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Causal discovery for fuzzy rule learning

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Abstract—In this paper, we focus on allying fuzzy logic, which is a suitable model for human-like information, and causality, which is a key concept for humans to generate knowledge from observations and to build explanations. If a fuzzy premise causes a fuzzy consequence, then acting on the fuzzy premise will have an impact on the fuzzy consequence. This is not necessarily the case for common fuzzy rules whose induction is based on correlation. Indeed, correlations may be due to some latent common cause of fuzzy premise and consequence. In this case, a change in the value of the fuzzy premise may not affect the fuzzy consequence as it should. We propose an approach to construct a set of causality-based fuzzy rules from crisp observational data. The idea is to identify causal relationships on the set of fuzzified inputs and outputs by well-known constraints-based causal discovery algorithms such as Peter-Clark and Fast Causal Inference. The causal discovery algorithms are combined with entropy-based conditional independent testing that avoids making hypotheses on the data distribution. Experiments are conducted to evaluate our approach in terms of ability to recover causal relationships between fuzzy sets in the presence of a latent common cause. The results illustrate the interest of our approach compared to a correlation-based approach and state-of-the-art approaches.

Index Terms—fuzzy rules, imperfect causality, constraint-based causal discovery, entropy.

I. INTRODUCTION

Fuzzy rules induction algorithms are usually based on statistical criteria such as correlation or examples covering. Although these algorithms can be efficient to predict the outputs, some studies also aim to provide insights about the mechanism that links inputs and outputs. Machine learning needs causal models of reality to reach the level of human intelligence [1]. Indeed, understanding causality is an essential feature of our understanding of the world [2]. Moreover, the fuzzy logic framework allows us to manipulate information expressed in natural language. Thus, a fuzzy representation of causal mechanisms would be very relevant in knowledge extraction applications requiring human understanding. This paper is motivated by the need to construct causality-based fuzzy rules. By causality-based fuzzy rules, we mean rules involving a fuzzy premise that is the cause of its fuzzy consequence. Such rules would not only have a predictive purpose but would also aim to provide insights concerning a system. Causality-based fuzzy rules would be useful in many scientific domains that require understandable causal

statements. For example, in the domain of diagnostic [3], to understand the causal relationships between disorders and symptoms or in the manufacturing domain [4], to identify causal links between the manufacturing parameters and the material property. In this context, the objective is to extract automatically these fuzzy rules from a crisp dataset. For example, in the manufacturing domain, it would be possible, to generate fuzzy causal statements such as “low viscosity of a liquid *causes* a low temperature of the liquid” from crisp observations of viscosity and temperature variables. Thus, this paper proposes an approach to identify causal links between fuzzy sets to learn fuzzy rules based on causality.

The remainder of this paper is organized as follows: section II introduces the most common framework to deal with causal discovery between crisp variables, and presents some state-of-the-art frameworks that have been proposed to handle imperfect causality. Our problem statement and motivations are exposed in section III. Our proposition of causal discovery between fuzzy variables is presented in section IV. Finally, in section V, the ability of our approach to recover causal links between fuzzy sets is evaluated on simulations and compared to state-of-the-art and correlation-based approaches.

II. BACKGROUND

In this section, we briefly describe the background of our work. We first introduce the causal discovery in the crisp case and we then talk about imperfect causality.

A. Causal discovery between crisp variables

One of the most common frameworks for describing causal mechanisms are Structural Causal Models (SCMs) [5]. They most often consist of structural equations, which specify the causal effects of each variable, and a causal graph, which is a causal interpretation of a Bayesian network [6]. The causal graph inherits the necessary and sufficient Markov condition that a variable is independent of its non-effects conditional on its direct causes. More formally, let us consider a causal directed acyclic graph $\mathbf{G} = (\mathcal{V}, \mathcal{E})$, where \mathcal{E} denotes the set of nodes, i.e., the set of variables involved in the causal mechanism, and \mathcal{V} denotes the set of edges representing the causal links between these variables. To illustrate conditional independence graphically, the notions of blocked path (sequence of edges) and d -separation have been introduced. A node C is said to block a path conditionally to a set of nodes S , if one of the following statements holds [6]: i) $C \in S$

and C is a chain $A \rightarrow C \rightarrow B$ or a fork $A \leftarrow C \rightarrow B$;
 ii) $C \notin S$, no descendant of C is in S and z is an inverted fork $A \rightarrow C \leftarrow B$. Two nodes X and Y in \mathcal{E} are said to be d -separated in \mathbf{G} by a subset S of \mathcal{E} if all paths between X and Y are blocked conditionally on S .

The study of causal mechanisms involves either recovering causal relationships from observed data (causal discovery) or inferring causal effects from a given causal graph (causal inference). In this paper, we focus on a causal discovery task. Causal discovery is a well-documented topic in the literature [6], [7]. There are three main families of causality discovery approaches, namely, the constraint-based (CB) [8]–[10], the score-based (SB) [11], and the functionally causal model-based (FCM) [12].

