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# Water transport properties of virtual fractal porous media: implications for the unsaturated transport properties of cement-based materials

Stéphane POYET \*

\* Université Paris-Saclay, CEA, Service d'Etude du Comportement des Radionucléides, F-91191, Gif-sur-Yvette, France

## Contact

- Postal address: CEA Saclay, DES/ISAS/DPC/SECR/LECBA, B158 PC25, F-91191 Gif-sur-Yvette cedex, France
- Mail: [stephane.poyet@cea.fr](mailto:stephane.poyet@cea.fr)

**Conflict of interest:** none

## Abstract

Cement-based materials are more and more recognized as fractal materials. From a practical point of view, fractality means that the pore size distribution can be described using fractal scaling law (i.e. power functions). Here, the simplest scaling law making use of a single fractal dimension was used to generate simple and virtual fractal porous media (based on bundles of parallel cylindrical pores). The capillary curve and permeability (relative and intrinsic) of the virtual porous media were then estimated through theoretical and numerical approaches. The results show that: (1) although bereft of any physical basis, van Genuchten equation bears some fractal information; (2) although highly flexible, van Genuchten equation fails to perfectly fit the capillary curves and can thus generate error in inverse analysis (for intrinsic permeability evaluation) and (3) there seems to be a relation between the intrinsic permeability and van Genuchten pressure parameter.

**Keywords:** Pore Size Distribution; Adsorption; Permeability; Fractal

## 1. Introduction

Concretes and cement-based materials are ubiquitous in construction because they are versatile and affordable. Even though they have been broadly used and studied for decades, they remain difficult to apprehend and describe because of their multiphase, multiscale and porous nature. For instance the pore-structure itself spans over several orders of magnitude in dimension - from the angstrom ( $10^{-10}$  m) up to the micrometer ( $10^{-6}$  m) or more - and it is still not possible today to obtain a clear view of the whole pore-structure even with the use of different experimental techniques [1,2]. The main consequence is that some concrete macroscopic properties that are strongly dependent on the pore-structure (such as transport properties) remain difficult to assess and often have to be tested experimentally.

It has been however known for years that the pores - or at least a fraction of them - exhibit fractal properties [3-11]. This was proven for the smaller pores (C-S-H pores) using small-angle x-ray (SAXS) or neutron scattering (SANS) [3,4,6,12-18]; for the larger pores using image

44 analysis (optical and/or electronic microscopy) [6–8,19] and of course using mercury intrusion  
 45 porosimetry (MIP) [20–26]. All these results are far from being consistent as there is a strong  
 46 scatter in the resulting fractal dimension values (they all lie between 1.0 and 3.5). Moreover it  
 47 has also been suggested recently, using MIP results, that the fractality of the pore-structure was  
 48 scale-dependent; the fractal dimension would not be not unique and there would exist at least  
 49 two dimension domains each having a different value of the fractal dimension [20–23,25,27].  
 50 The reader should keep in mind that MIP is a flawed technique that misallocates pore size and  
 51 can generate damage [28–32] (and then misestimate fractal dimensions); all the evaluations  
 52 obtained using MIP then only should be taken with great caution. However, the scale  
 53 dependence of the MIP fractal dimensions is consistent with the scatter obtained with the other  
 54 techniques. In the author’s mind, the cementitious materials are more than likely to be  
 55 multifractal materials although the corresponding scale domains and fractal dimensions remain  
 56 uncertain.

57 Pore fractality is an important feature because it means that the pore structure is similar at  
 58 different scales. From a mathematical point of view, this implies that the pore size distribution  
 59 should follow a simple scaling law in the form of  $r^{-D}$  where  $r$  is the pore radius and  $D$  the fractal  
 60 dimension (a constant) [33]. The pore-size distribution can then be described using with simple  
 61 power functions. It is the objective of this article to take advantage of using these simple  
 62 analytical equations that describe fractal pore-size distribution in order to generate virtual  
 63 porous media to be used as model materials to study the water transport properties of  
 64 unsaturated concretes.

65

## 66 **2. Unsaturated transport properties**

67 In low permeable unsaturated concretes, it was shown that water transport was mainly due to  
 68 the permeation of liquid water [34]. It is then possible to describe unsaturated water transport  
 69 using a single equation [35,36]:

$$70 \quad \phi \left( \frac{\partial S}{\partial P} \right) \frac{\partial P}{\partial t} = \text{div} \left[ \left( \frac{K k_r}{\eta} \right) \underline{\text{grad}}(P) \right] \quad (1)$$

71 where  $\phi$  is the porosity;  $S$  the saturation index;  $P$  pressure of liquid water in the pores (Pa);  $K$   
 72 intrinsic permeability ( $\text{m}^2$ );  $k_r$  relative permeability to water and  $\eta$  water viscosity (Pa s).

73 It is common practice to use the equation proposed by van Genuchten [37] to fit the capillary  
 74 curve (relationship between pressure  $P$  and saturation  $S$ ):

$$75 \quad \begin{cases} S(P) = \left[ 1 + \left( \frac{P}{P_0} \right)^{\frac{1}{1-m}} \right]^{-m} \\ P(S) = P_0 \left( S^{-\frac{1}{m}} - 1 \right)^{1-m} \end{cases} \quad (2)$$

76 where  $m$  and  $P_0$  are two positive constant,  $P_0$  is equivalent to a pressure (in Pa). The left-hand  
 77 term of equation (1) then writes:

$$78 \quad \phi \left( \frac{\partial S}{\partial P} \right) = - \frac{m \phi}{(m-1) P_0} \left( \frac{P}{P_0} \right)^{\frac{m}{1-m}} \left[ 1 + \left( \frac{P}{P_0} \right)^{\frac{1}{1-m}} \right]^{-1-m} \quad (3)$$

79 The relative permeability  $k_r$  is almost never characterized through experiments; rather it is  
 80 commonly evaluated using the model proposed by Mualem [38] that uses the capillary curve as  
 81 input data:

$$82 \quad k_r = \sqrt{S} \left[ \frac{\int_0^S \frac{du}{P(u)}}{\int_0^1 \frac{du}{P(u)}} \right]^2 \quad (4)$$

83 When van Genuchten equation [37] is used together with the model of Mualem [38], the well-  
 84 known following simple analytical expression aka known as Mualem-van Genuchten (MVG) is  
 85 obtained:

$$86 \quad \begin{cases} k_r(S) = \sqrt{S} \left[ 1 - \left( 1 - S^{\frac{1}{m}} \right)^m \right]^2 \\ k_r(P) = \left[ 1 + \left( \frac{P}{P_0} \right)^{\frac{1}{1-m}} \right]^{-\frac{m}{2}} \left( 1 - \left( \frac{P}{P_0} \right)^{\frac{1}{1-m}} \left[ 1 + \left( \frac{P}{P_0} \right)^{\frac{1}{1-m}} \right]^{-m} \right)^2 \end{cases} \quad (5)$$

87 Finally, it must be recalled that the value of the intrinsic permeability  $K$  is preferably assessed  
 88 through inverse analysis as those measured using water or gas are known to overestimate  
 89 flowrates [39].

