

Physics of $E \times B$ Staircases

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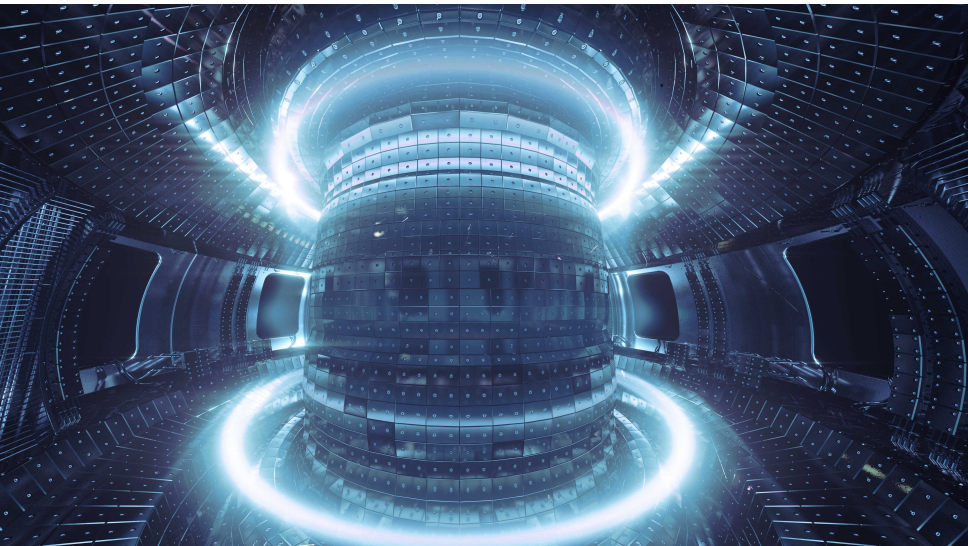
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Ackn.: D.W. Hughes & "Festival de Théorie" participants, Aix-en-Provence

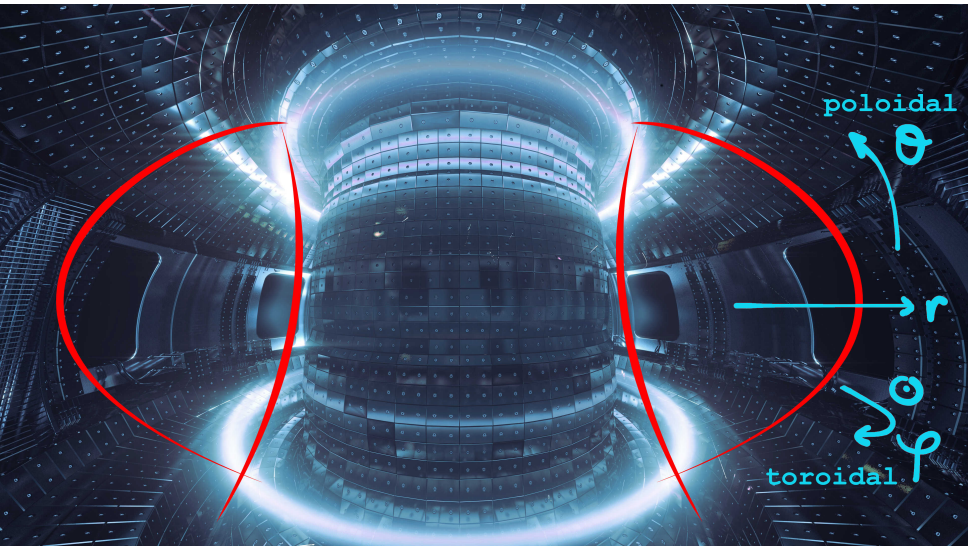
- since 2001: 12 editions [\sim 100 participants]
- *plasmas / neutral fluids / GFD / astrophysics / complex systems*
- *special mention to 2009 (5th) Festival & M. McIntyre's opening lecture*