The first family of methods exploits conditional independence relationships in the data to discover the underlying causal structure. These methods make the so-called *Causal Faithfulness Assumption*, i.e., the reciprocal of Markov condition: if $X \perp\!\!\!\perp Y|S$ in D , i.e., X and Y are independent given S , then X and Y are d -separated by S in \mathbf{G} . With the Markov and *faithfulness* assumptions, we have a one-to-one correspondence between the d -separations in the graph and the conditional independences in the data distribution. Let us introduce two well-known constraint-based causal discovery algorithms such as Peter-Clark (PC) [8] and Fast Causal Inference (FCI) [9], [10]. The PC algorithm was proposed to estimate a Markov equivalence class of DAGs, assuming that there are no unmeasured common causes and no selection variables. The first step of PC algorithm consists in the estimation of the skeleton, i.e., a non-oriented graph. The PC algorithm starts with a fully connected graph and performs a series of conditional independence tests to remove successively the edges corresponding to the d -separation. The orientation of the edges is based on the identification of v -structures, i.e., $A \rightarrow B \leftarrow C$. Indeed, the authors of [13] showed that two DAGs were Markov equivalent if and only if they share the same skeleton and the same v -structures. The FCI algorithm estimates a special class of Markov equivalence, called a partial ancestral graph, which permits to take into account the latent variable. In practice, FCI is a modification of PC that performs additional conditional independence tests due to the latent variables.

The second family of approaches, SB, consists in searching for the graphs maximizing the goodness of fit to the data distribution. The most commonly used fit score for this purpose is the Bayesian Information Criterion (BIC), but other scores have been proposed [14]. Once the score is defined, the search for an optimal graph is performed by heuristic methods [11].

Finally, the third family of causality discovery methods, FCM, aims at determining the orientation of edges in its process. To this end, these methods use the assumption that the effect noises should be independent of the causes. For example, in the case where we seek to identify whether X causes Y or whether Y causes X , the principle is to consider both possibilities, “ $X = f(Y, \epsilon_X)$ ” and “ $Y = f(X, \epsilon_Y)$ ”, under assumptions about the distribution of the data and the

functional relationship f , with ϵ_X and ϵ_Y being the resulting noises for these two configurations. These methods then search for an asymmetry between X and Y . In practice, to determine if X causes Y or if Y causes X , the two possible directions of the causal relationship are modeled. For example, if we obtain $Y \perp\!\!\!\perp \epsilon_X$ but $X \not\perp\!\!\!\perp \epsilon_Y$, we can then conclude that Y causes X .

B. Imperfect causality

For human interpretation, causal links are often expressed in vague language. This imperfection may concern the definitions of cause and effect, e.g. in the statement, “sleeping less causes unusual fatigue”. The imperfection may also concern the causal link itself, as in the statement, “sleeping less than 5 hours may cause unusual fatigue”. This article focuses on the search for causality without yet seeking to qualify the links. Thus, in this article, the imperfection concerns only the definitions of causes and effects. The SCMs introduced in section II-A are not suited to represent such imperfections. Indeed, since the nodes have a vague meaning, the distribution cannot be specified in an exact way. To overcome this problem and to introduce imperfection into the causality search, several alternative methods have been developed, the most common of which is the use of Kosko cognitive maps (KFCMs). KFCMs allow representing and dealing with imperfect causality of a dynamic system, i.e. a causal structure permitting cycles. Contrary to KFCMs, the acyclic framework is considered in this article. Indeed, we consider a straightforward pattern of causal relationships where a cause is always an input, and its effect is always an output. An alternative method developed by [15], [16] consists in formalizing the causal relationships based on a parsimonious coverage of the effects. To do this, the authors define relationship models for causality. This method defines possible relationships between causes and effects without guaranteeing their necessity. Its advantage is to consider sets of possible causes and their interactions. More formally, if we take the sets $\mathcal{X} = \{X_1, \dots, X_k\}$ and $\mathcal{Y} = \{Y_1, \dots, Y_k\}$, X_i can cause Y_j but not necessarily. However, this identification of possible causal links does not correspond to our study scope. Indeed, the objective is to obtain the causal link between fuzzy sets to ensure the relationship between the causes and the effect. Before the parsimonious covering theory was developed, Sanchez [3] had proposed the fuzzy relational methods in the domain of diagnosis problems. These methods were designed to represent the intensity of symptoms and disorders. These works are closer to our objective in this article. However, they assume that the symptoms are independent [17], which is not a guaranteed assumption in our work.

In the next section, our problem statement and our motivations are presented.

III. PROBLEM STATEMENT AND MOTIVATIONS

Let us suppose that crisp realizations of inputs and outputs variables are observed. The problem addressed in this paper is to generate imperfect causal knowledge from the realizations

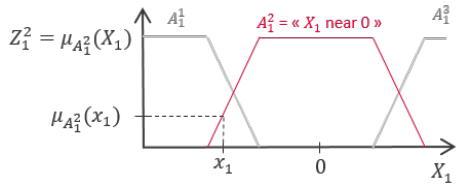


Fig. 1: Example of definition of a new variable Z_1^2 .

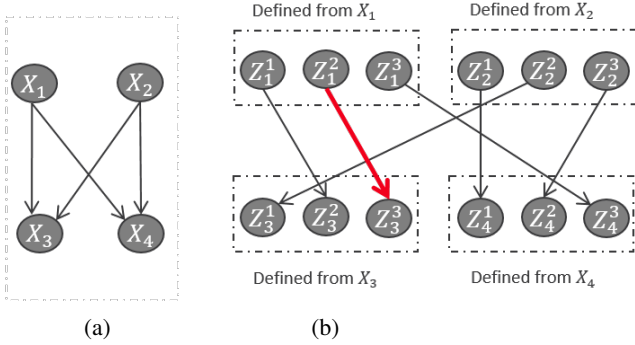


Fig. 2: Fictitious examples of causal graphs with the initial variables (a), with the new fuzzy variables (b).

under the following form: a list of causal relationships between fuzzy terms extracted from the inputs and outputs variables. More formally, let us denote the inputs and outputs variables involved in a causal structure as X_1, \dots, X_p , $p > 0$. From each initial random variable X_i , let us denote by $A_i^1, \dots, A_i^{m_i}$, $m_i > 2$, the fuzzy sets deduced. Then we define a new random variable for each fuzzy set A_i^j that describes the membership values to this fuzzy set by:

$$Z_i^j = \mu_{A_i^j}(X_i), \quad j = 1, \dots, m_i \text{ and } i = 1, \dots, p, \quad (1)$$

where $\mu_{A_i^j}$ denotes the membership function associated to the fuzzy set A_i^j .

