# Transport in Fusion Plasmas: Is the Tail Wagging the Dog?

Guilhem Dif-Pradalier, Philippe Ghendrih, Yanick Sarazin, Fabien Widmer, Yann Camenen, Xavier Garbet, Camille Gillot, Virginie Grandgirard, L. Vermare

# Transport in Fusion Plasmas: Is the Tail Wagging the Dog?

**#1: edge transport: new light on "shortfall" conundrum**

**#2: spontaneous confinement transition @separatrix**

G. Dif-Pradalier[1], Ph. Ghendrih[1], Y. Sarazin[1], F. Widmer[2], Y. Camenen[2], X. Garbet[1], C. Gillot[1], V. Grandgirard[1], L. Vermare[3]

[1]CEA, IRFM, France
[2]Aix Marseille Université, France
[3]LPP, Ecole Polytechnique, France

Ackn.: Festival de Théorie, Aix-en-Provence

# A major motivation: understand & reproduce whereby bifurcations to improved confinement occur

- spontaneous transitions come in many flavours: ITBs, $\{H; I; \dots\}$-modes
- common grounds: **turbulence** & self-reinforcing feedback
  - onset of differential rotation
  - steepening of $\nabla p_i$
  - electric field well (or hill) ➞ shear-induced bifurcation [Biglari PF 90, …]
- robustly observed (Tokamak, stellarator,…) upon modifications of edge/SOL operating **boundary conditions**

Global impact of localised boundary interactions is a classical problem

- fluids: Prandtl ; swirling flows [torque vs. velocity [Saint-Michel PRL 13]] ↔ **forcing**
- MFE: upstream [core & edge] impact of magn. connection to bound. [SOL/wall]

*[known importance of wall conditioning, recycling, etc.]*

take a step back ➞ focus on **low-confinement "L-mode" edge**
↳ **prerequisite** to understand **edge bifurcation(s)** *[Spoiler: interplay SOL ↔ edge ↔ core]*

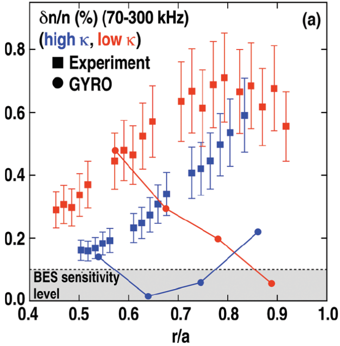
➤ Several (related) conundrums

❶ intrinsic problem [shortfall?] @plasma edge?

↳ edge often **linearly stable**

↳ where does edge turb. come from?

↳ propagation/contamination?




[Holland PoP 11]

# Some key questions: ❶ transition to turbulence? ❷ Causality in onset/sustainment of bifurcated state?

➤ Several (related) conundrums

❶ intrinsic problem [shortfall?] @plasma edge?

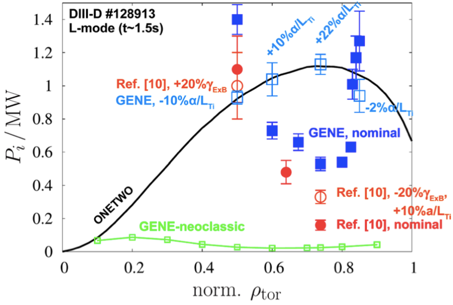
↳ edge often **linearly stable**

↳ where does edge turb. come from?

↳ propagation/contamination?

❷ important turb. properties:
all locally-determined?

❸ what presides over the onset of edge transport barrier? Mechanism(s)?




[Gorler PoP 14]

# What desirable/minimal set of ingredients?

- equil. gradient length $\sim$(few $\rho_i$) $\Rightarrow$ scale separations debatable in edge
  (gradient scale $\leftrightarrow F_{eq}$) $\not\gg$ (turb.scale $\leftrightarrow \delta f \equiv F - F_{eq}$)

- profiles $\equiv$ **large uncertainties** in edge; **poorly known** in SOL
  ↳ flux-driven desirable
  $\Rightarrow$ propagation of information on global scales
  $\Rightarrow$ add. symm.-breaking mechanisms edge turb.

