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Justine Labat, Francis Collino, Agnès Pujols, Muriel Sesques

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Justine Labat, Francis Collino, Agnès Pujols, Muriel Sesques. Décomposition de domaine sur des formulations intégrales surfaciques en électromagnétisme. SMAI 2021 - Dixième biennale des mathématiques appliquées et industrielles, SMAI; IMT (Institut de Mathématiques de Toulouse), Jun 2021, La Grande-Motte, France. cea-03274733

**HAL Id: cea-03274733**

**<https://cea.hal.science/cea-03274733>**

Submitted on 30 Jun 2021

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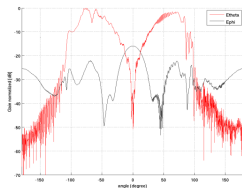
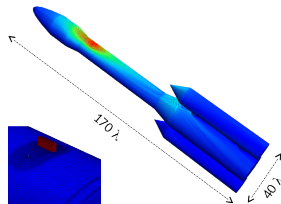
DE LA RECHERCHE À L'INDUSTRIE

## Décomposition de domaine sur des formulations intégrales surfaciques en électromagnétisme

Francis Collino   Justine Labat   Agnès Pujols   Muriel Sesques | CEA-CESTA

SMAI 2021

- Context : Solution of **electromagnetic** wave scattering problem by a **complex** object using a boundary element method
- Motivation : Accurate evaluation of **radar cross-sections**
  - ▶ Large-scale objects (in comparison with the wavelength)
  - ▶ Multi-scale phenomena

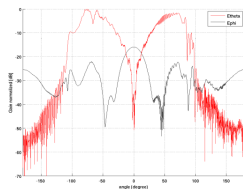
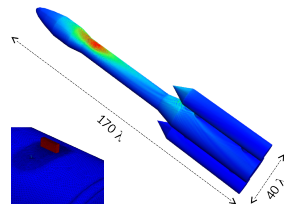


Example : Monopole antenna in the presence of a dielectric object on a launcher (ISAE Workshop, 2016)

- Context : Solution of **electromagnetic** wave scattering problem by a **complex** object using a boundary element method
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  - Large-scale objects (in comparison with the wavelength)
  - Multi-scale phenomena
- Difficulties : **High computational costs**
  - Number of degrees of freedom : 1 650 875
    - Approximately 100 000 000 in volume

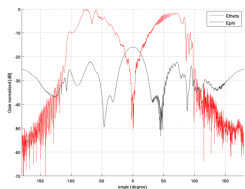
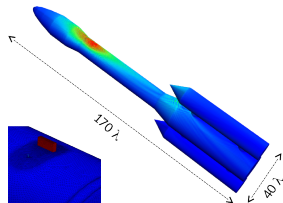
Solver	Number of CPU	Factorization (in CPU-H)
LL <sup>t</sup> (prediction)	15 000	151 435
Block low-rank	800	2 541
H-matrix	200	1 043

Hackbush (1999), Bebendorf (2008), . . .



Example : Monopole antenna in the presence of a dielectric object on a launcher (ISAE Workshop, 2016)

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  - ▶ Multi-scale phenomena
- Difficulties : **High computational costs**
- Objectives : Development of a **domain decomposition** method
  - ▶ Robust simulations for a **wide range** of frequencies
    - Solve very large problems (more than 10 000 000 of unknowns)
  - ▶ **Geometry-adaptative** strategies to handle multi-scale structures
    - Allow non-conformal meshes
  - ▶ Simulation-based engineering using **High Performance Computing**



Example : Monopole antenna in the presence of a dielectric object on a launcher (ISAE Workshop, 2016)

- Adaptation of volumical **discontinuous Galerkin** methods to surface problems

Peng, Hiptmair and Shao (2016), Messai and Pernet (2020)

	Volume	Surface	Comments
Locality	✓	✗	Iterative solution is inevitable
Restrictions	✓	✗	Restrictions of distributions are not easy to define
Traces	✓	✗	No trace theorem in $H_t^{-\frac{1}{2}}(\text{div}_\Gamma, \Gamma)$