Tokamak magnetised plasmas, a brief primer



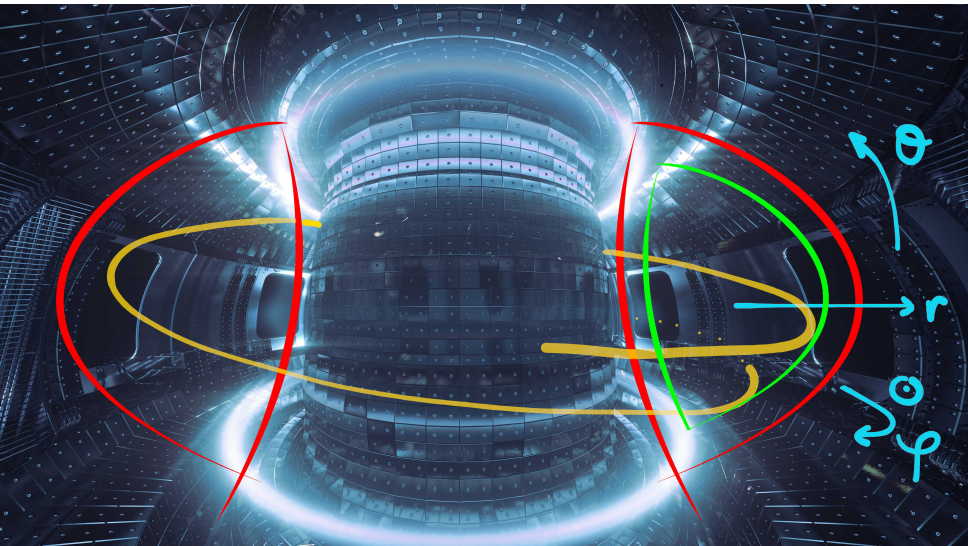
Visible light emission is where plasma is cold

Tokamak magnetised plasmas, a brief primer



Magnetic outer boundary of confined plasma

Tokamak magnetised plasmas, a brief primer



0th order: helical traject. [*many complications: rational q, trapping, stochasticity, ...*]

Turbulence is \sim quasi 2d [i.e. on flux surfaces $\simeq (\theta, \varphi)] \dots$ \rightarrow cross-field [r] transport key

\rightarrow Large free energy sources:

$$\hookrightarrow \nabla T, \nabla n, \nabla B, \nabla I, \dots$$

\rightarrow wealth of instabilities

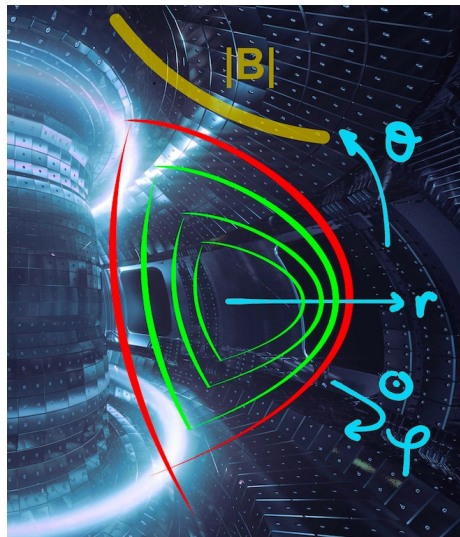
$$\hookrightarrow \text{MHD } (\mathbf{B}_0 + \delta\mathbf{B}) \ \& \ \boxed{\text{turb. } (\mathbf{E} = -\nabla\delta\phi)}$$

gyrokinetics

\rightarrow strong anisotropy [$\sim 2d$], i.e. $\frac{k_{\parallel}}{k_{\perp}} \sim 10^{-8}$

\rightarrow slow dyn. [$\ll \omega_c$] $\rightarrow \mathbf{v} \equiv v_{\parallel} \mathbf{b} + \boxed{\mathbf{v}_{\perp}}$

\blacktriangleright model cross-field transport



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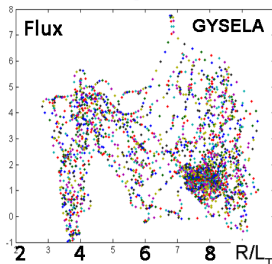
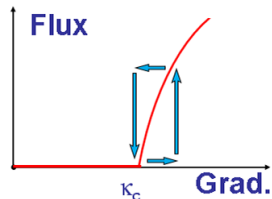
\rightarrow slow dyn. [$\ll \omega_c$] $\rightarrow \mathbf{v} \equiv v_{\parallel}\mathbf{b} + \boxed{\mathbf{v}_{\perp}}$

