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Tommaso Barani, Isabelle Ramière, Bruno Michel. Analysis of fabrication and crack-induced porosity migration in mixed oxide fuels for sodium fast reactors by the finite element method. *Journal of Nuclear Materials*, 2022, 558, pp.153341. 10.1016/j.jnucmat.2021.153341 . cea-03581232

HAL Id: cea-03581232

<https://cea.hal.science/cea-03581232v1>

Submitted on 19 Feb 2022

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Analysis of fabrication and crack-induced porosity migration in mixed oxide fuels for sodium fast reactors by the finite element method

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Abstract

We present an engineering-scale model for the migration of porosity in a fuel pellet experiencing a temperature gradient. The system of coupled pore advection and heat diffusion equations governing the problem is solved through a fixed-point iteration technique. The coupling between porosity and temperature fields is considered via the dependency of pore advection velocity on the local temperature and temperature gradient, and via the dependency of fuel thermal conductivity and of the volumetric power source on the local porosity. We employ the finite element method to discretize the resulting equations. As pure advection solutions obtained by this method are well-known to present spurious spatial oscillations, we introduce stabilization techniques in the pore advection equation. The proposed model is first tested against a benchmark problem representative for the conditions of an uranium-plutonium oxide fuel pellet irradiated in a sodium fast reactor. The results are compared to the those obtained by a model implemented in the BISON fuel performance code. The analysis shows how the results of the newly developed model are in line with those obtained by the reference model, and underlines a superior stability of the solution. The model is then applied to analyze the contribution of as-fabricated and crack-induced porosities in determining the fuel restructuring and in particular the central hole formation. A comparison to experimental data shows the impact of considering crack-induced porosity to predict the extent of the central void.

Keywords: Porosity Migration, Temperature gradient, Fixed-point iterations, Finite Element Method, Stabilization Techniques, Mixed Oxide Fuel

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1. Introduction

The combination of high temperatures and steep temperature gradients in the radial direction of fuel pellets irradiated in light-water or – mainly – fast reactors promotes a substantial restructuring of the as-fabricated microstructure [1]. The main phenomena governing such restructuring are sintering, grain growth, and void/pore migration. Focusing on (mixed) oxide fuel irradiated in fast reactors, such phenomena occur as fuel is brought to power and eventually result in the formation – proceeding from the outer part of the fuel towards the center – of an as-fabricated microstructure zone (i.e., where the temperatures are not high enough to promote the aforementioned phenomena), a zone marked by equiaxed grain growth, a zone marked by columnar grains oriented in the radial direction, and a central void [1, 2]. Each zone is characterized by a different density (and plutonium content), thus different bulk properties.

To properly analyze the performance of mixed oxide fuels in fast reactors, the phenomena listed above must be represented in the framework of the thermo-mechanical analysis of the fuel pin. In this work, our attention is drawn on modeling the pore migration mechanism from an engineering-scale perspective, i.e., in the framework of continuum mechanics. The local porosity influences a number of key properties, including fuel thermal conductivity, local power generation, and elastic properties [1, 3, 4], thus it is a dominant factor in determining the thermal condition in the fuel region.

A consensus arises in the literature about the leading mechanism for pore migration, which is attributed to transport via successive evaporation and condensation of the fuel on the pore surface at different temperatures [1, 2, 5–7]. In detail, the pores are normally filled with low-pressure, low-conducting gas species (e.g., CO₂ or He), which affects the local temperature gradient. Thus, the presence of the pore modifies the equilibrium partial pressure, which depends on the local matrix composition and temperature, inducing a preferred evaporation of some species from the hot zones and their condensation on the cold one. This transport mechanism, beside being responsible for the migration of the porosity, affects also the redistribution of actinides (namely, plutonium and americium) along the radius, since the chemical species containing these elements are less prone to evaporate and thus accumulates towards the hot pore interface as migration proceeds. For further details about this phenomenon, the reader is referred to [1, 2].

In the light of its importance in determining fuel performance of mixed oxide fuels in fast reactor conditions, models have been developed along the years [1, 5–7] and included in fuel performance codes to account for porosity migration [8–17]. Recent benchmark exercises [18] underlined the need to ameliorate models on pore migration, showing how the predictions on the central hole size are scattered and not seldom inaccurate. Moreover, in a recent work [19] it was proposed that the displacement of the free volumes due to the porosity migration contributes to the relocation strain of fuel. In particular, a 3D study realized in this work showed how the mass relocation through the evaporation/condensation along free crack surfaces can lead to a rigid body ra-

46 dial relocation displacement of the pellet fragment and then contribute to the
47 closure of the pellet-to-cladding gap.

48 The aforementioned codes resort either on finite differences/volumes method
49 (e.g., GERMINAL [15] and TRANSURANUS [8]) or on the finite element
50 method (e.g., CEDAR [13] and BISON [14, 17]) to solve the equations gov-
51 erning the migration of porosity. The solution of the (pure) advection equation
52 by the standard Galerkin Finite Element Method (G-FEM), resorting on a cen-
53 tered scheme for the discretization of gradient operator, is known to generate
54 spurious spatial oscillations (see for example [20]). Upwind schemes for the dis-
55 cretization of the gradient operator are known to remove this issue [21], but
56 they are generally not included in finite element libraries frameworks. Instead,
57 in the framework of standard G-FEM resorting on centered schemes, dedicated
58 stabilization techniques – acting as upwind schemes – can be adopted to ensure
59 a stable solution of such family of equations [21, 22].

60 In this work, we propose an original modeling framework for the coupled so-
61 lution of the pore migration and heat conduction equations by the finite element
62 method. The open-source library MFEM [23] is used as software platform. The
63 solution of the studied equations requires to solve a non-linear system, whose
64 solution is achieved by a fixed-point iteration algorithm. As for the stabilization
65 techniques, the solution of the pore advection equation is stabilized by two tech-
66 niques, namely the Streamline-Upwind (SU) and Streamline-Upwind/Petrov-
67 Galerkin (SUPG) schemes [22]. A critical comparison to a modern, finite-
68 element-based code (BISON) is presented and discussed on the example case
69 of as-fabricated porosity migration published in [14].

70 Since our solver enables an independent or coupled simulation of cracks-
71 induced and as-fabricated porosity migration and, to the best of our knowledge,
72 crack-induced migration has never been accurately simulated at the fuel scale,
73 we propose an analysis of the influence of cracks on the fuel restructuring pro-
74 cess. In particular, we present firstly a qualitative assessment of the interaction
75 between as-fabricated and crack-induced porosity, discussing the implications
76 on the central void formation. Finally, we provide a quantitative assessment by
77 analyzing an experiment carried out in the Phenix sodium fast reactor presented
78 in [19] and comparing the results obtained by the presented model against ex-
79 perimental results, underlining the impact of crack-induced porosity on the pre-
80 diction of the central hole extension. We underline that the focus of the present
81 work is more centered on the development of a physically grounded model de-
82 scribing pore migration and a corresponding consistent mathematical framework
83 to solve its governing equations in the framework of G-FEM. A thorough val-
84 idation, corroborated by sensitivity and uncertainty studies on the parameters
85 governing the pore migration and temperature distribution, is beyond the scope
86 of the present work and will be the object of future investigations.

87 The outline of the paper is as follows. In Section 2, we outline the mathe-
88 matical model developed to represent pore migration. In Section 3, we present
89 some stabilization techniques for the pore advection equations. In Section 4,
90 we present the comparison to the BISON results and critically analyze them.
91 In Section 5, we showcase the results on the crack influence on pore migration

92 and fuel restructuring, together with a preliminary assessment of the modeling
 93 framework against experimental data. Conclusions are drawn in Section 6.

94 2. Mathematical model

95 In this section, we present the equations governing the coupled temperature
 96 and porosity fields and the numerical scheme designed to solve the problem.

97 2.1. Governing equations and numerical solution scheme

98 The equations governing the temperature, $T(\mathbf{x}, t)$ ¹, and porosity, $p(\mathbf{x}, t)$,
 99 distribution are the energy conservation (heat conduction) equation and the
 100 pore advection equation, respectively, reading

$$\left\{ \begin{array}{l} \rho c_p \frac{\partial T}{\partial t} - \nabla \cdot [k(T, p) \nabla T] - q_v \frac{1-p}{1-p_0} = 0 \\ \frac{\partial p}{\partial t} + \nabla \cdot [\mathbf{v}(T)p] = 0 \end{array} \right. \quad (1a) \quad (1b)$$

101 where ρ (kg m^{-3}) is the fuel density, c_p ($\text{J kg}^{-1} \text{K}^{-1}$) is the heat capacity, T (K)
 102 is the temperature, k ($\text{W m}^{-1} \text{K}^{-1}$) is the thermal conductivity, q_v (W m^{-3}) is
 103 the volumetric heat source due to fissions, p_0 (/) is the as-fabricated (initial)
 104 porosity, p (/) is the current porosity, \mathbf{v} (m s^{-1}) is the pore velocity. This for-
 105 mulation enforces naturally the respect of the porosity physical bounds between
 106 zero (fully dense material) and one (void). In fact, when the porosity approaches
 107 one, the heat source is suppressed and therefore the temperature gradient be-
 108 comes null, in turn suppressing further pore migration, given its dependence
 109 on the temperature gradient (pore velocity equal to zero, cf. Eq. (9)). On the
 110 other hand, the lower bound for porosity is naturally enforced by the solution
 111 of the advection equation itself.

112 Given the coupled and non-linear nature of the problem, we conceived a
 113 fixed-point iteration scheme to achieve system (1) solution. Thanks to the
 114 fixed-point algorithm, the heat equation (1a) is linearized evaluating the thermal
 115 conductivity at the previous iteration temperature. Therefore, at each iteration,
 116 all the equations to be solved are linear (see Figure 1).

117 At each time step, the convergence check is carried out on the porosity
 118 solution, and consists in a mixed relative/absolute criterion, reading

$$\text{Max} (|p_{t+1}^{k+1} - p_{t+1}^k| - |p_{t+1}^k| \cdot \varepsilon_{rel} - \varepsilon_{abs})_{\text{Nodes}} < 0 \quad (2)$$

119 where ε_{rel} and ε_{abs} are the relative and absolute tolerances, respectively. This
 120 convergence criterion is preferred to a classical relative error check, since it
 121 eliminates the numerical complications arising when the porosity is close to
 122 zero and it automatically switches from relative to absolute error when needed.

¹With \mathbf{x} the spatial coordinates and t the time.