- magnetic connection to material boundaries
  ↳ expect $E_r$ shear $(\nabla p/n$ vs. $-\nabla T_e)$ $\Rightarrow$ incidence on transp. barrier onset?

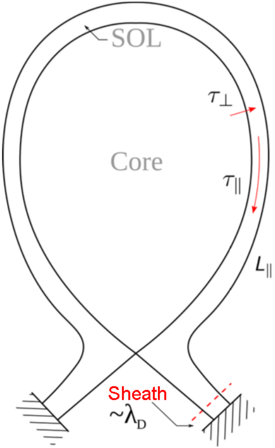
➤ this work: global organisation core $\leftrightarrow$ edge $\leftrightarrow$ SOL? *[Spoiler: strong interplay]*

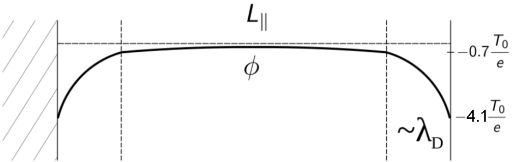
① **whereby edge transitions to / sustains a turbulent state?**

② **spontaneous transport barrier @separatrix**
↳ mechanisms whereby such bifurcations occur? causality?

[Dif-Pradalier, Communications Physics 2022]

# Accurate L–mode edge dynamics needs improved boundary conditions




$$\Delta\phi^{\text{sheath}} \underset{\text{fluid}}{\approx} \frac{T_e}{e}\frac{1}{2}\ln\left[\frac{m_e}{m_i}\left(1+\frac{T_i}{T_e}\right)\right] \underset{\text{(D)}}{\approx} -4.1\frac{T_e}{e}$$

[eg. Stangeby 2000]

Path followed:

simplified SOL physics within flux-driven GK...
...without resolving sheath (subgrid) & neutrals

- simplified limiter geom.
- simplified adiabatic electron response
- penalised approach
  - $F_s \to \mathbb{G}_{cold}$
  - $\phi \to \Delta\phi^{\text{sheath}}$

[Isoardi JCP10; Caschera Varenna 18]

# Minimally-relevant [first principles] model for L–mode edge dynamics



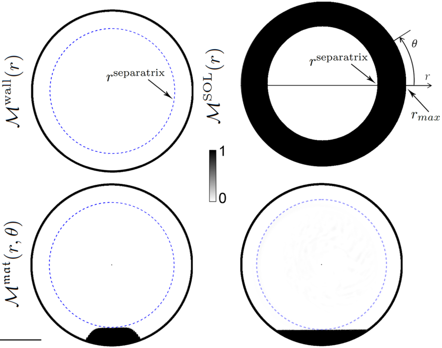
- $\dfrac{\mathrm{d}\bar{F}_s}{\mathrm{d}t} = \mathcal{C} + \mathcal{S} + \mathcal{D} - \nu\, \mathcal{M}^{\mathrm{mat}}(r,\theta)\,[\bar{F}_s - \mathbb{G}_{cold}]$

- $\Delta n_e = \rho + \frac{1}{n_{e_0}} \sum_s Z_s \nabla_\perp \cdot \left( \frac{n_{s_0}}{B\Omega_s} \nabla_\perp \phi \right)$

with $\frac{T_e}{e}\Delta n_e = \phi - \lambda\left[1 - \mathcal{M}^{SOL}(r)\right]\langle\,\phi\,\rangle_{\mathcal{FS}}$
$-\lambda \frac{e}{T_e}\Delta\phi^{\mathrm{sheath}}\left[\mathcal{M}^{SOL}(r) - \mathcal{M}^{\mathrm{mat}}(r,\theta)\right](T_e - T_e^{b.c.})$
$-\left[\mathcal{M}^{\mathrm{mat}}(r,\theta) - \mathcal{M}^{wall}(r)\right]\phi^{\mathrm{bias}}$

with $\rho(\mathbf{x},t) = \frac{1}{n_{e_0}} \sum_s Z_s \int \mathrm{d}^3 v\, \mathcal{J} \cdot (\bar{F}_s - \bar{F}_{s,eq})$

| Tokamak volume | Mask |
|---|---|
| All plasma | $1 - \mathcal{M}^{\mathrm{mat}}(r,\theta)$ |
| Closed field lines | $1 - \mathcal{M}^{\mathrm{SOL}}(r)$ |
| Scrape-Off Layer | $\mathcal{M}^{\mathrm{SOL}}(r) - \mathcal{M}^{\mathrm{mat}}(r,\theta)$ |
| Material boundaries | $\mathcal{M}^{\mathrm{mat}}(r,\theta)$ |
| Limiter only | $\mathcal{M}^{\mathrm{mat}}(r,\theta) - \mathcal{M}^{\mathrm{wall}}(r)$ |
| Wall only | $\mathcal{M}^{\mathrm{wall}}(r)$ |




