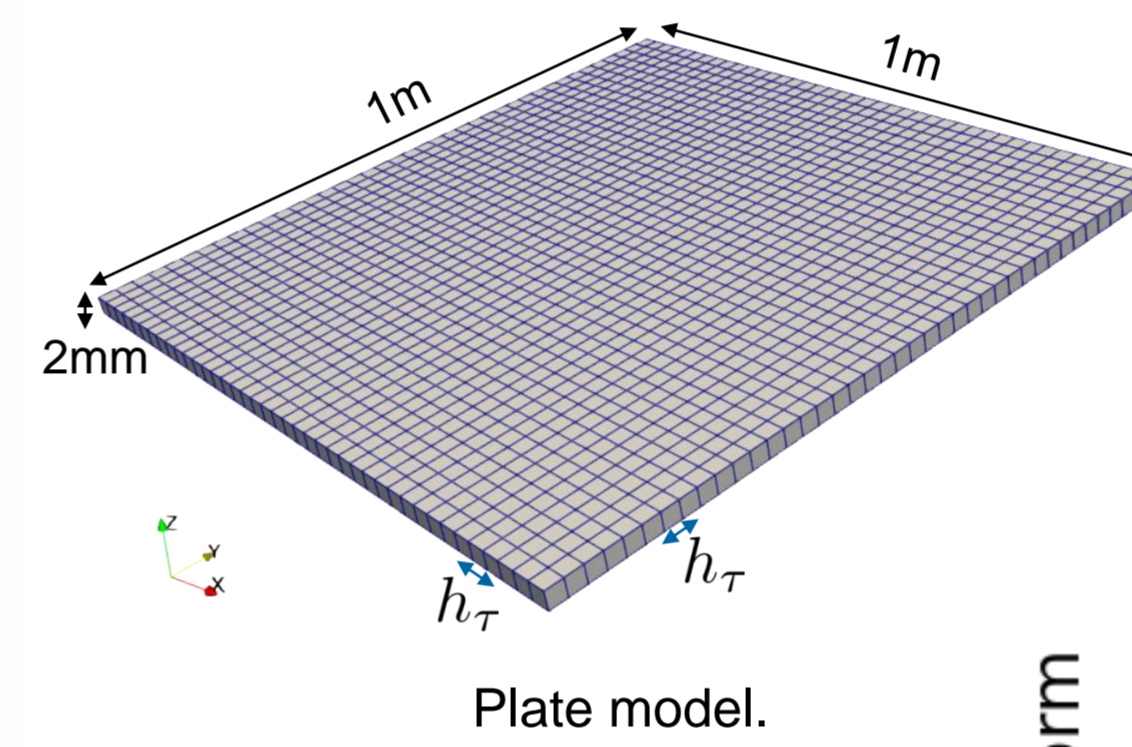
$$a(\mathbf{u}_0; w) = l_0(\mathbf{u}_0; w) \quad \text{where} \quad a(\mathbf{u}_0; w) = \int_{\hat{\Omega}} \boldsymbol{\Sigma}(\mathbf{u}_0) : \mathbf{D}_{\mathbf{u}} \mathbf{e}(\mathbf{u}_0) \cdot w \, d\hat{\Omega}, \quad l_0(\mathbf{u}_0; w) = \int_{\hat{\Omega}} \mathbf{f}_0 \cdot w \, J_0 \, d\hat{\Omega}.$$

Wave propagation problem (canceling δ^1 terms)

$$\frac{d^2}{dt^2} m(\mathbf{u}_0; \tilde{\mathbf{u}}, w) + \mathbf{D}_{\mathbf{u}} a(\mathbf{u}_0; \tilde{\mathbf{u}}, w) = \tilde{l}(\mathbf{u}_0; w) \quad \text{with} \quad \begin{cases} m(\mathbf{u}_0; \tilde{\mathbf{u}}, w) = \int_{\hat{\Omega}} \rho \tilde{\mathbf{u}} \cdot w \, J_0 \, d\hat{\Omega}, \\ \mathbf{D}_{\mathbf{u}} a(\mathbf{u}_0; \tilde{\mathbf{u}}, w) = \int_{\hat{\Omega}} \nabla_{\xi} \tilde{\mathbf{u}} : \tilde{\mathbf{C}}(\mathbf{u}_0) : \nabla_{\xi} w \, d\hat{\Omega}, \\ \tilde{l}(\mathbf{u}_0; w) = \int_{\hat{\Omega}} \tilde{\mathbf{f}} \cdot w \, J_0 \, d\hat{\Omega}. \end{cases}$$