Then, the problem consists in identifying causal relationships between the random variables $\{Z_i^j\}_{j=1, \dots, m_i, i=1, \dots, p}$ describing the values taken by the membership degrees of the fuzzy sets. In short, our problem statement consists in searching for a causal graph on fuzzy sets. Let us note that the considered imperfection concerns the causes and consequences but not the causal links.

In comparison with the standard causality discovery algorithms, the benefit of identifying such imperfect causal relations is that it refines the causality understanding while remaining highly intelligible. Let us illustrate this benefit through a fictitious example in the domain of manufacturing. In the example, realizations are observed for the following four random variables: $X_1 :=$ “Manufacturing parameter temperature”, $X_2 :=$ “Manufacturing parameter pressure”, $X_3 :=$ “Material strength property” and $X_4 :=$ “Elongation at Rupture”. Figure 1 shows the definition of the new random variable Z_1^2 in our fictional example. We see that with an observation x_1 of the random variable X_1 (temperature), we

can obtain an observation z_1^2 of the random variable Z_1^2 (temperature membership in the “near 0” state). The causal graph in Figure 2 is an example of a possible result if we applied a causality search method directly on the available random variables. In this graph, we have the manufacturing parameters X_1 and X_2 that both point to both properties X_3 and X_4 . This representation is accurate but does not inform us about the parameter values and properties involved in the causal relationships. In contrast, with an imprecise causal graph, as shown in Figure 2b, generated from the new variables, we would be able to extract more precise information about the underlying causal structure. For example, the colored and thick link tells us that the value of the degree of membership of the manufacturing parameter temperature (X_1) in the fuzzy set $A_1^2 =$ “near 0” has a direct causal effect on the value of the degree of membership of the property strength (X_3) in the fuzzy set $A_3^3 =$ “important”. In brief, the causal information can be formulated as “having a temperature near 0 has a direct causal effect on obtaining an important strength”. Thus, the crisp causal statement “temperature has a causal effect on strength” has been refined and expressed in an intelligible way.

Supposing that such an imperfect causal statement is available, we argue that it would be interpretable as a fuzzy rule. In this interpretation, the imperfect cause takes the role of a fuzzy premise. The imperfect effect is traduced by a positive or negative fuzzy consequence according to the sign of the correlation between the imperfect cause and effect. Let us go back to the imperfect causal statement seen in the previous example and assume a positive correlation between “temperature near 0” and “important strength”. Then we can formulate the following fuzzy rule: “If the temperature is near 0 then the strength is important”. In the end, answering our problem statement leads to a process for inducing causality-based fuzzy rules.

The advantage of the proposed causality-based fuzzy rule compared to common fuzzy rules is that it can be used not only for prediction but also for providing insights about the mechanism that links inputs and outputs. This is not necessarily the case for common fuzzy rules based on correlation. Indeed, correlations may be due to some latent common cause of fuzzy premise and consequence, thus acting on the fuzzy premise will have no impact on the fuzzy consequence. In the previous example of material manufacturing, latent variables can be the type and settings of manufacturing device used, the environment features, or the design of experiments. The causality-based fuzzy rules can also be used to generate general knowledge that can be reused as insight in other contexts (portability).

In this context, we propose an innovative approach to generate imperfect causal knowledge from crisp realizations presented in the remaining section.

In the next section, we propose a method for searching for the causal relationships between fuzzy variables.

IV. PROPOSED APPROACH

The main idea of the proposition is to adapt standard causal discovery search methods to the case where the random variables describe values of membership to fuzzy sets previously denoted $\{Z_i^j\}_{j=1,\dots,m_i, i=1,\dots,p}$. The fuzzy sets may be defined by the experts, by uniform extraction, or automatically extracted by learning methods like Fuzzy C-means clustering [18], [19].

Then, the observations for the initial random variables X_1, \dots, X_p are transformed as observations of the new variables $\{Z_i^j\}_{j=1,\dots,m_i, i=1,\dots,p}$. In the search for causal relationships between these new variables, some particularities must be considered. First, the new variables do not follow a known distribution. In addition, no hypothesis can be made about the functional relations between causes and effects. Another challenge is that the new variables are not all semantically independent as required when using causal research algorithm [20], [21]. Finally, some contextual information related to the inputs and outputs distinction must be incorporated. Indeed, in our setting, input fuzzy sets correspond to the set of possible causes and output fuzzy sets coincide with the set of possible effects. Thus, the orientations of the edges in the causal graph are also given. Let us describe our causal discovery procedure adapted to the above particularities.