- global domain: $0 \leqslant r/a \leqslant 1.3$
- Tore Supra #45 511
- $\rho_\star \approx 1/300$ → in the "local limit"

- no transport of mass
- transport of energy & momentum

→ "baseline" instab. ITG+PVG; extension to TKEs [see P. Donnel, *this conference*]

# Minimally-relevant [first principles] model for L–mode edge dynamics



- $$\frac{\mathrm{d}\bar{F}_s}{\mathrm{d}t} = \mathcal{C} + \mathcal{S} + \mathcal{D} - \nu\, \mathcal{M}^{\mathrm{mat}}(r,\theta)\,[\bar{F}_s - \mathbb{G}_{cold}]$$

- $$\Delta n_e = \rho + \frac{1}{n_{e0}} \sum_s Z_s \nabla_\perp \cdot \left( \frac{n_{s0}}{B\Omega_s} \nabla_\perp \phi \right)$$

with $$\frac{T_e}{e}\Delta n_e = \phi - \lambda\left[1 - \mathcal{M}^{SOL}(r)\right]\langle \phi \rangle_{\mathcal{FS}} - \lambda \frac{e}{T_e}\Delta\phi^{\mathrm{sheath}}\left[\mathcal{M}^{SOL}(r) - \mathcal{M}^{\mathrm{mat}}(r,\theta)\right](T_e - T_e^{b.c.}) - \left[\mathcal{M}^{\mathrm{mat}}(r,\theta) - \mathcal{M}^{wall}(r)\right]\phi^{\mathrm{bias}}$$

with $$\rho(\mathbf{x},t) = \frac{1}{n_{e0}} \sum_s Z_s \int \mathrm{d}^3 v\, \mathcal{J} \cdot (\bar{F}_s - \bar{F}_{s,eq})$$

- global domain: $0 \leqslant r/a \leqslant 1.3$
- Tore Supra #45 511
- $\rho_\star \approx 1/300$ ➞ in the "local limit"

**Systematic comparison TS#45511**

**#1 Flux-Driven & limiter**

$\updownarrow$

**#2 Flux-Driven & poloidally symm.**

$\mathcal{M}^{\mathrm{mat}}, \mathcal{M}^{SOL}, \mathcal{M}^{wall} \rightarrow 0$

$\forall t, r/a \geqslant 1 \quad \bar{F}_s \rightarrow cst \equiv F_{\#1}$

$\updownarrow$

**#3 Gradient-Driven & poloidally symm.**

$\mathcal{M}^{\mathrm{mat}}, \mathcal{M}^{SOL}, \mathcal{M}^{wall} \rightarrow 0$

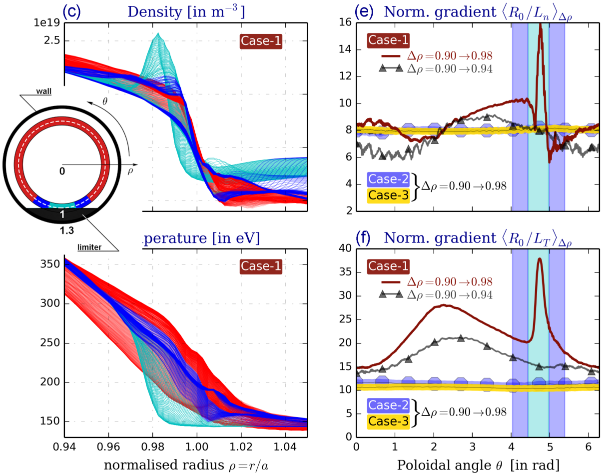
$\mathcal{S} \rightarrow 0$

$\forall r, t \quad \bar{F}_s \rightarrow cst \equiv F_{\#2}$

- no transport of mass
- transport of energy & momentum

➞ "baseline" instab. ITG+PVG; extension to TKEs

[see P. Donnel, *this conference*]




linear analysis of GYSELA profiles with local GKW

[Peeters CPC 09]