90

### 91 **3. Definition of a fractal porous medium (FPM)**

#### 92 **3.1. Pore-size distribution, porosity**

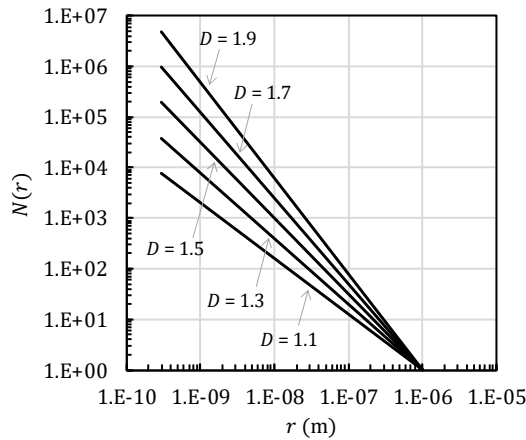
93 Here, the cementitious materials were considered as ordinary fractal materials, i.e. with a unique  
 94 fractal dimension. This choice clearly goes against what is commonly observed (i.e. different  
 95 fractal dimensions over different size ranges). This choice was made to favor the simplicity of  
 96 the approach in order to test and illustrate ideas, but this was done to the detriment of  
 97 representativeness... Let us consider a relative elementary volume (REV) of a virtual fractal  
 98 porous material in which the pores are constituted of straight, independent and parallel open-  
 99 ended cylinders of radius between  $R_m$  and  $R_M$  (with  $R_M \gg R_m$ ). The REV is a cube of dimension  
 100  $L_0$  in which the cumulated number of pores follows a fractal scaling [33,40–42]:

$$101 \quad \forall r \in [R_m; R_M] \quad N(\rho \geq r) = \left( \frac{r}{R_M} \right)^{-D} \quad (6)$$

102 where  $N$  is the cumulated number of pores of radius greater than  $r$  and  $D$  the fractal dimension  
 103 (with  $1 < D < 2$ ).

104 It is noticeable that, by definition, there is only one pore of radius  $R_M$ . Increasing the value of the  
 105 fractal dimension obviously increases the total number of pores but also the relative  
 106 contribution of the smallest pores (Figure 1).

107



108

109 *Figure 1: number of pores according to the fractal scaling ( $R_m = 3 \times 10^{-10} m$  and  $R_M = 1 \times 10^{-6} m$ )*

110

111 The number of pores of radius  $r$  is easily obtained by differentiation of eq. (1):

$$n(r) = |dN| = DR_M^D r^{-D-1} dr \quad (7)$$

The total porosity  $\emptyset$  of such a fractal porous medium is then:

$$\emptyset = \frac{1}{L_0^3} \int_{R_m}^{R_M} (\pi r^2 L_0) n(r) = \frac{\pi DR_M^D}{L_0^2} \int_{R_m}^{R_M} r^{2-D-1} dr = \frac{\pi DR_M^D}{(2-D)L_0^2} (R_M^{2-D} - R_m^{2-D}) \quad (8)$$

Expectedly, the value of the porosity depends on the pore-size interval  $[R_m; R_M]$ , fractal dimension  $D$  and also on the dimension of the REV  $L_0$ . The evaluation of the latter is not straightforward; in other words there is no evident exact method to evaluate  $L_0$ . Alternatively, an estimate can be obtained by considering that the porosity value ( $\emptyset_0$ ) is known (it usually lies between 30 and 40% for hardened cement pastes), the REV dimension  $L_0$  of a fractal hcp can then be estimated following:

$$\emptyset_0 = \frac{\pi DR_M^D}{(2-D)L_0^2} (R_M^{2-D} - R_m^{2-D}) \Rightarrow L_0 = \sqrt{\frac{\pi DR_M^D}{(2-D)\emptyset_0} (R_M^{2-D} - R_m^{2-D})} \quad (9)$$

Using  $\emptyset_0 = 35\%$ ;  $R_m = 3 \times 10^{-10}$  m;  $R_M = 1 \times 10^{-6}$  m and  $D = 1.5$  as input data, equation (4) yields  $L_0 \approx 5 \times R_M$  and then  $L_0 \propto R_M$ .

124

### 3.2. Water retention

The water retention curve of the FPM was evaluated using a simple approach that considers that water is retained due to (i) physical adsorption of water molecules on the pore walls and (ii) capillary condensation [43–46] (more details in [47]). Adsorption was assumed to result in a unique water layer (monolayer), the thickness of which ( $t_w$ ) was described using Langmuir's model [48]:

$$t_w(h) = \frac{kt_m h}{1+kh} \quad (10)$$

where  $h$  is the relative humidity (without unit);  $k$  Langmuir's equilibrium constant (without unit) and  $t_m$  the thickness of the complete monolayer (m). The value of these two parameters was set to  $k = 50$  and  $t_m = 3 \times 10^{-10}$  m (3 Å) following Morgan & Warren [49], Harkins & Jura [50], Hagymassy [51] and Yamanaka [52]. The pore saturation induced by adsorption is then:

$$S_a(h) = \frac{1}{\emptyset L_0^3} \int_{R_m}^{R_M} (2\pi r L_0) t_w(h) n(r) = 2t_w(h) \left( \frac{2-D}{1-D} \right) \left( \frac{R_M^{1-D} - R_m^{1-D}}{R_M^{2-D} - R_m^{2-D}} \right) \quad (11)$$

Capillary condensation was described using the well-known Kelvin equation that determines the maximal value of the pores radius  $r_K$  in which condensation occurs:

$$r_K(h) = -\frac{2M\sigma}{\rho RT \ln(h)} \quad (12)$$

where  $M$  is the water molar mass ( $18 \times 10^{-3}$  kg/mol);  $\sigma$  surface tension (0.072 N/m);  $\rho$  water density (998.3 kg/m<sup>3</sup> at 20°C);  $R$  ideal gas constant (8.3145 J/mol/K);  $T$  absolute temperature (273.15 K) and  $h$  relative humidity (without unit).

