Physics of E x B staircases

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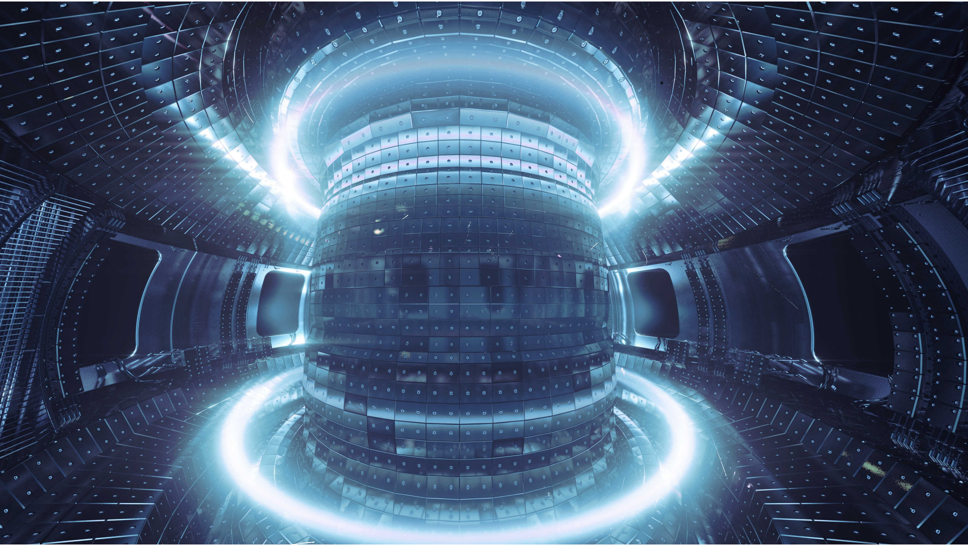
CEA, IRFM, France
[†]UCSD, La Jolla, USA
[⋆]M2P2, Marseille, France
[‡]ENEA, Frascati, Italy
[○]LPP, Palaiseau, France

Ackn.: D.W. Hughes & "Festival de Théorie" participants, Aix-en-Provence

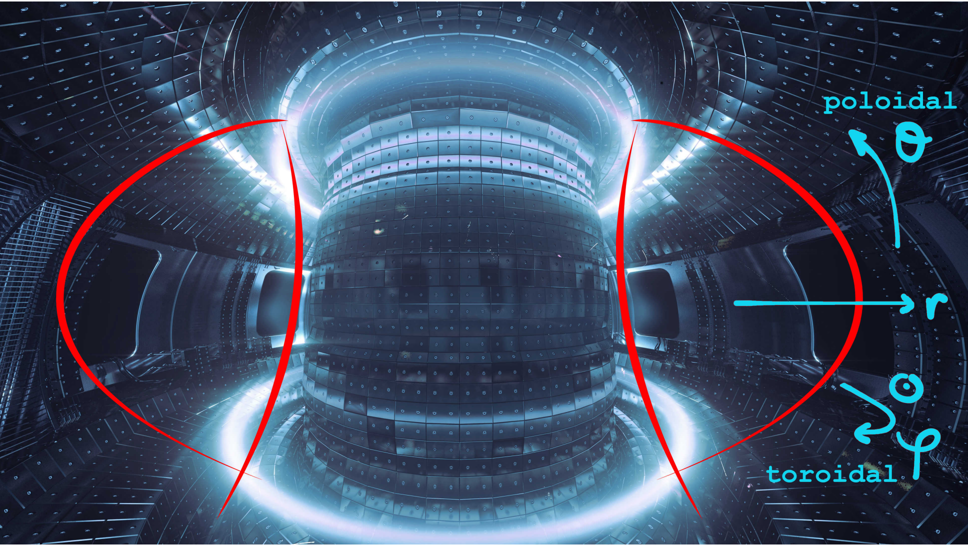
→ since 2001: 12 editions [$\sim$ 100 participants]
plasmas / neutral fluids / GFD / astrophysics / complex systems
special mention to 2009 (5th) Festival & M. McIntyre's opening lecture

