Physical challenges & numerical issues in controlled fusion plasmas

Yanick Sarazin

Physical challenges & Numerical issues in Controlled fusion plasmas

Y. Sarazin

CEA, IRFM, 13108 Saint-Paul-Lez-Durance, France.

Ack.: X. Garbet, Ph. Ghendrih, G. Dif-Pradalier

SMAI 23/06/2021 – mini-symposium "Mathematical modelling of thermonuclear fusion plasmas"

Outline

- Basics on controlled fusion & ITER project
- Physical challenges:
  - **Confinement**
  - **Heat & particle Exhaust**
  - **Disruptions**
- Focus on three critical issues:
  - **Fast particles**: hybrid MHD, turbulence
  - **Plasma-wall interaction**: the kinetic side
  - **Coulomb collisions**: towards an efficient (non-linear) operator

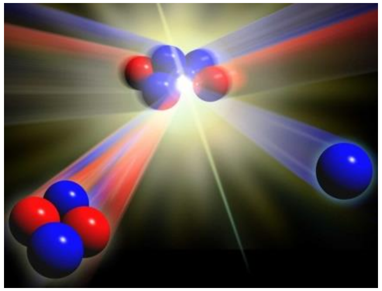
Fusion – the principles (I)

D	+	T	→	He	+	n

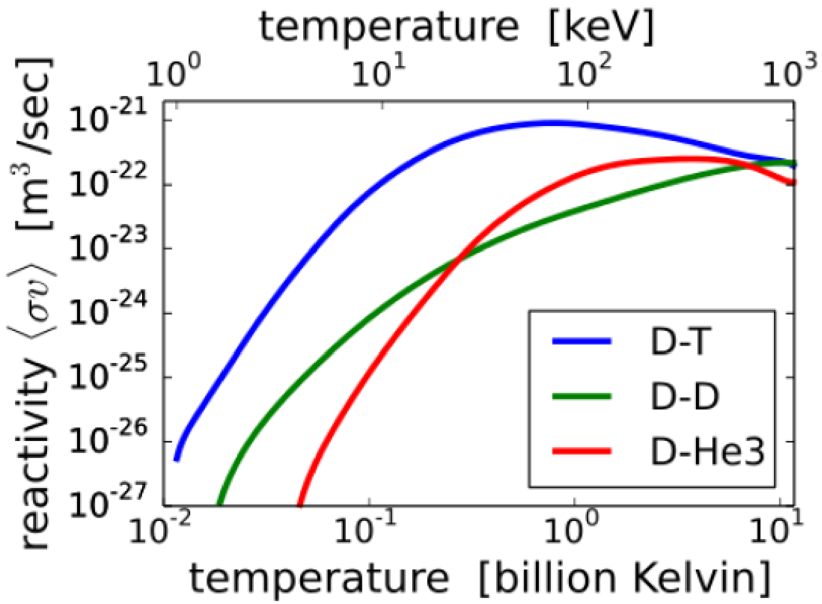
Hydrogen isotopes — Helium — Neutron

Stable — Unstable — stable φ & χ

- must be produced
- not key for ITER



- impact on device
- used to produce T
- not key for ITER



Threshold condition for fusion

$E_{th} \geq 10$ keV → **Plasma state**

→ interaction distance 10^{-13} m

Fusion – the principles (II)

T generation: $n + {}^{6\text{-}7}Li \rightarrow He + T\ (+n)$

$$D + T \rightarrow He + n$$

Plasma at 10-20 keV — 3.5 MeV — 14 MeV

Plasma heating then ash removal

Energy prod.

- **Power balance** at equilibrium: sources = losses

$$P_\alpha + P_{add} = \frac{W_{internal}}{\tau_E}$$

Internal energy ~ nT

α-heating $\sim n^2 \langle \sigma v \rangle_{DT}(T)$

additional heating

Energy confinement time

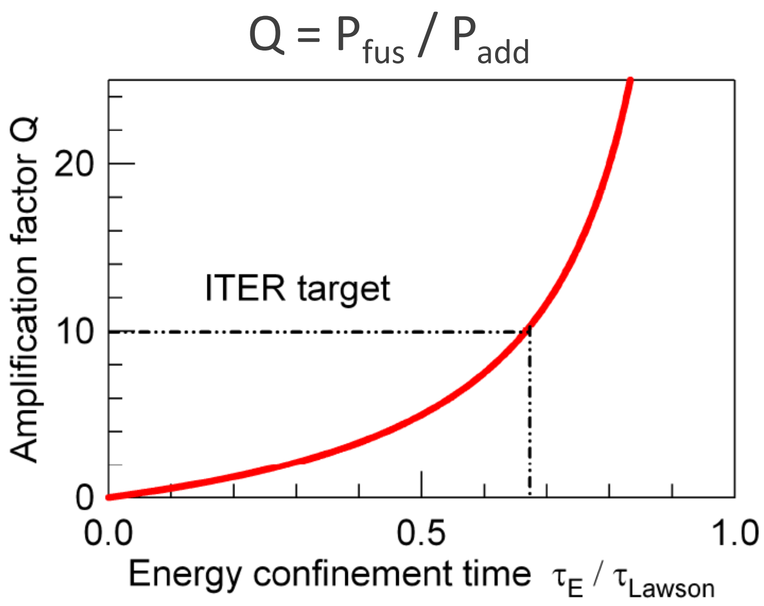
Towards ignition: the triple product (Lawson)

- **Self-heated Fusion** (ignition)

$$nT \;\tau_E \;>\; 6\;10^{21}\,\mathrm{m^{-3}.keV.s}$$

MHD stability — Turbulent transport

$$\tau_E = \frac{\text{energy content}}{\text{loss power}} \Rightarrow \text{governs fusion performance}$$



- **Magnetic confinement**

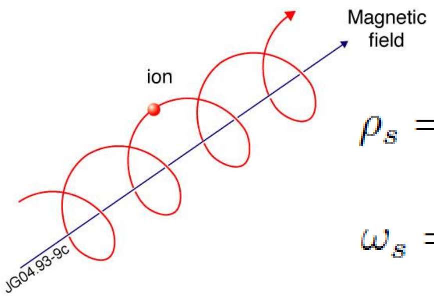
$T \approx 20$ keV	(threshold reaction)
$n \approx 10^{20}\mathrm{m^{-3}}$	($\beta = P_{kin}/P_{magn} < 1$)

$\Rightarrow$ Pressure ~ few atm ($P_{Sun} \sim 10^{11}$ atm)

$\Rightarrow$ Requires $\tau_E \approx 3$s

Particle confinement

- Particles confined by strong magnetic field B



ions

$$\rho_s = \frac{m_s v_\perp}{q_s B} \quad \sim 3.10^{-3}\ \text{m}$$

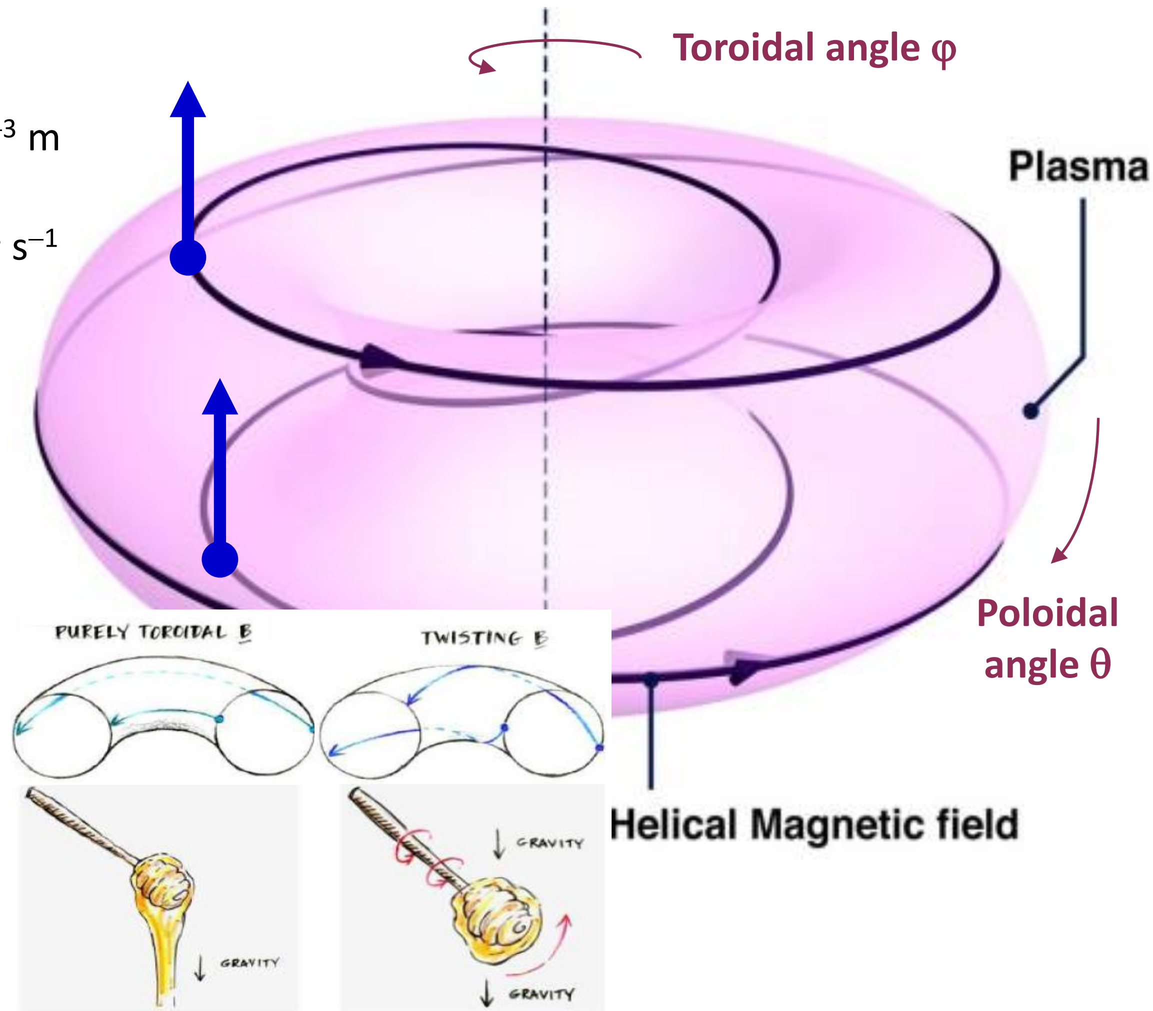
$$\omega_s = \frac{q_s B}{m_s} \quad \sim 3.10^{8}\ \text{s}^{-1}$$