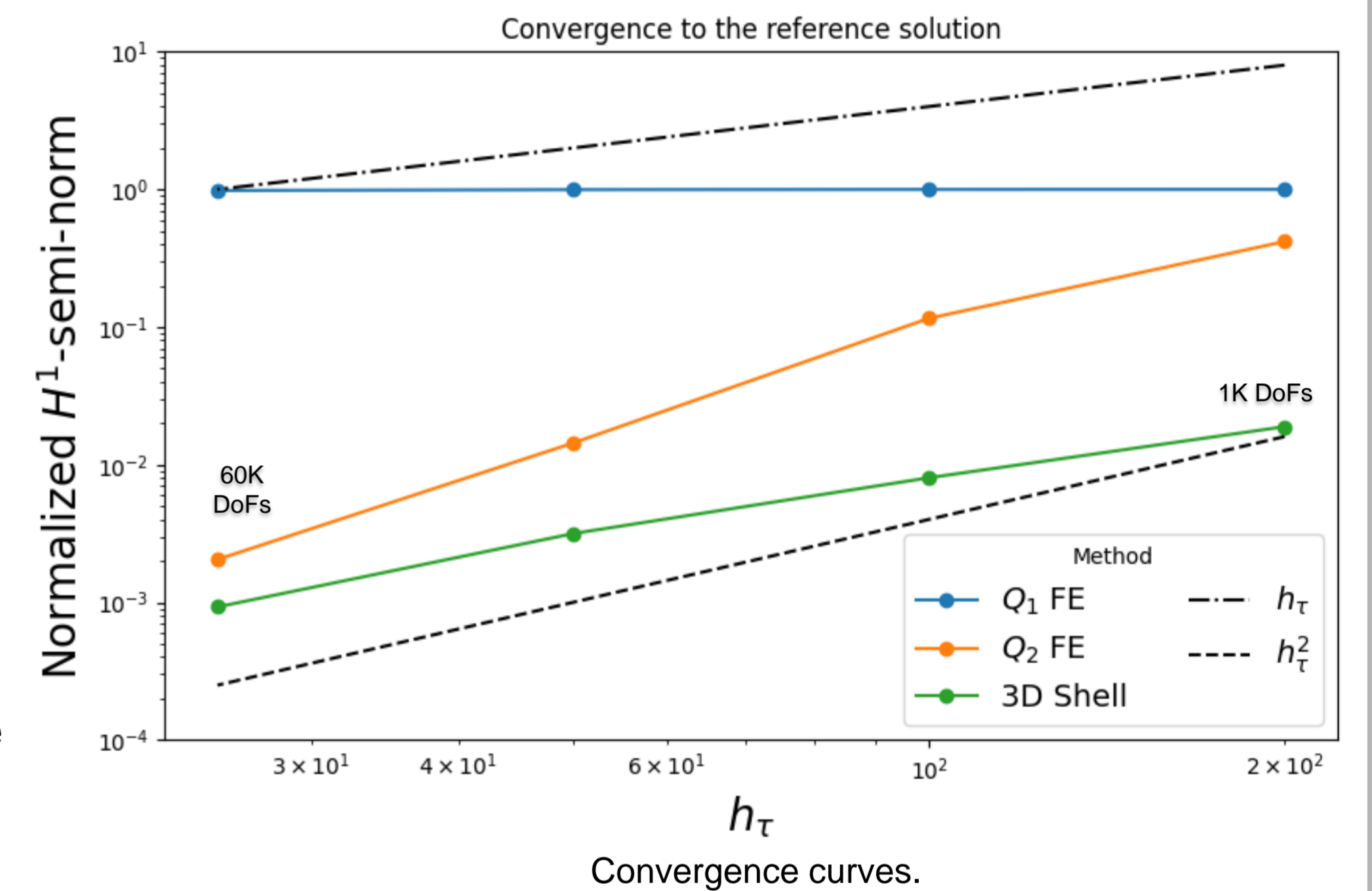
Numerical Locking

Quasi-static problem (solved using Newton-Raphson's method)



We use 3D Shell FE to overcome shear locking. We illustrate it by using the plate model (steel) and a reference solution computed with Q₂ FE* with h_τ = 12.5 mm.

- Q1 FE Locking: No convergence and it may leads to the interpretation that it is converged.
- Q2 FE Locking: Higher error when compared to Shell FE.