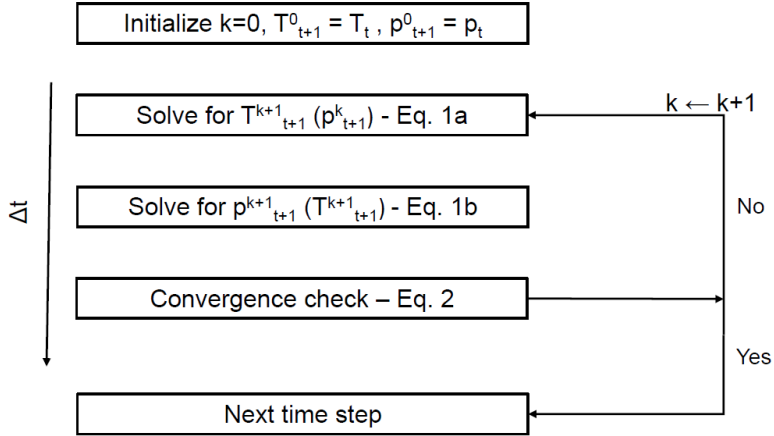


Figure 1: Sketch of the system solution scheme.

123 The numerical tool we choose to solve Eqs. (1) is MFEM [23], an open
 124 source collection of C++ libraries to solve partial differential equations (PDEs)
 125 via the finite element method. MFEM allows solving 1D, 2D, and 3D problems
 126 using different orders and types of finite elements. Moreover, it allows massive
 127 parallelization of the code. Various time integration schemes are available in
 128 the MFEM solver, both implicit and explicit.

129 It is worth underlying that the governing system considered in this work
 130 (Eqs. (1)) is conceptually similar to those proposed in previous works on the
 131 subject. Sticking to those employing the finite element method, and focusing
 132 on the most recent publications [14, 17] regarding the BISON fuel performance
 133 code, the main difference is found in the pore advection equation and in the
 134 numerical strategy to couple energy and pore advection equations.

135 In fact, in the BISON model a term equal to $(1 - p)$ multiplies the pore
 136 velocity, representing the suppression of pore migration when the void is formed.
 137 Nevertheless, this term does not have a physical ground, i.e., the governing
 138 physics is artificially manipulated to suppress pore migration when full “void”
 139 is achieved. Indeed, this is an unnecessary constraint, since the temperature
 140 gradient naturally vanishes when the porosity equals one, and therefore pore
 141 migration is automatically suppressed in this case, because the pore migration
 142 velocity is directly proportional to the temperature gradient in the matrix [1, 5–
 143 7].

144 On the numerical aspect, the solution of the governing system in BISON is
 145 sought through the Jacobian-Free Newton-Krylov (JFNK) method, which en-
 146 ables a fully-coupled solution of the problem, and considering an implicit time
 147 integration, whereas we consider a fixed-point iteration scheme and an explicit
 148 time integration. The JFNK method should be regarded, in general, as a leading
 149 method to solve coupled non-linear PDEs, in light of its positive sides in terms

150 of fast non-linear convergence, scalability, and parallelization possibility [24].
151 For the considered coupled system, which is well-posed, fixed-point iterations
152 converge in few steps. This algorithm can be easily implemented in every com-
153 putational framework and does not require the estimation of any Jacobian-like
154 matrix, which is generally time consuming to build and computationally ex-
155 pensive to handle. For example, the results presented in Section 4 have been
156 obtained on a personal computer. Finally, for the time integration scheme, im-
157 plicit schemes theoretically ensure a numerical stability of the solution in time
158 independently from the time step [20]. At the same time, since the physical
159 phenomena occurring in the nuclear fuel might exhibit fast intrinsic dynamics,
160 employing too large time steps would result in inaccurate solutions. Hence,
161 there is not significant differences in time steps that can be employed using
162 explicit and implicit schemes.

163 3. Stabilization of the pore advection equation

164 Advection-dominated equations solved by the Galerkin Finite Element Method
165 (G-FEM), employing a centered scheme for the discretization of the differential
166 operators, are known to be unstable [20, 22], i.e., to exhibit spurious spatial
167 oscillations in the solution. This problem, classically encountered in other fields
168 such as fluid mechanics, has been encountered also in previous works analyzing
169 the pore migration by the finite element method [14]. In the latter work, a
170 workaround is introduced by adding to the pure advection equation a laplacian
171 term multiplied by a constant diffusivity, which is thought to be representative
172 for pore (bulk) diffusivity. Such a correction is questionable, both on a physical
173 and mathematical perspective. On the one hand, the typical size of fabrica-
174 tion pores (in the micrometric range) is such that surface and bulk diffusion
175 processes are strongly inhibited [25] and could be deemed irrelevant compared
176 to the transport by evaporation/condensation. On the other hand, inserting
177 a constant diffusivity in the advection equation results in a distortion of the
178 solution with respect to the correct one (e.g., [22]).

179 To overcome the aforementioned problems, one can change the discretization
180 scheme for the gradient (namely, opting for upwind schemes) or use dedicated
181 stabilization techniques for centered schemes. In this work, we implemented
182 in MFEM two classical stabilization techniques for the pore advection equa-
183 tion, namely the Streamline Upwind (SU) and the Streamline Upwind/Petrov-
184 Galerkin (SUPG) schemes [22, 26]. These techniques consist in modifying in
185 a consistent manner an advection equation, introducing dedicated stabilization
186 terms and allowing for a stable solution also in the framework of the G-FEM.
187 An interesting observation is that these stabilization techniques share with the
188 upwind discretization of the gradient an essential equivalence, i.e., they result in
189 the addition of a diffusion term to the centered discretization of the advection
190 equation [21]. Employing the SU or SUPG techniques is generally favored, since
191 it allows remaining in the framework of standard finite elements.

192 *3.1. Streamline upwind*

193 The streamline upwind (SU) method consists in adding an artificial diffusion
 194 term optimally chosen to balance the G-FEM intrinsic (negative) diffusivity,
 195 yielding an exact nodal solution. The idea is to add a diffusivity in the “flow”
 196 direction such that the cell Peclet number becomes equal to 1, i.e., only the
 197 “useful” amount of diffusivity is introduced. Accordingly, eq. (1b) is modified
 198 as follows

$$\frac{\partial p}{\partial t} + \nabla \cdot (\mathbf{v}p) = \nabla \cdot \overline{\overline{K}} \nabla p \quad (3)$$

199 where $\overline{\overline{K}}$ ($\text{m}^2 \text{s}^{-1}$) is a second order tensor defined as (e.g., in a bi-dimensional
 200 case)

$$\overline{\overline{K}} = \frac{h^e}{2|\mathbf{v}|} \begin{bmatrix} v_i v_i & v_i v_j \\ v_j v_i & v_j v_j \end{bmatrix} \quad (4)$$

201 where v_i and v_j are the component of the velocity along the generic i and j
 202 direction, h^e is the finite element size, and $|\mathbf{v}|$ the magnitude of the velocity.

203 One can notice how this method is *consistent*, since it depends explicitly
 204 on the mesh size, thus approaches zero when the mesh size diminishes. Since
 205 it is dependent on the local velocity and on the element size, it introduces the
 206 correct amount of diffusivity in the whole domain. It is deemed superior to the
 207 approach employed in [14] albeit being practically so simple. It is worthwhile
 208 underlying how this approach can be plugged on the advection equation directly
 209 in the strong formulation. Nonetheless, for un-stationary problems or stationary
 210 problems with non uniform source terms, this stabilization technique is well-
 211 known to be too diffusive, see Appendix A and [22].

212 *3.2. Streamline upwind/Petrov-Galerkin*

213 To introduce the SUPG stabilization technique, we pass to the weak form
 214 of the advection equation (1b). Being p the generic trial function, w the test
 215 function, and Ω the considered domain, we write

$$\int_{\Omega} w \frac{\partial p}{\partial t} d\Omega + \int_{\Omega} w \nabla \cdot (\mathbf{v}p) d\Omega = 0 \quad (5)$$

216 Being Ω^{el} a partition of the domain, we add to the LHS of the equation
 217 above a term of the form

$$r(p, w) = \sum_{el=1}^{n_{el}} \int_{\Omega^{el}} \mathcal{P}(w)^e \tau^e \mathcal{R}^e(p) d\Omega^{el} \quad (6)$$

218 where $\mathcal{P}(w)$ is an operator applied to the test function and $\mathcal{R}(p)$ the residual
 219 of the PDE we are solving. Plugging the aforementioned term in equation (5),

220 being the operator $\mathcal{P}(w)$ the skew-symmetric part of the advection operator,
 221 we obtain the SUPG formulation²

$$\int_{\Omega'} \frac{\partial p}{\partial t} (w + \tau \mathbf{v} \cdot \nabla w) + \int_{\Omega'} \nabla \cdot [\mathbf{v} p] (w + \tau \mathbf{v} \cdot \nabla w) = 0 \quad (7)$$

222 The upstream parameter τ is defined as $\tau = h^e/2|\mathbf{v}|$, where h^e is the finite ele-
 223 ment partition size. In the case of a divergence free velocity, the $\mathcal{P}(w)$ operator
 224 applied on the divergence term in association with the upstream parameter is
 225 similar to the SU formulation. However, it must be underlined that while the
 226 SU stabilization can be induced directly in the strong formulation of the ad-
 227 vection equation, the full SUPG formulation can be only included in the weak
 228 formulation of the problem due to the required modification of the mass ma-
 229 trix. A verification of the implementation of the SUPG method is presented
 230 in [Appendix A](#).

231 4. Analysis of as-fabricated porosity migration

232 To assess the results of the modeling framework exposed above, we present
 233 the simulation results on a test-case firstly introduced in [14]. The results are
 234 compared to those published in the aforementioned work, showing how the re-
 235 sults obtained by our model are in line with those obtained by BISON, yet
 236 showcasing a superior stability in the solution thanks to the employed stabi-
 237 lization techniques. In addition, a comparison on the same test-case using the
 238 two different stabilization techniques presented above is showed, to demonstrate
 239 how the techniques are mostly equivalent on the case of interest.

240 4.1. Setup of the calculations

241 The computational mesh is a circular sector of radius 2.675 mm spanning
 242 $\pi/8$ in the angular direction. We considered meshes, especially for the com-
 243 parison to the BISON results, having different densities, namely 50 and 100
 244 intervals in the radial direction. Non-conforming meshes are employed in this
 245 section, trying to preserve an aspect ratio close to the unity of the elements,
 246 which are of quadrilateral, first order type. The oxygen-to-metal ratio is taken
 247 equal to 1.975 and the plutonium content equal to 20 wt.%.

248 For the heat conduction equation, a uniform initial temperature equal to 623
 249 K is considered. A time-varying Dirichlet boundary condition is applied on the
 250 outer surface, linearly varying from 623 to 1300 K over a time period of 10^4 s.
 251 Over the same time, the linear power is brought to 500 W cm^{-1} . A zero flux

²From there on, we will employ the notation

$$\int_{\Omega'} = \sum_{el=1}^{n_{el}} \int_{\Omega^{el}}$$

252 boundary condition (homogeneous Neumann boundary condition) is enforced
 253 at the other surfaces, enabling axisymmetry.