- Closed field lines

$\Rightarrow$ **particle vertical drift**

$\Rightarrow$ helicity to compensate

- Magnetic field lines twist around tori called magnetic surfaces



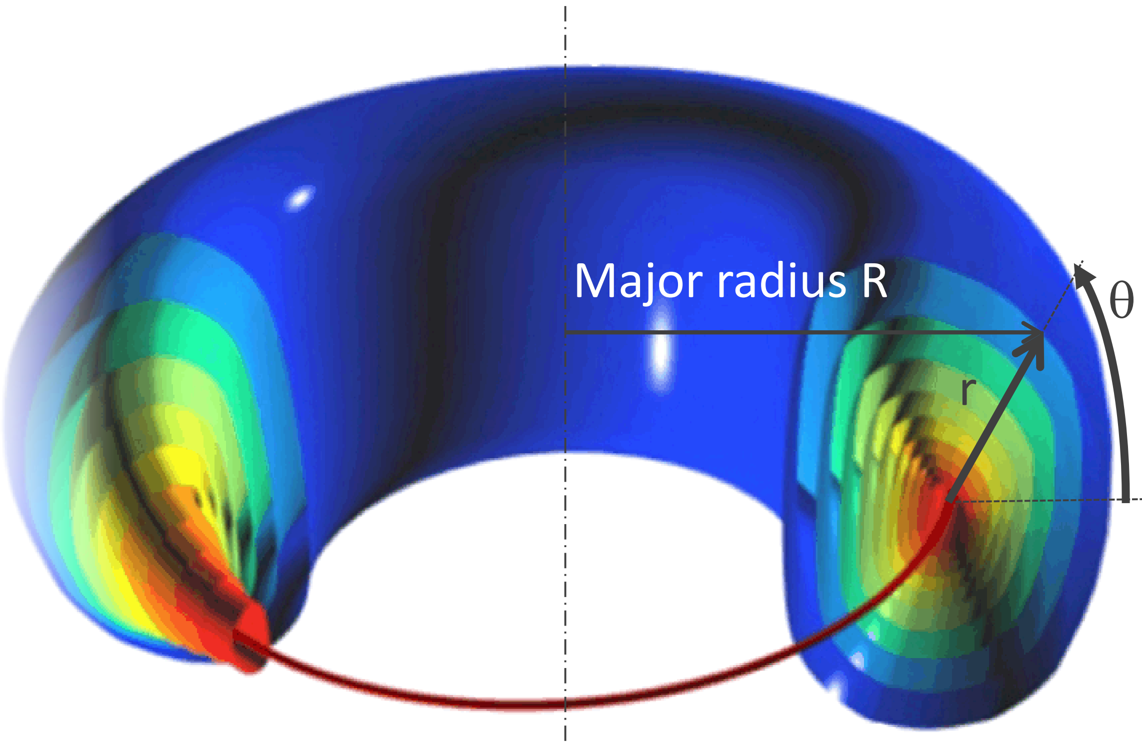
MHD confinement

- **Strong anisotropy** of the transport: // to **B** >> ⊥ to **B**
  ⇒ Fast homogenization of magnetic flux surfaces

- Magnetic surfaces are (at leading order) isobar, iso-temperature, iso-current

- **Lorentz force** balances **pressure gradient**

$$\mathbf{j} \times \mathbf{B} = \nabla p$$



Nested magnetic surfaces

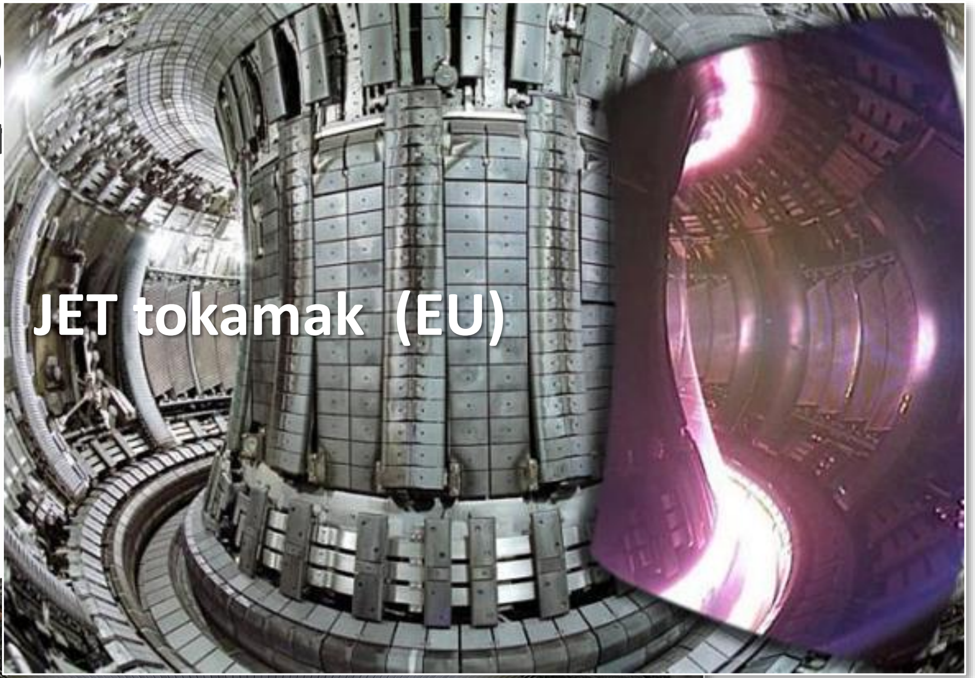
MHD confinement

- **Strong anisotropy** of the transp
  $\Rightarrow$ Fast homogenization of magn

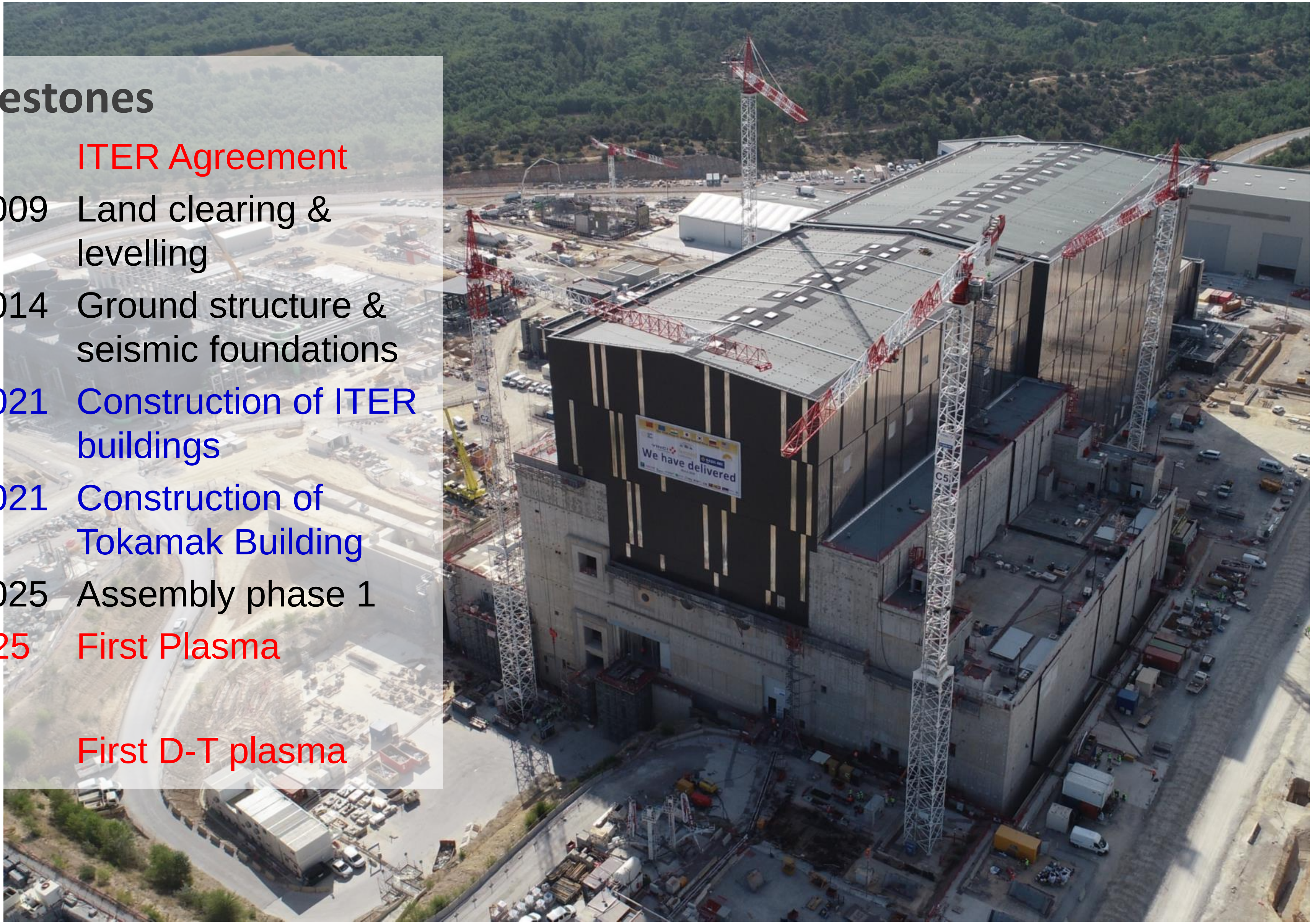
- Magnetic surfaces are (at leading order) isobar, iso-temperature, iso-current

