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***Abstract*—The problem of gain estimation of a superdirective dipole-based end-fire array is discussed in this contribution. The current method to compute the gain, for a given element radiation efficiency, is based on the array factor (AF) theory. This work is intended to show that an equivalent formulation can be done using the Spherical Wave Expansion (SWE). Besides the interest in validating the theory, the main objective is a better understanding of the radiation and attenuation phenomena that occur in compact and superdirective arrays. The limits in their practical implementations are imposed by the high sensitivity of the system. The SWE theory provides more information in the expression of the radiated field, thus unfolding the possibility to address the problem with lower sensitive solutions.**

***Index Terms*— compact arrays, superdirectivity, small antennas, end-fire arrays.**

I. INTRODUCTION

The maximization of directivity in end-fire arrays has been demonstrated in previous studies using array factor (AF) theory [1]–[5] and more recently Spherical Wave Expansion (SWE) [6]. It is possible to show through numerical and full-wave simulations that the two methods are equivalent. By mean of the SWE method, upper bounds for the maximum directivity of end-fire arrays of P Huygens-sources and P electrical-dipoles have been founded as P^2+2P and of $P^2+P-1/2$, respectively [7]. The use of the SWE theory is mainly limited when losses are considered. To this purpose, the definition of the dissipation factor is re-addressed to express TE and TM spherical modes power dissipation, depending on the radiation loss and the size of the minimum sphere enclosing the array. The synthesis of superdirective arrays consists in determining the optimal feed for each elements, while their distance tends to zero.

The very compact dimension of superdirective arrays leads to a significant impact of the mutual-coupling effect and enhanced losses. This study presents a model for an accurate evaluation of the gain, aiming to show that the preliminary phase of theoretical synthesis of superdirective arrays can be carried out by using exclusively the SWE theory. Moreover, the numerical sensitivity affecting the problem diverges as the inter-elements distance tends to zero. Having a larger set of functions describing the radiated field, such as in the case of the SWE theory, offers the possibility to decrease the sensitivity. Following the concept of gain, its definition in AF theory and SWE is discussed in Section II. Then, Section III explains the methodology used to validate the proposed model, with numerical and full-wave simulations results. Finally, in section IV conclusions are drawn.

II. GAIN DEFINITION

Similarly to the directivity, the gain is a useful parameter to measure the directional capabilities of the antenna (array), but taking into account antenna(s) efficiency. This figure of merit provides additional information on how the antenna is able to convert the input power into output power. According to [8], the absolute gain of an antenna is the ratio between the field intensity in a given direction and the total accepted input power, defined as

$$G(\theta_0,\phi_0) = \frac{|\vec{E}(\theta_0,\phi_0)|^2}{2\eta_0(P_{rad}+P_{loss})} \tag{1}$$

where η_0 is the free-space impedance, r is the radial distance of the measured point and $\vec{E}(\theta_0,\phi_0)$ is the electric field in the chosen direction. The denominator represents the total accepted power sum of radiated power P_{rad} and the P_{loss} expressing the amount of power that is not radiated and dissipates on the resistive part of the antenna. In the case of an array, the computation of gain becomes more complicated. The effect of mutual coupling should be taken into account additionally to the losses of each element. Parameters as loss resistance, efficiency factor, quality factor or dissipation factor bring equivalent results in the evaluation of losses, but depending on the mathematical approach used is more convenient the use of one instead of another.

A. Array Factor (AF) Theory

The literature on superdirective arrays is quite large [1]–[5]. In the general case of P array elements, with the AF theory the field is

$$E_{tot}(\theta,\phi) = \sum_p A_p \cdot f_p(\theta,\phi) \cdot e^{jkrr_p} \tag{2}$$

the $A_p e^{jkrr_p}$ terms are the amplitude and phase coefficients related to the elements feed and phase shift due to positioning. The $f_p(\theta,\phi)$ is the far-field pattern of the source

p. Regarding the energy balance of the antenna array, the radiated power loss can be calculated as follows

$$P_{loss} = \frac{1}{2}\sum_{n=1}^{P} R_{loss,n} \left|I_n\right|^2 \tag{3}$$

where I_n is the maximum of the current running through the *n-th* element, and $R_{loss,n}$ is the loss resistance of the *n-th* element. Similarly, the radiated power P_{rad} is given by

$$P_{rad} = \frac{1}{2}\sum_{m=1}^{P}\sum_{n=1}^{P} I_m I_n^* R_{mn} \tag{4}$$

where the $R_{m,n}$ are the mutual resistances for the *n-th* and *m-th* elements. The currents I_n are proportional to the feeding coefficients A_n of the array, thus the terms $h_{mn} = \nu R_{mn}$ can be defined, with ν a proportional factor between the feeding coefficients and the currents I_n. Hence, the total array gain is

$$G = \frac{\left|\sum_{m=1}^{P}\sum_{n=1}^{P} a_m a_n^* e^{jk\hat{r}\cdot r_m} e^{-jk\hat{r}\cdot r_n}\right|^2}{\sum_{m=1}^{P}\sum_{n=1}^{P} a_m a_n^* h_{mn} + \sum_{n=1}^{P}\left|a_n\right|^2 h_{loss,n}} \tag{5}$$

at equation (5) is applied the normalization

$$a_n = A_n \left|f_n(\theta,\phi)\right| \tag{6}$$

$$h_{m,n} = \frac{H_{m,n}}{f_n(\theta,\phi)\cdot f_n(\theta,\phi)^*} \tag{7}$$

The h_{mn} are elements of $\mathbf{H}$, a $P\times P$ matrix consisting of all the self-and mutual resistances. The losses of each element are represented by $h_{loss,n}$, added only to the respective self-resistance.