254 As for the pore advection equation, a uniform initial condition with a poros-
 255 ity equal to 0.15 is set. The boundary conditions enforced in this equation are
 256 tricky. In fact, the pure advection equation does not induce “physically” sound-
 257 ing natural boundary conditions and its solution normally involves enforcing
 258 Dirichlet BC at the “inlet” of the domain [22]. On the other hand, adding a
 259 stabilization technique such the ones introduced in Section 3 induces a “diffu-
 260 sive” flux natural boundary condition, which can be more easily justified from a
 261 physical perspective. Considering the SU stabilization technique, the naturally-
 262 induced weak boundary condition has the form of a homogeneous Neumann
 263 boundary condition

$$\int_{\Gamma} w \left[(\overline{K} \nabla p) \cdot \mathbf{n} \right] d\Gamma = 0 \quad (8)$$

264 which takes into account both velocity (through the diffusion-like coefficient)
 265 and normal porosity gradient, while the latter is the only term considered in [14].

266 The relative and absolute tolerances in the fixed-point iteration scheme are
 267 set to 10^{-6} and 10^{-8} , respectively. The time integration was carried out consid-
 268 ering a time step of 1 s and an explicit forward Euler time integration scheme.

269 The respect on the Courant cell condition (i.e., $Co = \sum_{i=1}^N \frac{v_i \Delta t}{\Delta x_i} < 1$ for every
 270 cell) is controlled at each time step, since the velocity varies with time.

271 4.2. Model parameters

272 The pore velocity expression is known to be a very uncertain yet pivotal
 273 property for the assessment of pore migration [27]. Different correlations have
 274 been developed in the years, for both UO_2 and $(U, Pu)O_2$ [1, 5–7, 28–31], whose
 275 results showcase a substantial scattering [27]. More sophisticated approaches,
 276 taking into account the vapor pressures of the different species found in the
 277 vapor phase, are illustrated in [15, 17].

278 In this work, the pore velocity is evaluated according to Sens [7], as was done
 279 already in a previous work on the subject [14], reading

$$|\mathbf{v}| = c_0 (c_1 + c_2 T + c_3 T^2 + c_4 T^3) \Delta H_s P_{0,s} \exp \left\{ -\frac{\Delta H_s}{RT} \right\} T^{-2.5} |\nabla T| \quad (9)$$

280 where c_0 , c_1 , c_2 , c_3 , and c_4 are constants, ΔH_s ($J \text{ mol}^{-1}$) is the heat of vapor-
 281 ization, $P_{0,s}$ is a material parameter, and R ($J \text{ mol}^{-1} \text{ K}^{-1}$) is the universal gas
 282 constant.

283 The pore velocity depends in principle on the temperature gradient across
 284 the pore itself, but a relationship to the temperature gradient across the matrix
 285 has been classically considered in the literature, to couple the solution of the heat
 286 conduction equation directly to the pore advection equation. In this way, when
 287 the central hole is formed and the heat generation is suppressed (cf. equation
 288 (1a)), the gradient flattens and naturally suppresses the pore advection.

The thermal conductivity is accounted for considering the correlation proposed by Kato and coworkers [32], discarding the correction terms accounting for Am and Np contents (since we are not investigating minor actinides bearing (U, Pu)O₂) and replacing the porosity correction term by the Maxwell-Eucken model [33], which is a more appropriate way to calculate the thermal conductivity of a two-species mixture as it is modeled in this case ³. Thus, the Kato correlation for the temperature-dependent part reads

$$k(T) = \frac{1}{(2.713 \cdot x + 1.595 \times 10^{-2}) + (2.493 - 2.625 \cdot x) \times 10^{-4} \cdot T + \frac{1.541 \times 10^{11}}{T^{5/2}} \exp\left(-\frac{1.522 \times 10^4}{T}\right)} \quad (10)$$

289 whereas the complete correlation reads

$$k(T, p) = k(T) \frac{k_{He} + 2k(T) - 2p(k(T) - k_{He})}{k_{He} + 2k(T) + p(k(T) - k_{He})} \quad (11)$$

290 where x (/) is the deviation from stoichiometry and k_{He} (W m⁻¹ K⁻¹) is the
 291 thermal conductivity of the pores. As far as the thermal conductivity calculation
 292 is concerned, we assume the pore to be filled with helium, for which we take
 293 a representative, constant value, equal to 0.69 W m⁻¹ K⁻¹. The correction of
 294 thermal conductivity on burnup is not considered in the present study, since we
 295 are analyzing only very short irradiation histories. A modification of Eq. (11)
 296 to account for burnup would be straightforward and could rely, for example, on
 297 the recent work published by Magni and coworkers [34]. It is worth noticing
 298 that redistribution of plutonium would occur and influence the local thermal
 299 conductivity (directly and affecting the oxygen-to-metal ratio), but we are not
 300 accounting for this phenomena in the proposed model since we focus more on
 301 providing a robust and consistent modeling of pore migration, rather than a
 302 comprehensive fuel behavior module.

303 4.3. Comparison of the SU and SUPG techniques

304 The results obtained on the test-case employing the SU and SUPG stabili-
 305 zation techniques are reported in Figure 2. Two different mesh densities are
 306 considered, namely 50 (Fig. 2a) and 100 (Fig. 2b) elements.

307 The results are in line with theoretical expectations. We can see how the
 308 spurious oscillations classically encountered when solving the advection equation
 309 by the G-FEM are removed by both techniques. In fact, the difference between
 310 the two techniques is minimal, with the results obtained by the SU method
 311 being slightly more diffusive than those obtained by the SUPG method. Sub-
 312 stantial differences between these techniques arise when the initial condition is

³Other approaches for the porosity correction of the thermal conductivity are available in the open literature (e.g., see [4]). Indeed, the correction employed here allows to directly pass from the conductivity of pure oxide to the conductivity of pure gas (helium) when the central hole forms, without the need of introducing step-wise thresholds.

313 not uniform in the domain, or when space-varying source terms are considered
 314 (see [Appendix A](#)). In this case, all of the previous aspects are not met. The
 315 influence of the inclusion of velocity divergence in the SUPG stabilization can
 316 be however seen on the size of the peak at the interface between the “central
 317 void” and the “columnar grain” region. It results in a smoother solution with
 318 respect to the SU one. Moreover, one can see how increasing the mesh density
 319 leads the two solutions to being slightly closer and smoother, as expected from
 320 the finite element theory.

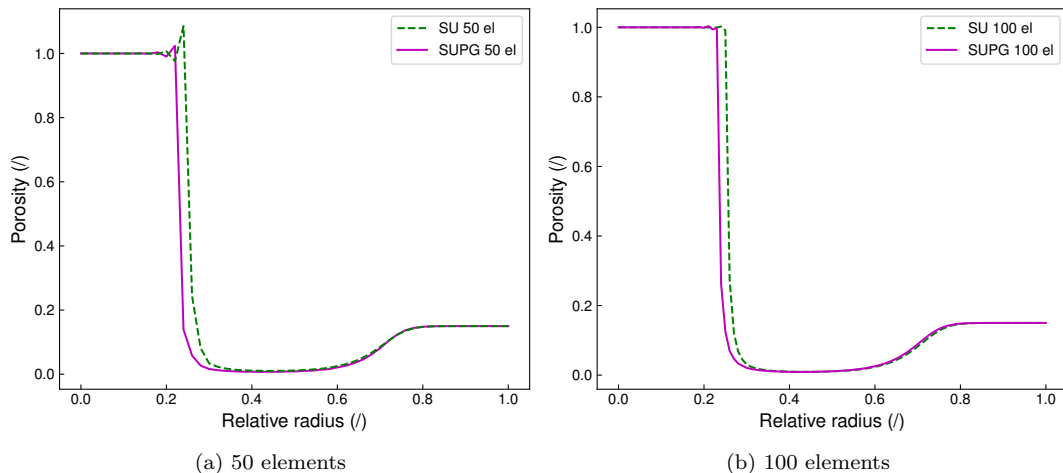


Figure 2: Comparison of the results in terms of porosity as a function of the local radius obtained with MFEM and considering the SUPG and SU stabilization techniques, using respectively 50 (Figure 2a) and 100 element (Figure 2b) meshes. The overshooting of the porosity with respect to its physical bound (i.e., the peak greater than one) employing the coarser mesh (Figure 2a) is a numerical artifact. In fact, it is a result of the steep gradient of the pore velocity across a single mesh element and its discretization in the framework of G-FEM.

321 The few remaining oscillations found where the porosity is subject to a very
 322 steep variation can be mitigated by the mesh refinement. Indeed, they will
 323 always appear due to (i) the element-wise, steep gradient of the pore velocity
 324 and given that G-FEM relies on the support of the test functions taking into
 325 account all neighboring nodal contributions, and (ii) due to the non divergence-
 326 free nature of the physical problem.

327 4.4. Analysis of calculation results

328 The SU stabilization is the technique chosen to obtain the results presented
 329 in this work, if not stated otherwise. The choice is due to the fact that, as men-
 330 tioned above, the results obtained by SU and SUPG techniques are very close
 331 for the problem of interest, with both the techniques successfully removing the
 332 spurious oscillations in the solution of the pore advection equation. Moreover,

333 as can be seen from the mathematical formulation, the SU method does not
 334 modify the mass matrix of the associated algebraic problem, whereas the SUPG
 335 modifies it (the time derivative of the porosity is multiplied by the gradient of
 336 the test function). The results herein presented are obtained employing a com-
 337 putational mesh having 100 radial elements, and considering an explicit Euler
 338 (forward Euler) time integration scheme.

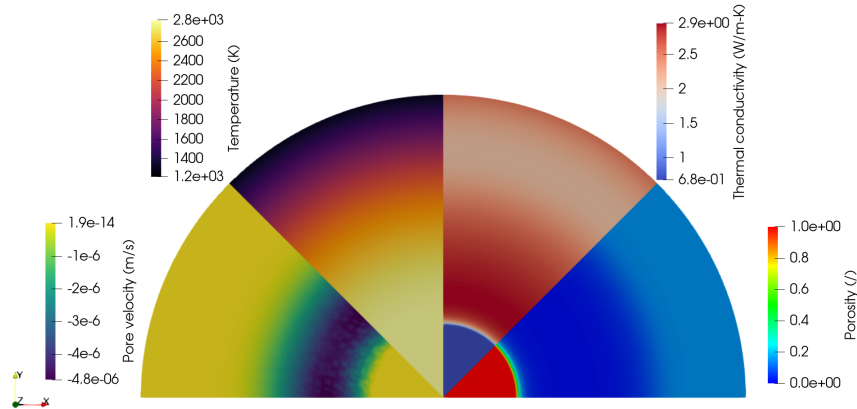


Figure 3: Anti-clockwise, contour plots of porosity, thermal conductivity, temperature, and pore velocity (radial component) after 10^4 s. The various interdependences can be appreciated.