- **Lorentz force** balances **pressure gradient**

$$\mathbf{j} \times \mathbf{B} = \nabla p$$

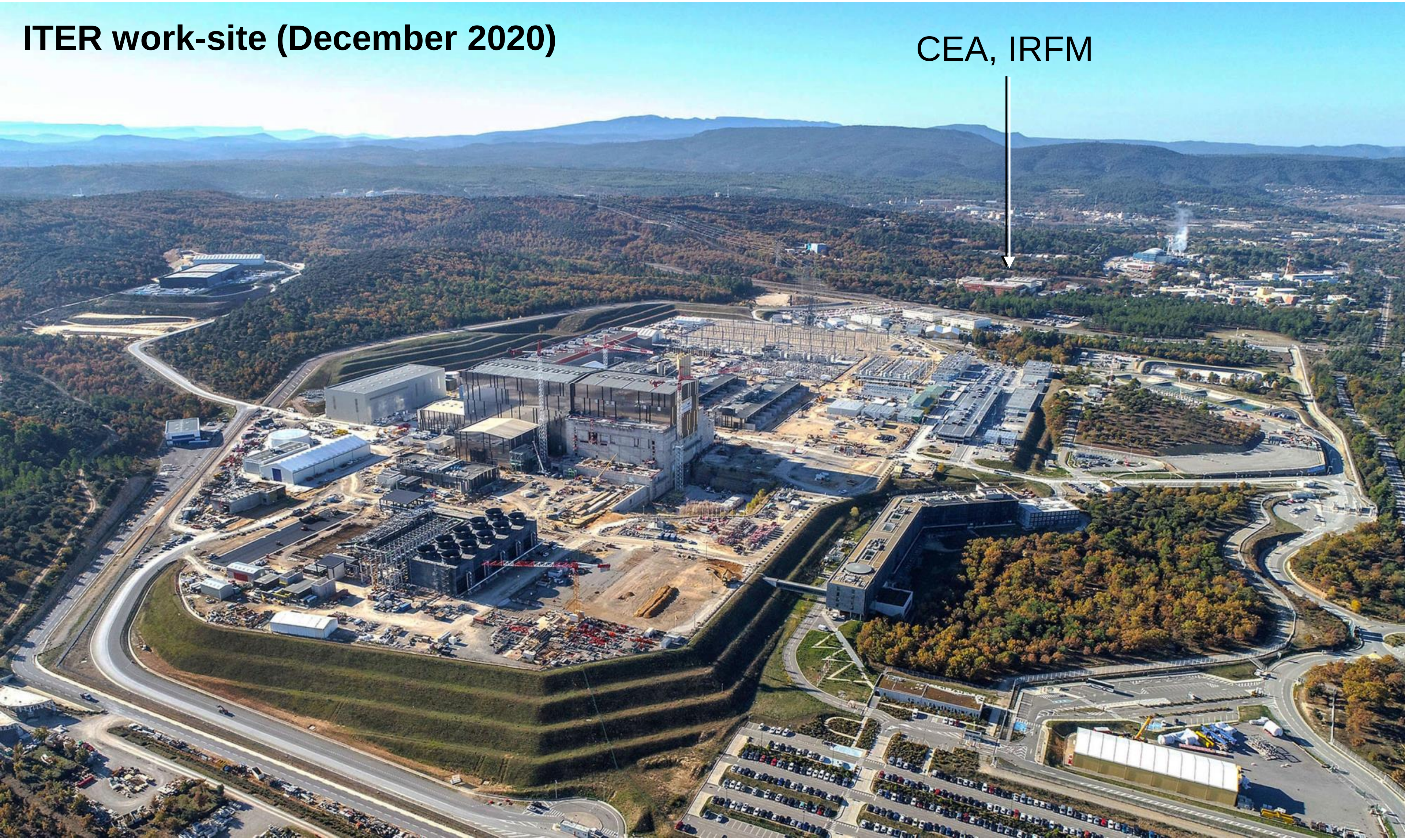


ITER



Milestones

2006	ITER Agreement
2007-2009	Land clearing & levelling
2010-2014	Ground structure & seismic foundations
2010-2021	Construction of ITER buildings
2014-2021	Construction of Tokamak Building
2018-2025	Assembly phase 1
Dec 2025	First Plasma
…	
203X	First D-T plasma


ITER work-site (December 2020)
CEA, IRFM

Outline

- Basics on controlled fusion & ITER project
- Physical challenges:
  - **Confinement**
  - **Heat & particle Exhaust**
  - **Disruptions**
- Focus on three critical issues:
  - **Fast particles**: hybrid MHD, turbulence
  - **Plasma-wall interaction**: the kinetic side
  - **Coulomb collisions**: towards an efficient (non-linear) operator

Plasma-wall interaction → the divertor

- Interface with plasma facing components: in general, hot & diluted plasma

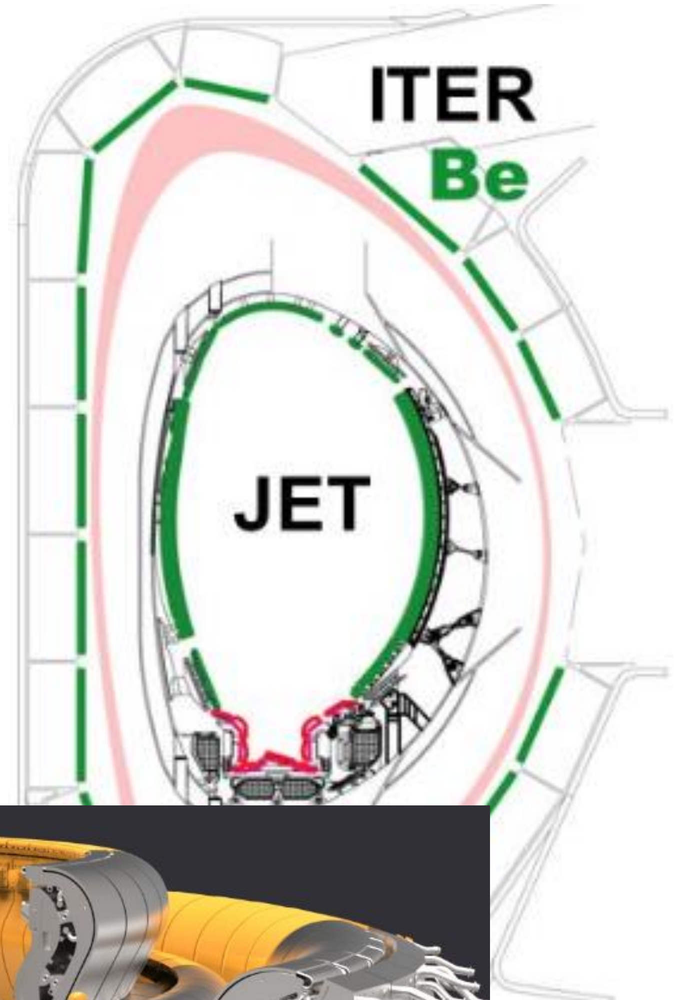
  ⇒ emission of **impurities**

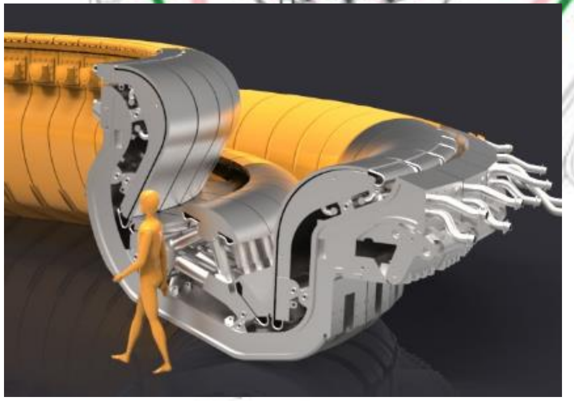
  ⇒ inefficient **pumping**

- Preferable solution: **dense, cold & radiating plasma**

  → appropriate device = **divertor**

  - Heat exhaust (~10 $MW.m^{-2}$)
  - Pumping ashes (He)
  - Controlling impurity influx
  - Fueling the plasma with D-T
  - Resilience to transient heat loads