B. Spherical Wave Expansion (SWE) theory

As already shown in our previous works [6], for the study and synthesis of superdirective array the SWE theory allows expressing the radiated field as a combination of a rich set of functions. These are discriminated by the index $s = 1,2$ for TE or TM modes, $n = 1,..,N$ the spherical mode order, and $m = -n,..,n$ the azimuthal oscillation. Considering an array of P sources, in far-field condition, the total electric field is

$$E_{tot}(\theta,\phi) = \sum\nolimits_{smn} \vec{K}_{smn}(\theta,\phi) \sum\nolimits_{smn,p} \alpha_p Q_{smn,p} \tag{8}$$

the $\vec{K}_{smn}(\theta,\phi)$ are the spherical far-field functions and $Q_{smn,p}$ the spherical coefficients as defined in [9], α_p the complex coefficients expressing the element feed. Then, substituting (8) in (1) the gain in SWE is written as

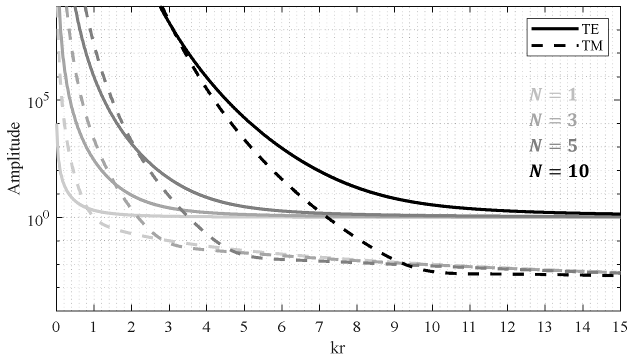

Fig. 1. Dissipation factor for TE and TM spherical modes of order *N=1,3,5,10* as a function of the electrical size *kr*. The material losses are normalized to 1.

$$G(\theta_0,\phi_0) = \frac{\left|\sum_{smn,p} \alpha_p Q_{smn,p} \vec{K}_{smn}(\theta_0,\phi_0)\right|^2}{\sum_{smn}\left|Q_{smn}\right|^2 + P_{loss}} \tag{9}$$

where the term $\sum_{smn}\left|Q_{smn}\right|^2$ is proportional to the total radiated power, according to [9]. In the literature, an upper bound for the radiation efficiency in the case of infinitesimal antennas can be found in [10], or calculation of the efficiency for given small antennas geometry by using the equivalent circuit [11]. Besides that, the calculation of the P_{loss} term using SWE theory has not been shown yet in a practical case.

Harrington in [12] addressed the calculation of the maximum gain for a spherical metallic shield of radius R considering it as a discontinuity in the medium for the characteristics impedances of the spherical TE and TM modes, which are out-traveling from the center of the sphere. The conclusion is that the losses are, at radius R, given by the ratio between the real part of the complex impedance of the metal η_c and of the vacuum η_0 when the maximum mode order $N_{\max} \approx R$. For a generic value of $N_{\max}$ and/or a radius kr the losses can be quantified applying the general definition of the dissipation factor as

$$D_n = \frac{\mathrm{Re}\{\eta_c\}}{2\eta_0}\left[\left|F_n(kr)\right|^2 + \left|F_n{}'(kr)\right|^2\right] \tag{10}$$

where the metal losses are associated to the attenuation of the spherical Hankel functions $F_n(kr) = kr\cdot h_n^{(2)}(kr)$, describing the radial amplitude dependence kr of the spherical waves, for each mode order n. Fig. 1 displays the dissipation factor normalized to $2\eta_0/\mathrm{Re}\{\eta_c\}$. The components of the TE and TM modes are picked separately to remark the different attenuation associated. As stated by Harrington in [12], because of the orthogonality of energy and power the total power is $P_{rad} + P_{loss} = \sum_n P_{rad,n}\cdot(1+D_n)$. From the considerations on the orthogonality of power, the gain can be expressed as

$$G(\theta_0,\phi_0)=\frac{\left|\sum_{smn}Q_{smn}\overline{K}_{smn}(\theta_0,\phi_0)\right|^2}{\sum_{smn}\left|Q_{smn}\cdot\sqrt{(1+D_n)}\right|^2} \tag{11}$$

To the best knowledge of the authors, the definition of gain as in (11) has not been provided yet. The material losses in (10) expressed by $\mathrm{Re}\{\eta_c\}/2\eta_0$ are defined for the ideal case above discussed. In the practical case, this term is substituted by the normalized loss resistance of each radiating element.