Tokamak magnetised plasmas, a brief primer



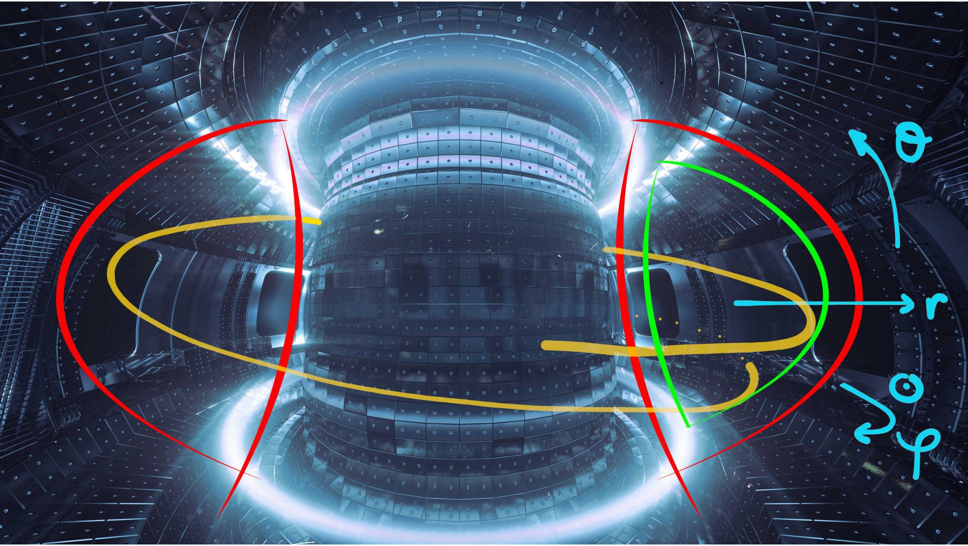
Visible light emission is where plasma is cold

Tokamak magnetised plasmas, a brief primer



Magnetic outer boundary of confined plasma

Tokamak magnetised plasmas, a brief primer



0^{th} order: helical traject. *[many complications: rational q, trapping, stochasticity, ...]*

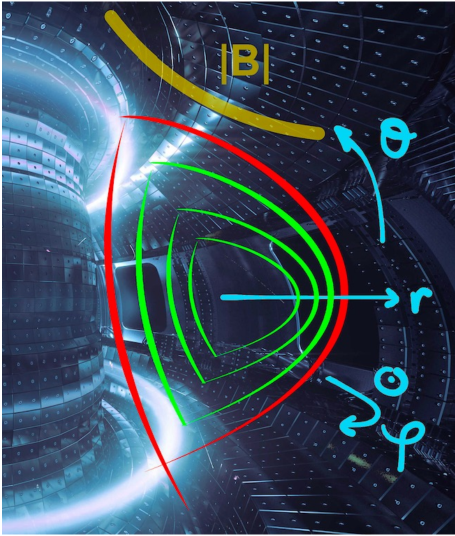
→ Large free energy sources:
↳ ∇T, ∇n, ∇B, ∇I, ...

→ wealth of instabilities
↳ MHD $(\mathbf{B}_0 + \delta\mathbf{B})$ & $\underbrace{\boxed{\text{turb. } (\mathbf{E} = -\nabla\delta\phi)}}_{\text{gyrokinetics}}$

→ strong anisotropy $[\sim 2d]$, i.e. $\frac{k_\parallel}{k_\perp} \sim 10^{-8}$

→ slow dyn. $[\ll \omega_c]$ → $\mathbf{v} \equiv v_\parallel \mathbf{b} + \boxed{\mathbf{v}_\perp}$

➤ model cross-field transport



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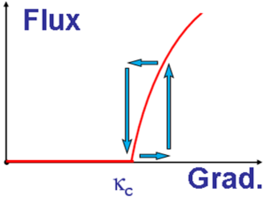
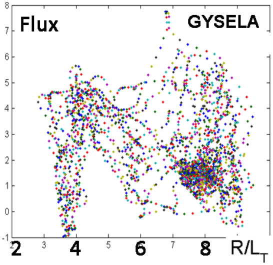
➤ model cross-field transport

→ slow drive & fast relax. processes
↳ slow modes key ➠ near-marginal operation





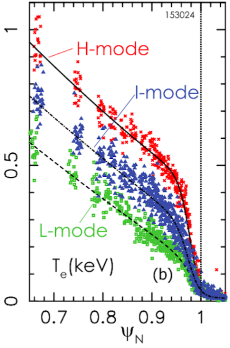
➤ **non-linear saturation & control?** transport barriers & staircases

transport barriers required

❶ internal: q profile tailoring (**B** field geom.), rotation, ...