## w/o Limiter

$\rho \leqslant .92$: ITG/TEM unstable

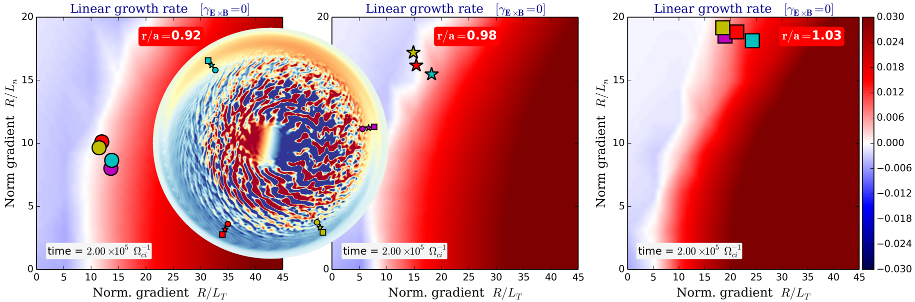
$\rho \geqslant .92$: outer edge **stable**

## with Limiter

- narrow $(r, \theta)$ region DW/interchange unstable
- rest of edge is stable

➤ interesting situation: turbulence organisation in edge?

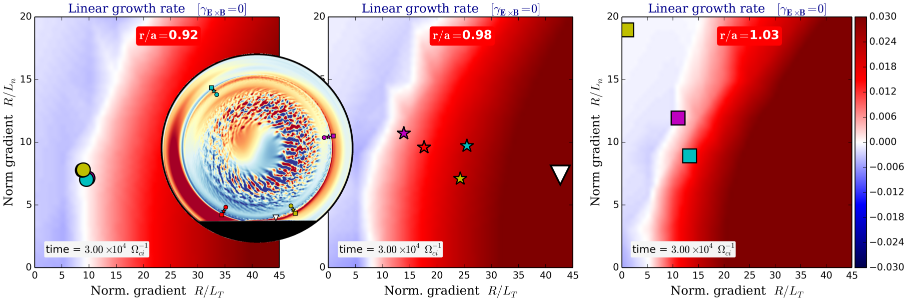
**<u>No Limiter</u>** $\equiv$ polo. symmetric cases




<u>major caveats:</u> $\mathbf{E} \times \mathbf{B} = 0$ & poloidally (parallel) homogeneity

<u>long story short:</u> ITG & TEM unstable $r/a \leqslant 0.92$ ; outer edge **stable** $r/a \geqslant 0.92$

**WITH Limiter** $\equiv$ (large) localised free energy sources



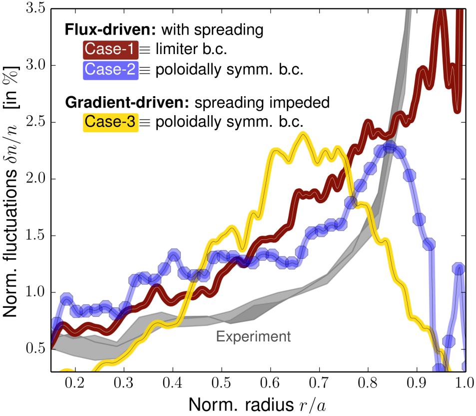

when edge instab. starts: *profiles ~same @LFS midplane w/ or w/o limiter*

- narrow $(r,\theta)$ region DW/interchange unstable ; PVG marginally unstable
- rest of edge is stable [marginally-unstable @steady-state]

➠ interesting situation: turbulence organisation in edge? investigate $\delta n/n$

- reasonable trend with limiter;
- shortfall without;
- spreading ["contamination"] key

[Mattor; Garbet; Hahm; . . . ; Singh]