Plasma-wall interaction → the divertor

- Interface with plasma facing components: in general, hot & diluted plasma

  ⇒ emission of **impurities**

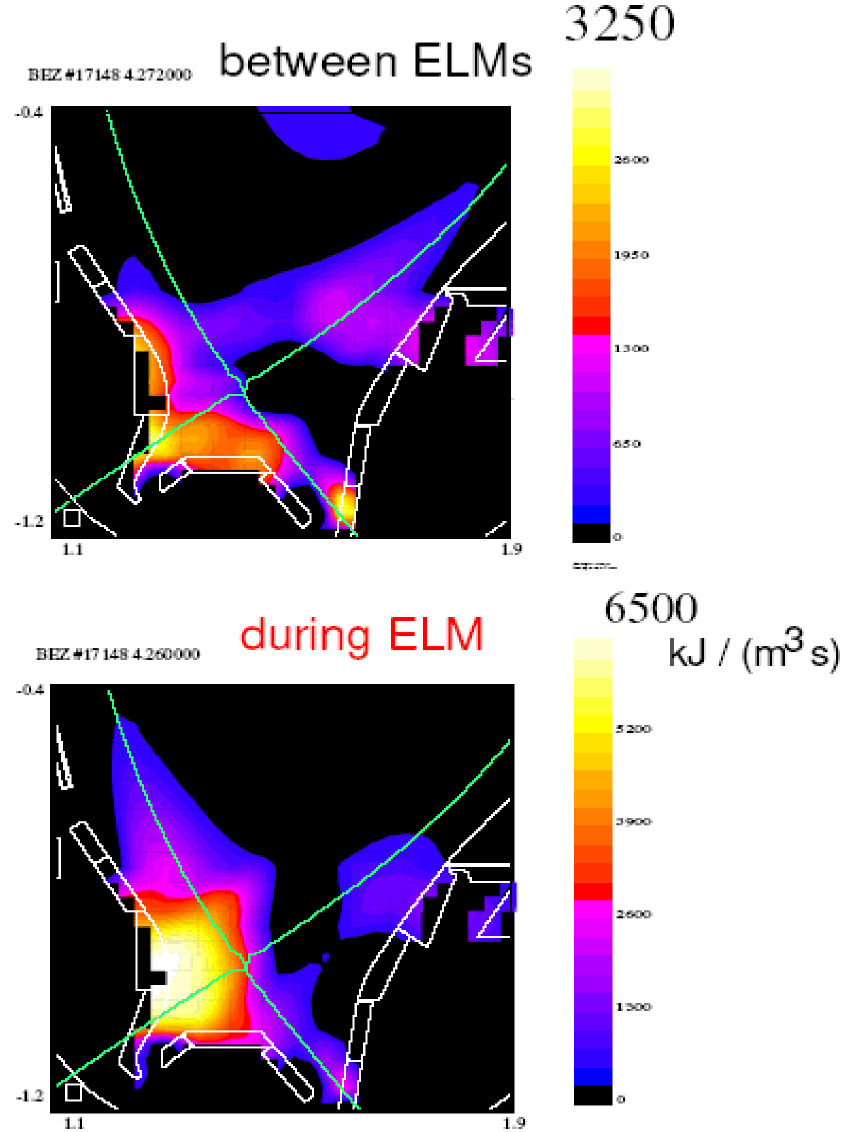
  ⇒ inefficient **pumping**

- Preferable solution: **dense, cold & radiating plasma**

  → appropriate device = **divertor**

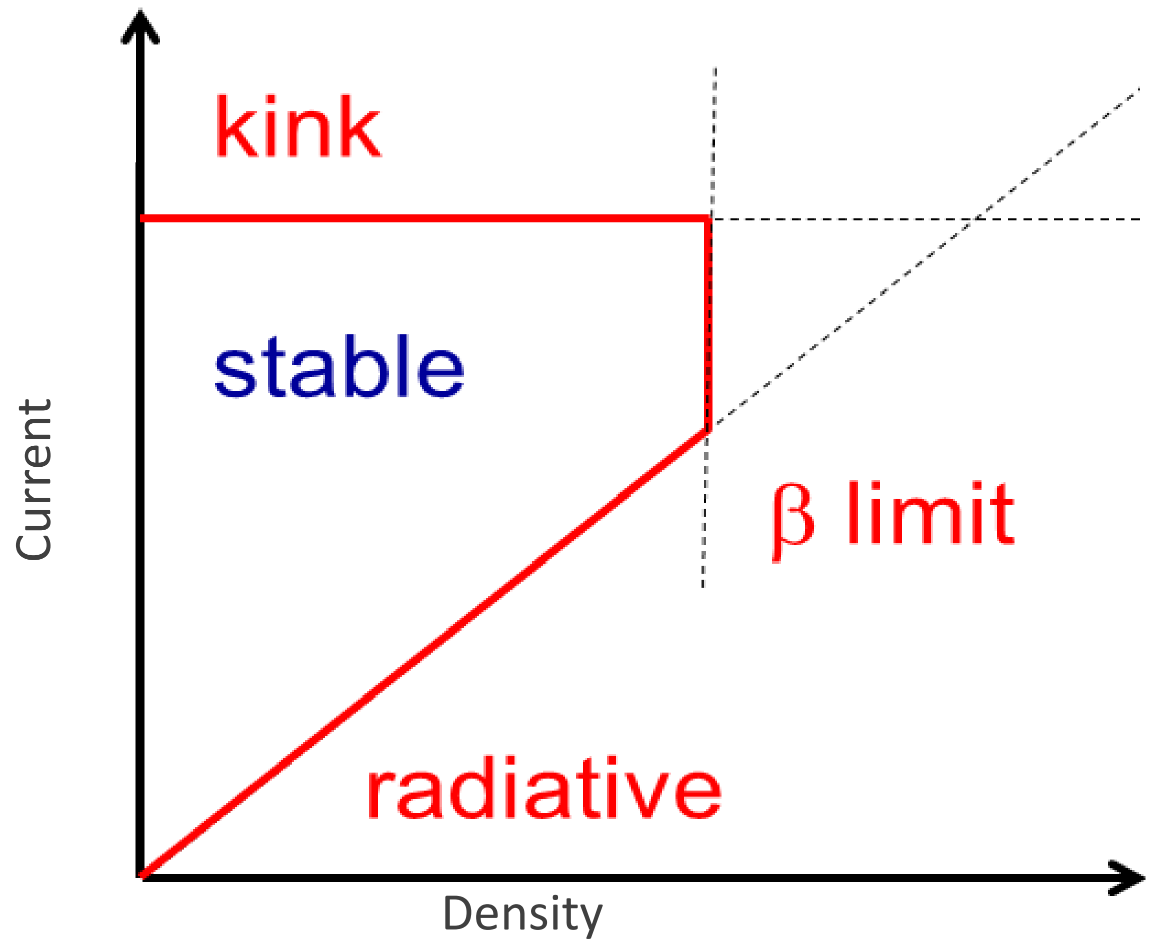
  - Heat exhaust (~10 $MW.m^{-2}$)
  - Pumping ashes (He)
  - Controlling impurity influx
  - Fueling the plasma with D-T
  - Resilience to transient heat loads

Asdex-Upgrade



- **Magneto-hydrodynamic (MHD) instabilities** driven by
  - Pressure
  - Current
  - Radiative losses

- **Dynamics** of MHD instabilities
  - Steady-state
  - Oscillations
  - Disruption



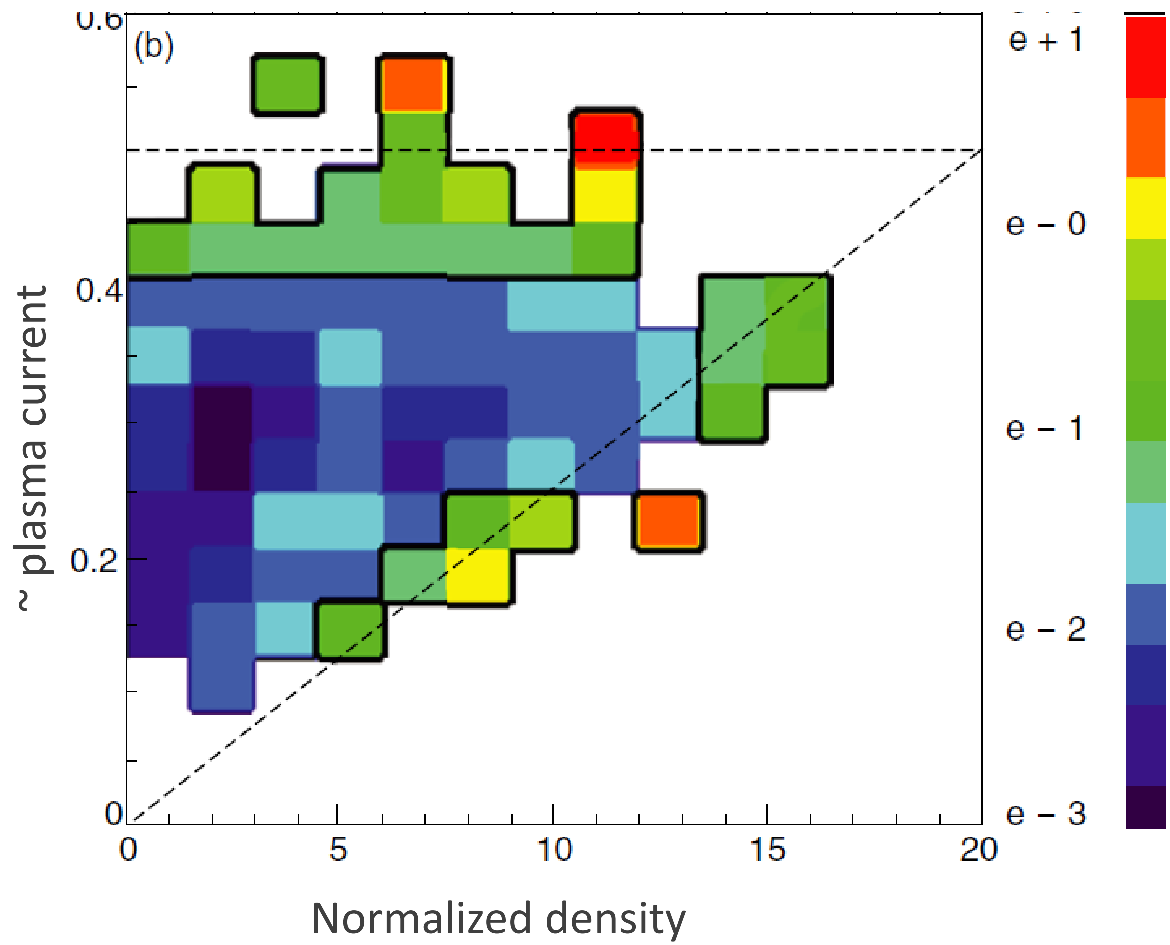
Operational domain → large scale instabilities

- **Magneto-hydrodynamic (MHD) instabilities** driven by
  - Pressure
  - Current
  - Radiative losses

- **Dynamics** of MHD instabilities
  - Steady-state
  - Oscillations
  - Disruption