\blacktriangleright model cross-field transport

\rightarrow slow drive & fast relax. processes

\hookrightarrow **slow modes** key \Rightarrow **near-marginal** operation

\blacktriangleright **non-linear saturation & control?** transport barriers & staircases



transport barriers required

- ① internal: q profile tailoring (**B** field geom.), rotation, ...
- ② external: helped revitalise fusion programme [Wagner PRL 82]
 - ↳ boundary-influenced bifurcations/transitions
 - ↳ come in many flavours (H-, I-, QH-modes, ...)
 - ↳ often prone to deleterious instabilities (MHD)

common grounds? ≡ shear feedback

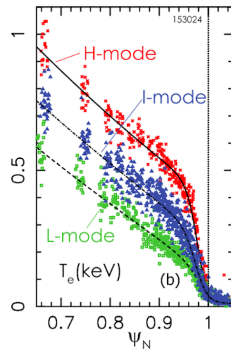
Nonlin. sat. turb. #1: large-scale modes & zonal flows

[e.g. Diamond PPCF 05]

- minimal inertia & minimal damping
- benign repository for free energy

NB: better **one large barrier** or **many small** ones?

➤ other nonlinear saturation mechanisms? → **evolution of vorticity** key

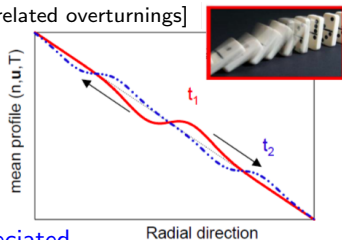


[Hubbard NF 16]

$$\underbrace{\partial_t \langle v_y \rangle}_{\text{ZF}} = \underbrace{-\partial_x \langle v_x v_y \rangle}_{\text{Reynolds force}} = \langle v_{E \times B} \text{ PV} \rangle, \text{ with } \left\{ \begin{array}{l} \left(\frac{\partial}{\partial t} + u_g \cdot \nabla \right) \left[\underbrace{h - \nabla^2 h + \ln \frac{H_0}{f}}_{\text{planetary PV}} \right] = 0 \\ \left(\frac{\partial}{\partial t} + u_{E \times B} \cdot \nabla \right) \left[\underbrace{\phi - \nabla^2 \phi + \ln \frac{n_0}{\omega_{c,i}}}_{\text{plasma PV}} \right] = 0 \end{array} \right.$$

another means for NL sat. #2: mean gradient dynamics

- ↳ turbulence **spreading, fronts, avalanches** [≡ correlated overturnings]
- ↳ role of forcing & bound. conditions
- ↳ ubiquitous in modelling;
- ↳ elusive experimentally... increasing body of experimental evidence [e.g. Ida NF 16; Hahm KPS 18]



precise importance still **not fully appreciated**
 emphasise multi-scale (**fluctuation**) ↔ (**mean profile**) interplay
 distrib. of scales $\Delta_{[aval.]} \in ((\rho_i \rightarrow \mathcal{L}_{system})) \rightarrow$ potentially **BAD**

However:

The name of the game so far? \equiv study secondary patterns

[focus on ZFs and (much less) fronts/avalanches]

natural **secondary patterns** for DW/interchange turbulence. . . mutually-exclusive:

- zonal flows: benign, shear feedback, transport barrier
- “avalanches”: detrimental, potentially scale-independent mixing

The name of the game so far? \equiv study secondary patterns

[focus on ZFs and (much less) fronts/avalanches]

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coexistence?

\exists in \neq parameter space?

\exists alternately in time?

→

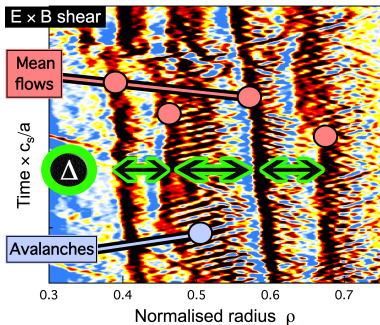
not really: ZFs & avalanches
co-exist near-marginality

separation in space →

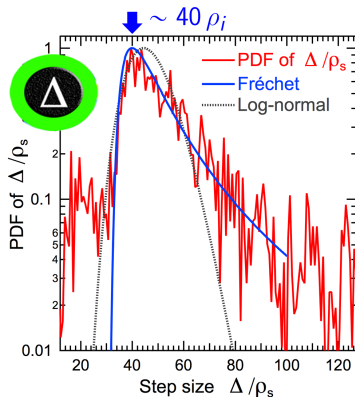
emergent mesoscale [distrib. of scales]