**Proxy for spatial turb. spreading:**

$$\left(\frac{\partial}{\partial t} + \bar{\mathbf{v}}_{E,\theta}\ \frac{\partial}{\partial \theta}\right)(nI) + \nabla \cdot \Gamma_I = \mathcal{P} - \mathcal{D}$$

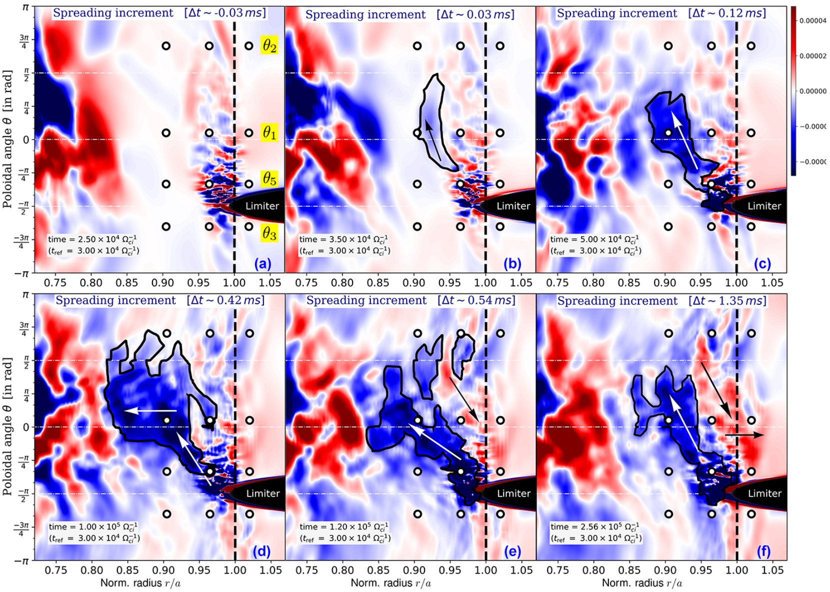
$$\Gamma_I(r,\theta,t) \equiv \left\langle \int \mathrm{d}^3 v (\mathbf{v}_{ExB} \cdot \nabla r) \frac{\tilde{f}^2}{F_M} \right\rangle$$

↳ investigate systematic spreading increments $\Delta S \equiv \Gamma_I(r,\theta,t) - \Gamma_I(r,\theta,t_{\mathrm{ref}})$

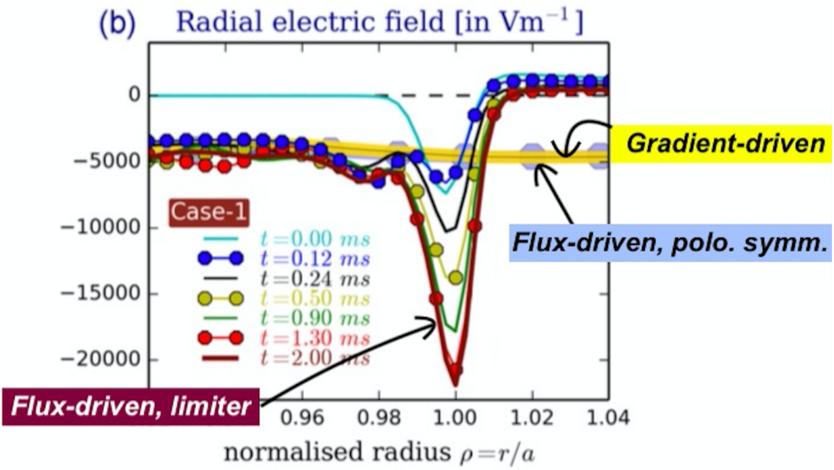
$\Delta S \geqslant 0 \quad \rightarrow$ radially-outward fluxes of turbulence intensity