339 Figure 3 reports a collection of the results obtained at the end of the con-
 340 sidered time period, highlighting some of the main quantities governing the
 341 problem. Detail analyses of such quantities are reported in Figures 4 and 5.
 342 The coupled nature of the variables and the parameters naturally arises in such
 343 results. For example, in Figure 4 the pore advection velocity is reported, to-
 344 gether with the two quantities mostly governing it, the temperature and the
 345 temperature gradient. It can be appreciated that up to 0.2 relative radius,
 346 the temperature gradient is null (since the porosity is equal to one and there
 347 is not heat generation), and this is suppressing the pore migration. Thermal
 348 conductivity of the pore/oxide “mixture” and its dependencies on porosity and
 349 temperature can be glimpsed in Figure 5. It can be noticed the synergic effect
 350 of porosity and temperature on such a property: in fact, the maximum values
 351 are reached in the outer part, when the temperature is low enough to dominate
 352 the effect of the local porosity and allow for an efficient transport mechanism,
 353 and around 0.3 relative radius, where the almost absence of porosity due to its
 354 migration and the high temperatures result in a high conductivity. Moreover,
 355 where the porosity reaches one, i.e., in the central void, the constant value of
 356 the thermal conductivity corresponds to the helium conductivity. It is worth
 357 underlining that we do not include in this analysis the plutonium redistribu-
 358 tion [1, 2, 27], which would surely affect the radial power profile, hence the

359 temperature.

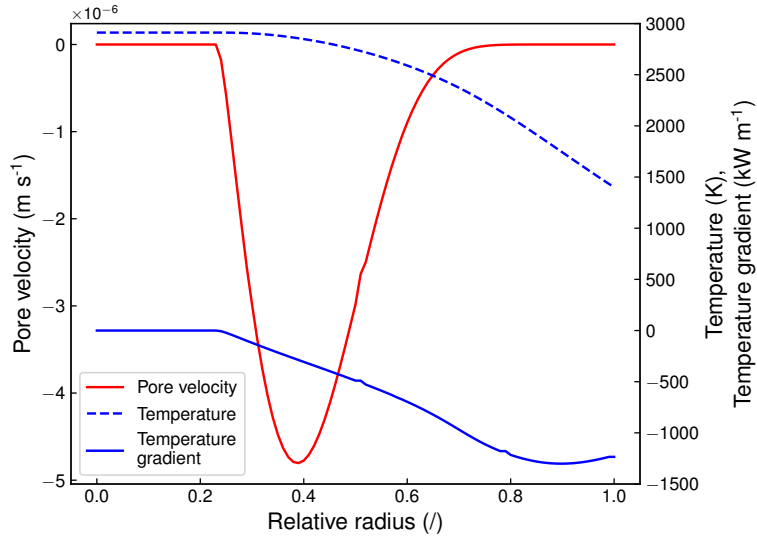


Figure 4: Pore velocity (component in the radial direction), temperature and temperature gradient as a function of the relative radius.

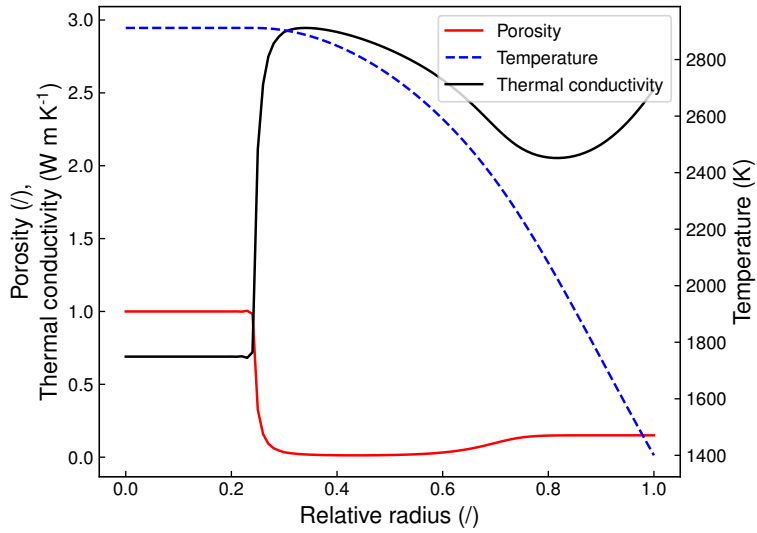


Figure 5: Thermal conductivity, porosity, and temperature as a function of the relative radial position at the end of the test-case.

360 *4.5. Comparison to the BISON calculations*

361 The result of the presented model have been compared to those presented
362 in [14] obtained using BISON. In particular, we are interested in comparing the
363 results on the porosity obtained with different mesh densities.

364 The solutions are compared in Figure 6. As mentioned above, the SU sta-
365 bilization is employed in this section. Overall we can see that the solutions are
366 in good agreement. Indeed, we can underline how the solutions obtained using
367 the coarsest mesh (50 elements), showed in Figure 6a shows different degrees of
368 stability, the one obtained with our approach demonstrating a superior stabil-
369 ity, in both the void and the restructuring zone. As discussed in Section 4.3,
370 the little oscillations observed in our results near the interface between void
371 and bulk are inherently due to the G-FEM formulation. Let us mention that
372 these oscillations remain substantially constant in time and follow the interface
373 void/bulk.

374 Our approach surpasses the one reported in [14] for two main reasons. First,
375 we employ a mathematically consistent stabilization technique, based on the
376 SU method, whereas in BISON a constant diffusivity is included, to change the
377 PDE nature from hyperbolic to elliptic and to limit the spurious oscillations in
378 the solution of the pore advection equation. This approach, which resembles
379 the SU formulation from a practical perspective, is not consistent (i.e., it does
380 not vanish when the mesh size tends to zero) from the finite element perspective
381 and needs to be tuned based on the user experience. Second, we solve for a more
382 correct equation governing the pore advection, not including the term $(1 - p)$ as
383 done in the BISON formulation [14, 17], which the authors claim to be needed
384 to suppress pore advection when the porosity approaches zero. In fact, the
385 coupling between the pore advection and energy equation with the expression of
386 the parameters as reported, together with a proper stabilization of the solution,
387 is guaranteeing the observation of the physical porosity limit. We postulate
388 that this term is responsible for the difference between our solution and the one
389 obtained by the BISON code, since the velocity magnitude is multiplied by a
390 factor smaller than one, thus “reducing the migration” of porosity with respect
391 to the case where it is not considered (as in our formulation).

392 **5. Analysis of crack-induced porosity migration**

393 The role played by cracks as a source of lenticular pore has been outlined
394 by several authors in the literature [2, 7], despite no models elucidating the
395 physical mechanism are available at the moment. In a recent work [19], such
396 mechanism was assumed to play a substantial role in the relocation of fuel at
397 beginning of life. In addition, the healing of cracks by distillation of heavy
398 metal components has been theorized and observed in $(U, Pu)O_2$ [35, 36]. In
399 this section, we present a qualitative analysis on the interaction of as-fabricated
400 and crack-induced porosity, to draw more general conclusions on their impact on
401 fast reactor pellet performance. Moreover, we present a preliminary assessment
402 of the model by comparing its predictions to an experimental results relative to a

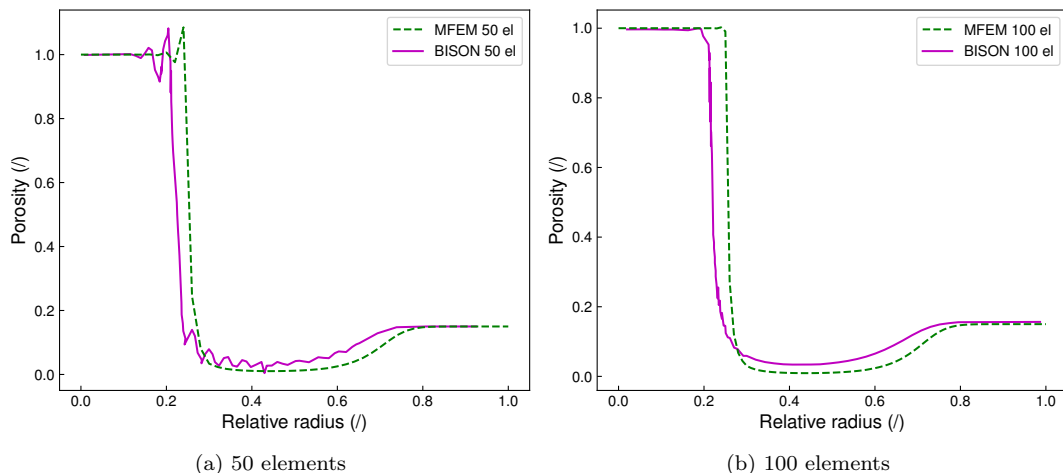


Figure 6: Comparison of the results in terms of porosity as a function of the local radius obtained with MFEM and considering the SU stabilization technique to the one published in [14], using respectively 50 (Figure 6a) and 100 element (Figure 6b) meshes.

403 fuel pin irradiated in the Phenix sodium fast reactor, underlying the interaction
 404 and synergies of as-fabricated and crack-induced porosity.

405 *5.1. Analysis of the interaction between as-fabricated and crack-induced porosity*

406 In this work, we do not aim at directly accounting at the microscopic scale
 407 the physical phenomena governing the porosity transport due to the presence
 408 of cracks. Rather, we aim at demonstrating how such mechanisms could in
 409 principle be included in the present modeling framework. In particular, we
 410 seek a quantification of the contributions to the central hole formation arising
 411 from as-fabricated and crack-induced porosities. To qualitatively assess these
 412 mechanisms, we considered again the test-case analyzed in Section 4 in three
 413 different initial conditions:

- 414 a) A homogeneously dispersed porosity, with no cracks, accounting for a 15%
 415 void fraction (the same analyzed in Section 4.4);
- 416 b) A crack having a thickness such that the void fraction in the domain is
 417 equal to that of the first point;
- 418 c) A combination of the previous two, having a total porosity equal to the
 419 double of the previous cases.

420 The mesh considering the crack is a conforming mesh, constituted of trian-
 421 gular, first order elements. The crack pattern considered is a simplification of
 422 that developing in real conditions. The related assumptions are taken according
 423 to [37], i.e., only radial cracks developing under the first rise to power are con-
 424 sidered, and axial and circumferential cracks are not accounted. The pellet is

425 supposed to split in 8 fragments spanning 22.5 degrees in the angular direction.
 426 The crack thickness is calculated in order to reach the desired volumetric void
 427 (i.e., 15%). The computational domain has the same radius and angular span
 428 as the one considered in the previous section. The mesh density varies along the
 429 radius and in the angular direction, to properly represent the interface between
 430 the crack and the fuel pellet. The meshes are reported in [AppendixB](#).

431 In Figure 7 we report the initial condition of case **b)** in the upper half part
 432 of the figure and the solution at the end of simulation in the lower half part
 433 (reflected for the sake of representation). We can appreciate the migration of
 434 the porosity from the crack to the center of the pellet, originating the central
 435 hole. The crack healing (in the restructured zone) and concurrent central hole
 436 formation is coherent with experimental observation on irradiated (U, Pu)O₂
 437 fuel in sodium fast reactors [1, 19].