- For 10⁻² of relative error, Shell FE took 2secs while Q₂ FE took 3mins.

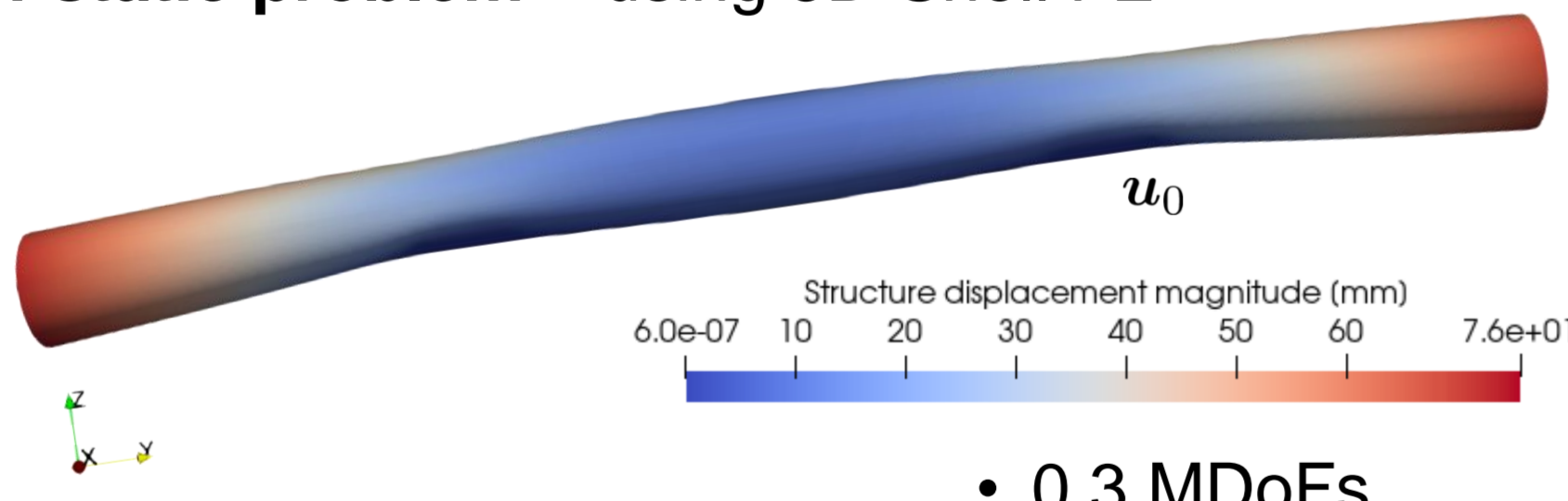


*Order 2 Hexahedral finite elements.

The reference computation had 0.4MDoFs and took 53 minutes to compute.

Simulation toolchain/pipeline and performances

Quasi-static problem – using 3D Shell FE



$$a(\mathbf{u}_0; w) = l_0(\mathbf{u}_0; w)$$

- 0.3 MDoFs.
- 6 Newton iterations.
- CPU time: 5 minutes.
- Memory Peak: 4.7GB

A steel pipe subjected to a bending force is modelled.