$\Delta S \leqslant 0 \quad \rightarrow$ inward fluxes of turb. intensity

# Limiter-borne fluctuations contaminate → edge & core then LCFS ↔ core cyclic equilibration



# Spontaneous poloidal symm. breaking @LCFS density shelf & T grad.→ persistent transport barrier



# Vorticity balance $E_r'(r,\theta)$ to probe mechanisms for barrier build-up & sustainment

- track vorticity production [GK eq. + **E** × **B** velocity + gyroaverage [eg. Sarazin PPCF 2021]]

$$\boxed{\partial_t \langle \Omega_r \rangle + \frac{1}{r}\partial_r r \left[ \left\langle v_{Er}\Omega_r \right\rangle + \left\langle v_{\star r}\Omega_r \right\rangle - \left\langle v_{\star\theta}\frac{1}{r}\partial_\theta E_r \right\rangle \right] = r.h.s}$$

$$r.h.s \approx -\partial_t \langle \Omega_\theta \rangle - \frac{\partial_\theta}{r}\langle (v_{E\theta} + v_{\star\theta})\Omega_\theta \rangle - \partial_r \frac{\partial_\theta}{2r}\langle v_{E\theta}^2 \rangle + \frac{\partial_\theta}{2r^3}\partial_r \langle r^2 v_{Er}^2 \rangle - \frac{\partial_\theta}{r}\left\langle v_{\star r}\frac{1}{r}\partial_\theta v_{E\theta} \right\rangle$$

$$\Omega_r \approx -\partial_r E_r \qquad \& \qquad \Omega_\theta = \partial_\theta^2 \phi / r^2$$

$$v_{Er} = -\partial_\theta \phi / r \qquad \& \qquad v_{E\theta} = \partial_r \phi = -E_r$$

$$v_{\star r} = -\partial_\theta p_\perp / r \qquad \& \qquad v_{\star\theta} = \partial_r p_\perp$$

# Vorticity balance $E_r'(r,\theta)$ to probe mechanisms for barrier build-up & sustainment

- track vorticity production [GK eq. $+ \mathbf{E} \times$

$$\partial_t \langle \Omega_r \rangle + \frac{1}{r}\partial_r r \Big[ \Big\langle v_{Er}\Omega_r \Big\rangle + \Big\langle v_{\star r}\Omega_r \Big\rangle - \Big\langle$$

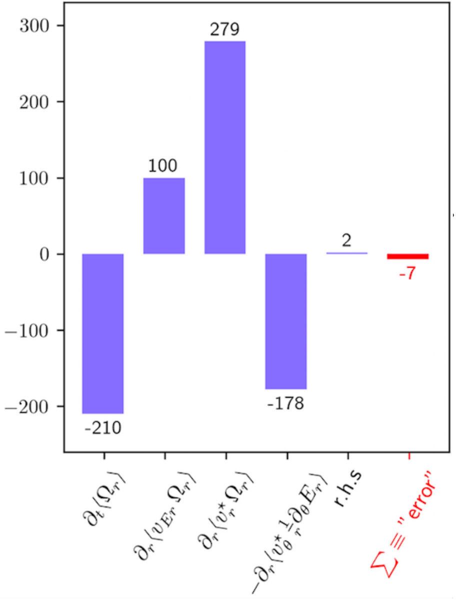
$$r.h.s \approx -\partial_t \langle \Omega_\theta \rangle - \frac{\partial_\theta}{r} \langle (v_{E\theta} + v_{\star\theta})\Omega_\theta \rangle - \partial$$

$$\Omega_r \approx -\partial_r E_r \quad \& \quad \Omega_\theta = \partial_\theta^2 \phi /$$

$$v_{Er} = -\partial_\theta \phi / r \quad \& \quad v_{E\theta} = \partial_r \phi$$

$$v_{\star r} = -\partial_\theta p_\perp / r \quad \& \quad v_{\star\theta} = \partial_r$$

➤ accurate vorticity balance



# Vorticity balance $E_r'(r,\theta)$ to probe mechanisms for barrier build-up & sustainment



- track vorticity production [GK eq. + $\mathbf{E}\times\mathbf{B}$ velocity + gyroaverage [eg. Sarazin PPCF 2021]]