III. Simulation Results

To validate the proposed gain model, results from numerical and full-wave simulations are reported in this section. The AF and SWE theories are used to calculate the gain for superdirective end-fire arrays of 2 and 3 electrical dipoles. The directivity estimation and optimization is discussed in [6], and results obtained are in very good agreement by comparing the two theories considered. The evaluation of gain in the case of maximum directivity for end-fire arrays has been already studied with the array factor theory, and results validated via full-wave simulations and measurements [7]. The scope is to show that the same study can be entirely carried-out with the SWE theory. The numerical simulations are performed in MATLAB using infinitesimal dipoles model for the radiated field. The chosen size for the infinitesimal dipoles, due to its impact in the gain definition according to (10), is chosen to be $\lambda/(2\pi)$ as defined by Wheeler in [13].

Assuming all elements having an equal radiation efficiency and radiation pattern, the values of loss resistance are set to 0.01 and 0.05, i.e. efficiency of 99% and 95% respectively. These values are used in (10) to express the material losses. Then, the gain for optimal directivity is calculated for d approaching zero and results displayed in Fig. 2. The SWE gain presents higher attenuations for larger spacing of the elements comparing to the AF theory, and lower for d close to 0. This may be the results of the spherical modes distribution, which by increasing the electrical size of the array, according to the expression $N_{\max}=10+kr_0$, cause that higher order modes appear, which are heavily attenuated by the dissipation factor. In both array configuration the chosen losses are compared with the lossless case $r_{loss}=0$ which returns the directivity and a perfect match in the comparison.

As for the numerical case, full-wave simulations of half-wave electrical dipoles array are performed in CST MS. The conducting material is copper ($\sigma\approx 5\cdot 10^7\,S/m$) and the loss resistance is calculated by the simulator, where the mutual coupling effect is taken into account. The far-field is extracted and the directivity optimized for d approaching zero. The optimal solutions are used for post-processing the field in the full-wave simulator. Hence, the results displayed in Fig. 3 compare the gain from full-wave simulation (reference) and the gain calculated on the imported field using the SWE and AF theories. The values of gain calculated

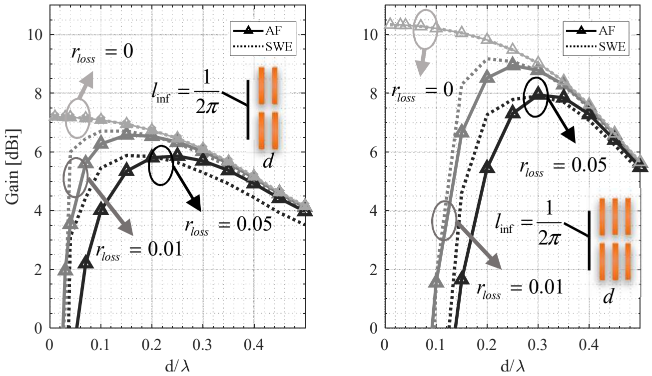

Fig. 2. Theoretical gain for 2 (left) and 3 (right) dipoles arrays. The AF and SWE theories are compared using different r_{loss} and for d which tends to 0.

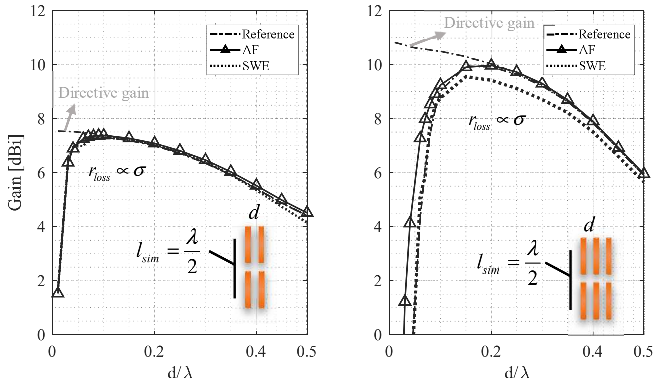

Fig. 3. Simulated gain for 2 (left) and 3 (right) dipoles arrays. The full-wave simulated gain (reference) is compared with the SWE (dotted line) and the AF (triangle-marked). The simulated directive gain is plot as reference.

with two theories are in good agreement for both array configurations.

IV. Conclusions

This work provides a new method for antenna or array gain estimation based on the SWE theory, proposed as an alternative to the well-known AF theory. The author wants to stress the fact that the purpose of this study goes beyond the theoretical validation of the model, but aims to provide more information on the radiation properties of a given radiating structure. In the specific case of directivity optimization problem, the introduction of losses in the spherical coefficients matrix $Q_{smn,loss}$ reduce its conditioning, extremely high when shorts inter-element distance are imposed in the array, and consequently a net decrease of the sensitivity of the system. Furthermore, once determined the correct model for the gain, this could be used to perform gain optimizations of the array, which has a great interest in many applications.

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