❷ external: helped revitalise fusion programme [Wagner PRL 82]

↳ boundary-influenced bifurcations/transitions
↳ come in many flavours (H-, I-, QH-modes, ...)
↳ often prone to deleterious instabilities (MHD)



[Hubbard NF 16]

common grounds? ≡ shear feedback

Nonlin. sat. turb. #1: large-scale modes & zonal flows

[e.g. Diamond PPCF 05]

- minimal inertia & minimal damping
- benign repository for free energy

NB: better one large barrier or many small ones?

➤ other nonlinear saturation mechanisms? → **evolution of vorticity** key

$$\underbrace{\partial_t \langle v_y \rangle}_{\text{ZF}} = \underbrace{-\partial_x \langle v_x \, v_y \rangle}_{\text{Reynolds force}} = \langle v_{Ex} \, \boxed{\text{PV}} \rangle, \text{ with}$$

$$\left(\frac{\partial}{\partial t} + u_g \cdot \nabla\right) \Big[\underbrace{h - \nabla^2 h + \ln \frac{H_0}{f}}_{\text{planetary PV}} \Big] = 0$$

$$\left(\frac{\partial}{\partial t} + u_{\mathbf{E}\times\mathbf{B}} \cdot \nabla\right) \Big[\underbrace{\phi - \nabla^2 \phi + \ln \frac{n_0}{\omega_{c,i}}}_{\text{plasma PV}} \Big] = 0$$

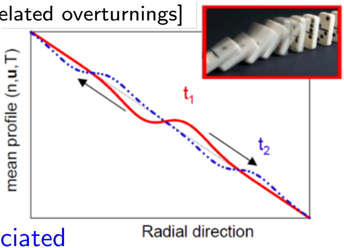
another means for NL sat. #2: **mean gradient** dynamics

↳ turbulence spreading, fronts, avalanches [≡ correlated overturnings]

↳ role of forcing & bound. conditions

↳ ubiquitous in modelling;

↳ elusive experimentally. . . increasing body of experimental evidence [e.g. Ida NF 16; Hahm KPS 18]



However:

precise importance still not fully appreciated

emphasise multi-scale (fluctuation) ↔ (mean profile) interplay

distrib. of scales $\Delta_{\text{[aval.]}} \in ((\rho_i \to \mathcal{L}_{system}))$ → potentially BAD

The name of the game so far? $\equiv$ study <u>secondary patterns</u>

[focus on ZFs and (much less) fronts/avalanches]

natural **secondary patterns** for DW/interchange turbulence. . . mutually-exclusive:

- zonal flows: benign, shear feedback, transport barrier
- "avalanches": detrimental, potentially scale-independent mixing

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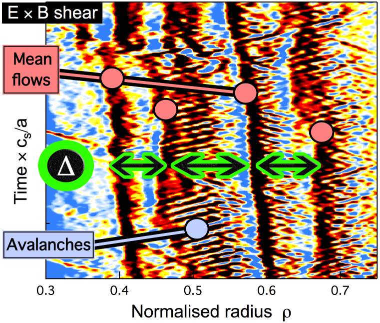
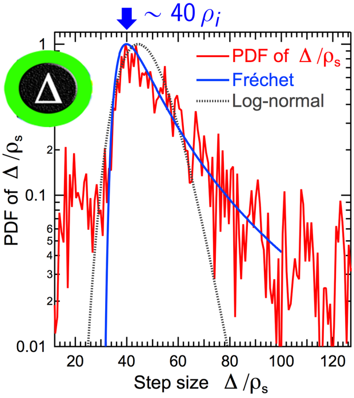
coexistence?

$\exists$ in $\neq$ parameter space?
$\exists$ alternately in time?
$\rightarrow$ not really: ZFs & avalanches co-exist near-marginality

separation in space $\rightarrow$ emergent mesoscale [distrib. of scales]
tertiary structure: the $\mathbf{E} \times \mathbf{B}$ staircase

[Dif-Pradalier, PRE 10, PRL 15 & NF 17] [Kosuga, PRL13 & PoP 14] [Ashourvan PRE 16 & PoP 17] [Rath PoP 16, 18, 21, 22], [Hornung NF 17], [Wang NF 18, 20], [Qi NF 21], [Milovanov PRE 21], [Qinghao NF 22], ...