tertiary structure: the $\mathbf{E} \times \mathbf{B}$ staircase

[Dif-Pradalier, PRE 10, PRL 15 & NF 17] [Kosuga, PRL13 & PoP 14] [Ashourvan PRE 16 & PoP 17] [Rath PoP 16, 18, 21, 22], [Hornung NF 17], [Wang NF 18, 20], [Qi NF 21], [Milovanov PRE 21], [Qinghao NF 22], ...

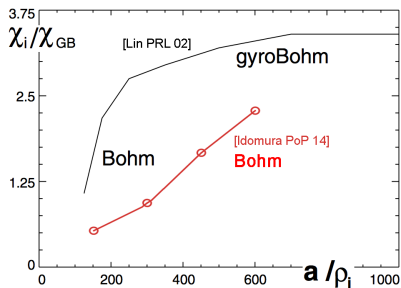
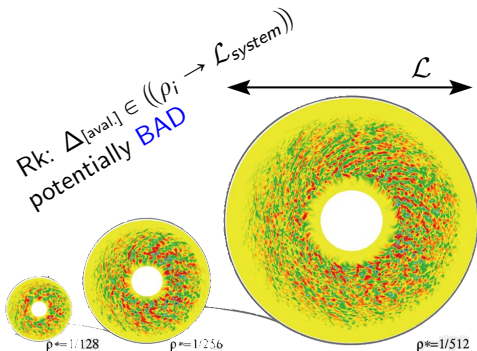


[Dif-Pradalier, PRE 10 & NF 17]

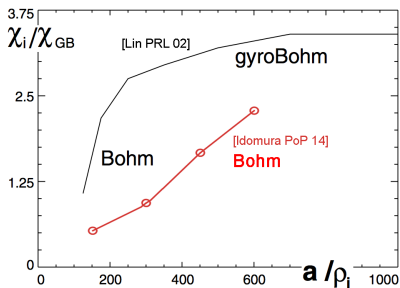
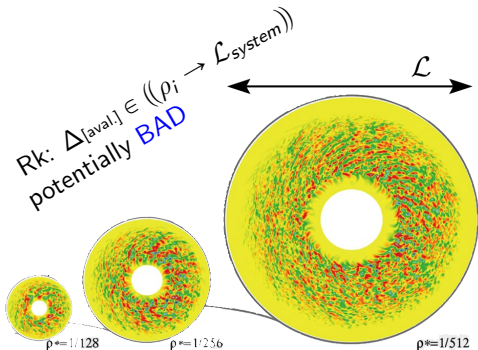


- 1) relevant scale is Δ : $Q = \int \mathcal{K}(r, r') \nabla T(r') dr'$; $\mathcal{K}(r, r') = \frac{S}{\pi} \frac{-\Delta/2}{(\Delta/2)^2 + (r-r')^2}$
- 2) barriers for transport @'avalanche' scale $\Delta \sim \text{const. } \forall \text{ size}$ **yet still** $\exists \Delta \sim \mathcal{L}$
tamed $\propto \rho_i$ untamed
- 3) existence: near marginal stability ; robust: kin. elec. response & E-M fluct.

[Rath, PoP 21 & 22]



confinement $D \sim v_* l_{mix} \rightarrow$ **relevant l_{mix} ?** $\equiv \{l_c \text{ (micro)}; \Delta \text{ (meso)}; \mathcal{L} \text{ (macro)}\}$



confinement $D \sim v_* \ell_{mix} \rightarrow$ **relevant ℓ_{mix} ?** $\equiv \{\ell_c \text{ (micro)}; \Delta \text{ (meso)}; \mathcal{L} \text{ (macro)}\}$

possible resolution:

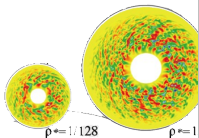
[Dif-Pradalier NF 17]

distance to **marginal stability matters**

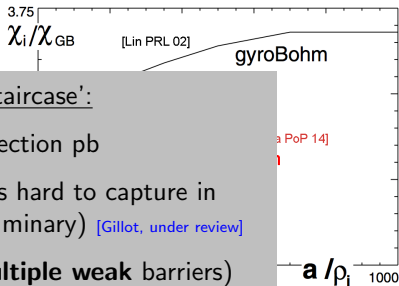
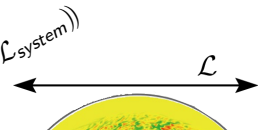
with staircase $\equiv \ell_{mix} \sim \Delta \text{ (meso)} \propto \rho_i \rightarrow$ gyro-Bohm

w/o staircase $\equiv \ell_{mix} \rightarrow \mathcal{L}_{system} \rightarrow$ Bohm

Rk: $\Delta_{[aval.]} \in ((\rho_i \rightarrow \mathcal{L}_{system}))$
 potentially **BAD**



confinement \mathcal{L}



tertiary pattern 'E x B staircase':

1. transport as scale-selection pb
2. near-marginal regimes hard to capture in reduced models (preliminary) [Gillot, under review]
3. (single **large**) vs (**multiple weak** barriers)

- experimental evidence?
- theoretical understanding?

possible resolution:

[Dif-Pradalier NF 17]

distance to **marginal stability matters**

with staircase $\equiv \ell_{mix} \sim \Delta_{(meso)} \propto \rho_i \rightarrow$ gyro-Bohm

w/o staircase $\equiv \ell_{mix} \rightarrow \mathcal{L}_{system} \rightarrow$ Bohm

$\mathbf{E} \times \mathbf{B}$ staircase as tertiary structure \rightarrow natural outcome of DW/interchange turb.

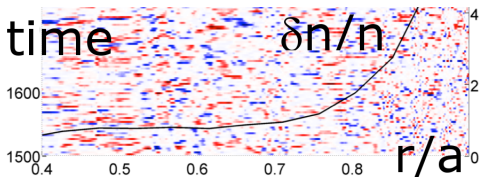
\hookrightarrow if so, why direct observation so scarce?

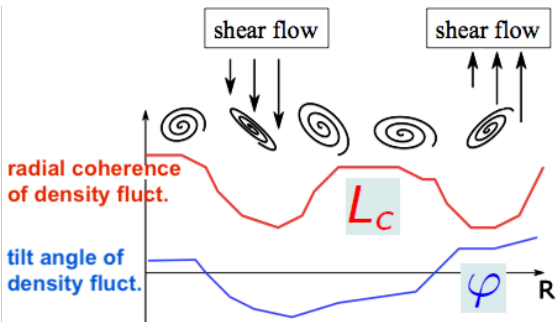
- ▶ \tilde{p} fluct. difficult, ∇p better \rightarrow **lack (r,t) resolution**
- ▶ avalanches controversial (despite progress)
- ▶ $\tilde{\phi}$ or flows better \rightarrow direct ZF measurement notoriously difficult [Fujisawa PRL 04]
+ dynamics

▶ **a possible way forward:** fast-swept relectometry

- non-perturbative
- full **profile**
- **high (r,t) resolution**

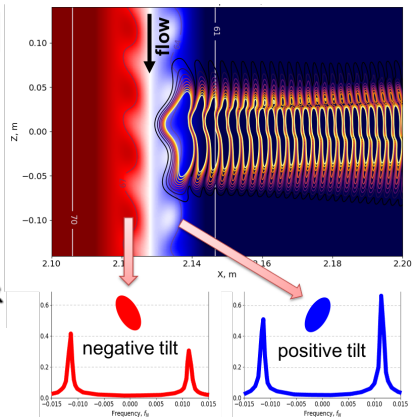
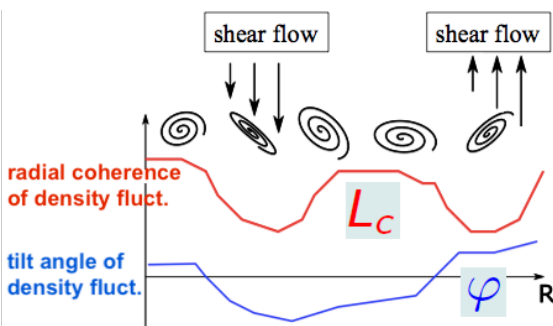
[Clairet RSI 10, Hornung PPCF 13]





Expected staircase signatures:

- ▶ local minima of the fluctuations size,
- ▶ sudden variation of the tilt angle, etc.