The constraint-based family of causal discovery algorithms has been adopted. Indeed, FCMs would require making assumptions on the functional links between the variables. Score-based are not designed for the case of latent variables. The constraint-based methods allow considering latent variables and avoiding assumptions on the functional links by using appropriate hypothesis tests. Moreover, they offer the possibility to take into account contextual information during the estimation of the causal graph. Hence, we selected the two well-known constraint-based algorithms PC and FCI presented in the previous section. As for the conditional independence testing, a procedure called Kernel-based Conditional Independence test [22], has been designed to make no hypothesis on the distribution of the variables, nor on the functional relations between them. This procedure based on conditional independence characteristics expressed in terms of cross-covariance operator [23]. However, the kernel-based conditional independence test performances depend on the adequacy between the kernel form and the sample distribution. To circumvent this problem, we adapt the discovery causality research by integrating the entropy notion. We then propose another procedure based on Stochastic complexity-based Conditional Independence criterium (SCI) [24]. In this procedure, conditional mutual information is used as a measure for conditional independence. If the conditional mutual information of X and Y given Z is null, i.e.,

$$I(X; Y|Z) := H(X|Z) - H(X|Z, Y) = 0, \quad (2)$$

then the two random variables X and Y are statistically independent given Z . The authors's conditional independence test is based on an approximation of conditional mutual

information using stochastic complexity [24]. Since the SCI procedure is designed for discrete data, the data are discretized to form equal frequency bins.

To refine the causality research, we integrate into our approach different constraints, in particular, to avoid the problem of semantic dependency when searching for causal relations between the new variables, which is known for perturbing the causality search. Pure redundancy happens if two variables are logically or mathematically inter definable [20]. The case of pure redundancy could happen in our situation if for two fuzzy sets A and B , we have $\forall x$,

$$\mu_A(x) = 1 - \mu_B(x). \quad (3)$$

This case of pure redundancy is avoided by setting the number of extracted fuzzy sets per initial variable greater than 2. By construction, there will remain some kind of dependency between the extracted fuzzy sets from the same initial variable. To facilitate the search for causal links, the PC or FCI algorithm is given the information that no causal relation are allowed between fuzzy sets extracted from the same initial variable. Thus, instead of starting from a fully connected graph, not allowed edges are withdrawn before running the causality search algorithm.

Similarly, contextual information is taken into account by withdrawing the not allowed edges. Finally, after running the causality search algorithm, the orientations of the found edges are all reoriented in the direction of inputs toward outputs.

The complexity of our approach depends on the causal discovery algorithm employed. Let us denote the adjacency set, i.e., set of adjacent nodes, of A in \mathbf{G} by $\mathbf{Adjencies}(\mathbf{G}, A)$. In the case of classical PC algorithm, the complexity is $O(\nu n)$, where $\nu = \max(p^q, p^2)$ and q is the maximal size of adjacency sets [25]. In our fuzzy version of PC algorithm, the p initial variables are replaced by the new variables $\{Z_i^j\}_{j=1,\dots,m_i, i=1,\dots,p}$ which increases the complexity. However it is significantly reduced with the removal of edges between new variables defined from the same initial variable or the edges that are not allowed. Taking the fictitious example of Figure 2b, where causality is assumed to be oriented from the inputs X_1, X_2 towards the outputs X_3, X_4 , we get $q = 6$, instead of $q = 11$. Our proposition of fuzzy causal discovery instantiated with PC algorithm is described in algorithm 1. Since the orientation of the edges is known to be from the inputs to the outputs, we restrict the graph construction to the skeleton estimation phase of PC.

V. APPLICATION TO FUZZY RULE INDUCTION

In this section, experiments are conducted on simulated fuzzy sets to evaluate the ability of the proposed approach to recover causal relationships between fuzzy sets. First, the design of the simulations is described. Then, the proposed approach performances are evaluated and compared to state-of-the-art alternative procedures.

Algorithm 1 *fuzzy pc SCI*

Data: n observations of X_1, \dots, X_p

Result: Fuzzy causal graph \mathbf{G}

```
1: for  $i = 1, \dots, p$  do
2:   Extract  $m_i > 2$  fuzzy sets  $A_i^1, \dots, A_i^{m_i}$  from  $X_i$ 
3:   for  $j = 1, \dots, m_i$  do
4:     Defined a new variable  $Z_i^j = \mu_{A_i^j}(X_i)$ 
5:     Deduce  $n$  observations of the  $Z_i^j$  new variables
6:   end for
7: end for
8: Discretize the  $n$  new observations of
    $\{Z_i^j\}_{j=1, \dots, m_i, i=1, \dots, p}$  to form equal frequency bins
9:  $\mathbf{G} \leftarrow$  full connected graph on  $\{Z_i^j\}_{j=1, \dots, m_i, i=1, \dots, p}$ 
10: for  $i = 1, \dots, p$  do
11:   Withdraw edges  $A - B$  in  $\mathbf{G}$  such that  $A, B \in$ 
      $\{Z_i^j\}_{j=1, \dots, p}$ 
12: end for
13: Withdraw the other edges not allowed by the contextual
     information
14:  $k \leftarrow 0$ 
15: repeat
16:   repeat
17:     Select a pair of adjacent nodes  $A, B$ , in  $\mathbf{G}$  and a set
     of nodes  $S \in \text{Adjencies}(\mathbf{G}, A) \setminus \{B\}$  such that  $|S| = k$ 
18:     if  $A \perp\!\!\!\perp B | S$  with the SCI criterium then
19:       Withdraw edge  $A - B$ 
20:     end if
21:   until All  $A, B$  and  $S$  have been tested
22:    $k \leftarrow k + 1$ 
23: until For all pairs of adjacent nodes  $A, B$ ,
      $|\text{Adjencies}(G, A) \setminus \{B\}| < k$ 
```