[Dif-Pradalier, PRE 10 & NF 17]

1) relevant scale is Δ: $Q = \int \mathcal{K}(r,r')\nabla T(r')\,\mathrm{d}r'$; $\mathcal{K}(r,r') = \frac{S}{\pi}\frac{-\Delta/2}{(\Delta/2)^2+(r-r')^2}$

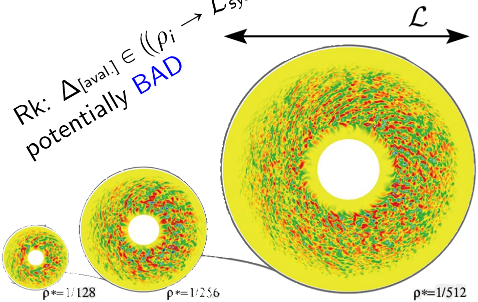
2) barriers for transport @‘avalanche’ scale $\underbrace{\Delta \sim \text{const.}\ \forall\ \text{size}}_{\text{tamed } \propto \rho_i}$ **yet still** $\exists$ $\underbrace{\Delta \sim \mathcal{L}}_{\text{untamed}}$

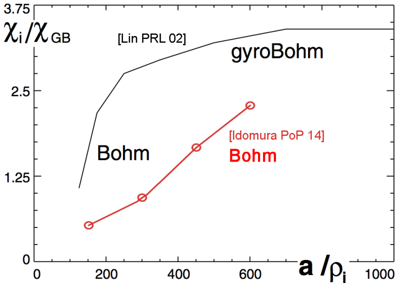
3) <u>existence:</u> near marginal stability ; <u>robust:</u> kin. elec. response & E-M fluct.

[Rath, PoP 21 & 22]

Why care? *1) bigger should be better...*

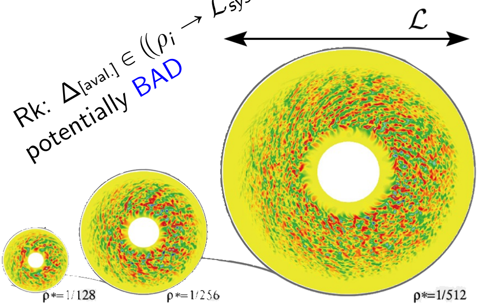
Rk: $\Delta_{[aval.]} \in ((\rho_i \to \mathcal{L}_{system}))$ potentially BAD

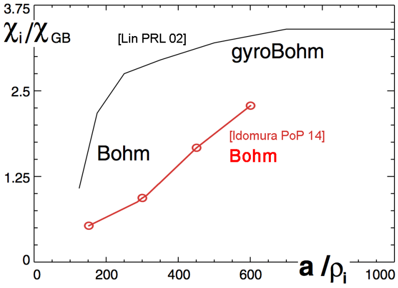




confinement $D \sim v_\star \ell_{mix} \rightarrow$ **relevant ℓ_{mix}?** $\equiv \{\ell_c$ (micro) ; Δ (meso) ; $\mathcal{L}$(macro)$\}$

Rk: $\Delta_{[aval.]} \in ((\rho_i \to \mathcal{L}_{system}))$ potentially BAD





confinement $D \sim v_\star \ell_{mix}$ ➞ **relevant ℓ_{mix}?** $\equiv \{\ell_c$ (micro) ; Δ (meso) ; $\mathcal{L}$(macro)$\}$

possible resolution:

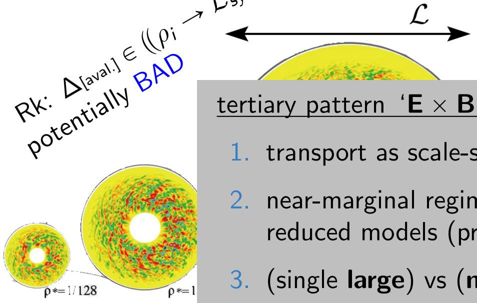
[Dif-Pradalier NF 17]