The reader must keep in mind that the presence of the adsorbed film reduces the apparent radius of the pores and thus condensation occurs in pores of radius lower than ( $r_K + t_w$ ) instead of  $r_K$ . The pore saturation induced by capillary condensation is then:

$$S_c(h) = \frac{1}{\emptyset L_0^3} \int_{R_m}^{r_K+t_w} (\pi r^2 L_0) n(r) - \frac{1}{\emptyset L_0^3} \int_{R_m}^{r_K+t_w} (2\pi r L_0) n(r) t_w(h) = \left[ \frac{(r_K+t_w)^{2-D} - R_m^{2-D}}{R_M^{2-D} - R_m^{2-D}} \right] - 2t_w(h) \left( \frac{2-D}{1-D} \right) \left[ \frac{(r_K+t_w)^{1-D} - R_m^{1-D}}{R_M^{2-D} - R_m^{2-D}} \right] \quad (13)$$

Eventually, the saturation is the sum of the two contributions (adsorption and condensation):

$$148 \quad S(h) = S_c(h) + S_a(h) = \left[ \frac{(r_K + t_w)^{2-D} - R_m^{2-D}}{R_M^{2-D} - R_m^{2-D}} \right] + 2t_w(h) \left( \frac{2-D}{1-D} \right) \left[ \frac{R_M^{1-D} - (r_K + t_w)^{1-D}}{R_M^{2-D} - R_m^{2-D}} \right] \quad (14)$$

149 The saturation proves to be a simple analytical function, with only two terms that depend on the  
 150 pore-size interval  $[R_m; R_M]$  and fractal dimension  $D$ . It is noteworthy that when  $(r_K + t_w)(h) =$   
 151  $R_M$  all the pores are filled with water and then  $S = 1$ . Otherwise, when  $(r_K + t_w)(h) = R_m$  there  
 152 is no condensed water but the saturation is not nil because of the presence of the adsorbed film  
 153 on the pore walls and so  $S = 2t_w(h) \left( \frac{2-D}{1-D} \right) \left( \frac{R_M^{1-D} - R_m^{1-D}}{R_M^{2-D} - R_m^{2-D}} \right) > 0$ .

154

### 155 3.3. Intrinsic permeability

156 To compute the permeability of the FPM, one must start with Poiseuille law that gives the flow  
 157 through a cylinder of radius  $r$  and length  $L_0$ :

$$158 \quad q(r) = \frac{\pi r^4}{8\eta} \left( \frac{\Delta P}{L_0} \right) \quad (15)$$

159 where  $\mu$  is the dynamic viscosity of water ( $1.0016 \times 10^{-3}$  Pa s at 20 °C) and  $\Delta P$  the pressure  
 160 difference between the inlet and outlet of the cylinder.

161 The flow through all the pores of the fractal porous medium is then the sum of the contributions  
 162 of all the pores:

$$163 \quad Q_t(r) = \int_{R_m}^{R_M} \left[ \frac{\pi r^4}{8\eta} \left( \frac{\Delta P}{L_0} \right) \right] n(r) = \frac{\pi}{8\eta} \left( \frac{\Delta P}{L_0} \right) D R_M^D \int_{R_m}^{R_M} r^{3-D} dr = \frac{\pi D R_M^D}{8\eta(4-D)} \left( \frac{\Delta P}{L_0} \right) (R_M^{4-D} - R_m^{4-D}) \quad (16)$$

164 One can easily calculate the apparent permeability  $K_e$  of the fractal porous medium considering  
 165 that:

$$166 \quad Q_t(r) = \frac{K_e}{\eta} L_0^2 \left( \frac{\Delta P}{L_0} \right) = \frac{\pi D R_M^D}{8\eta(4-D)} \left( \frac{\Delta P}{L_0} \right) (R_M^{4-D} - R_m^{4-D}) \Rightarrow K_e = \frac{\pi D R_M^D}{8(4-D)L_0^2} (R_M^{4-D} - R_m^{4-D}) \quad (17)$$

167 It is clear here that the pores were assumed to be straight (without tortuosity); this was done on  
 168 purpose to make the model as simple as possible and also because pore-tortuosity is already  
 169 accounted for in Mualem model [38] and there was no use of accounting for it twice. The reader  
 170 should however be informed that it is possible to include tortuosity using a supplementary  
 171 fractal scaling law for the pore length  $L_0$  [53–58].

172 As for the porosity, the permeability of the FPM is a function of the pore-size interval  $[R_m; R_M]$ ,  
 173 fractal dimension  $D$  and also on the dimension of the REV  $L_0$ . Yet, remembering that  $L_0 \propto R_M$  and  
 174 that  $R_M \gg R_m$  it becomes obvious that  $K_e \propto R_M^2$ . This result is not surprising as it appears  
 175 similar to the model proposed by Katz & Thompson based on percolation theory [59,60]:

$$176 \quad K_e = \frac{F}{226} d_c^2 \propto d_c^2 \quad (18)$$

177 where  $F$  is the formation factor ( $0 < F < 1$ ) that accounts for pore structure conformation and  
 178 tortuosity and  $d_c$  is the critical pore entry diameter that corresponds to the inflexion point in the  
 179 intrusion curve obtained using MIP. In other words,  $d_c$  is the greatest pore diameter that allow  
 180 entry to the porous network and it seems reasonable that  $d_c \approx 2R_M$ .