A. Design of simulations

The simulations are designed to experiment the ability for recovering a causal structure among fuzzy sets. The candidate approaches will be given the simulated fuzzy sets to estimate a causal graph. The estimated causal graph will be compared to the true causal graph that was used to generate the fuzzy sets. The simulations design can then be described in two steps. First, we simulate a true causal structure. Then, we generate the membership values to fuzzy sets corresponding to the true causal structure. The causal structure considered to play the *true causal graph* are all constructed on the same following pattern: four initial crisp variables, among which two are inputs X_1, X_2 and two outputs X_3, X_4 . From each initial variable, three fuzzy sets are considered. We obtain 12 new variables $\{Z_i^j\}_{j=1,2,3, i=1,2,3,4}$ describing the memberships of the 12 fuzzy sets. The latter 12 variables are the nodes of the true causal graph.

The edges are chosen in order to correspond to a list of fuzzy rules. Then, the simulation of the *true fuzzy causal graph* consists in defining the rule base. While all the linguistic terms of each input and output linguistic variables have not been used at least once (to ensure the perfect coverage of the data), a rule

is created. In this paper, we do not consider conjunctions and disjunctions yet, so building the rule base consists in finding a bijection between the terms of the inputs and the terms of the outputs.

The causal graph represented in Figure 2b is an example of a simulated *true fuzzy causal graph*.

Let us now describe the generation of realizations for the variables $\{Z_i^j\}_{j=1,2,3, i=1,2,3,4}$. We considered two types of simulations, some with a latent common cause of both the inputs X_1 and X_2 , noted H ($X_1 \not\perp\!\!\!\perp X_2$ but $X_1 \perp\!\!\!\perp X_2 | H$), and some without any latent variable ($X_1 \perp\!\!\!\perp X_2$). In the case of no latent variable, n observations of X_1 and X_2 were obtained as realizations of two standard normal distributions: $X_1 \sim \mathcal{N}(0, 1)$ and $X_2 \sim \mathcal{N}(0, 1)$. In the case of the latent variable, we considered a standard normal latent variable $H \sim \mathcal{N}(0, 1)$ and used it to generate n observations of X_1 and X_2 with the following additive relations: $X_1 \sim \mathcal{N}(0, 1) + H$ and $X_2 \sim \mathcal{N}(0, 1) + H$.

Once the n observations of the input variables are generated, we automatically create a strong partition of their universe. It is based on membership functions whose number is chosen randomly within a range that is specified as hyper-parameter. The membership functions are triangular except the first and last ones that are semi-trapezoidal functions. The outputs are processed the same way. Then, we compute the output values regarding the input values. For now, without loss of generality of our approach, we use a simple Mamdani system whose aggregation and defuzzification functions can be set as hyper-parameters (by default the Max aggregation and the centroid defuzzification).

We tested the abilities of several approaches to recover the *true fuzzy causal graph* from the n realizations of $\{Z_i^j\}_{j=1,2,3, i=1,2,3,4}$. The list of the tested approaches is given below :

- *fuzzy pc SCI* and *fuzzy fci SCI* denotes our proposed approaches. The data are discretized to form equal frequency categories. Then the causal research algorithms PC or FCI are performed using the entropy-based SCI Conditional Independence test [24].
- *fuzzy pc gauss* and *fuzzy fci gauss* consist in performing the causal research algorithms PC or FCI considering the conditional independence test based on Fisher z-transformation of the partial correlation [26].
- *fuzzy pc KCI* and *fuzzy fci KCI*, similarly consist in performing the causal research algorithms PC or FCI, but this time using the Kernel-based Conditional Independence test [22].
- *corr τ* denotes the approach based on the correlations consisting in adding all allowed edges between nodes with correlation higher than τ . Since the distributions are unknown, the correlations are computed with the Kendall rank correlation coefficient.
- *random* consists in adding each allowed edge with a probability 0.5.

To compare the performances of all these approaches, we use a precision score (percentage of edges found that are

correct) and recall (percentage of true edges that are found). More formally, the precision is defined by:

$$precision = \frac{truepositive}{truepositive + falsepositive} . \quad (4)$$

The recall is defined by:

$$precision = \frac{truepositive}{truepositive + falsenegative} . \quad (5)$$

In our context, a true positive is a simulated edge found by the method. A false positive corresponds to a predicted edge that is not simulated. A false negative is a simulated edge that is not predicted by a method.

B. Influence of the latent variable

With the simulation design described above, we consider two cases: with and without a latent variable H . The precision and recall results are illustrated in Figure 3 on 500 simulations with $n = 300$ observations for our proposed approaches *fuzzy pc SCI*, *fuzzy fci SCI*, the approaches *fuzzy pc gauss*, *fuzzy fci gauss* and the *corr τ* approaches instantiated with τ varying in $[0, 1]$ and *random*. Additional results are detailed in the case of a latent variable by the boxplots of the precision and recall scores given in Figure 4.

The few simulations ($\leq 0.3\%$) for which one of the approaches failed to estimate a causal graph were removed from the results (where one of the variables is too close from a constant). For our approaches *pc SCI* and *fci SCI*, discretization was set to form 20 categories of equal frequencies. Several discretization sizes have been tested. The bin number retained corresponds to the one which obtains the best results.

These figures show that the *random* approach reaches the expected precision and recall. Indeed, since each edge is added with a probability 0.5, and there are always 6 true edges to find among 36 possible edges, one can deduce that the expected precision is $1/6$ and the expected recall $1/2$.