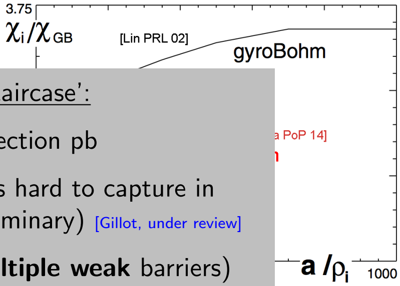
distance to marginal stability matters

with staircase $\equiv \ell_{mix} \sim \Delta$ (meso) $\propto \rho_i$ ➞ gyro-Bohm

w/o staircase $\equiv \ell_{mix} \to \mathcal{L}_{system}$ ➞ Bohm

Rk: $\Delta_{[aval.]} \in ((\rho_i \to \mathcal{L}_{system}))$
potentially BAD





tertiary pattern '**E** × **B** staircase':

1. transport as scale-selection pb
2. near-marginal regimes hard to capture in reduced models (preliminary) [Gillot, under review]
3. (single **large**) vs (**multiple weak** barriers)

➤ experimental evidence?

➤ theoretical understanding?

confinement L ... ; $\mathcal{L}_{(\text{macro})}\}$

possible resolution: [Dif-Pradalier NF 17]

distance to marginal stability matters

with staircase $\equiv \ell_{mix} \sim \Delta_{(\text{meso})} \propto \rho_i$ ➞ gyro-Bohm

w/o staircase $\equiv \ell_{mix} \to \mathcal{L}_{system}$ ➞ Bohm

E $\times$ **B** staircase as tertiary structure ➞ natural outcome of DW/interchange turb.

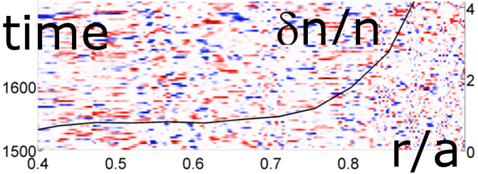
↳ if so, why direct observation so scarce?

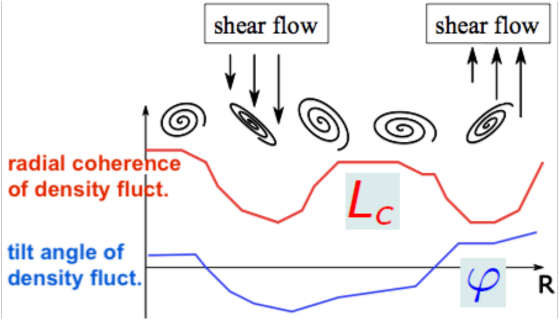
- $\tilde{p}$ fluct. difficult, ∇p better ➞ **lack (r,t) resolution**
- avalanches controversial (despite progress)
- $\tilde{\phi}$ or flows better ➞ direct ZF measurement notoriously difficult [Fujisawa PRL 04]
  + dynamics

➤ **a possible way forward:** <u>fast-swept relectometry</u>

- non-perturbative
- full **profile**
- **high (r,t)** resolution

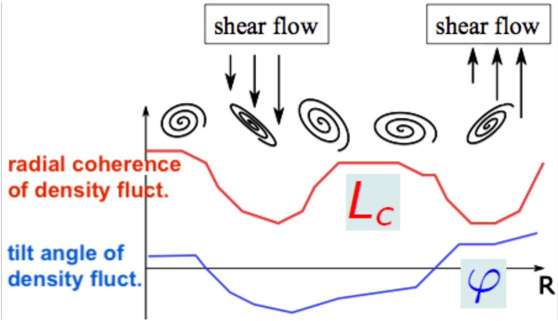
[Clairet RSI 10, Hornung PPCF 13]

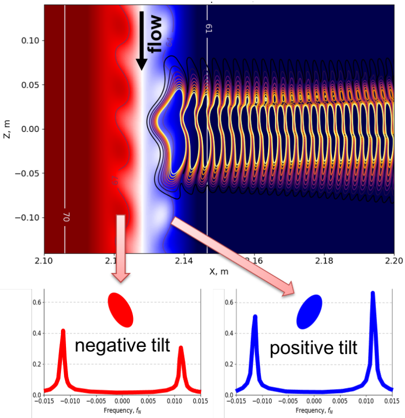




Expected staircase signatures:

- local minima of the fluctuations size,
- sudden variation of the tilt angle, etc.





Expected staircase signatures:

- local minima of the fluctuations size,
- sudden variation of the tilt angle, etc.

confirmation from synthetic diagn.