Disruptivity at JET

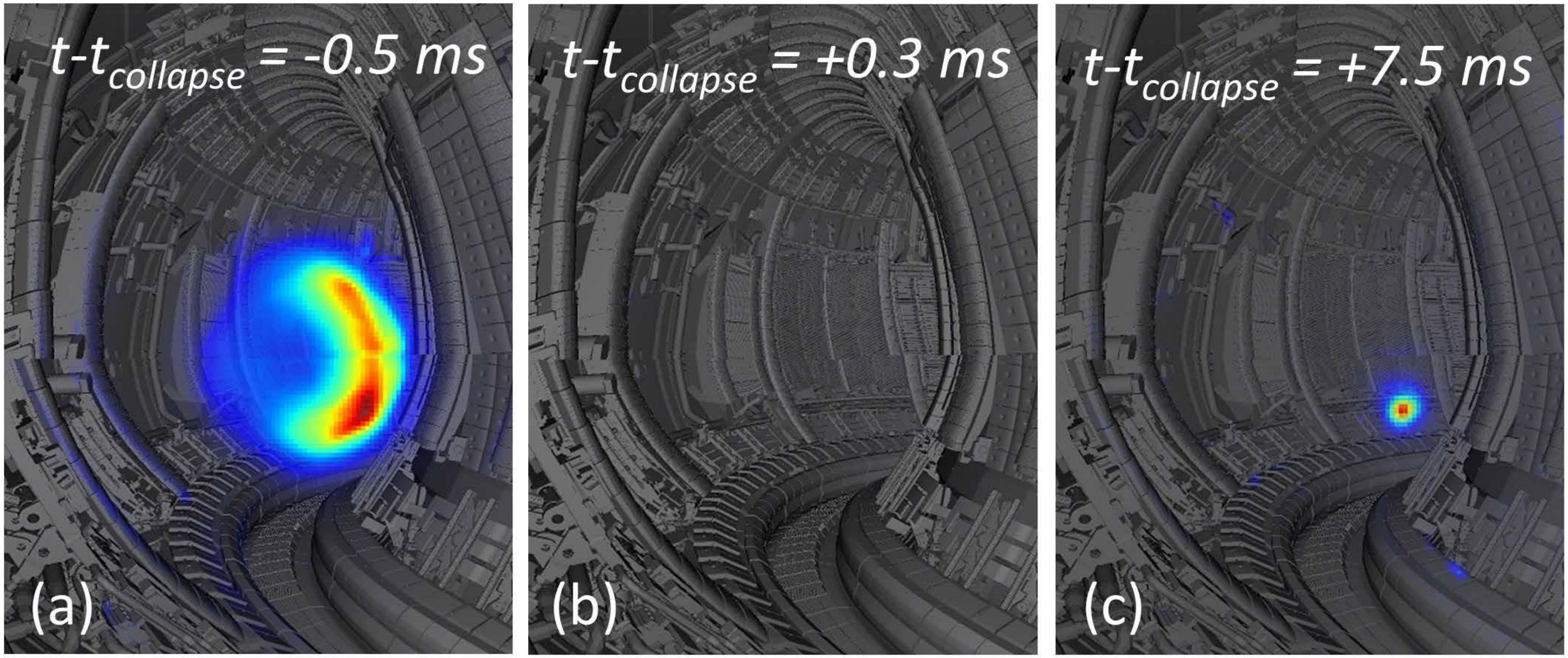
De Vries et al. (2009)



Controlling runaway electrons

- **Disruptions** = sudden (few ms) loss of confinement
  - Huge forces on plasma vessel
  - Runaway Electron beam $v_e \sim c \rightarrow$ up to 100 MJ in ITER
- **Strategies**: detection, avoidance and/or mitigation

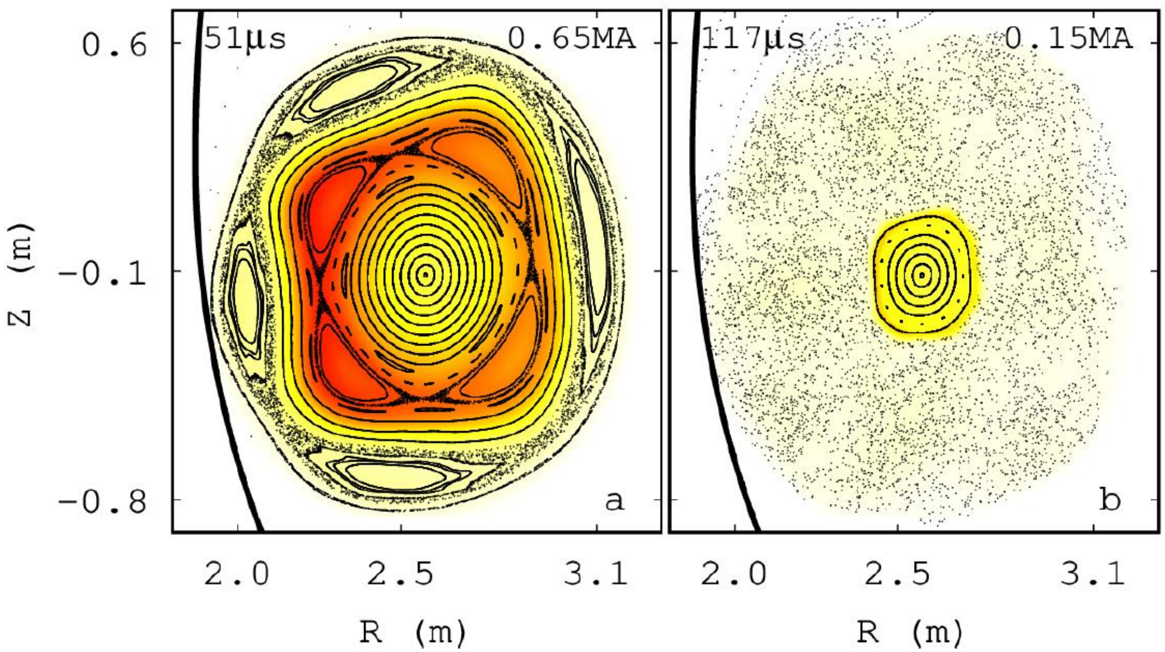
Mitigation of RE at JET with shattered D2 pellets [Reux et al., PRL 2021]



Controlling runaway electrons

- **Disruptions** = sudden (few ms) loss of confinement
  - Huge forces on plasma vessel
  - Runaway Electron beam $v_e \sim c \rightarrow$ up to 100 MJ in ITER
- **Strategies**: detection, avoidance and/or mitigation

Mitigation of RE at JET with shattered D2 pellets [Reux et al., PRL 2021]



Tearing MHD mode

⇓

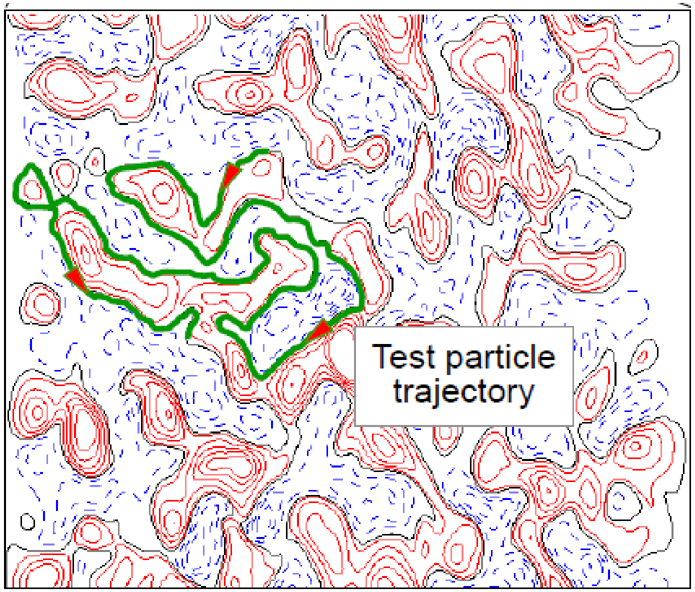
Stochastization of field lines

⇓

RE no longer confined in 100µs

Performance ⇒ towards transport control

- **Several transport channels**
  - **Heat** (τ_E) → electrostatic turbulence
  - α **particles, runaway electrons** → electromagnetic modes
  - **Impurities** → turbulence & collisions



- **Key physics**:
  Resonant and/or non-resonant coupling between particles and EM fields

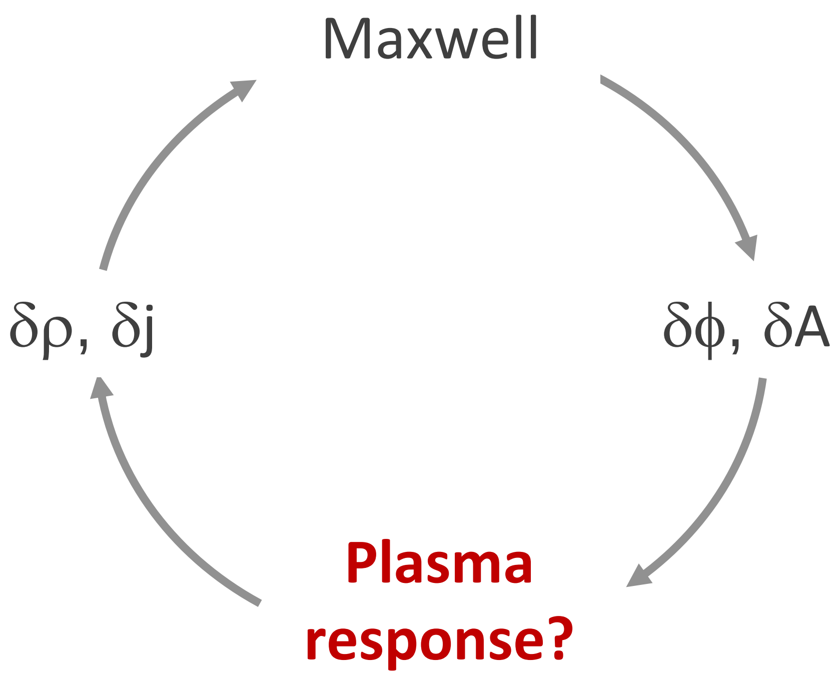
- **Numerical issues**:
  - Model description: fluid moments and/or adiabatic vs. (gyro)kinetic → coupling issues
  - Mix of slow & fast particles: thermal particles vs. α or runaway electrons, ions vs. electrons → resolution issues

Self-consistent description of plasma

Quasi-neutrality: $\rho = 0$ (OK for $k\lambda_D \ll 1$)

Maxwell-Ampère: $\nabla \times \mathbf{B} = \mu_0 \, \mathbf{j}$

Fluctuations of charge & current densities



Fluctuations of electro-magnetic field

3 main descriptions:
- MHD / Multi-Fluid
- (Gyro-) Kinetic

Kinetic vs. fluid & MHD

- **(Gyro)Kinetic**:
  - Vlasov-like equation
  - Sources & sinks
  - Collisions

$$\mathrm{d}_t F_s = S + C(F_s)$$

- (5)6D + time: $F_s(\mathbf{x}, \mathbf{v}, t)$
- Integrals on velocity to get charge & current densities