The approaches by correlation *corr τ* perform differently following the considered threshold. In terms of precision, the best performances are reached for τ around 0.6. When τ is greater than 0.7, the more τ increases, the more both precision and recall scores decrease. This is explained by the fact that keeping only the edges associated with a very high correlation means keeping fewer and fewer edges. In the other way, the smaller τ is chosen, the lower the precision is, while the recall increases towards 1. Thus, the smaller τ , the more edges will be added, until we get the fully connected graph (6×6 edges) at $\tau = 0$. In this extreme case, all the real edges will be discovered so the recall will be maximal ($= 1$). On the other hand, among the 6×6 edges found, only 6 will be correct, hence a very low precision at $1/6$. When introducing the latent variable H , we observe a clear decrease in terms of precision and a slight decrease in terms of recall.

Let us now focus on the approaches that are designed for considering causality : *pc gauss*, *fci gauss*, and our approach *pc SCI* and *fci SCI*. We observe that compared to the correlation-based approaches, all these approaches are more able to maintain their performances in terms of precision

and recall when the latent variable is introduced. We see that *pc gauss* and *fci gauss* are not competitive with the correlated-based approaches, these low performances are due to the fact that the conditional independence test based on Fisher z-transformation of the partial correlation is designed for Gaussian data which is not a realistic assumption in our simulation. Finally, our proposed approach *pc SCI* and *fci SCI* are the only ones to reach precision scores greater than 0.6. The consistency of our results is illustrated in the boxplots in Figure 4. These graphs highlight that our method gives the best performances for the precision (while the recall had to be improved). In the context of our example given in Section III (the discovery of new material), precision is preferred to select the most relevant manufacturing parameters for defining the rules to predict the material properties. Furthermore, the results also show a similarity between PC and FCI despite the presence of a latent variable. This resemblance can be explained by the fact that, in the simulation settings, the latent variable is a common cause of both initial inputs X_1 and X_2 . However, in our procedure, we do not allow edges between inputs. Thus, the ability of FCI to recover the presence of latent variables between fuzzy inputs is not noticeable in our settings.

C. Influence of the number of observations

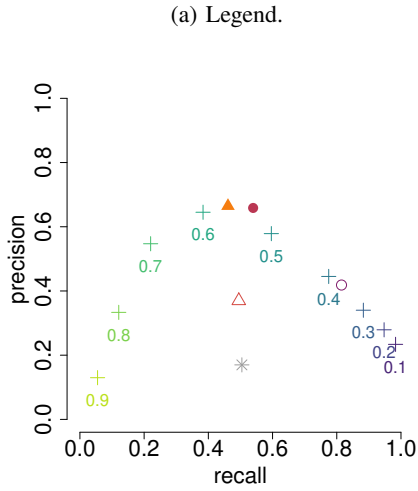
Let us now study the influence of the number of observations on the approaches performances. The means scores of precision and recall according to n are presented in Figure 5 for all the approaches. Note that the approaches *pc KCI* and *fci KCI* were only tested for $n = 100$ for computational reasons. As for the previous experiments, few simulations were removed. However, for all n considered, the percentage of removed simulations was less or equal to 3% of all simulations.

We observe that the correlation-based approaches remain rather stable in terms of precision and recall except for $\tau \leq 0.7$ where the precision decreases with higher n . The approaches *pc gauss*, *fci gauss* loose precision when n increases. The approaches *pc KCI*, *fci KCI* were only tested for $n = 100$ because they are in practice computationally demanding. These approaches gave (for $n = 100$) both precision and recall scores lower than our proposed approaches *pc SCI*, *fci SCI*. This low performance is explained by the fact that the default Gaussian kernel is not adapted to our simulated data.

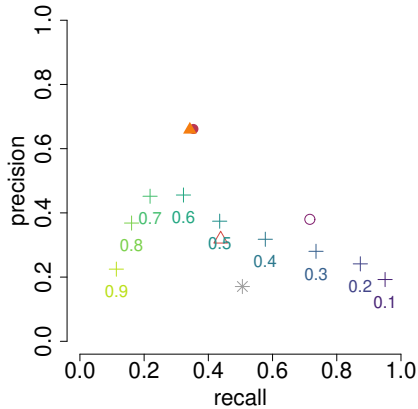
The proposed approaches both gain precision when n increases up to 300 observations, then remain stable. In terms of recall, our approaches are not sensible to n .

To summarize these experiments, the proposed approaches reach the greater precision scores in the case where a latent common cause is involved in the causal structure. Without such a latent variable, our results stay competitive with the correlation-based approaches. Our approach is sensitive to the number of observations in terms of precision but stays

- fuzzy pc gauss
- △ fuzzy fci gauss
- fuzzy pc SCI 20 + corr τ
- ▲ fuzzy fci SCI 20
- * random



(b) Without latent variable.



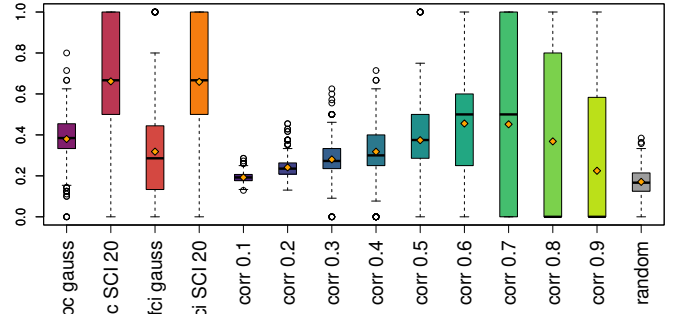
(c) With latent variable H .