[Medvedeva 22]

Main predictions recovered

❶ quasi-regularly spaced local minima turb. correlation lengths. . .

❷ . . . sudden tilt in fluctuations angle

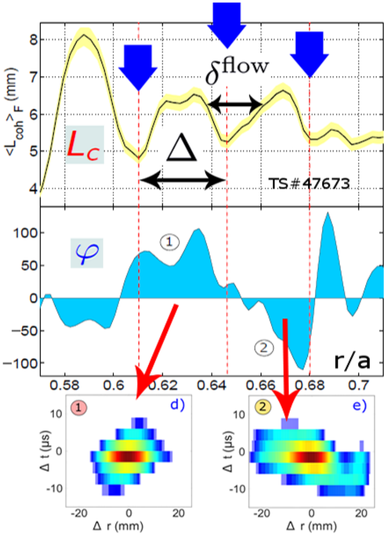
❸ $\delta^{\text{flow}} \approx 11 \pm 4\rho_s$ [Hahm PPCF 00 ; Fujisawa PRL 04]

❹ $\Delta \sim 40\rho_s$ & mesoscale [$\sim 30 \rightarrow 100\rho_s$]

❺ weakly correlated to q profile

Fast-sweep. reflecto. finds robust staircase signatures in Tore Supra

[Dif-Pradalier, PRL 15; Hornung NF 17]



prediction

experiment

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❷ . . . sudden tilt i

❸ $\delta^{\text{flow}} \approx 11 \pm 4$

❹ $\Delta \sim 40\rho_s$ &

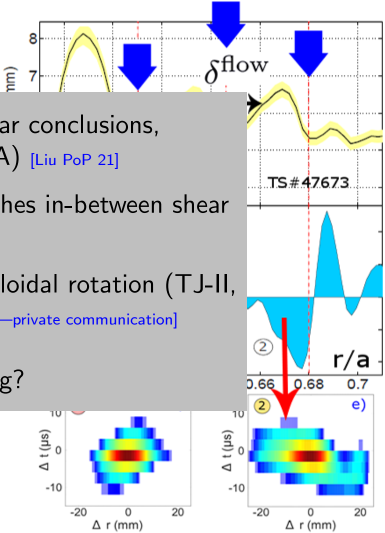
❺ weakly correlat

1. similar techniques, similar conclusions, different machine (HL2A) [Liu PoP 21]
2. identification of avalanches in-between shear layers (Kstar) [Choi, NF 19]
3. direct measurements poloidal rotation (TJ-II, stellarator) [Van Milligen, NF —private communication]

➤ theoretical understanding?

Fast-sweep. reflecto. finds robust staircase signatures in Tore Supra

[Dif-Pradalier, PRL 15; Hornung NF 17]



prediction

experiment

If we are to better understand this, we need simple models

- robust control parameters, so far: near-marginal & low-collisional; I_p?
- quantitative theory still in its infancy, yet interesting ideas:

generic → **plasma-specific**

- phase slip
  [Guo PRL 17]
- time delay
  [Kosuga PRL 13 & PoP 14]
- bistability
  [Balmforth JFM 98]

- wave trapping
  [Garbet PoP 21]
- NL Schrödinger
  [Milovanov PRE 21]

- resonant/non-resonant
  [Qinghao NF 22]

generic setup:

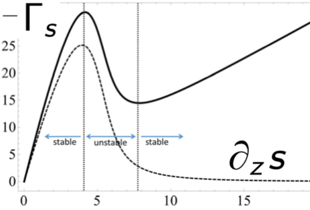
uniform density gradient [along z] $+$ fluid is stirred
robust outcome ➞ well-mixed layers with sharp interfaces

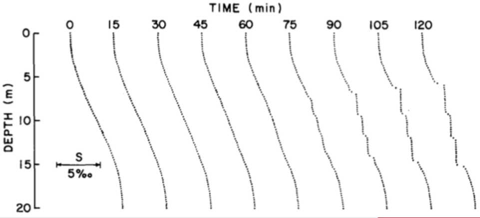
[Phillips DSRO 72; Posmentier JPO 77]

fecund modelling framework:

➤ 1d diffusion: $\partial_t s = -\partial_z \Gamma_s$, with flux $\Gamma_s = f(\partial_z s)$

↳ $\partial_t s = -\underbrace{\frac{\partial \Gamma_s}{\partial_{\partial_z s}}}_{>0 \,\Rightarrow\, \text{possible instability}} \partial_z^2 s$