181 Also note that equation (17) can alternatively be rewritten to include porosity  $\phi$  as suggested by  
 182 Guarracino [61], this presents the major advantage to flush the cumbersome parameter  $L_0$ :

$$183 \quad K_e = \frac{\phi}{8} \left( \frac{2-D}{4-D} \right) \left( \frac{R_M^{4-D} - R_m^{4-D}}{R_M^{2-D} - R_m^{2-D}} \right) \Rightarrow K_e \propto \phi R_M^2 \quad (19)$$

184

185 **3.4. Relative permeability**

186 Mualem’s model [38] was used to evaluate the relative permeability  $k_r$  of the FPM (eq. 4). Even  
 187 though the equations used here are quite simple, it was not possible to derive an analytical  
 188 expression for the function  $P(S)$ ; because of the two different exponents  $(2 - D)$  and  $(1 - D)$  in  
 189 eq 14). Alternatively,  $k_r$  was evaluated using the trapezoidal rule of Riemann sums following the  
 190 original proposition of Mualem [38,62]. The saturation interval  $[0;1]$  was discretized into  $n =$   
 191  $10^4$  intervals forming a non-uniform grid with  $u_0 = 0$  (dry state) and  $u_n = 1$  (saturated state)  
 192 and the numerator in eq. (4) then wrote:

193 
$$\begin{cases} \int_0^S \frac{du}{P(u)} = \sum_{i=1}^m (u_i - u_{i-1}) \left( \frac{1}{P_i} + \frac{1}{P_{i-1}} \right) \\ \text{with } 1 \leq m \leq (n - 1) \mid S = u_m \end{cases} \quad (20)$$

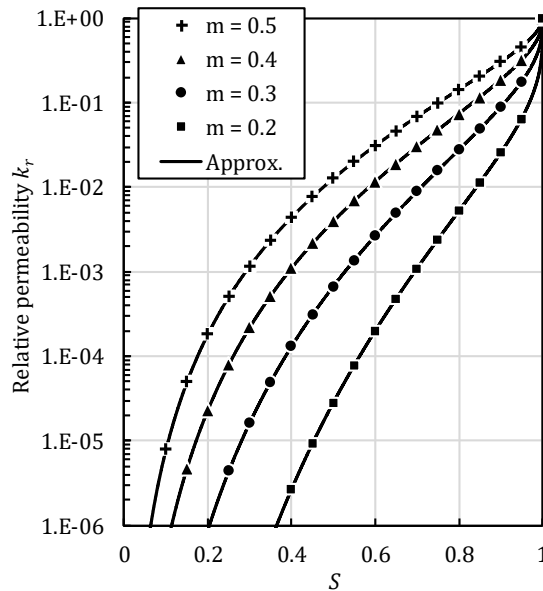
194 where  $u_i$  is the saturation value of the  $i^{\text{th}}$  interval of the non-uniform grid and  $P_i$  the  
 195 corresponding value of the water pressure (MPa).

196 It is obvious that  $P_n^{-1} = \infty$ , so the denominator in eq. (20) cannot be evaluated numerically. To  
 197 overcome this difficulty, the surface area of the last trapezoid was not accounted in the  
 198 summation and the relative permeability was then calculated as follows:

199 
$$k_r(S) = \sqrt{u_m \left[ \frac{\sum_{i=1}^m (u_i - u_{i-1}) \left( \frac{1}{P_i} + \frac{1}{P_{i-1}} \right)}{\sum_{i=1}^{n-1} (u_i - u_{i-1}) \left( \frac{1}{P_i} + \frac{1}{P_{i-1}} \right)} \right]^2} \quad \text{with } 1 \leq m \leq (n - 1) \quad (21)$$

200 The quality of the numerical approximation was tested by comparing the relative permeability  
 201 curves evaluated using eq. (21) to those obtained using usual MVG analytical equation (eq. 5).  
 202 The results are presented in Figure 2. The numerical approximation scheme did provide an  
 203 almost perfect match with the analytical equation despite the fact that one trapezoid surface  
 204 area was neglected.

205



206  
 207 *Figure 2: comparison between numerical approximation (eq. 21) and MVG analytical equation (eq. 5)*

208

## 209 4. Results

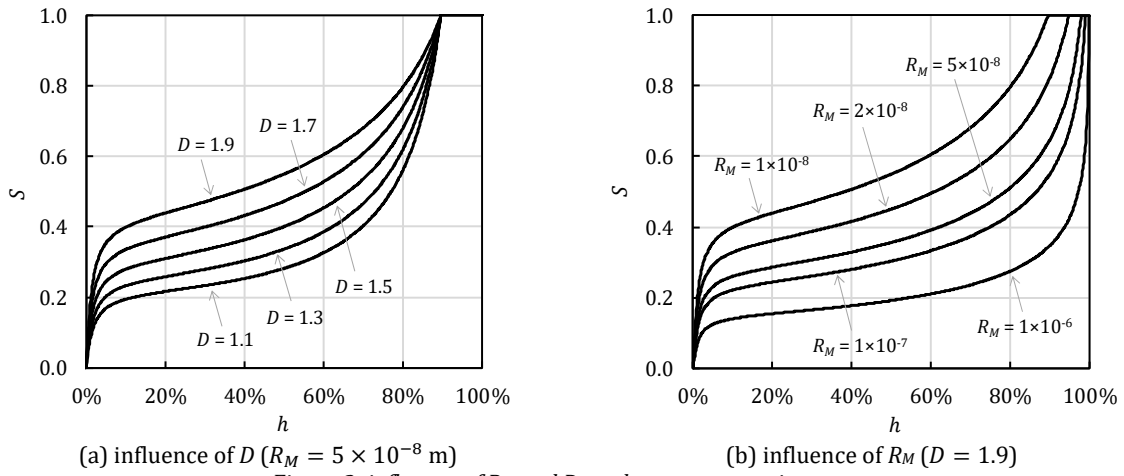
### 210 4.1. Water retention

211 Figure 3 presents some water retention curves that were obtained for the FPM using eq. (14)  
212 and the following data:

- 213 • the minimal pore radius  $R_m$  was assumed to be constant and equal to the monolayer  
214 thickness to allow the edification of a monolayer in the smallest pores ( $R_m = t_m = 3 \times$   
215  $10^{-10}$  m);
- 216 • the maximal pore radius  $R_M$  was varied between 10 nm and 1  $\mu\text{m}$  ( $1 \times 10^{-8}$  m  $\leq R_M \leq$   
217  $1 \times 10^{-6}$  m);
- 218 • the fractal dimension was varied between 1.1 and 1.9 ( $1.1 < D < 1.9$ ).

219 The reader can observe that the water retention curves of the virtual FPM are of type IV [63] and  
220 that they are quite similar to the desorption isotherms of common cement-based materials [64–  
221 68]. Figure 3(a) underscores the influence of the fractal dimension  $D$  and Figure 3(b) the  
222 influence of the maximal pore radius  $R_M$ . Increasing the fractal dimension  $D$  mainly results in  
223 increasing the saturation value at low RH (*i.e.* the monolayer) because it controls the pore  
224 number distribution and the thus the relative contribution of the smaller pores (Figure 1). The  
225 maximal pore radius  $R_M$  exhibits two distinct effects: firstly decreasing the value of  $R_M$  increases  
226 the length of the plateau at high RH through Kelvin equation (7) and secondly changing the value  
227 of  $R_M$  also impacts the pore number distribution (and then the contribution of the smaller pores)  
228 through equation (1).