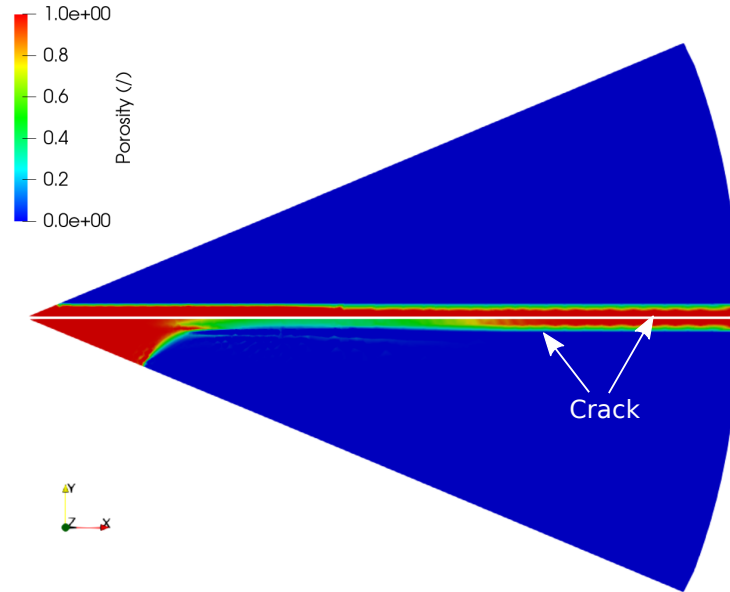


Figure 7: Initial condition (upper half) and final results (lower half) on the porosity distribution starting from a crack in the radial direction. For the sake of clarity, we underline that each circular sector represent a different condition.

438 The final configuration of cases **a)** and **b)** are reported in Figure 8. In
 439 this case, we are considering the same initial void fraction in the two circular
 440 sectors herein represented, but in the upper half, the initial porosity is homoge-
 441 neously distributed in the volume, whereas in the lower half the initial porosity
 442 is distributed as in Figure 7. It can be seen that the resulting central holes have
 443 different radii, with the one obtained for the homogeneous porosity being larger.

444 Case **c)** is compared to case **a)** in Figure 9, with the former summing up to a

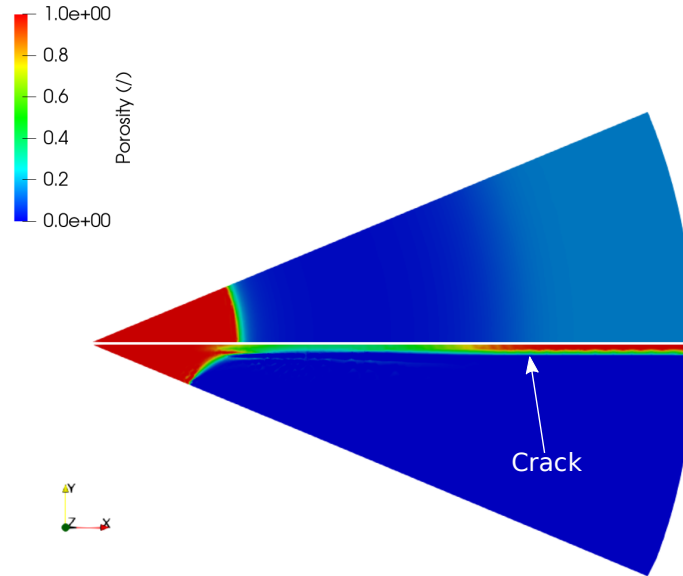


Figure 8: Final results on the porosity redistribution starting from the same volumetric void fraction distributed homogeneously in the domain (upper half) and in a crack (lower half). For the sake of clarity, we underline that each circular sector represents a separate case.

445 30% of void fraction in the domain. This would be the situation occurring as the
 446 fuel is brought to power and undergoes cracking. In this case, the central hole
 447 results larger than in the aforementioned ones, due to the synergic contribution
 448 of the two porosity types. Yet, we can observe how the radius of the central void
 449 is less than sum of the individual contributions brought about considering the
 450 different porosities separately. This is expected and it is a direct consequence
 451 of the non-linear coupling between the equations governing temperature and
 452 porosity.

453 It is worth spending some comments on the results herein showed. First,
 454 the developed model has the original capability, compared to the state of the
 455 art, of computing the migration of crack-induced porosity through a direct rep-
 456 resentation of the crack itself and of its shape, rather than using an equivalent,
 457 homogenized porosity dispersed in the fuel. Second, the capability of correctly
 458 estimating the central hole extension is strictly dependent on the possibility of
 459 modeling the migration of crack-induced porosities, since it has been found that
 460 they can play a major role in determining its size [19]. In this sense, the model
 461 and the solvers developed in this work substantially surpass the state of the art
 462 capabilities of fuel performance codes.

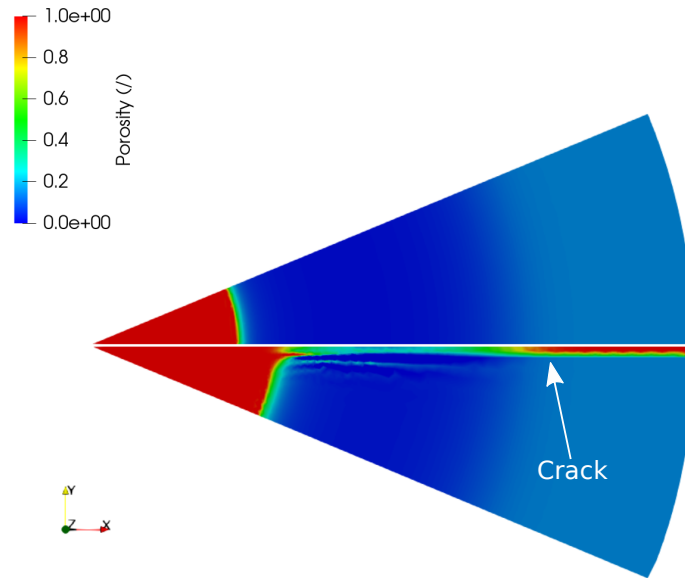


Figure 9: Final results on the porosity distribution, considering an initial condition having both as-fabricated and crack-induced porosity (lower half) and only as-fabricated porosity (upper half). For the sake of clarity, we underline that each circular sector represents a separate case.

463 5.2. Preliminary assessment against experimental results

464 The validation of the model developed in this work can be carried out only after
 465 its inclusion in the framework of a fuel performance code, since at the present
 466 status a large number of important phenomena governing the fuel behavior are
 467 not included in the modeling framework considered. Nonetheless, we present a
 468 preliminary assessment of the model against experimental data regarding the
 469 central hole formation. In particular, we analyze a fuel pellet included in a pin
 470 irradiated into the Phenix sodium fast reactor. The fuel pellet has an initial
 471 radius of 2.716 mm and an initial porosity of 4.1%. Other details are reported
 472 in [19], in which the analysed case is referred to as “Fuel Pin 1”. The choice
 473 of this pin is due to the “low” discharge burnup of the pin (around 1% at.),
 474 which limits the impact of burnup-dependent phenomena (such as fuel swelling,
 475 constituents redistribution, or chemical speciation), not taken into account at
 476 the present moment by our model, on the final geometry of the fuel pellet. The
 477 goal of this exercise is to underline the importance of both as-fabricated and
 478 crack-induced porosity in determining the extent of fuel central void.

479 The setup of the case was carried out as follows. The fuel pellet experi-
 480 enced a linear heat rate around 400 W cm^{-1} throughout the whole irradiation.
 481 The fuel external temperature as a function of time was extracted from the

482 PLEIADES/GERMINAL simulation of the pin presented in [19] and is reported
 483 in Figure 10. Two configurations are herein considered. The first one considers
 484 only the migration of as-fabricated porosity, thus the associated computational
 485 mesh consists of a circular sector spanning 22.5 degrees. The second one, which
 486 considers both as-fabricated and crack-induced porosity, has a geometry similar
 487 to that reported in the previous section, i.e., to the circular section representa-
 488 tive for the fuel pellet is added a surface representative for the crack. The crack
 489 pattern is again those considered at the beginning of irradiation (e.g., see [37])
 490 with straight, radial cracks. As for the crack thickness, we consider that the 8
 491 fragments are fully displaced and in contact with the cladding, thus the initial
 492 internal void surface/volume due to the gap is conserved and transfered into
 493 the cracks⁴ For both configurations, conforming and *a priori* refined first order
 494 triangular elements are used (see details in AppendixB).

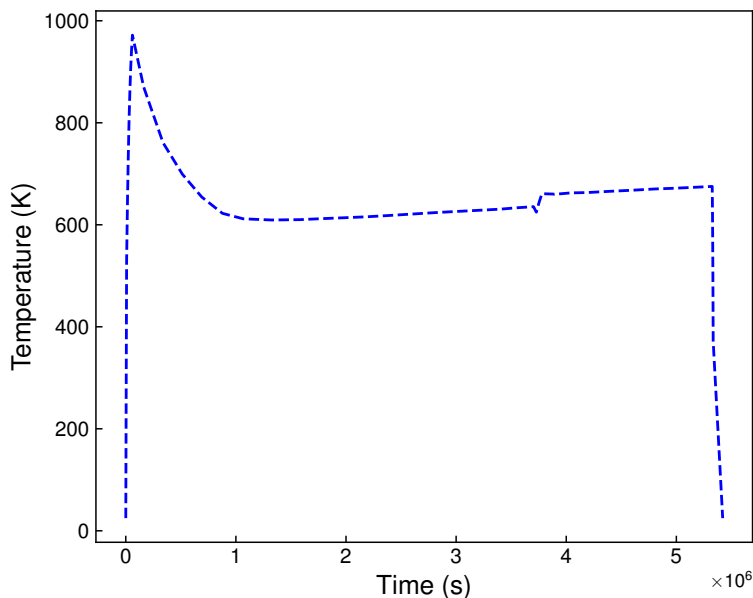


Figure 10: Fuel external temperature as a function of time for the fuel pellet considered in this assessment exercise.

495 The results at the end of the simulations are compared in Figure 11. As it is
 496 noticeable, the configuration considering the migration of the porosity induced
 497 by cracks⁵ yields results that are closer to the experimental data then the one

⁴It is worth to notice that one can in principle choose a different number of fragments and a different angle to model the cracking pattern. In this case, the thickness of the crack would need to be adjusted to preserve the total free volume.

⁵The 1D plot reported here refers to a radius forming an angle with the x axis equal to 11

498 considering only as-fabricated porosity. This result is in line with what was
 499 already shown in a previous work on the same [19] fuel pin, for which considering
 500 solely the migration of as-fabricated porosity was not enough to correctly assess
 501 the extent of the central void. As for the determination of the columnar grain
 502 region, the present model does not directly model its development, but can be
 503 associated to the part of the fuel pellet where the porosity is less than 2%. If this
 504 value is considered as a threshold, the predictions by the present model are in
 505 line with the experimental data reported in Figure 11. The corresponding results
 506 on the entire computational domains are reported in Figure 12. Indeed, it must
 507 be underlined that this result is just a first assessment of the model capabilities,
 508 and that a more rigorous validation to a more consistent set of experimental
 509 data is needed to draw definitive conclusions. The analysis will be possible as
 510 the model will be included in a future version of the PLEIADES/GERMINAL
 511 fuel performance code.