- **Fluid & MHD**:
  - Conservation equations of particles, momentum, energy

$$\rho d_t \mathbf{V} = -\nabla P + \mathbf{J} \times \mathbf{B} + \nu \nabla^2 \mathbf{V}$$

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \frac{\eta}{\mu_0} \nabla^2 \mathbf{B}$$

$$d_t = \partial_t + \mathbf{V} \cdot \nabla$$

+ eq on pressure P

- 3D + time: V(**x**,t), P(**x**,t), …

Both descriptions require High Performance Computing

E.g. GK ion turbulence in ITER: $a/\rho_{ci} \sim 10^3$ & $\omega_{turb}\tau_E \sim 10^5$ $\rightarrow$ $\sim 3.10^{11}$ grid points, ~20 Mh.core

Outline

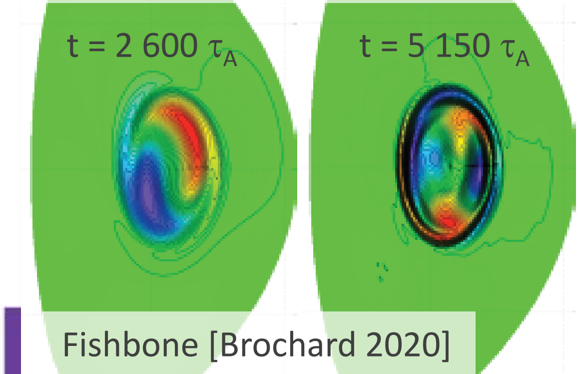
- Basics on controlled fusion & ITER project
- Physical challenges:
  - Confinement
  - Heat & particle Exhaust
  - Disruptions
- Focus on three critical issues:
  - **Fast particles**: hybrid MHD, turbulence
  - **Plasma-wall interaction**: the kinetic side
  - **Coulomb collisions**: towards an efficient (non-linear) operator

MHD valid at high frequency & large scale

- Ideal MHD approach valid if
  $\omega >> k.v$ & $k_\perp \rho << 1$

XTOR-K

[Lütjens 2010; Nicolas 2012; Brochard 2020]



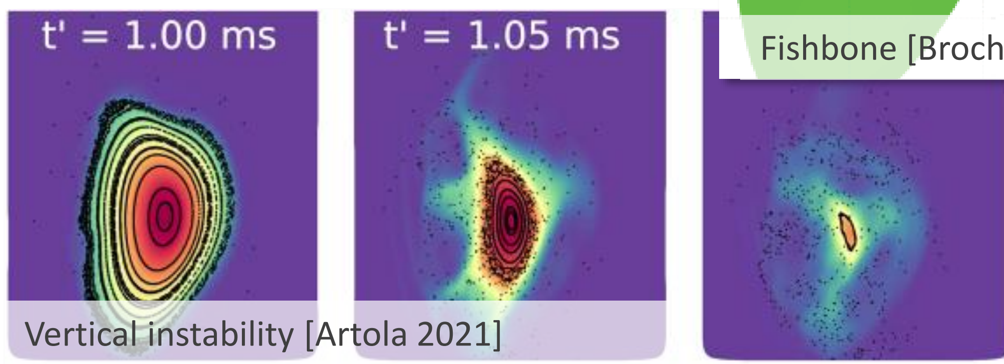
Fishbone [Brochard 2020]

- Works well for ideal global modes ($\omega\, \tau_A \sim 1$)

$$\tau_A = a/v_A \sim \beta^{1/2}\, a/v_{th}$$

JOREK

[Huysmans 2007; Czarny 2008; Hoelzl 2021]



Vertical instability [Artola 2021]

Numerical bottlenecks:

- Implicit schemes to filter out fast time scales
- Small resistivity ($\eta_{ITER} \sim 10^{-9}$) $\rightarrow$ small scales ($\sim \eta^{1/2}$)
- Nonlinear dynamics is important (e.g. crashes, …)

} Requires many modes

$\Downarrow$

Inversion of (very) large sparse matrices

Beyond MHD → two fluid effects

- Accounting for departures from MHD flow is essential for quantitative predictions → Ion flow $\mathbf{V}_i = \mathbf{V} + \mathbf{V}_* + \dots$

MHD flow $\mathbf{V} = \dfrac{\mathbf{E}\times\mathbf{B}}{B^2} + V_\parallel \mathbf{b}$ Diamagnetic flow $\mathbf{V}_* = \dfrac{\mathbf{B}\times\boldsymbol{\nabla} P_i}{enB^2}$

$$mn\mathrm{d}_t\mathbf{V}_i = -\boldsymbol{\nabla} P_i + en(\mathbf{E} + \mathbf{V}_i\times\mathbf{B}) - \boldsymbol{\nabla}.\boldsymbol{\Pi}_{gv}$$

- "Gyroviscous cancellation": gyroviscous stress tensor Π_{gv} cancels out with diamagnetic dynamic [Hinton-Horton '71; Hazeltine,'85]

$$mn\{\partial_t\mathbf{V}_* + [(\mathbf{V}+\mathbf{V}_*).\boldsymbol{\nabla}]\mathbf{V}_* + \mathbf{b}(\mathbf{V}_*.\boldsymbol{\nabla})V_\parallel\} = -\boldsymbol{\nabla}.\boldsymbol{\Pi}_{gv} - \boldsymbol{\nabla}\chi$$

Residual pressure $\chi = (P_i/2\omega_c)\,\mathbf{b}.\boldsymbol{\nabla}\times\mathbf{V}_{\perp i}$ ($\sim \rho_* P_i \ll P_i$) incorprated in P_{total}

- Modified density, momentum, … eqs

$$d_t\rho + \rho\nabla\cdot\mathbf{V} + \nabla\cdot(\rho\mathbf{V}_*) = 0$$

$$\rho d_t\mathbf{V} + \rho\,(\mathbf{V}_*\cdot\nabla)\,\mathbf{V}_\perp = -\nabla P + \mathbf{J}\times\mathbf{B}$$

OK in slab geometry
More complex in toroidal

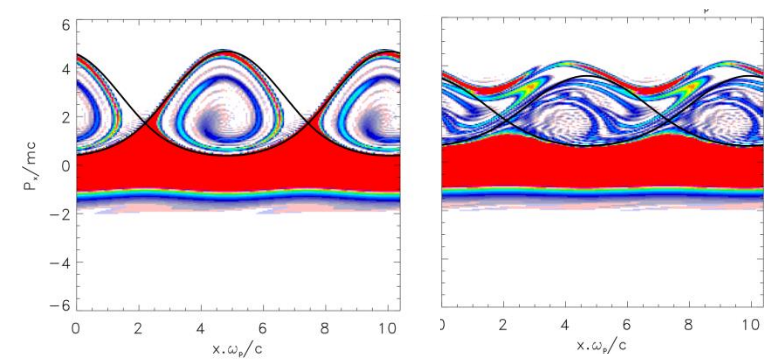
[Ramos,'05]

Limitations of fluid/MHD → Hybrid codes

- **Fast particles interact resonantly with (E,B) fields:**
  - modify MHD modes
  - drive Energetic Particle driven modes

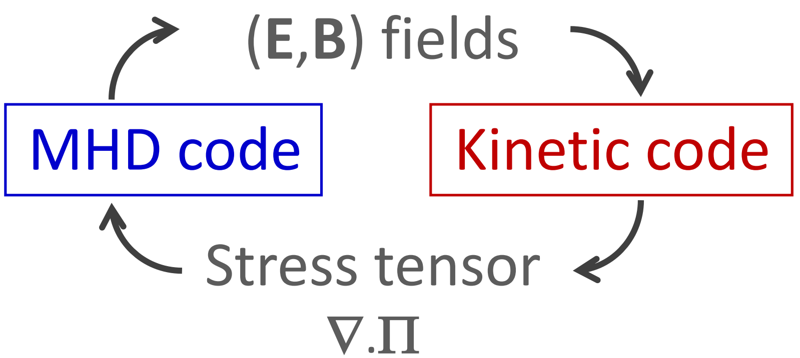
  ⇒ Track wave-particle resonances

  ⇒ kinetic description



- **Hybrid MHD codes:**
  - Thermal particles → MHD
  - Fast particles → (gyro)kinetic

  ⇒ Coupling via stress tensor



E.g. CGL expression of Π :

[Chew, Goldberger, Low '56]

F_h → pressures

$$\Pi_{hot} = P_{\|}\mathbf{bb} + P_{\perp}\left(\mathbf{I} - \mathbf{bb}\right)$$

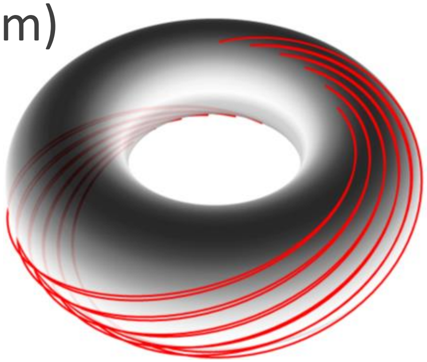
$$P_{\|} = \int d^3\mathbf{v} F_h m v_{\|}^2$$

$$P_{\perp} = \int d^3\mathbf{v} F_h \frac{1}{2} m v_{\perp}^2$$

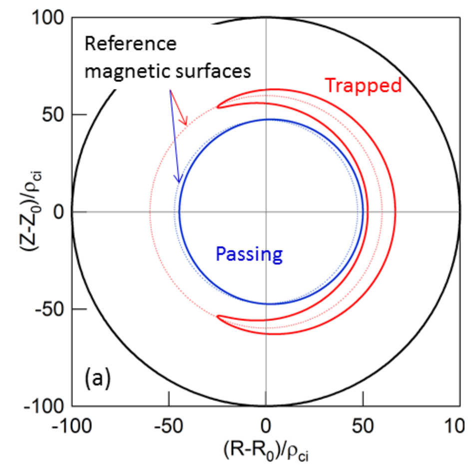
[Park '92; Briguglio '98; Lütjens '15]

Challenging electron-ion turbulence simu.