229



230

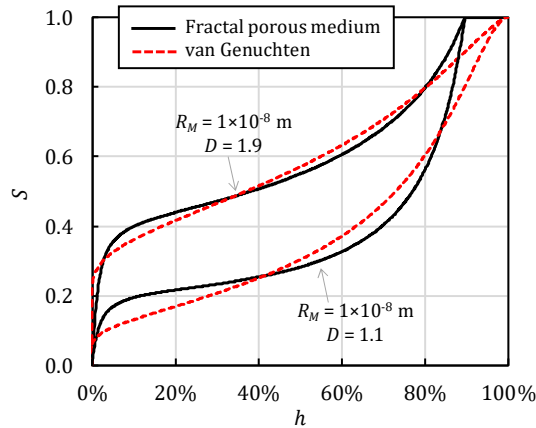
Figure 3: influence of  $R_M$  and  $D$  on the water retention curve

231

232 All the water retention curves were fitted using van Genuchten equation (18) using a classical  
233 least-square minimization process. It must be mentioned here that, despite its deemed great  
234 adaptability, van Genuchten equation (2) did not provide a good fit for all the water retention  
235 curves. This was particularly obvious with small values of  $R_M$  that generated large plateaus at  
236 high RH that could not be described using van Genuchten (Figure 4), because van Genuchten  
237 implicitly assumed that the pore size distribution is unbounded and that  $S = 1$  only when  $P \rightarrow$   
238  $\infty$ . Actually, this observation proved to be valid for all the tested cases (Figure 12), this was  
239 already observed and discussed elsewhere [62].

240





241

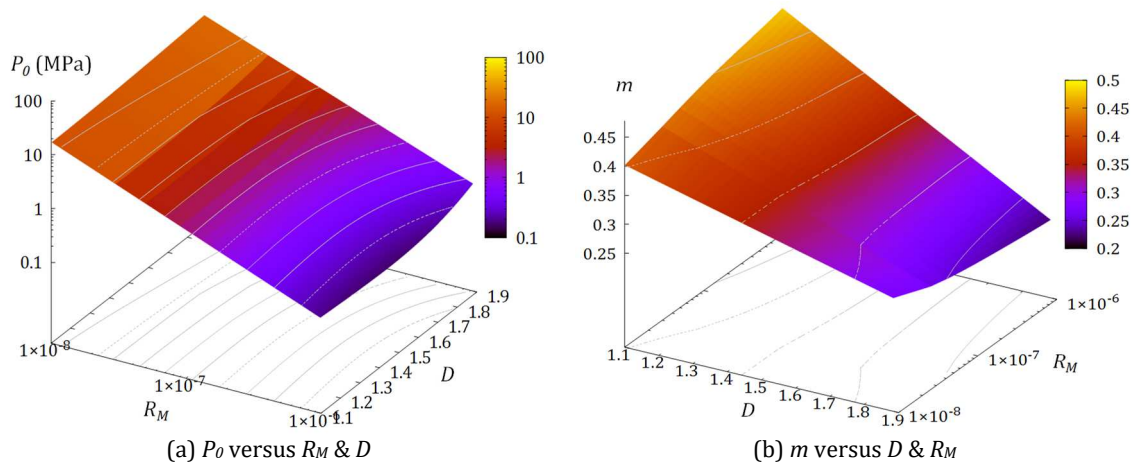
242

Figure 4: examples of poor fits obtained using van Genuchten equation

243

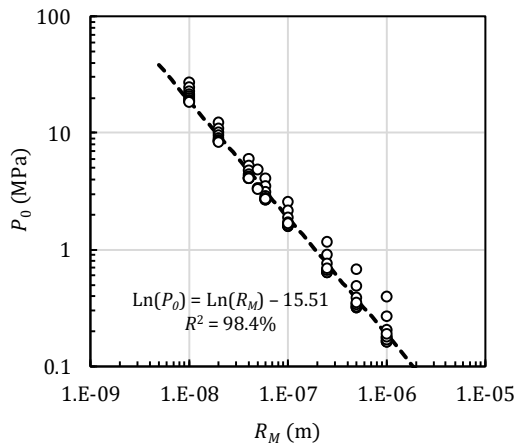
244 The values of the two parameters  $P_0$  and  $m$  were found to depend on both fractal dimension  $D$   
 245 and maximal pore radius  $R_M$  (Figure 5) but further examination of these results unexpectedly  
 246 showed that the exponent  $m$  was more or less a function of the fractal dimension  $D$  alone and  
 247 that the pressure  $P_0$  was a function of the maximal pore radius  $R_M$  alone (Figure 6). Roughly  
 248 speaking, the  $m$ -exponent is a measure of the fractal dimension ( $m \propto -D$ ) whereas the bubbling  
 249 pressure  $P_0$  is a measure of the bigger pores dimension ( $P_0 \propto R_M^{-1}$ ). This result is surprising  
 250 because van Genuchten equation was proposed to provide an analytical solution to Mualem's  
 251 model without any physical basis. Note that the similarities between van Genuchten model and  
 252 fractal approaches were already noted and discussed by Hunt [69].