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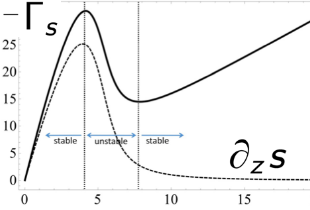
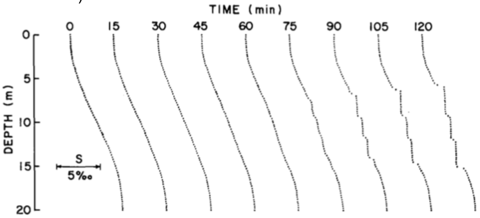


➤ issues: [Pruzina PRF 22]
ill-posed $\lim_{k\to\infty} \gamma_{lin}(k) = \infty$
interfaces $\to \infty$ thin
link absent $(s \ or \ \Gamma_s) \leftrightarrow (\tilde{v} \ or \ \tilde{v}^2)$
physics of $\Gamma_s = f(\partial_z s)$?



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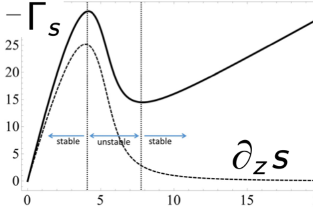

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[Balmforth JFM 98]
➤ [Ashourvan PRE16]
[Guo PPCF19]

$\Gamma_s = -\ell\, \tilde{v}\, \partial_z s$
$\ell = g(\tilde{v}\,;\, \partial_z s)$ interpolates btw
2 typical scales; #1: **<u>intrinsic</u>** (stirring) ; #2: **<u>emergent</u>**

Bi-stability may generate staircase-like patterns
—Idea after [Balmforth *et al.*, J. Fluid. Mech. 98]

- start: H–W eqs [Ashourvan PRE16, PoP 17; Guo PPCF19]

$$d_t\nabla^2\phi = \eta\nabla_\parallel^2[\log n - \phi] + \mu\nabla_\perp^4\phi$$
$$d_t \log n = \eta\nabla_\parallel^2[\log n - \phi] + D\nabla_\perp^2 n$$

- expand vorticity, density & PV$\equiv n-u$ in terms of mean/fluct.

$$\partial_t\bar\zeta + \partial_x\underbrace{\langle\tilde v_x\tilde\zeta\rangle}_{\text{flux}} = \text{dissip.} + (\text{ext.production})$$

- key: mixing of PV ➞ $\zeta = PV = q$
  - assume diffusive closure $\langle\tilde v_x\tilde q\rangle \equiv -D\partial_x\bar q$
  - dimensional args: $D \sim \ell^2\,\tilde q^2/\alpha_\parallel$
- what prescription for ℓ ?
  $\exists$ at least 2 scales in model:

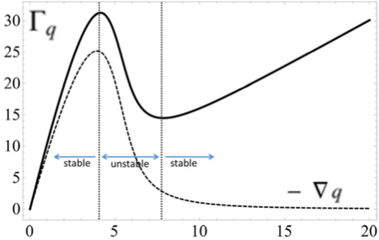
❶ length scale $\ell_F \equiv$ constant forcing for turb.

❷ Rhines scale $\ell_R \equiv$ cross-over from: [Malkov PRF 19, Pruzina PRF 22]

- eddy-like $\tau_{ov}^{-1} \sim \tilde v/\ell_R \sim \tilde q$
- wave-like $\omega_{DW} \sim \frac{k}{1+k^2}v_D \sim \frac{k}{1+k^2}\nabla\bar n \sim \ell_R\nabla\bar q$
- equating both: $\ell_R \sim \frac{\tilde q}{\nabla\bar q}$

- 'combined' mixing length
  $\frac{1}{\ell^2} \equiv \frac{1}{\ell_F^2}\left(1+\frac{\ell_F^2}{\ell_R^2}\right)^\beta \Rightarrow$ flux:

$$\Gamma_q \sim \frac{1}{[1+\ell_F^2(\nabla\bar q)^2/\tilde q^2]^\beta}\nabla\bar q$$