Fig. 3: Mean scores of precision against mean scores of recall for 6 approaches presented in the legend (3a). Results over 485 simulations in the case where there is no latent variable (3b) and over 495 simulations in the case with a latent variable (3c).

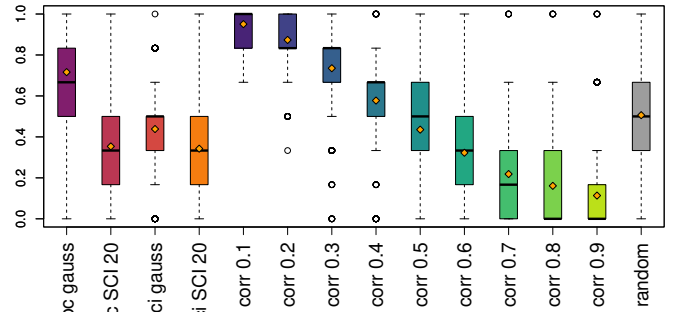
competitive with the other approaches even for a small number of observations.

VI. CONCLUSION

Standard fuzzy rules would tend to associate premises with consequences if the correlations are strong between premises and consequences. However, a correlation should not be interpreted as causality. Indeed a correlation could be explained by the presence of a common latent cause. This paper proposes an innovative framework to represent causal statements where the definitions of the cause and the effect are expressed by



(a) Precision



(b) Recall

Fig. 4: Boxplots of precision (4a) and recall (4b) scores. Diamond shape inside boxplots correspond to the mean scores presented in Fig 3.

fuzzy sets. Straightforwardly, these statements allow defining causality-based fuzzy rules. Compared to standard fuzzy rules, the causality-based fuzzy rules are not only designed for prediction. They also allow a better understanding of a system. Indeed causality is a key notion to reach the level of human intelligence. Thus, our rules can be useful in many scientific domains where it is necessary to provide insights concerning a system.

The proposed approach enables the generation of causality-based fuzzy statements from crisp observations. It performs constraints-based causal discovery algorithms like PC or FCI, combined with entropy-based conditional independence testing before data discretization. The causality research is refined to consider the different constraints specific to the fuzzy rule generation context, by preventing some causal links.

Experiments on simulations were performed, in the presence of a latent common cause and without any latent variable. The ability to recover the causal links was evaluated in terms of precision and recall for our approach and alternative ones. The proposed approach obtained competitive scores of precision and recall in the case where no latent variable is involved. In the case where there is a latent common cause, our approach obtained better performances in terms of precision, than the considered state-of-the-art and correlation-based approaches.

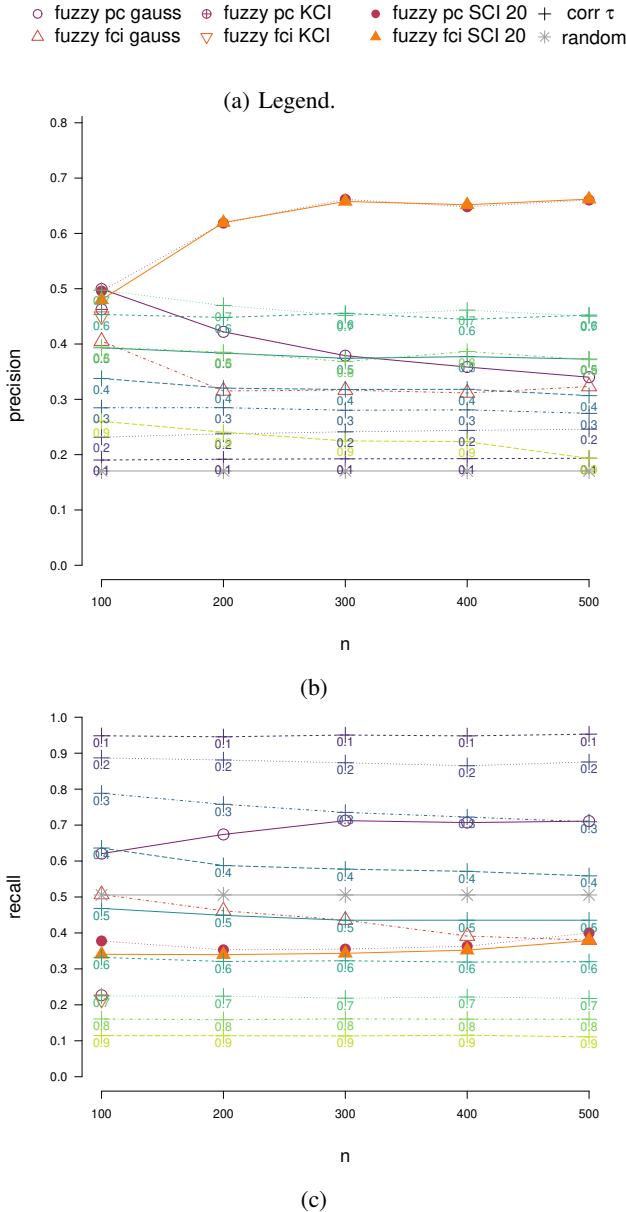


Fig. 5: For the 8 approaches presented in the legend (5a), mean scores of precision 5b and recall 5c as a function of the number of observations n . All results over at least 485 simulations.