253

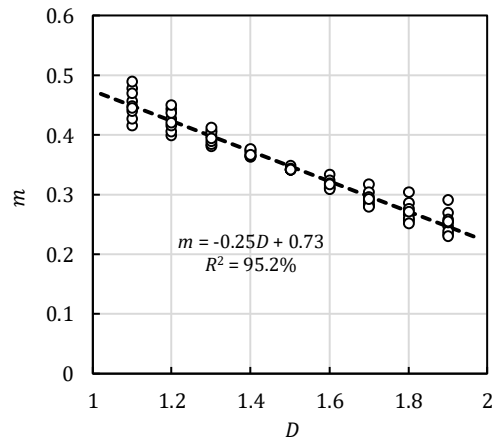


254

Figure 5: influence of  $R_M$  and  $D$  on van Genuchten parameters ( $m$ ,  $P_0$ )



(a)  $P_0$  versus  $R_M$  ( $1.1 \leq D \leq 1.9$ )



(b)  $m$  versus  $D$  ( $1 \times 10^{-8} \text{ m} \leq R_M \leq 1 \times 10^{-6} \text{ m}$ )

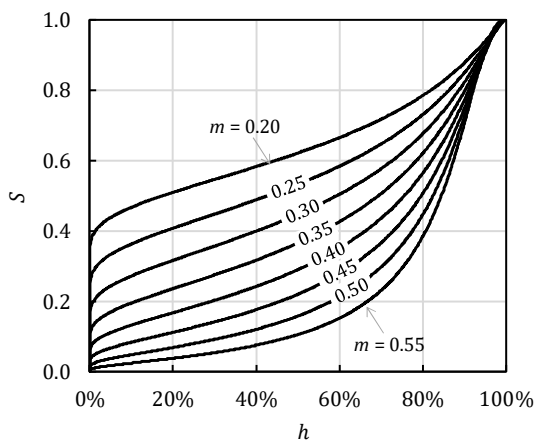
255

Figure 6: compilation of all the results from Figure 5 showing that  $m \propto -D$  and  $P_0 \propto R_M^{-1}$

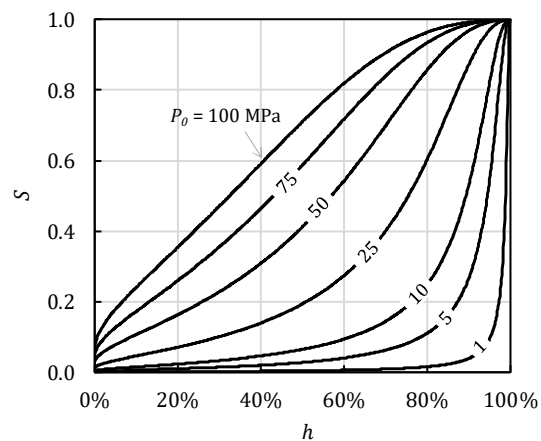
256

257 To go further in studying the “fractal nature” of van Genuchten equation, Figure 7 depicts the  
 258 influence of the two parameters  $m$  and  $P_0$  on the water retention curve. It is plain to see that  $m$   
 259 mainly controls the saturation at low RH (Figure 7-a) as is the case with the fractal dimension  $D$   
 260 (Figure 3-a). The same holds true for the pressure  $P_0$  (Figure 7-b) and the maximal pore radius  
 261  $R_M$  (Figure 3-b) that generate a plateau at high RH and also impact the saturation at low RH.

262



(a) Influence of  $m$  ( $0.15 < m < 0.55$ )



(b) Influence of  $P_0$  ( $1 < P_0 < 100 \text{ MPa}$ )

263

Figure 7: influence of  $P_0$  and  $m$  on van Genuchten water retention curves

264

## 265 4.2. Intrinsic permeability

266 Figure 8 shows that the intrinsic permeability  $K_e$  of the FPM (eq. 17) was ranging between  
 267  $10^{-18} \text{ m}^2$  and  $10^{-15} \text{ m}^2$ . Despite the fact that no tortuosity was introduced in the description, the  
 268 latter results did present the same order of magnitude as the permeability values measured  
 269 using gas [70–72]. The reader should keep in mind that the values obtained using water are  
 270 much lower than using gas [70,73]. This may be due to the change in microstructure induced by  
 271 the drying (necessary for the gas permeability tests) [74,75] and also to electrical interactions  
 272 between the solvent (water is a polar molecule) and the pore walls that limit the mobility of  
 273 water and then decreases permeability [76–80]. As for tortuosity, water-substrate interactions  
 274 were not accounted for here; so gas permeability results were the most appropriate to compare  
 275 with.

276

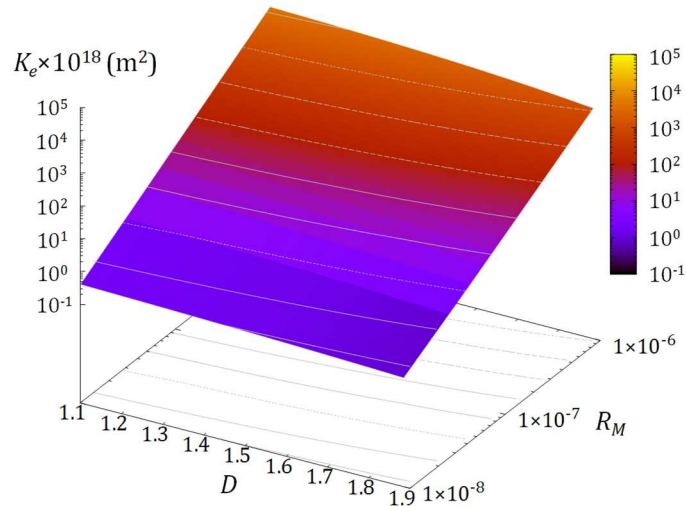


Figure 8: influence of  $D$  and  $R_M$  on the intrinsic permeability  $K_e$  of the fractal porous medium

277

278

279

280 Examining Figure 8, it is plain to see that the intrinsic permeability  $K_e$  of the FPM is mostly  
 281 influenced by the maximal pore radius  $R_M$ . Remembering that the pressure  $P_0$  was a measure of  
 282  $R_M$  then a relation between  $K_e$  and  $R_M$  should exist:  $K_e \propto P_0^{-2}$ . Note that this was already noted  
 283 and verified by Guarracino [61] for a large soil dataset.

284 In an attempt to check the existence of such a relation for cement-based materials, some  
 285 permeability results obtained using inverse analysis were plotted in Figure 9 and some piece of  
 286 corresponding information was reported in Table 1. It is plain to notice that all the results were  
 287 obtained by the same author: it was a deliberate choice (but this could called into question) not  
 288 to include the results obtained by other researchers because it would have induced too much  
 289 scatter as inverse analysis is known to be a highly sensitive method [36,62]. Also note that the  
 290 results also include data obtained using the cup-method as it was shown to give similar results  
 291 to inverse analysis [81]. The results include non-carbonated and carbonated concretes and  
 292 pastes that yielded variations in  $P_0$  and  $K$  of two and four orders of magnitude respectively.