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confirmation from synthetic diagn.

[Medvedeva 22]

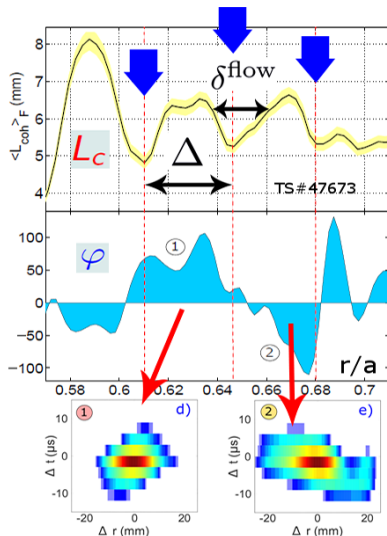
Main predictions recovered

- ① quasi-regularly spaced local **minima** turb. correlation lengths. . .
- ② . . . sudden **tilt in fluctuations angle**
- ③ $\delta^{flow} \approx 11 \pm 4 \rho_s$ [Hahm PPCF 00 ; Fujisawa PRL 04]
- ④ $\Delta \sim 40 \rho_s$ & **mesoscale** [$\sim 30 \rightarrow 100 \rho_s$]
- ⑤ **weakly correlated** to q profile

Fast-sweep. reflecto. finds robust staircase signatures in Tore Supra

[Dif-Pradalier, PRL 15; Hornung NF 17]

prediction



experiment

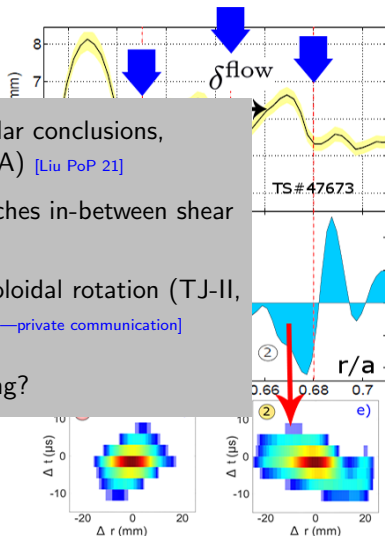
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- ① quasi-regularly correlated shear layers
- ② ... sudden tilt in shear layers
- ③ $\delta^{\text{flow}} \approx 11 \pm 4 \rho_s$
- ④ $\Delta \sim 40 \rho_s$ & $\Delta t \sim 10 \rho_s$
- ⑤ weakly correlated shear layers

1. similar techniques, similar conclusions, different machine (HL2A) [Liu PoP 21]
 2. identification of avalanches in-between shear layers (Kstar) [Choi, NF 19]
 3. direct measurements poloidal rotation (TJ-II, stellarator) [Van Milligen, NF —private communication]
- theoretical understanding?

Fast-sweep. reflecto. minus robust staircase signatures in Tore Supra

[Dif-Pradalier, PRL 15; Hornung NF 17]



prediction

experiment

If we are to better understand this, we need simple models

- robust control parameters, so far: near-marginal & low-collisional; I_p ?
- quantitative theory still in its infancy, yet interesting ideas:

generic

plasma-specific

- phase slip

[Guo PRL 17]

- time delay

[Kosuga PRL 13 & PoP 14]

- bistability

[Balmforth JFM 98]

- wave trapping

[Garbet PoP 21]

- NL Schrödinger

[Milovanov PRE 21]

- resonant/non-resonant

[Qinghao NF 22]



generic setup: | uniform density gradient [along z] + fluid is stirred
robust outcome \rightarrow well-mixed layers with sharp interfaces

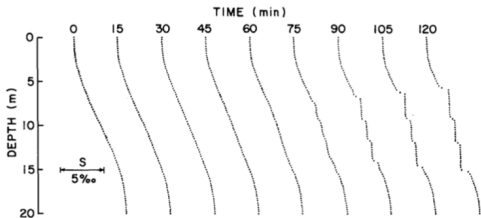
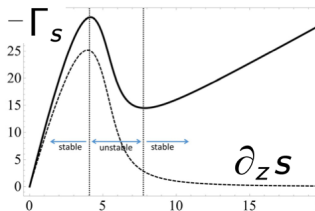
[Phillips DSRO 72; Posmentier JPO 77]

fecund modelling framework:

► 1d diffusion: $\partial_t s = -\partial_z \Gamma_s$, with flux $\Gamma_s = f(\partial_z s)$

$$\hookrightarrow \partial_t s = - \underbrace{\frac{\partial \Gamma_s}{\partial \partial_z s}}_{> 0} \partial_z^2 s$$

$> 0 \Rightarrow$ possible instability



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ill-posed $\lim_{k \rightarrow \infty} \gamma_{lin}(k) = \infty$

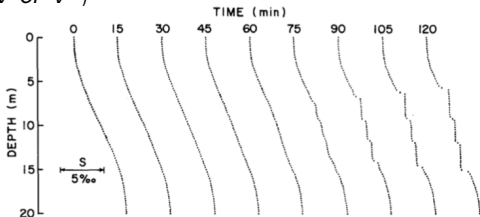
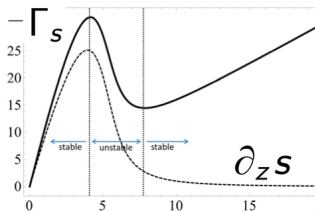
interfaces $\rightarrow \infty$ thin

link absent (s or Γ_s) \leftrightarrow (\tilde{v} or \tilde{v}^2)

physics of $\Gamma_s = f(\partial_z s)$?

► issues:

[Pruzina PRF 22]



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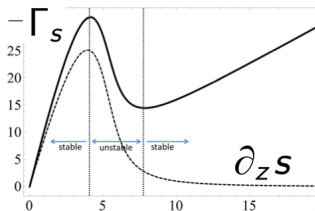
► [Ashourvan PRE16]

[Guo PPCF19]

$$\Gamma_s = -\ell \tilde{v} \partial_z s$$

$\ell = g(\tilde{v}; \partial_z s)$ interpolates btw

2 typical scales; #1: **intrinsic** (stirring) ; #2: **emergent**



—Idea after [Balmforth *et al.*, J. Fluid. Mech. 98]

- start: H–W eqs [Ashourvan PRE16, PoP 17; Guo PPCF19]

$$d_t \nabla^2 \phi = \eta \nabla_{\parallel}^2 [\log n - \phi] + \mu \nabla_{\perp}^4 \phi$$

$$d_t \log n = \eta \nabla_{\parallel}^2 [\log n - \phi] + D \nabla_{\perp}^2 n$$

- expand vorticity, density & PV $\equiv n - u$ in terms of mean/fluct.

$$\partial_t \bar{\zeta} + \partial_x \underbrace{\langle \tilde{v}_x \tilde{\zeta} \rangle}_{\text{flux}} = \text{dissip.} + (\text{ext. production})$$

- key: mixing of PV $\rightarrow \zeta = PV = q$

- ▶ assume diffusive closure $\langle \tilde{v}_x \tilde{q} \rangle \equiv -D \partial_x \bar{q}$
- ▶ dimensional args: $D \sim \ell^2 \tilde{q}^2 / \alpha_{\parallel}$

- what prescription for ℓ ?
 \exists at least 2 scales in model:

① length scale $\ell_F \equiv$ constant forcing for turb.

② Rhines scale $\ell_R \equiv$ cross-over from:

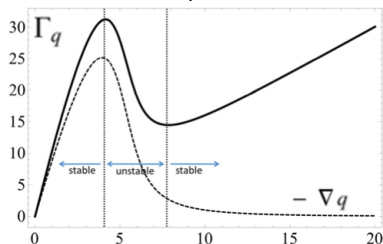
[Malkov PRF 19, Pruzina PRF 22]