- Electrostatic turbulence mainly at ion scale (few $\rho_i \sim$ cm)



- Yet, kinetic electrons are mandatory:
  - Modify Ion Temperature Gradient (ITG) turbulence
  - **Trapped electrons** $\rightarrow$ Trapped Electron Modes
  - **Passing electrons** $\rightarrow$ ETG turb., Electromagnetic effects



- **Numerical issues**: $m_D/m_e \approx 3\,672$

  $\Rightarrow \rho_e \sim \rho_i/60$, $v_{th,e} \sim v_{th,i} \times 60 \Rightarrow$ **memory, CPU time**

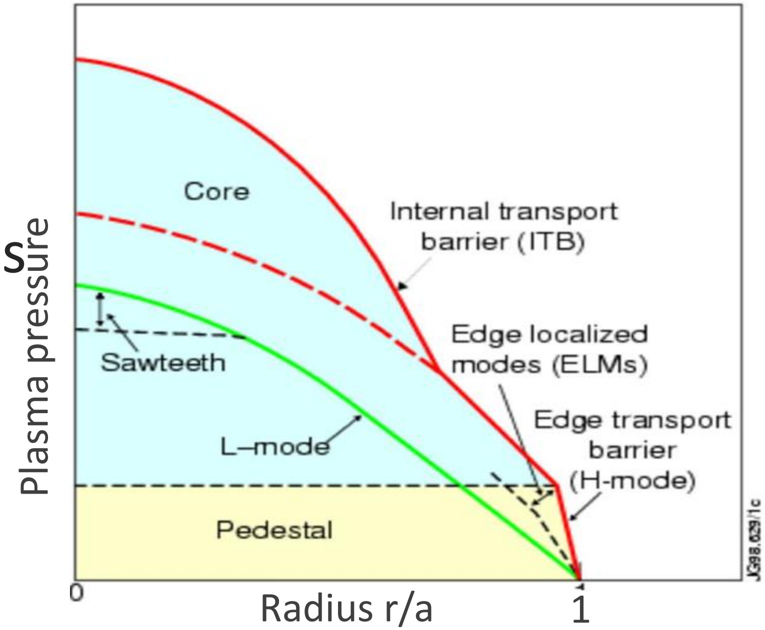
- **Staggered approach:** [Idomura 2016; Lanti 2020]
  - Trapped electrons $\rightarrow$ Kinetic
  - Passing electrons $\rightarrow$ Boltzmann response (or fluid)

Particle conservation

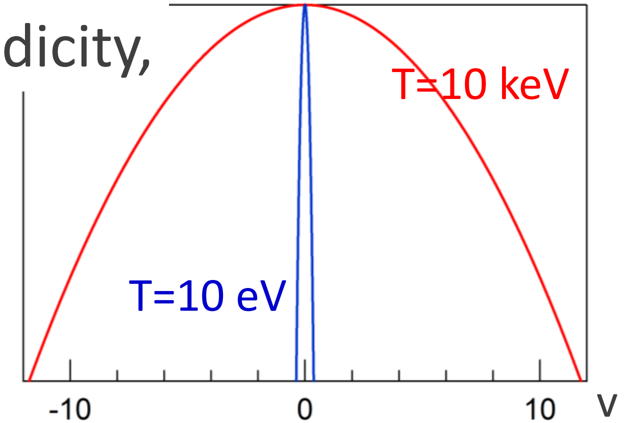
$$\underbrace{n_{e,trapped}}_{\int_{trapped} d^3v\, F_e} + \underbrace{n_{e,passing}}_{\propto f_p \exp(e\,\delta\phi/T_e)} = \underbrace{n_i}_{\int d^3v\, J[F_i] + n_{pol}}$$

Confinement: core-edge coupling

- In tokamaks, global confinement much dependent on edge plasma:
  - Edge transport barrier = spontaneous reduced turbulent transport
  - Critical role of sheared electric field $v_{E\theta} = -E_r/B \rightarrow$ vortex stretching (already in L-mode)

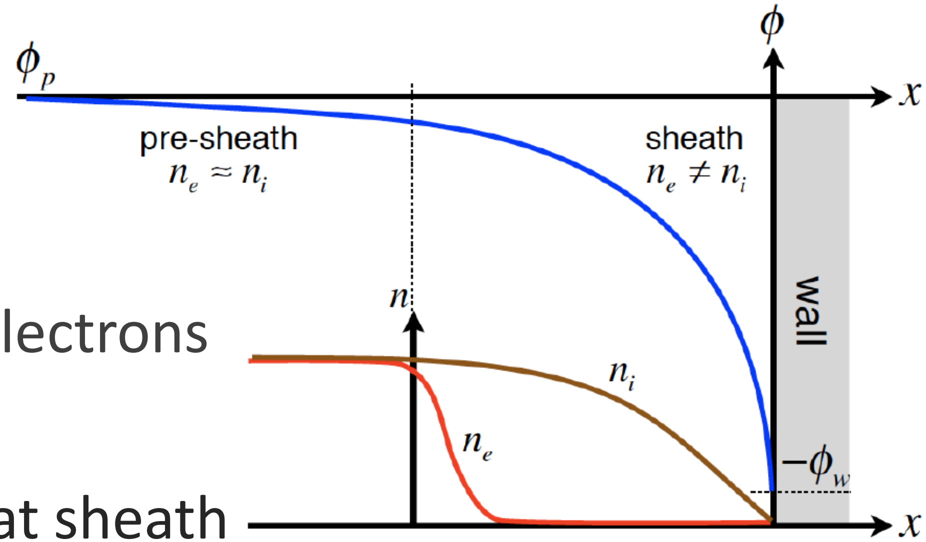


- Key challenges when addressing core & edge plasma transport:
  - Closed vs. open field lines $\Rightarrow$ loss of periodicity, plasma-wall interaction
  - Complex 3D geometry
  - Huge variations of temperature (10 keV $\rightarrow$ a few eV)



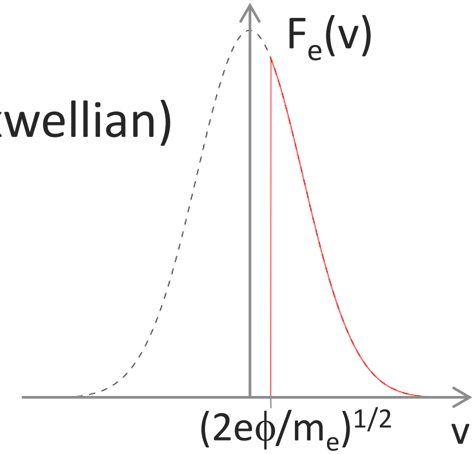
Physics of plasma-wall interaction = // BC

- **Mainly governed by // motion**: [Bohm 1949; …; Riemann 1991; Stangeby 2000; Ghendrih 2011]
  - Different electron & ion inertia
    - ⇒ Space electric charge @ wall
  - Large $E_{//}$ in the sheath
    - ⇒ Accelerates ions / confines electrons
    - ⇒ Vanishing current in the wall
  - Bohm (fluid) criterion: $v_i = \pm c_s$ at sheath entrance



[Martinez-Sanchez 2015]

- **Physical & numerical issues**:
  - Strongly distorted distribution functions (non-Maxwellian)
  - Broad range of scales: $\underset{\approx 100\text{m}}{L_{//} \approx 2\pi qR} >> \lambda_{sheath} \approx \underset{\approx 10^{-4}\text{m}}{\lambda_D}$
  - Kinetic Bohm criterion?
  - Important role of reflected ions



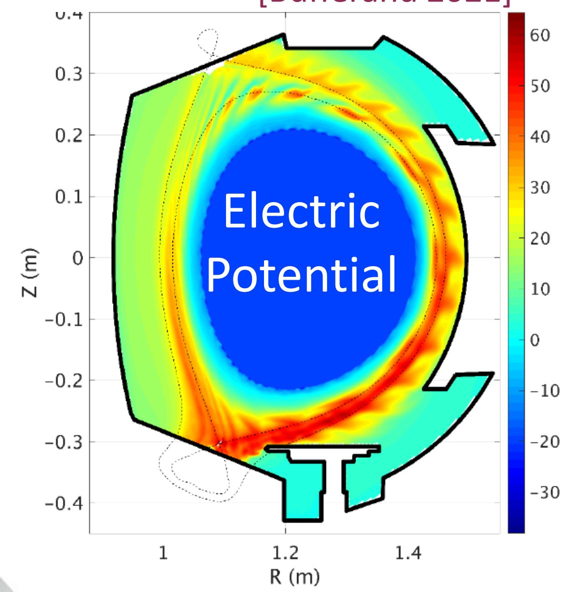
PWI using immersed boundary conditions

- **Immersed boundary via penalization technique:**
  - Additional Krook terms: $dF/dt = -\nu(F - F_{target})$
  - On-off mask (tanh) for wall / plasma
  - → Flexible, experience in fluid
  - → Unknown territory in kinetic

[Isoardi 2010; Paredes 2014]

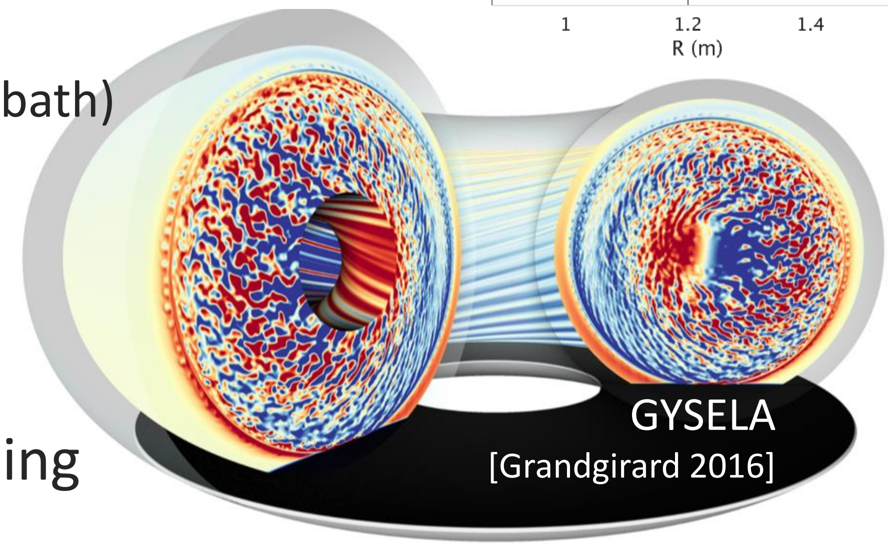
SOLEDGE3X-EIRENE

[Bufferand 2021]