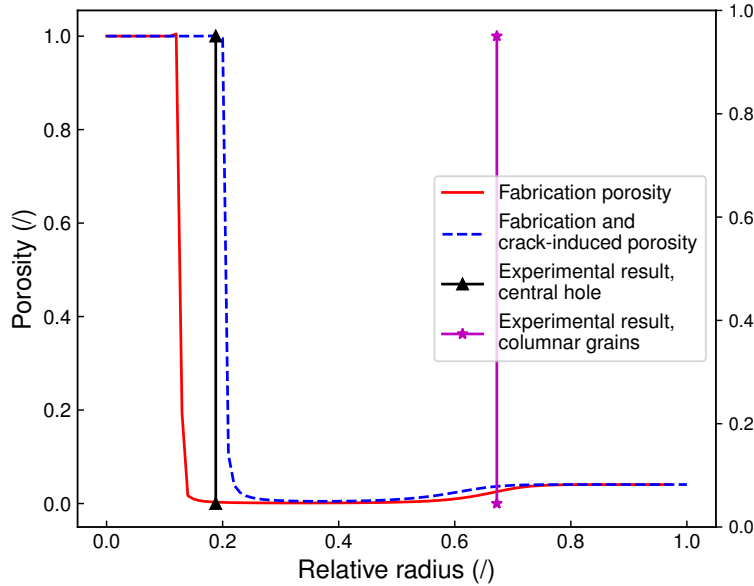


Figure 11: Comparison of experimental results to model predictions, considering only as-fabricated porosity and including crack-induced porosity.

512 It is worth noticing that the underlying assumption is that we discard the
 513 mechanisms governing pores nucleation from cracks, and that the velocity ex-
 514 pression employed to describe the migration of as-fabricated pores remains valid
 515 in this case. That is, we implicitly assume that pore are nucleated at the
 516 crack surface, driven by the circumferential component of the velocity, and then

degrees.

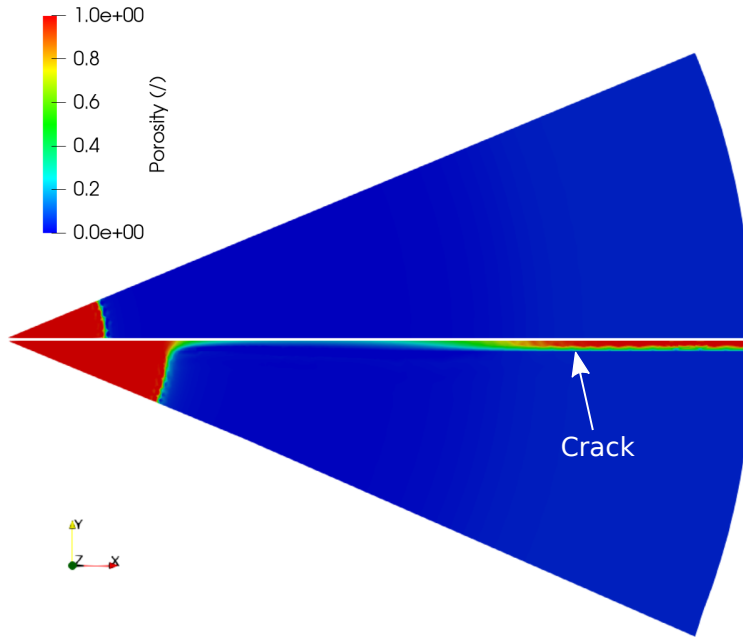


Figure 12: Contour plot of the porosity distribution in the two configurations, considering only as-fabricated porosity (upper half) and including crack-induced porosity (lower half). For the sake of clarity, we underline that each circular sector represent a different result.

517 transported in the circumferential and radial directions with the same velocity
 518 equation as the as-fabricated porosity. Under this modeling assumption, the
 519 pore velocity is also determining the rate at which crack-induced porosity is
 520 nucleated. A more realistically modeling framework would consider at the same
 521 time the evaporation of components from the “hot” segment of the crack, their
 522 diffusion in the vapor phase, and their subsequent condensation in the “cold”
 523 part. The inclusion of such phenomena in this framework is left as a future
 524 development of the present work.

525 **6. Conclusions**

526 In this work, we proposed an original modeling framework for the coupled so-
527 lution of the pore advection and heat conduction equations by the finite element
528 method. The model is intended to describe the porosity migration in $(U,Pu)O_2$
529 fuel irradiated in fast reactor conditions and considers the interdependencies among
530 the solution variables and the parameters governing the problem. We imple-
531 mented a numerical solver for the problem in MFEM, an open-source library
532 for PDEs solution by the FEM. The solution algorithm includes a simple but
533 robust fixed-point strategy combined to an explicit time solver. The modeling
534 framework includes original and consistent stabilization techniques with respect
535 to the state of the art in fuel performance codes for the pore advection equa-
536 tions, namely the Streamline-Upwind and Streamline-Upwind/Petrov-Galerkin
537 techniques.

538 The developed finite element model has been compared to a model included
539 in the fuel performance code BISON, based as well on the finite element method.
540 The analysis was based on a test-case representative for the conditions experi-
541 enced by a fuel pellet irradiated in a sodium fast reactor. The results obtained by
542 our model, in terms of porosity distribution in the fuel pellet, are in agreement
543 with those obtained by BISON. Moreover, the employed stabilization technique
544 for the pore advection equation eliminates the spurious oscillations encountered
545 in the BISON simulation when employing a coarse mesh, demonstrating the
546 improvement brought about by the model developed in this work. Despite rely-
547 ing on the introduction of an “artificial” diffusion term, the proposed numerical
548 framework enables us to introduce a mathematically consistent term in the
549 equations, which is not inducing errors in the solution.

550 The model has been applied to the study of the porosity migration con-
551 sidering different types of porosity, namely including crack-induced alongside
552 as-fabricated porosity. The analysis showed how the extension of the central
553 void due to the migration of these different types of porosity is different, and
554 combining the two types of porosity we demonstrate how the resulting cen-
555 tral void is larger. This analysis is applied also to a fuel pin irradiated in the
556 Phenix sodium fast reactor, and we underline the importance of considering
557 crack-induced porosity in the assessment of the model against an experimental
558 case.

559 Overall, the model we are presenting in this work, with respect to those
560 available in the state of the art, includes on one hand a more rigorous, dedicated
561 mathematical treatment of the spurious oscillations found in the solution of
562 the pore advection equation in state-of-the-models utilizing the finite element
563 method. On the other, accounting for crack-induced porosity stands out as a
564 unique capability of the developed model with respect to the ones available in
565 the open literature, and paves the way to its application in the study of crack
566 healing in sodium fast reactor mixed oxides fuel.

567 Future developments of the outlined modeling framework encompass a re-
568 assessment of pore velocity and its study from a microscopic point of view, to
569 derive a novel and robust behavioral law on this important parameter. More-

570 over, the inclusion of the equations governing the plutonium, americium, and
571 oxygen redistribution is of interest, in order to account for the effects of such
572 quantities on the thermal and porosity solutions. Finally, once these modeling
573 advancements will be available, we envisage a validation of the model against
574 separate effect tests focused on the pore velocity model, e.g., comparing to the
575 data from the Am-1 experiment [38], and a integral validation of the pore mi-
576 gration model, once it will be included in the GERMINAL/PLEIADES fuel
577 performance code, against other Phenix irradiation data.

578 Acknowledgments

579 This work has been done in the framework of a cooperative program between
580 CEA, FRAMATOME and EDF, devoted to the development of the fuel elements
581 for GEN IV Reactors.

582 Appendix A. SUPG solver verification

583 To verify the correctness of the SU/SUPG implementation in MFEM, we
584 compared the solutions obtained by our solvers against test-cases reported in
585 the open literature as reference problems for the SUPG method development
586 [22]. In particular, a steady-state case with a source term and a transient case
587 with only the internal evolution are presented. The details about these test-
588 cases are reported in [22]. It is worth underlining that the implementation of
589 the SU method does not call for a particular solver, since the term induced
590 by this stabilization technique is a diffusion operator, whose discretization is
591 already available in MFEM.

592 The results for the steady-state one are presented in Figure A.13. The test-
593 case consists in considering a pure advection problem with constant advection
594 velocity and a source term, which is reported in the figure. We consider a 1D
595 mesh and impose a Dirichlet boundary condition at the inlet – i.e., at $x = 0$.
596 The results are in agreement with those proposed by Brooks and Hughes [22].

597 The test-case on the unsteady solution is taken again from Brooks and
598 Hughes [22], and the results are reported in Figure A.14. The initial condi-
599 tion is a classic cosine hill, natural boundary conditions are enforced on the two
600 ends of the 1D domain. A pure advection problem with a constant and unitary
601 velocity oriented towards the positive x axis is considered. The time integration
602 is carried out through an explicit Euler scheme, imposing a time step so that
603 the Courant number is 0.5. As one can see, the solution obtained with our
604 implementation is in line with that reported in the original paper for all the
605 considered time steps. For the sake of comparison, the solution obtained by
606 the SU stabilization technique is reported in Figure A.15, considering the same
607 initial condition, geometry, and parameters. As one could see, the performance
608 of the SU technique when considering a non-uniform initial condition is poorer
609 than the SUPG.

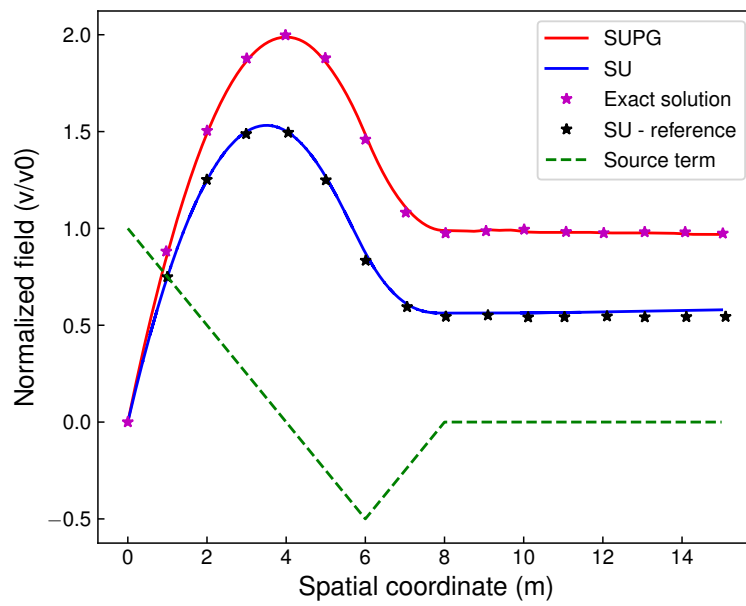


Figure A.13: Comparison of the reference and calculated solution employing SUPG techniques in the steady state test-case. The results obtained with the SU stabilization are also reported for the sake of comparison.