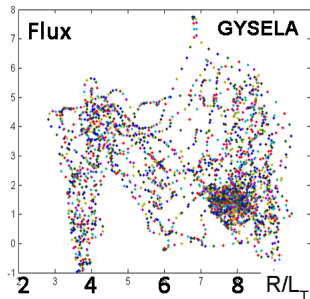
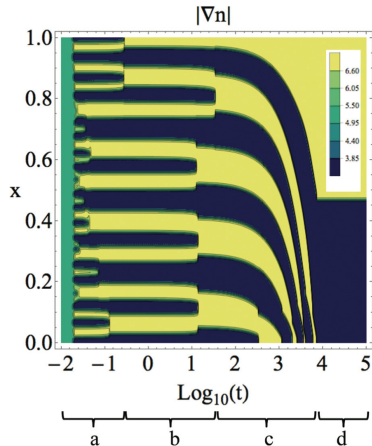
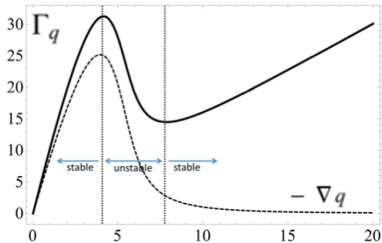
- ▶ eddy-like $\tau_{ov}^{-1} \sim \tilde{v} / \ell_R \sim \tilde{q}$
- ▶ wave-like $\omega_{DW} \sim \frac{k}{1+k^2} v_D \sim \frac{k}{1+k^2} \nabla \bar{n} \sim \ell_R \nabla \bar{q}$
- ▶ equating both: $\ell_R \sim \frac{\tilde{q}}{\nabla \bar{q}}$

- ‘combined’ mixing length

$$\frac{1}{\ell^2} \equiv \frac{1}{\ell_F^2} \left(1 + \frac{\ell_F^2}{\ell_R^2} \right)^{\beta} \Rightarrow \text{flux:}$$

$$\Gamma_q \sim \frac{1}{[1 + \ell_F^2 (\nabla \bar{q})^2 / \tilde{q}^2]^{\beta}} \nabla \bar{q}$$





- stability of shear layers? (log. time scale!)
- Competition mergers/re-formation?
→ what evidence from experiments?

generic

plasma-specific



- phase slip

[Guo PRL 17]

- time delay

[Kosuga PRL 13 & PoP 14]

- bistability

[Balmforth JFM 98]

- wave trapping

[Garbet PoP 21]

- NL Schrödinger

[Milovanov PRE 21]

- resonant/non-resonant

[Qinghao NF 22]

— heat avalanches \leftrightarrow traffic flow

— time delay btw heat flux & temperature gradient \leftrightarrow driver response time

— modified Burgers equation \rightarrow clustering instability

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- DW/ZFs \equiv (WKE) \leftrightarrow (zonal mom. cons.) \rightarrow radially-periodic NL solutions
[nonlinear solution WKE \rightarrow wave action density \rightarrow Reyn. stress \rightarrow ZF]
- instability \rightarrow (i) wave trapping due to refraction by ZFs
- (ii) passing waves propagate at group velocity \leftrightarrow avalanches
- staircase motif \equiv interaction btw (i) & (ii)

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— staircase \equiv wavepacket coupled NL oscillators (node=ZF; spring=avalanche)

— **assume**: NLSE with subquadratic nonlinearity

→ (i) localisation + (ii) vibrations with white Lévy noise \leftrightarrow avalanche

— staircase motif \equiv dynamic equil. @ "edge" localisation[i]/delocalisation[ii]

→ predict event-size distribution of plasma avalanches \leftrightarrow GYSELA

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- mixing: through wave-particle resonances \leftrightarrow part. orbits leave footprint SC
- tuning/detuning by ZFs of part. orbits \rightarrow resonant or non-resonant transport
- $\ell \in [\ell_{forcing}; \ell_0]$ in [Balmforth JFM 98] \leftrightarrow 2 states of mixing: resonant/nonresonant
- $\mathcal{D}_r \neq \mathcal{D}_{nr} \rightarrow$ Turing patterns \equiv staircase

Propensity

- large sources of free energy, slowly-driven, fast relax. → near-marginal
 - concentrate energy & momentum → ZFs, barriers for transport & bifurcations
 - intermittent relaxations of free energy content → 'avalanches'/fronts,...
-
- $\mathbf{E} \times \mathbf{B}$ staircase solves pattern coexistence pb
 - ▶ Transport problem ↔ scale selection problem
 - ↳ what physics sets intensity & scale Δ ? → can we predict & control?
 - ↳ multiple interplaying scales/instabilities? role of avalanches?
 - ↳ implications for modelling?
 - ▶ Triggered by theory/modelling, beginning of experimental observations
 - ▶ General understanding of layering/anti-diffusion captivating yet still elusive
 - Amends understanding of transport in plasma core
 - ▶ altern. route → improved confinement? [≡ single large vs. many weak barriers]