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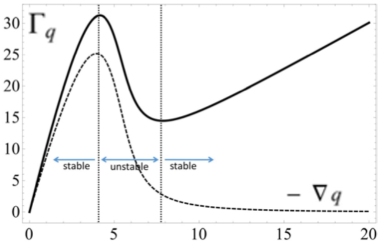

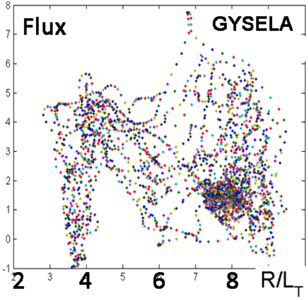

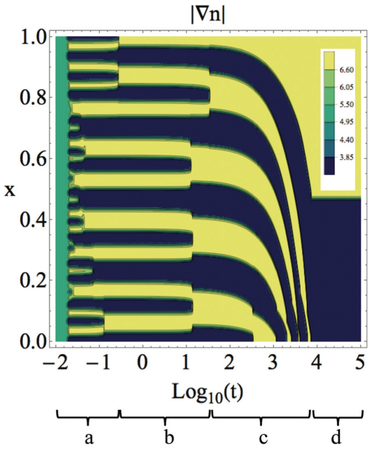

- stability of shear layers? (log. time scale!)
- Competition mergers/re-formation?
  - $\rightarrow$ what evidence from experiments?

Underlying principles?

generic → **plasma-specific**

- phase slip
  [Guo PRL 17]
- time delay
  [Kosuga PRL 13 & PoP 14]
- bistability
  [Balmforth JFM 98]

- wave trapping
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- NL Schrödinger
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- resonant/non-resonant
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— heat avalanches ↔ traffic flow
— time delay btw heat flux & temperature gradient ↔ driver response time
— modified Burgers equation → clustering instability

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— DW/ZFs $\equiv$ (WKE) $\leftrightarrow$ (zonal mom. cons.) $\rightarrow$ radially-periodic NL solutions
[nonlinear solution WKE $\rightarrow$ wave action density $\rightarrow$ Reyn. stress $\rightarrow$ ZF]
— instability $\rightarrow$ (i) wave trapping due to refraction by ZFs
— (ii) passing waves propagate at group velocity $\leftrightarrow$ avalanches
— staircase motif $\equiv$ interaction btw (i) & (ii)

generic → **plasma-specific**

- phase slip
  [Guo PRL 17]
- time delay
  [Kosuga PRL 13 & PoP 14]
- bistability
  [Balmforth JFM 98]
- wave trapping
  [Garbet PoP 21]
- NL Schrödinger
  [Milovanov PRE 21]
- resonant/non-resonant
  [Qinghao NF 22]

— staircase ≡ wavepacket coupled NL oscillators (node=ZF; spring=avalanche)
— assume: NLSE with subquadratic nonlinearity
  ➞ (i) localisation + (ii) vibrations with white Lévy noise ↔ avalanche
— staircase motif ≡ dynamic equil. @"edge" localisation[i]/delocalisation[ii]
  ➞ predict event-size distribution of plasma avalanches ↔ GYSELA

Underlying principles?

generic **plasma-specific**

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— mixing: through wave–particle resonances $\leftrightarrow$ part. orbits leave footprint SC
— tuning/detuning by ZFs of part. orbits $\rightarrow$ resonant or non-resonant transport
— $\ell \in [\ell_{forcing}\,;\ \ell_O]$ in [Balmforth JFM 98] $\leftrightarrow$ 2 states of mixing: resonant/nonresonant
$\mathcal{D}_r \neq \mathcal{D}_{nr} \rightarrow$ Turing patterns $\equiv$ staircase

Propensity

- large sources of free energy, slowly-driven, fast relax. ➞ near-marginal
- concentrate energy & momentum ➞ ZFs, barriers for transport & bifurcations
- intermittent relaxations of free energy content ➞ 'avalanches'/fronts,...

➤ **E** × **B** staircase solves pattern coexistence pb

- Transport problem ↔ scale selection problem
  - ↳ what physics sets intensity & scale Δ? ➞ can we predict & control?
  - ↳ multiple interplaying scales/instabilities? role of avalanches?
  - ↳ implications for modelling?
- Triggered by theory/modelling, beginning of experimental observations
- General understanding of layering/anti-diffusion captivating yet still elusive

➤ Amends understanding of transport in plasma core

- altern. route → improved confinement? [≡ single large vs. many weak barriers]