Dimensions: 2.94m, Ø 0.19m and 8mm thick.

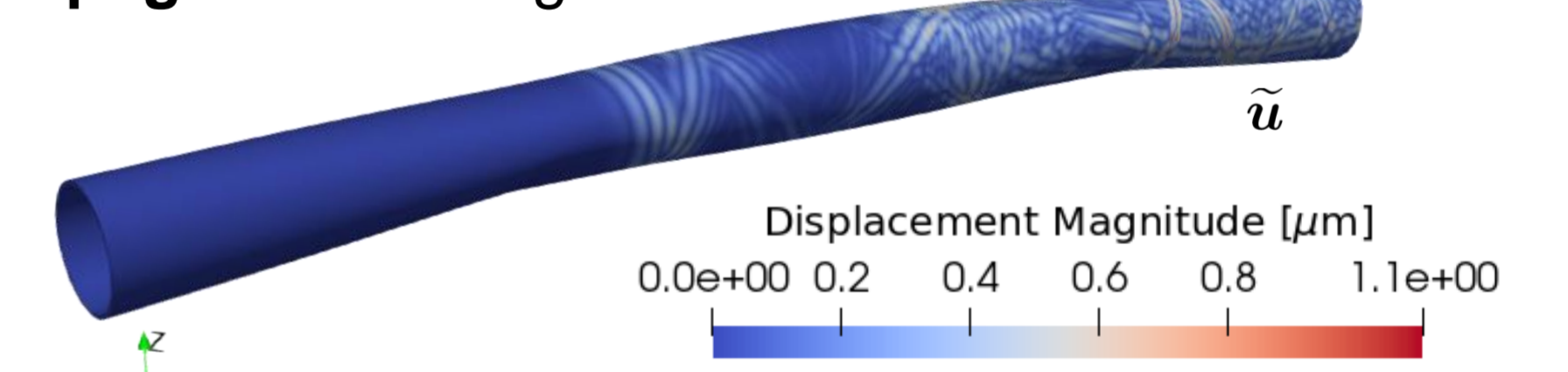
Excitation frequency: 80kHz

Mesh + \mathbf{u}_0

Interpolate and compute the tangent stiffness on the wave propagation mesh.

The computations were done in a laptop with a 2.3 GHz 8-Core processor and 32 GB RAM.

Wave propagation – using SFEM



$$\frac{d^2}{dt^2} m(\mathbf{u}_0; \tilde{\mathbf{u}}, w) + \mathbf{D}_{\mathbf{u}} a(\mathbf{u}_0; \tilde{\mathbf{u}}, w) = \tilde{l}(\mathbf{u}_0; w)$$