293 All the points almost align along the curve  $K(P_0) = AP_0^{-2}$  (with  $A \approx 3.0 \times 10^{-19} \text{ m}^2$ ) but with  
 294 some scatter that remained encouragingly less than one order of magnitude. It must be recalled  
 295 that the method proposed by Katz and Thompson [59,60] that is commonly used to evaluate the  
 296 intrinsic permeability based on MIP tests [82–84] is known to present large differences (up to  
 297 two orders of magnitude for low-permeable materials) [85–88]. Of course, there are not enough  
 298 data to judge the existence and quality of the proposed correlation and more results would be  
 299 necessary and the readers are invited to test this correlation using their own results. Yet, such a  
 300 correlation would be very useful because it would allow the intrinsic permeability to be  
 301 estimated using the water retention curve (similarly to the relative permeability through  
 302 Mualem model) and then reduce the number of tests needed to characterize the water transport  
 303 properties. Finally, the reader should keep in mind that such a correlation would be fluid  
 304 dependent.

305

306

Table 1: permeability results plotted in Figure 9

Reference	Materials	Composition/remarks	Method
Poyet et al. (2011) [36]	Concrete	Old concrete, highly porous and permeable Unknown composition	Inverse analysis
Drouet et al. (2015) [89]	Paste	4 different binders, $w/c = 0.40$	Inverse analysis
Auroy et al. (2015) [81]	Paste	Same pastes as Drouet et al. (2015) [89]	Inverse analysis + cup-method
Poyet et al. (2019) [90]	Concrete	Low permeable OPC concrete ( $w/c = 0.42$ )	Inverse analysis

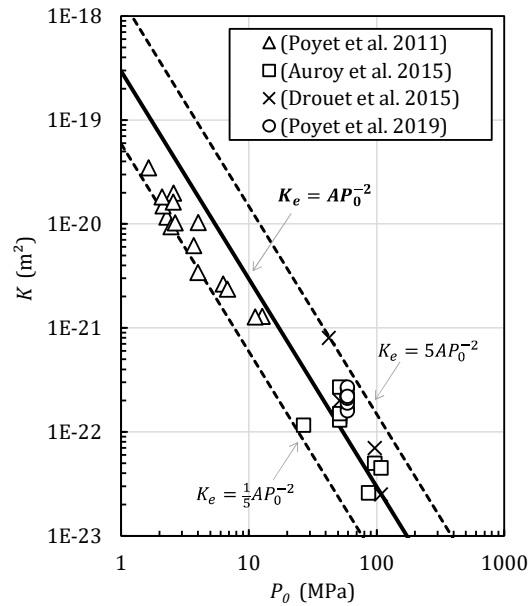


Figure 9: intrinsic permeability  $K$  versus  $P_0$ , from [36], [89], [81] and [90]

308

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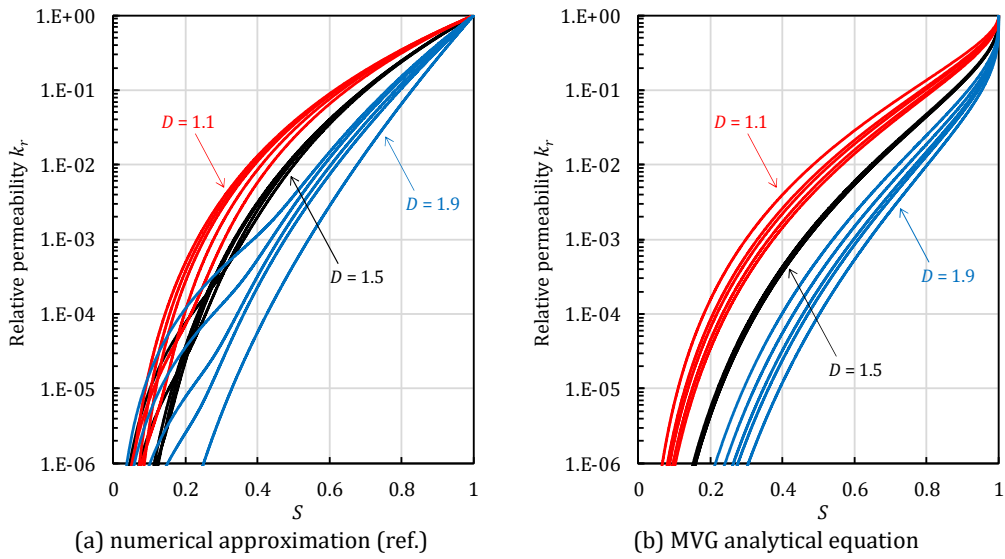
310

### 311 4.3. Relative permeability

312 Figure 10 compares the relative permeability of some fractal porous media obtained using  
 313 Mualem's model (eq. 4) to those obtained using van Genuchten equation (eq. 5). For the sake of  
 314 clarity, only three values of  $D$  (1.1, 1.5 and 1.9) are displayed and for each five values of  $R_M$  ( $10^{-8}$ ,  
 315  $2 \times 10^{-8}$ ,  $5 \times 10^{-8}$ ,  $10^{-7}$  and  $10^{-6}$  m). It was clear that the relative permeability  $k_r$  of the FPM (Figure  
 316 10-a) was a function of  $D$  and  $R_M$ , although the fractal dimension appeared to be the most  
 317 influential parameter. It should be mentioned that the unusual change of slope occurring in  
 318 some curves below 0.4 in saturation ( $D = 1.9$ ) was due to the strong knee that was obtained for  
 319 small  $R_M$  values (see Figure 3 and Figure 4).

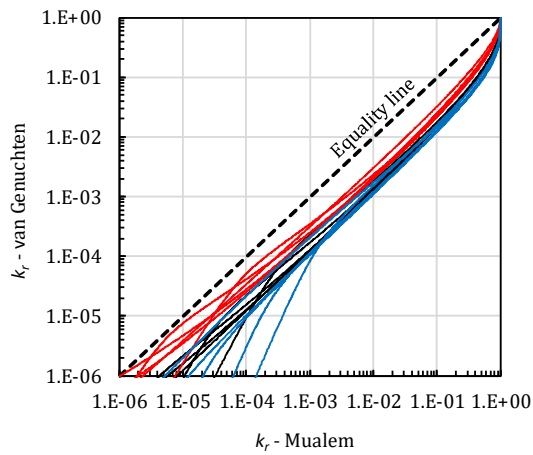
320 The use of van Genuchten equation (Figure 10-b) did not appear to be satisfactory as it clearly  
 321 led to underestimate the  $k_r$  values (Figure 11). This underestimation was due to the limited  
 322 fitting ability of van Genuchten equation that was not able to perfectly fit the water retention  
 323 curves of all the FPM. This was already noted for small  $R_M$  values because of the plateau at high  
 324 saturation (Figure 4) but this conclusion was found to be valid in all cases, even for high values  
 325 of  $R_M$ . The isotherms obtained using van Genuchten all showed differences with those of the FPM  
 326 (Figure 12) even though they could not be detected using the naked eye in most cases. The point  
 327 was that these differences, even small, were responsible through Mualem's model, of the  
 328 underestimation by one order of magnitude that was observed in Figure 11. This last point is  
 329 important because it sheds light on the subject of the permeability of cement-based materials  
 330 and more specifically on the fact the permeability values to be used in simulations (through  
 331 inverse analysis) are smaller than those measured using water [39]. Part of these differences  
 332 might be explained by van Genuchten equation and its limited fitting ability because inverse  
 333 analysis is known to be very dependent on the input data and the models that are used [62].