The first perspective of this work is to apply our proposition to a real application in collaboration with experts. Then, another task will be to work on the fuzzy set extraction step. We plan to optimize the interpretability and causal relevance of fuzzy sets. We also plan to consider the joint effects that correspond to the case of “and”/“or” combinations in the fuzzy premises. However, this introduction of the joined effect in our procedure of fuzzy causal discovery would lead to an increase in complexity that would need to be optimized.

- [1] J. Pearl, “Theoretical impediments to machine learning with seven sparks from the causal revolution,” *arXiv preprint arXiv:1801.04016*, 2018.
- [2] C. P. Agueda, “Causality in science,” *Pensamiento Matemático*, no. 1, p. 12, 2011.
- [3] E. Sanchez, “Solutions in composite fuzzy relation equations: application to medical diagnosis in brouwerian logic,” in *Readings in Fuzzy Sets for Intelligent Systems*, pp. 159–165, Elsevier, 1993.
- [4] H. Hajri, J.-P. Poli, and L. Boudet, “Towards monotonous functions approximation from few data with gradual generalized modus ponens: Application to materials science,” in *2021 IEEE 33rd International Conference on Tools with Artificial Intelligence (ICTAI)*, pp. 796–800, IEEE, 2021.
- [5] J. Pearl, *Causality*. Cambridge university press, 2009.
- [6] R. Guo, L. Cheng, J. Li, P. R. Hahn, and H. Liu, “A survey of learning causality with data: Problems and methods,” *ACM Computing Surveys (CSUR)*, vol. 53, no. 4, pp. 1–37, 2020.
- [7] C. Glymour, K. Zhang, and P. Spirtes, “Review of causal discovery methods based on graphical models,” *Frontiers in genetics*, vol. 10, p. 524, 2019.
- [8] P. Spirtes, C. N. Glymour, R. Scheines, and D. Heckerman, *Causation, prediction, and search*. MIT press, 2000.
- [9] D. Colombo, M. H. Maathuis, M. Kalisch, and T. S. Richardson, “Learning high-dimensional directed acyclic graphs with latent and selection variables,” *The Annals of Statistics*, pp. 294–321, 2012.
- [10] P. L. Spirtes, C. Meek, and T. S. Richardson, “Causal inference in the presence of latent variables and selection bias,” *arXiv preprint arXiv:1302.4983*, 2013.
- [11] D. M. Chickering, “Optimal structure identification with greedy search,” *Journal of machine learning research*, vol. 3, no. Nov, pp. 507–554, 2002.
- [12] S. Shimizu, P. O. Hoyer, A. Hyvärinen, A. Kerminen, and M. Jordan, “A linear non-gaussian acyclic model for causal discovery,” *Journal of Machine Learning Research*, vol. 7, no. 10, 2006.
- [13] P. Bonissone, M. Henrion, L. Kanal, and J. Lemmer, “Equivalence and synthesis of causal models,” in *Uncertainty in artificial intelligence*, vol. 6, p. 255, Elsevier Science & Technology, 1991.
- [14] G. Schwarz *et al.*, “Estimating the dimension of a model,” *Annals of statistics*, vol. 6, no. 2, pp. 461–464, 1978.
- [15] D. Dubois and H. Prade, “Fuzzy relation equations and causal reasoning,” *Fuzzy sets and systems*, vol. 75, no. 2, pp. 119–134, 1995.
- [16] D. Dubois and H. Prade, “A glance at causality theories for artificial intelligence,” in *A Guided Tour of Artificial Intelligence Research*, pp. 275–305, Springer, 2020.
- [17] D. Dubois and H. Prade, “An overview of ordinal and numerical approaches to causal diagnostic problem solving,” *Abductive reasoning and learning*, pp. 231–280, 2000.
- [18] J. Dunn, “A graph theoretic analysis of pattern classification via tamura’s fuzzy relation,” *IEEE Transactions on Systems, Man, and Cybernetics*, no. 3, pp. 310–313, 1974.
- [19] J. C. Bezdek, “Objective function clustering,” in *Pattern recognition with fuzzy objective function algorithms*, pp. 43–93, Springer, 1981.
- [20] D. Malinsky and D. Danks, “Causal discovery algorithms: A practical guide,” *Philosophy Compass*, vol. 13, no. 1, 2018.
- [21] P. Spirtes and R. Scheines, “Causal inference of ambiguous manipulations,” *Philosophy of Science*, vol. 71, no. 5, pp. 833–845, 2004.
- [22] K. Zhang, J. Peters, D. Janzing, and B. Schölkopf, “Kernel-based conditional independence test and application in causal discovery,” *arXiv preprint arXiv:1202.3775*, 2012.
- [23] K. Fukumizu, F. R. Bach, and M. I. Jordan, “Dimensionality reduction for supervised learning with reproducing kernel hilbert spaces,” *Journal of Machine Learning Research*, vol. 5, no. Jan, pp. 73–99, 2004.
- [24] A. Marx and J. Vreeken, “Testing conditional independence on discrete data using stochastic complexity,” in *The 22nd International Conference on Artificial Intelligence and Statistics*, pp. 496–505, PMLR, 2019.
- [25] M. Kalisch and P. Bühlmann, “Robustification of the pc-algorithm for directed acyclic graphs,” *Journal of Computational and Graphical Statistics*, vol. 17, no. 4, pp. 773–789, 2008.
- [26] M. Kalisch and P. Bühlmann, “Estimating high-dimensional directed acyclic graphs with the pc-algorithm,” *Journal of Machine Learning Research*, vol. 8, no. 3, 2007.