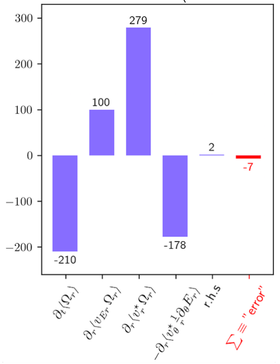
$$\boxed{\partial_t\left\langle\Omega_r\right\rangle + \frac{1}{r}\partial_r r\left[\left\langle v_{Er}\Omega_r\right\rangle + \left\langle v_{\star r}\Omega_r\right\rangle - \left\langle v_{\star\theta}\frac{1}{r}\partial_\theta E_r\right\rangle\right] = r.h.s}$$

$$r.h.s \approx -\partial_t\left\langle\Omega_\theta\right\rangle - \frac{\partial_\theta}{r}\left\langle(v_{E\theta}+v_{\star\theta})\Omega_\theta\right\rangle - \partial_r\frac{\partial_\theta}{2r}\left\langle v_{E\theta}^2\right\rangle + \frac{\partial_\theta}{2r^3}\partial_r\left\langle r^2 v_{Er}^2\right\rangle - \frac{\partial_\theta}{r}\left\langle v_{\star r}\frac{1}{r}\partial_\theta v_{E\theta}\right\rangle$$

$$\Omega_r \approx -\partial_r E_r \quad \& \quad \Omega_\theta = \partial_\theta^2\phi/r^2$$

$$v_{Er} = -\partial_\theta\phi/r \quad \& \quad v_{E\theta} = \partial_r\phi = -E_r$$

$$v_{\star r} = -\partial_\theta p_\perp/r \quad \& \quad v_{\star\theta} = \partial_r p_\perp$$




➤ accurate vorticity balance

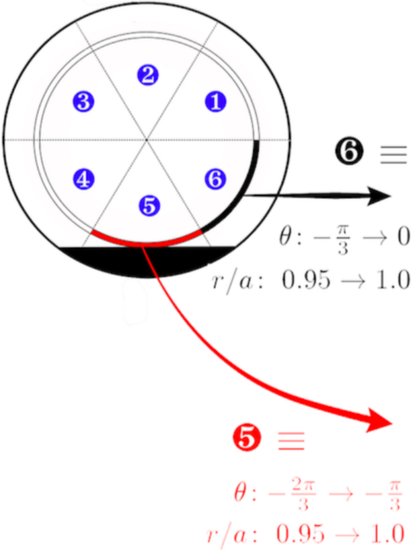
➤ But. . .
- no statement about dynamical relevance
- assess causality in barrier build-up?

➤ **"Transfer Entropy"** ≡ net directional flow of information

↔ **causality** [Schreiber PRL 00;. . . VanMilligen NF 14; Nicolau PoP 18]

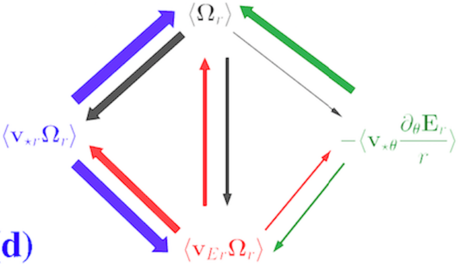
# Pressure inhomogeneities key: forecasts Reynolds force & contributes propagating field inhomogeneities






no fluctuations

(b)




(d)

# Conclusions: SOL–edge–core interplay informs "shortfall" conundrum & transp. barrier onset/sustainment

- penalised limiter ➞ simplified SOL   miss $e^-$ $\|$ dynamics; convection & neutrals
  ↳ free energy injection in narrow region ➞ turb. redistribution key

- interplay SOL, edge & core   [Dif-Pradalier, Communications Physics 2022]
  ➞ spontaneous persistent **transport barrier** $\equiv E_r$ build-up;
  ➞ **with limiter:** **no "shortfall"** $\equiv$ clarifies spreading controversy
  ❶ poloidal asymmetry (cold region. . . )
  ❷ edge-to-core then ❸ cyclic edge→core & core→edge
  ➞ **w/o limiter:** edge stable & **strong "shortfall"** in NM'sL

- Turbulence **not only locally driven** by local gradients but 'nonlocally' controlled by **fluxes of turb. activity**, primarily (though not exclusively) coming **from the edge** & mediated thru **interplay with material bound.**

- transp. barrier build-up ➞ $\langle v_{Er}\Omega_r\rangle$, $\boxed{\langle v_{\star r}\Omega_r\rangle}$ & $\langle v_{\star\theta}\frac{\partial_\theta E_r}{r}\rangle$ **key to $E_r$ growth**