334



335 Figure 10: comparison between the relative permeability  $k_r$  curves obtained using either the numerical approximation  
 336 (eq. 21) or the MVG analytical expression (eq. 5)

337



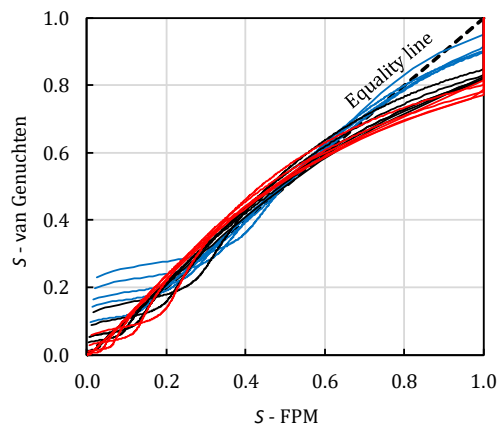
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Figure 11: comparison between the  $k_r$  values obtained using the numerical approximation (eq. 21) or the MVG analytical  
 equation (eq. 5)

341



342

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344

Figure 12: comparison of the capillary curves obtained of the virtual FPM (eq. 14) and fitted using van Genuchten  
 equation (eq. 2)

## 346 5. Conclusion

347 The pore-structure of cement-based materials is more and more recognized as being fractal. The  
 348 mathematical meaning of fractality is the existence of a simple analytical pore scaling law (in the  
 349 form of a power function). Using this scaling law and considering a single fractal dimension, it  
 350 was easy to generate virtual and simple fractal porous media that were then used to study some  
 351 important water transport properties (intrinsic and relative permeability and water retention  
 352 curve) and the associated Mualem model and van Genuchten equations. It must be recalled here  
 353 that due to their simplicity (single fractal dimension) the virtual fractal porous media may not be  
 354 representative of the real cement-based materials.

355 Firstly, van Genuchten equation was used to fit the capillary curves that were generated  
 356 considering physical adsorption and capillary condensation. It appeared that, although bereft of  
 357 any physical basis, van Genuchten equation may be considered as bearing some physical  
 358 information because its two parameters were closely related to the fractal porous media  
 359 properties: the exponent  $m$  was a measure of the fractal dimension ( $D$ ) whereas the pressure  $P_0$   
 360 was a measure of the maximal pore radius ( $R_M$ ).

361 Secondly, the capillary curves were used to evaluate the relative permeability (making use of  
 362 Mualem model). Although it is known to be flexible, van Genuchten equation did not provide  
 363 perfect fits and the differences between the fitted capillary curves and those of the fractal porous  
 364 media were sufficient to generate significant error: the relative permeability values were  
 365 systematically underestimated up to one order of magnitude. This partly explains why the  
 366 intrinsic permeability values obtained using inverse analysis are always lower than the  
 367 measured ones. This also confirms the strong dependence of inverse analysis upon the models  
 368 and data used.

369 Finally, the equations regarding permeability of the fractal porous media suggested that there  
 370 should exist a relation between the intrinsic permeability and van Genuchten parameter  $P_0$ .  
 371 Analysis of some results of the author tended to confirm the existence of such relationship even  
 372 though the number of results are not sufficient to conclude. Such a relationship would prove  
 373 extremely useful because it could help reducing the number of tests needed to describe water  
 374 transport.

375

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 378 (<https://wxmaxima-developers.github.io/wxmaxima/>)

379

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606 **Nomenclature**

- 607 •  $R_m$  minimal pore radius (m)
- 608 •  $R_M$  maximal pore radius (m)
- 609 •  $D$  fractal dimension (without unit)
- 610 •  $N$  cumulated number of pores (without unit)
- 611 •  $r$  pore radius (m)
- 612 •  $n$  number of pores of radius  $r$  (without unit)
- 613 •  $\emptyset$  porosity (without unit)
- 614 •  $S$  saturation ratio (without unit)
- 615 •  $S_a$  saturation ratio due to adsorption (without unit)
- 616 •  $S_c$  saturation ratio due to condensation (without unit)
- 617 •  $P$  liquid water pressure (Pa)
- 618 •  $t$  time (s)
- 619 •  $K$  intrinsic permeability (m<sup>2</sup>)
- 620 •  $k_r$  relative permeability to water (without unit)
- 621 •  $\eta$  dynamic water viscosity (Pa s)
- 622 •  $m$  constant of van Genuchten equation (without unit)
- 623 •  $P_0$  constant of van Genuchten equation (Pa)
- 624 •  $t_w$  thickness of the water layer adsorbed on the pore walls (m)
- 625 •  $t_m$  thickness of the water monolayer (m)
- 626 •  $k$  constant of Langmuir model (without unit)
- 627 •  $h$  relative humidity (without unit)
- 628 •  $r_K$  Kelvin radius (m)
- 629 •  $M$  water molar mass (kg/mol)
- 630 •  $\sigma$  water surface tension (N/m)
- 631 •  $\rho$  water density (kg/m<sup>3</sup>)
- 632 •  $R$  universal gas constant (J/mol/K)
- 633 •  $T$  absolute temperature (K)
- 634 •  $q$  flowrate through a pore of radius  $r$  (m<sup>3</sup>/s)
- 635 •  $Q_t$  flowrate through all the pores (m<sup>3</sup>/s)
- 636 •  $\Delta P$  pressure difference applied to the pores (Pa)
- 637 •  $F$  formation factor (without unit)
- 638 •  $d_c$  critical pore diameter (m)

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