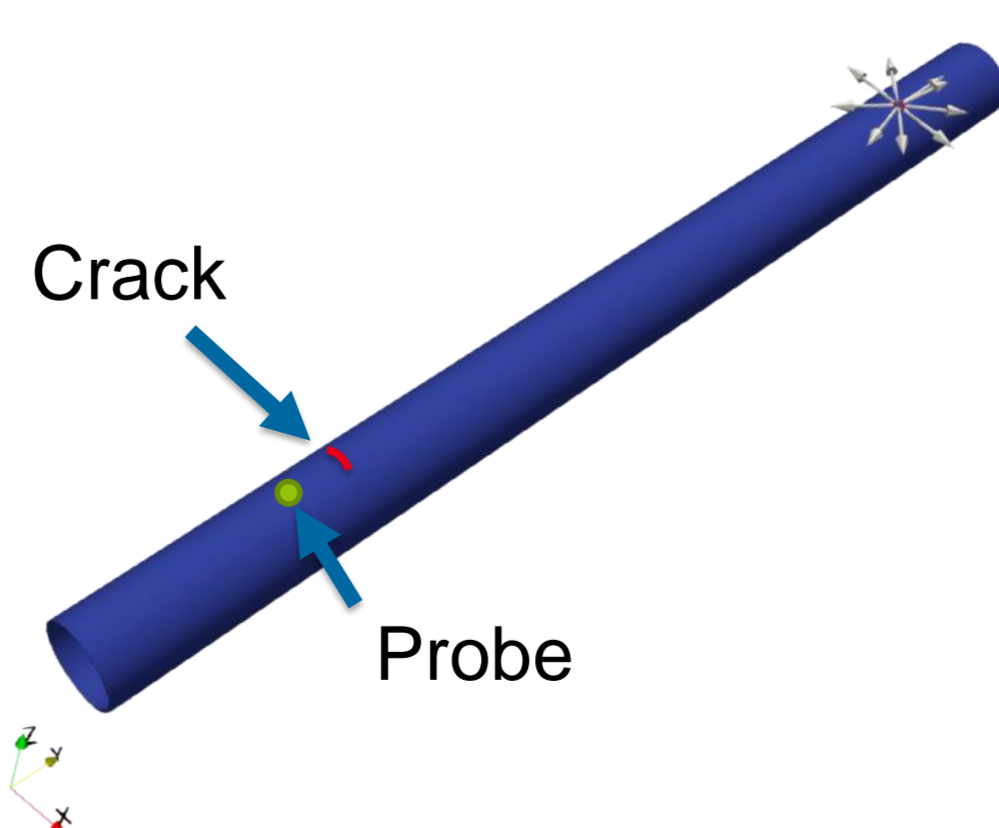
- 11 MDoFs.
- 5091 timesteps.
- CPU time: 20 mins.
- Memory Peak: 2.8GB

Numerical Illustration of the effects of structural loading (predeformation)

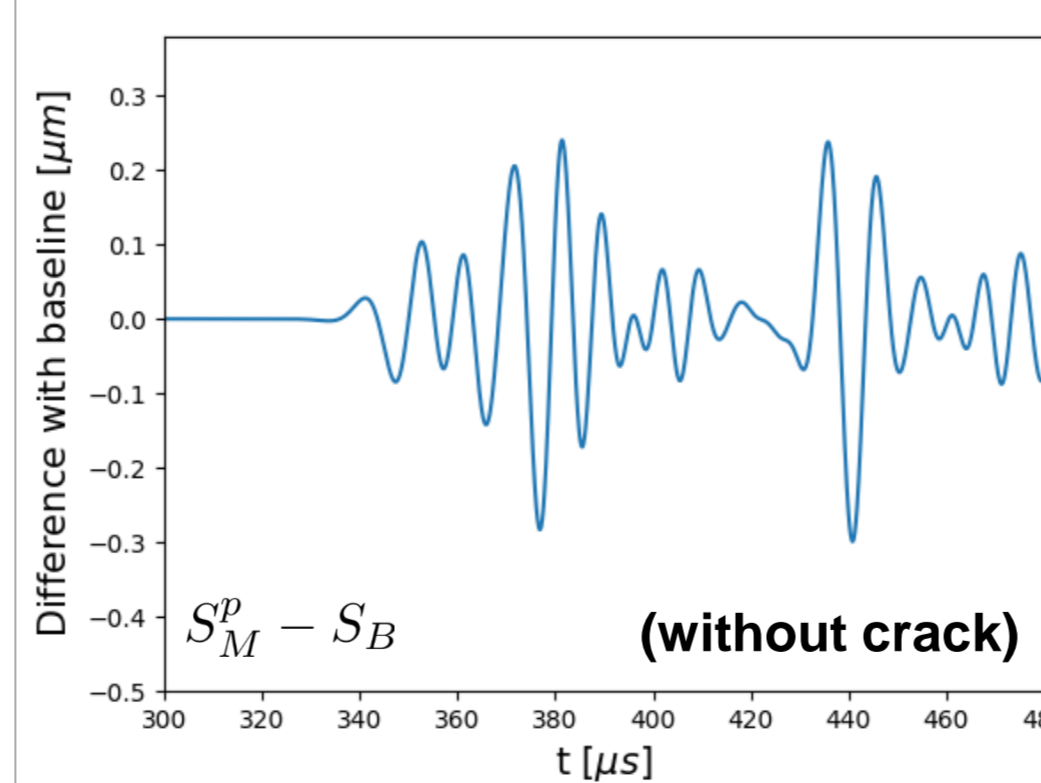
We illustrate the effects of predeformation in damage detection using the baseline method. The baseline represents the ultrasonic measurement in a healthy structure (without cracks). Using simulation we compare two baselines (without and with predeformation) using two measurements (without and with crack) that were simulated, respectively:

$$S_B, S_B^p, S_M^p \text{ and } S_M^{p,c}$$

Model:

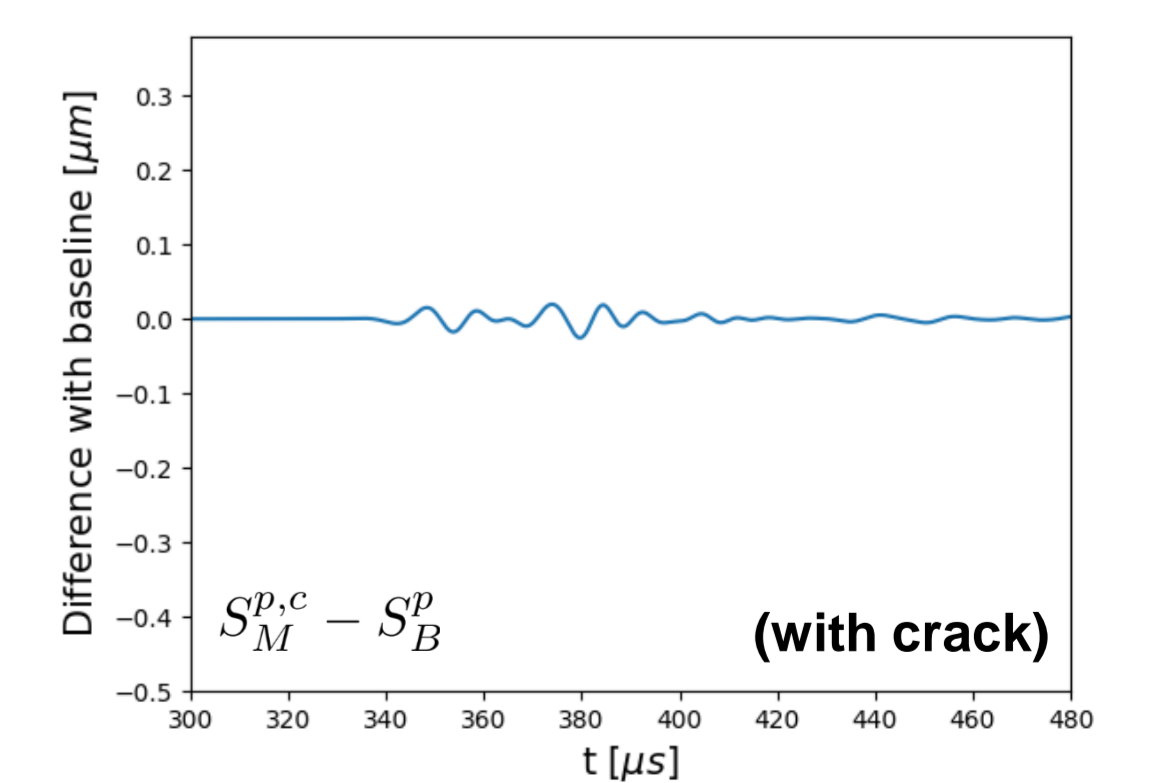


Baseline not considering predeformation



Predeformation influence may masks the defect.

Baseline considering predeformation



The crack influence is distinguished.

Conclusion: A wave propagation model in predeformed structures is presented and used to illustrate the importance of modeling efforts to consider such operational conditions.

Perspectives: Experimental validation of the model and its use with inverse methods for deformation reconstruction using ultrasonic measurements.

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