- **Wall penalization:**
  - ∞ reservoir of particles
  - ∞ reservoir of energy (thermal bath)
  - ∞ conductor

- **Plasma in wall:**
  - ∞ / partial recombination
  - ϕ=0 grounded potential / biasing
  - ∞ momentum sink



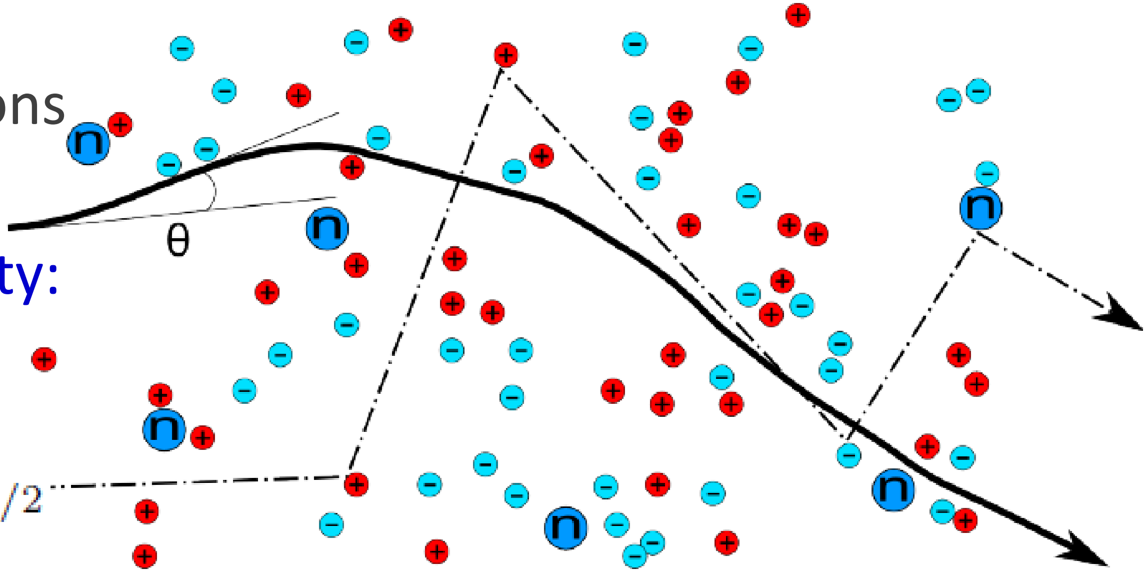
[Caschera 2018; Dif-Pradalier 2020]

Weak collisions… Yet essential

- **Dilute & hot plasma:**
  - Most Coulomb collisions are grazing
  - Weak core collisionality:

$$\nu_* = \nu_{coll}\tau_{\|} \ll 1$$

$$\nu_{coll} \sim nT^{-3/2} \qquad \tau_{\|} \sim L_{\|}T^{-1/2}$$



- **Critical issues governed by collisions** (NS vs. Euler):
  - Relaxation towards Maxwellian
  - Flow damping: **friction** on trapped particles (→ transition between turbulent regimes, …), **Zonal Flow** damping (→ turbulence saturation)
  - Impurity transport, especially high Z + possible synergy turb./coll.
  - Momentum & Energy exchanges between species

General form of collision operator

- **Landau formulation** (1936) of the Fokker-Planck operator
  (limit of small angle scattering events & cut-off at Debye shielding distances)

$$\frac{\partial f_a}{\partial t} = \sum_b \mathcal{C}_{ab}(f_a, f_b)$$

Coulomb logarithm: $\ln\Lambda = \ln\left(\frac{\lambda_D}{b_0}\right) \gg 1$

$$\mathcal{C}_{ab}(f_a, f_b) = \frac{\partial}{\partial \mathbf{v}} \cdot \left\{ \frac{2\pi e_a^2 e_b^2 \ln\Lambda}{m_a} \int \mathrm{d}^3\mathbf{v}' f_a(\mathbf{v}) f_b(\mathbf{v}') \frac{\mathbb{I}u^2 - \mathbf{u}\otimes\mathbf{u}}{u^3} \cdot \left(\frac{1}{m_a}\frac{\partial \ln f_a(\mathbf{v})}{\partial \mathbf{v}} - \frac{1}{m_b}\frac{\partial \ln f_b(\mathbf{v}')}{\partial \mathbf{v}'} \right) \right\}$$

$$\mathbf{u} = \mathbf{v} - \mathbf{v}'$$

  - Particle, momentum & energy **conservation** $\rightarrow$ exchanges between species
  - Satisfies Boltzmann **H-theorem** (1872): Entropy production & $C(f_{Maxwell})=0$

[Helander-Sigmar'05, Hirschman-Sigmar '76 '77 '81, Hinton-Hazeltine '76, Hinton '83, Rosenbluth-Hazeltine-Hinton '72, Abel '08, Barnes '09, Donnel '19]

Towards a nonlinear collision operator

- Usually **linearized**: $f_s = f_{M,s} + \delta f_s$ with $\delta f_s \ll f_{M,s}$

$$\mathcal{C}_{ab}(f_a, f_b) \approx \mathcal{C}_{ab}(f_{Ma}, f_{Mb}) + \mathcal{C}_{ab}(\delta f_a, f_{Mb}) + \mathcal{C}_{ab}(f_{Ma}, \delta f_b)$$

Test particle op. Field op.

- **Full operator** required at the edge (strong fluctuations, departure $F_{Maxwell}$)

$$\mathcal{C}_{ab}(f_a, f_b) = -\frac{4\pi e_a^2 e_b^2 \ln\Lambda}{m_a m_b}\frac{\partial}{\partial \mathbf{v}} \cdot \left(\frac{m_a}{m_b}\frac{\partial h_b}{\partial \mathbf{v}} f_a - \frac{1}{2}\frac{\partial^2 g_b}{\partial \mathbf{v}\partial \mathbf{v}} \cdot \frac{\partial f_a}{\partial \mathbf{v}} \right)$$

$$\nabla_v^2 h_b = -4\pi f_b \qquad \nabla_v^2 g_b = 2 h_b$$

Rosenbluth potentials (1957)

- **Physical** issues:
  - Conservation properties
  - Neoclassical results $df_a/dt = C_{ab} \rightarrow \chi_{neo}, V_{\theta,neo}, \Gamma_{Z,neo}, \ldots$
- **Numerical** issues:
  - Whatever mass ratio m_a/m_b and charge states e_s
  - To be adapted to actual choice of variables, e.g. $(v, v_\parallel/v)$, $(v_\parallel, \mu)$
  - Moderate CPU time $\rightarrow$ parallelization

Conclusions

- First principal simulations are crucial for ITER (pulse preparation, post-pulse analysis)
- At the limits of current HPC supercomputers (memory & CPU time)
- A hierarchy of complementary approaches

Reduced 1D models	**3D Fluid, MHD**	**5D Gyrokinetic**	**6D kinetic**
(D,V), QuasiLinear, K-ε, …		Eulerian, PIC/MC, SL	

- Numerical bottlenecks: coupling $\neq$ approaches, core $\rightarrow$ wall
- Validity limitation of physical models $\Rightarrow$ numerical issues
- Other issues: optimization of 3D magnetic configurations (stellarators), free boundary magnetic equil., …

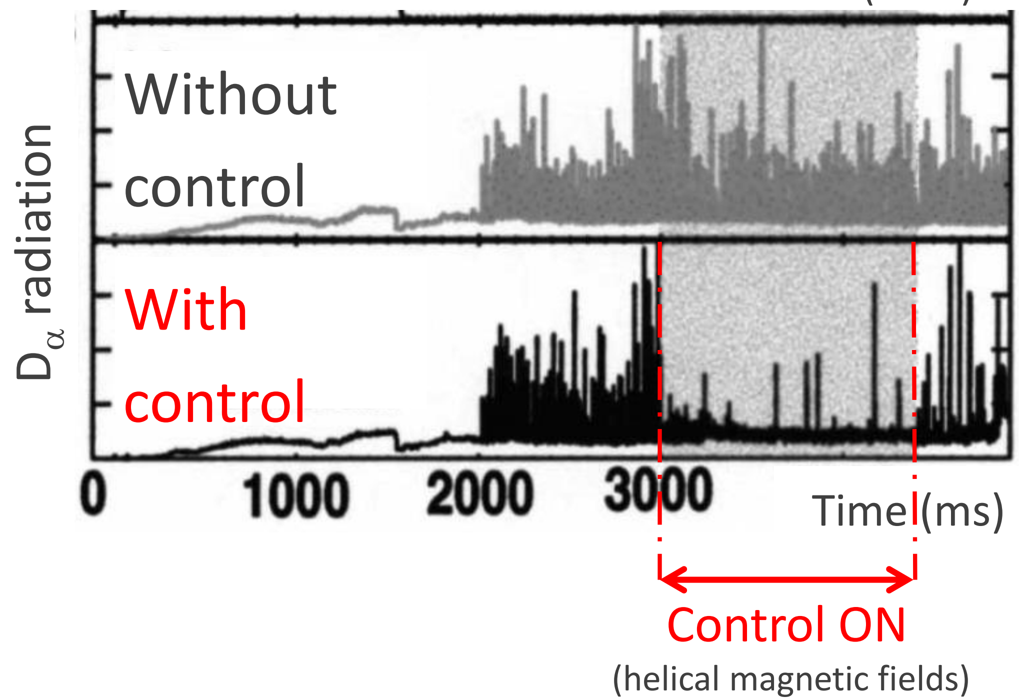
Inputs from Maths is essential!

Back-up slides

Controlling relaxation oscillations

Relaxation oscillations in JET (ELMs)

Evans et al. PRL (2004)



- Pressure / current exceeds instability threshold → fast relaxation → recovery
- Bursts of particle & heat fluxes
- Control is mandatory: via external coils or localized heating / current