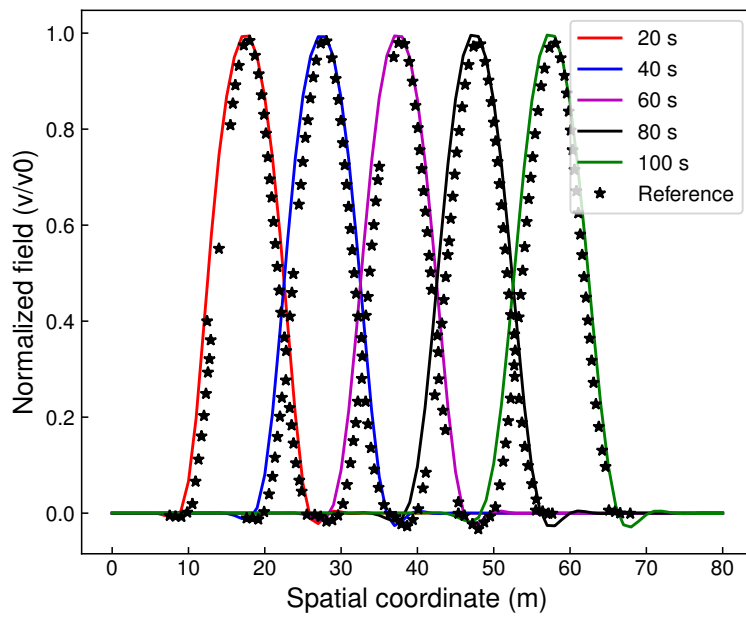


Figure A.14: Comparison of the reference solution (triangles) and calculated solution employing the SUPG stabilization after 20 s, 40 s, 60 s, 80 s, and 100 s.

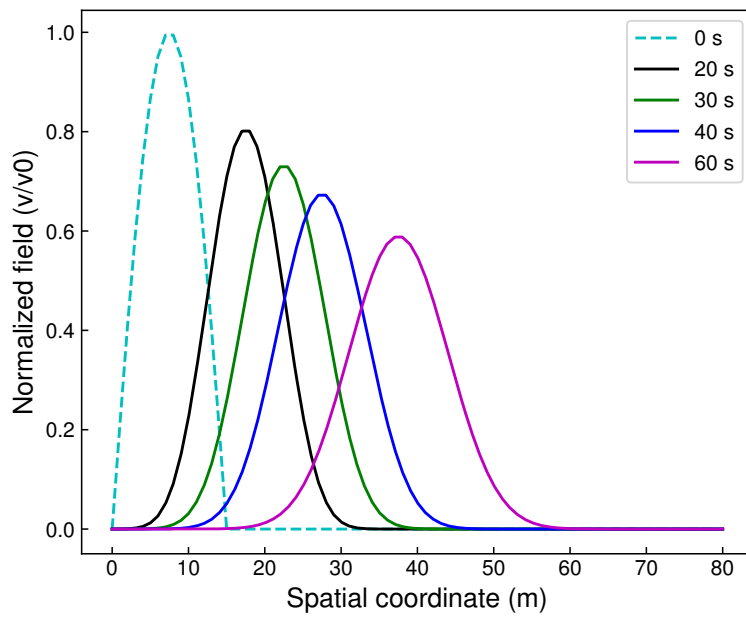


Figure A.15: Solution calculated on the test-case obtained considering an cosine-hill initial condition and employing the SU stabilization technique.

610 **AppendixB. Computational meshes**

611 In this appendix, we report the computational meshes employed in this work.
612 The non-conforming meshes employed in Sections 3 and 4, having 50 and 100
613 intervals in the radial directions, are reported in Figures B.16. The mesh is
614 created such that the aspect ratio of the elements is as closest as possible to
unity, and include a triangular central element.

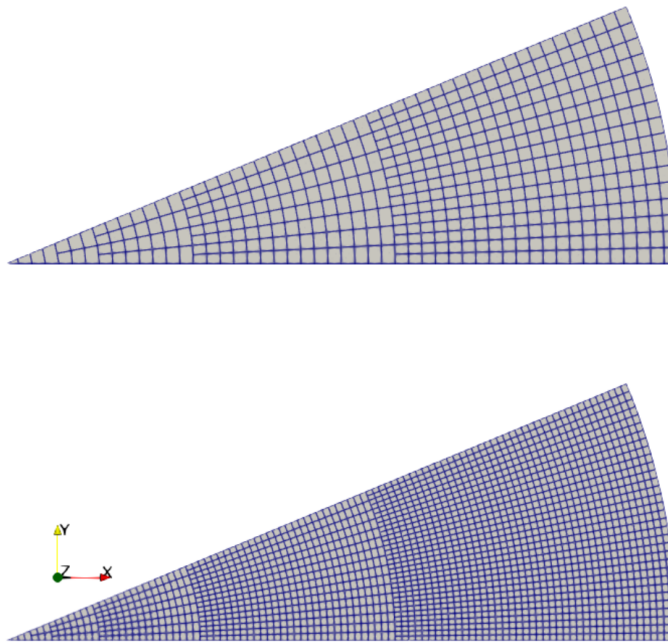


Figure B.16: Non conforming computational meshes having 50 and 100 intervals in the radial direction.

615 As for Section 5, different meshes are employed. In these cases, the mesh is
616 conforming, and *a priori* refinement is applied in order to guarantee the solution
617 is converged in mesh size. Hence, finer cells are employed at the central part
618 of the pellet and close to the crack, where the steepest porosity gradients are
619 expected. Triangular elements are employed in this case. An example of the
620 mesh for the cracked domain is reported in Figure B.17, whereas the same mesh
621 without the crack is the reported in Figure B.18.
622

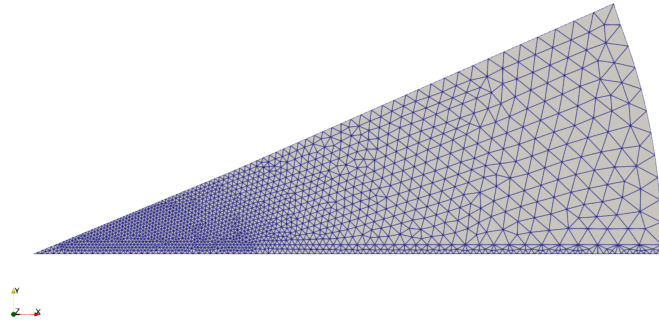


Figure B.17: Computational mesh for the cases including the crack.

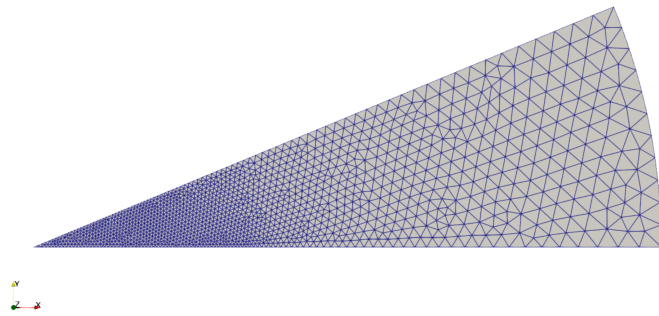


Figure B.18: Computational mesh for the cases having an un-cracked domain.

623 **References**

- 624 [1] D. Olander, Fundamental aspects of nuclear reactor fuel elements, 1976.
625 [doi:10.1016/0022-3115\(77\)90226-4](https://doi.org/10.1016/0022-3115(77)90226-4).
- 626 [2] M. Welland, Matter transport in fast reactor fuels, in: R. Konings,
627 R. Stoller (Eds.), Comprehensive Nuclear Materials, Elsevier, 2020, pp.
628 630–676.
- 629 [3] M. Pelletier, Y. Guérin, Fuel Performance of Fast Spectrum Oxide Fuel,
630 in: R. J. Konings, R. E. Stoller (Eds.), Comprehensive Nuclear Materials
631 (Second Edition), Second Edition, Elsevier, Oxford, 2020, pp. 72–105. [doi:](https://doi.org/10.1016/B978-0-12-803581-8.11690-X)
632 <https://doi.org/10.1016/B978-0-12-803581-8.11690-X>.
- 633 [4] D. Staicu, Thermal Properties of Irradiated UO₂ and MOX, in: R. J.
634 Konings, R. E. Stoller (Eds.), Comprehensive Nuclear Materials (Second
635 Edition), Second Edition, Elsevier, Oxford, 2020, pp. 149–172. [doi:https:](https://doi.org/10.1016/B978-0-12-803581-8.11726-6)
636 [//doi.org/10.1016/B978-0-12-803581-8.11726-6](https://doi.org/10.1016/B978-0-12-803581-8.11726-6).
- 637 [5] F. Nichols, Theory of columnar grain growth and central void formation
638 in oxide fuel rods, Journal of Nuclear Materials 22 (2) (1967) 214–222.
639 [doi:https://doi.org/10.1016/0022-3115\(67\)90031-1](https://doi.org/10.1016/0022-3115(67)90031-1).
- 640 [6] F. Nichols, Pore migration in ceramic fuel elements, Journal of Nu-
641 clear Materials 27 (2) (1968) 137–146. [doi:https://doi.org/10.1016/](https://doi.org/10.1016/0022-3115(68)90118-9)
642 [0022-3115\(68\)90118-9](https://doi.org/10.1016/0022-3115(68)90118-9).
- 643 [7] P. Sens, The kinetics of pore movement in UO₂ fuel rods, Journal of Nuclear
644 Materials 43 (3) (1972) 293–307.
- 645 [8] K. Lassmann, TRANSURANUS: a fuel rod analysis code ready for use,
646 Journal of Nuclear Materials 188 (1992) 295–302.
- 647 [9] J. Melis, L. Roche, J. Piron, J. Truffert, GERMINAL — A computer code
648 for predicting fuel pin behaviour, Journal of Nuclear Materials 188 (1992)
649 303–307. [doi:https://doi.org/10.1016/0022-3115\(92\)90488-7](https://doi.org/10.1016/0022-3115(92)90488-7).
- 650 [10] T. Ozawa, T. Abe, Development and Verifications of Fast Reactor Fuel
651 Design Code CEPTAR, Nuclear Technology 156 (1) (2006) 39–55.
- 652 [11] A. Karahan, J. Buongiorno, Modeling of thermo-mechanical and irradiation
653 behavior of mixed oxide fuel for sodium fast reactors, Journal of Nuclear
654 Materials 396 (2) (2010) 272–282.
- 655 [12] S. E. Lemehov, F. Jutier, Y. Parthoens, B. Vos, S. Van Den Berghe, M. Ver-
656 werft, N. Nakae, MACROS benchmark calculations and analysis of fission
657 gas release in MOX with high content of plutonium, Progress in Nuclear
658 Energy 57 (2012) 117–124, nuclear Materials: Selected articles from the E-
659 MRS 2011 Spring Meeting. [doi:https://doi.org/10.1016/j.pnucene.](https://doi.org/10.1016/j.pnucene.2011.12.010)
660 [2011.12.010](https://doi.org/10.1016/j.pnucene.2011.12.010).

- 661 [13] T. Uwaba, T. Mizuno, J. Nemoto, I. Ishitani, M. Ito, Development of a
662 mixed oxide fuel pin performance analysis code "CEDAR": Models and
663 analyses of fuel pin irradiation behavior, *Nuclear Engineering and Design*
664 280 (2014) 27–36. doi:[https://doi.org/10.1016/j.nucengdes.2014.
665 08.032](https://doi.org/10.1016/j.nucengdes.2014.08.032).
- 666 [14] S. Novascone, P. Medvedev, J. W. Peterson, Y. Zhang, J. Hales, Modeling
667 porosity migration in LWR and fast reactor MOX fuel using the finite
668 element method, *Journal of Nuclear Materials* 508 (2018) 226–236.
- 669 [15] M. Lainet, B. Michel, J.-C. Dumas, M. Pelletier, I. Ramière, GERMINAL,
670 a fuel performance code of the PLEIADES platform to simulate the in-pile
671 behaviour of mixed oxide fuel pins for sodium-cooled fast reactors, *Journal*
672 *of Nuclear Materials* 516 (2019) 30–53. doi:[https://doi.org/10.1016/
673 j.jnucmat.2018.12.030](https://doi.org/10.1016/j.jnucmat.2018.12.030).
- 674 [16] Y. Sukjai, K. Shirvan, Enhancing FRAPCON fuel performance code for
675 physical phenomena at high temperature and high burnup, *Journal of Nu-*
676 *clear Materials* 517 (2019) 113–127.
- 677 [17] T. Ozawa, S. Hirooka, M. Kato, S. Novascone, P. Medvedev, Develop-
678 ment of fuel performance analysis code, BISON for MOX, named Okami:
679 Analyses of pore migration behavior to affect the MA-bearing MOX fuel
680 restructuring, *Journal of Nuclear Materials* 553 (2021) 153038. doi:<https://doi.org/10.1016/j.jnucmat.2021.153038>.
- 681 [18] L. Luzzi, T. Barani, B. Boer, L. Cognini, A. Del Nevo, M. Lainet, S. Leme-
682 hov, A. Magni, V. Marelle, B. Michel, D. Pizzocri, A. Schubert, P. Van
683 Uffelen, M. Bertolus, Assessment of three European fuel performance
684 codes against the SUPERFACT-1 fast reactor irradiation experiment, *Nu-*
685 *clear Engineering and Technology* doi:[https://doi.org/10.1016/j.net.
686 2021.04.010](https://doi.org/10.1016/j.net.2021.04.010).
- 687 [19] M. Temmar, B. Michel, I. Ramiere, N. Favrie, Multi-physics modelling of
688 the pellet-to-cladding gap closure phenomenon for SFR fuel performance
689 codes, *Journal of Nuclear Materials* 529 (2020) 151909. doi:[https://doi.
690 org/10.1016/j.jnucmat.2019.151909](https://doi.org/10.1016/j.jnucmat.2019.151909).
- 691 [20] A. Quarteroni, *Numerical Models for Differential Problems*, MS&A,
692 Springer Milan, 2010.
- 693 [21] K. Bathe, *Finite Element Procedures*, Prentice Hall, Englewood Cliffs, NJ,
694 1996.
- 695 [22] A. Brooks, T. Hughes, Streamline upwind/Petrov-Galerkin formulations for
696 convection dominated flows with particular emphasis on the incompressible
697 Navier-Stokes equations, *Computer Methods in Applied Mechanics and*
698 *Engineering* 32 (1) (1982) 199–259.
- 699

- 700 [23] R. Anderson, J. Andrej, A. Barker, J. Bramwell, J.-S. Camier, J. Cervený,
701 V. Dobrev, Y. Dudouit, A. Fisher, T. Kolev, W. Pazner, M. Stowell, V. To-
702 mov, I. Akkerman, J. Dahm, D. Medina, S. Zampini, MFEM: A modular fi-
703 nite element methods library, *Computers & Mathematics with Applications*
704 81 (2021) 42–74. doi:<https://doi.org/10.1016/j.camwa.2020.06.009>.
- 705 [24] D. Knoll, D. Keyes, *Jacobian-free Newton–Krylov methods: a survey of*
706 *approaches and applications*, *Journal of Computational Physics* 193 (2)
707 (2004) 357–397. doi:<https://doi.org/10.1016/j.jcp.2003.08.010>.
708 URL [https://www.sciencedirect.com/science/article/pii/](https://www.sciencedirect.com/science/article/pii/S0021999103004340)
709 [S0021999103004340](https://www.sciencedirect.com/science/article/pii/S0021999103004340)
- 710 [25] F. Nichols, Kinetics of diffusional motion of pores in solids: A review,
711 *Journal of Nuclear Materials* 30 (1) (1969) 143–165.
- 712 [26] R. Codina, Comparison of some finite element methods for solving the
713 diffusion-convection-reaction equation, *Computational methods in applied*
714 *mechanics and engineering* 156 (1998) 185–210.
- 715 [27] V. Di Marcello, V. Rondinella, A. Schubert, J. van de Laar, P. Van
716 Uffelen, *Modelling actinide redistribution in mixed oxide fuel for sodium*
717 *fast reactors*, *Progress in Nuclear Energy* 72 (2014) 83–90, symposium E
718 @ E-MRS 2013 SPRING MEETING Scientific basis of the nuclear fuel
719 cycle. doi:<https://doi.org/10.1016/j.pnucene.2013.10.008>.
720 URL [https://www.sciencedirect.com/science/article/pii/](https://www.sciencedirect.com/science/article/pii/S0149197013002011)
721 [S0149197013002011](https://www.sciencedirect.com/science/article/pii/S0149197013002011)
- 722 [28] W. J. Lackey, F. J. Homan, A. R. Olsen, Porosity and actinide redistri-
723 bution during irradiation of (u, pu)₂O₂, *Nuclear Technology* 16 (1) (1972)
724 120–142.
- 725 [29] S. Guarro, D. Olander, *Actinide redistribution due to pore migration in*
726 *hypostoichiometric mixed-oxide fuel pins*, *Journal of Nuclear Materials*
727 57 (2) (1975) 136–144. doi:[https://doi.org/10.1016/0022-3115\(75\)](https://doi.org/10.1016/0022-3115(75)90253-6)
728 [90253-6](https://doi.org/10.1016/0022-3115(75)90253-6).
729 URL [https://www.sciencedirect.com/science/article/pii/](https://www.sciencedirect.com/science/article/pii/S0022311575902536)
730 [0022311575902536](https://www.sciencedirect.com/science/article/pii/S0022311575902536)
- 731 [30] C. Clement, Analytic solutions to mass transport equations for cylindrical
732 nuclear fuel elements, *Journal of Nuclear Materials* 68 (1) (1977) 54–62.
733 doi:[https://doi.org/10.1016/0022-3115\(77\)90216-1](https://doi.org/10.1016/0022-3115(77)90216-1).
734 URL [https://www.sciencedirect.com/science/article/pii/](https://www.sciencedirect.com/science/article/pii/S0022311577902161)
735 [0022311577902161](https://www.sciencedirect.com/science/article/pii/S0022311577902161)
- 736 [31] C. Clement, M. Finnis, *Plutonium redistribution in mixed oxide (U, Pu)O₂*
737 *nuclear fuel elements*, *Journal of Nuclear Materials* 75 (1) (1978) 193–200.
738 doi:[https://doi.org/10.1016/0022-3115\(78\)90044-2](https://doi.org/10.1016/0022-3115(78)90044-2).
739 URL [https://www.sciencedirect.com/science/article/pii/](https://www.sciencedirect.com/science/article/pii/S0022311578900442)
740 [0022311578900442](https://www.sciencedirect.com/science/article/pii/S0022311578900442)

- 741 [32] M. Kato, K. Maeda, T. Ozawa, M. Kashimura, Y. Kihara, Physical prop-
742 erties and irradiation behavior analysis of Np- and Am-Bearing MOX Fu-
743 els, *Journal of Nuclear Science and Technology* 48 (4) (2011) 646–653.
744 [doi:10.1080/18811248.2011.9711745](https://doi.org/10.1080/18811248.2011.9711745).
- 745 [33] A. Eucken, Allgemeine gesetzmäßigkeiten für das wärmeleitvermögen ver-
746 schiedener stoffarten und aggregatzustände, *Forschung auf dem Gebiet des*
747 *Ingenieurwesens A* 11 (1940) 6–20.
- 748 [34] A. Magni, T. Barani, A. Del Nevo, D. Pizzocri, D. Staicu,
749 P. Van Uffelen, L. Luzzi, [Modelling and assessment of thermal](#)
750 [conductivity and melting behaviour of mox fuel for fast reac-](#)
751 [tor applications](#), *Journal of Nuclear Materials* 541 (2020) 152410.
752 [doi:https://doi.org/10.1016/j.jnucmat.2020.152410](https://doi.org/10.1016/j.jnucmat.2020.152410).
753 URL [https://www.sciencedirect.com/science/article/pii/](https://www.sciencedirect.com/science/article/pii/S0022311520310187)
754 [S0022311520310187](https://www.sciencedirect.com/science/article/pii/S0022311520310187)
- 755 [35] D. R. O’Boyle, R. O. Meyer, Redistribution of Uranium and Plutonium in
756 mixed-oxide fuels during irradiation, in: *Panel Meeting on the Behaviour*
757 *and Chemical State of Fission Products in Irradiated Fuels*, IAEA, Vienna,
758 Austria, 1972, pp. 41–66.
- 759 [36] D. R. Olander, The kinetics of actinide redistribution by vapor migration in
760 mixed oxide fuels (I). By cracks, *Journal of Nuclear Materials* 49 (1973/74)
761 21–34.
- 762 [37] J. Sercombe, B. Michel, C. Riglet-Martial, O. Fandeur, Modeling of Pellet
763 Cladding Interaction, in: R. J. Konings, R. E. Stoller (Eds.), *Comprehen-*
764 *sive Nuclear Materials (Second Edition)*, 2nd Edition, Elsevier, Oxford,
765 2020, pp. 417–465.
- 766 [38] K. Tanaka, S. Miwa, S. ichi Sekine, H. Yoshimochi, H. Obayashi, S. ichi
767 Koyama, Restructuring and redistribution of actinides in Am-MOX fuel
768 during the first 24h of irradiation, *Journal of Nuclear Materials* 440 (1)
769 (2013) 480–488. [doi:https://doi.org/10.1016/j.jnucmat.2013.01.](https://doi.org/10.1016/j.jnucmat.2013.01.351)
770 [351](https://doi.org/10.1016/j.jnucmat.2013.01.351).