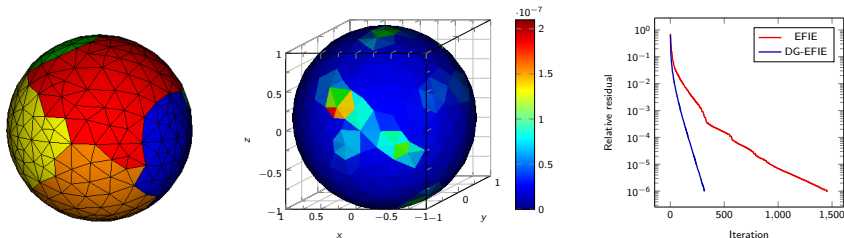
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- Why not to directly use an iterative solver on BEM ?

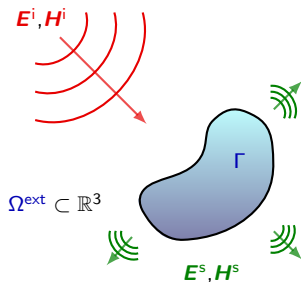
- ▶ Main motivation : **non-conformal** meshes
- ▶ EFIE simulation preconditioned using domain partitioning on a low-frequency case



- 1 Discontinuous Galerkin-based surface domain decomposition method
  - Model problem
  - Discontinuous formulation
  - Discrete formulation
  - Iterative procedure
  
- 2 Numerical results
  - Accuracy of solutions
  - Convergence of the iterative solver
  - Computational costs
  
- 3 Conclusion and perspectives



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- Using a time-harmonic dependence in  $\exp(+i\omega t)$

$$\nabla \times \mathbf{E}^s + i\kappa Z_0^{-1} \mathbf{H}^s = 0 \quad \text{in } \Omega^{\text{ext}}$$

$$\nabla \times \mathbf{H}^s - i\kappa Z_0 \mathbf{E}^s = 0 \quad \text{in } \Omega^{\text{ext}}$$

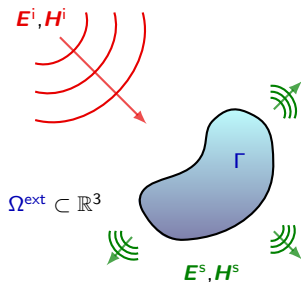
$$\mathbf{n} \times \mathbf{E}^s = -\mathbf{n} \times \mathbf{E}^i \quad \text{on } \Gamma$$

$$\lim_{|\mathbf{x}| \rightarrow \infty} |\mathbf{x}| (Z_0 \mathbf{H}^s \times \hat{\mathbf{x}} - \mathbf{E}^s) = 0 \quad \text{unif. in } \hat{\mathbf{x}} = \frac{\mathbf{x}}{|\mathbf{x}|}$$

$\kappa$  : wave-number,  $Z_0$  : impedance coefficient in vacuum

- $\Omega^{\text{ext}}$  : Exterior domain
- $\Gamma$  : Scattering surface
- $\mathbf{E}^i, \mathbf{H}^i$  : Incident fields
- $\mathbf{E}^s, \mathbf{H}^s$  : Scattered fields
- $\mathbf{E}, \mathbf{H}$  : Total fields

$$\mathbf{E} = \mathbf{E}^i + \mathbf{E}^s \quad \mathbf{H} = \mathbf{H}^i + \mathbf{H}^s$$



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- Using the Stratton-Chu formulas for  $\mathbf{x} \in \Omega^{\text{ext}}$

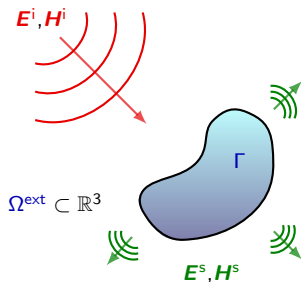
$$\mathbf{E}^s(\mathbf{x}) = -i\kappa \mathcal{T}\mathbf{J}(\mathbf{x}) \quad \text{and} \quad \mathbf{H}^s(\mathbf{x}) = \frac{1}{Z_0} \mathcal{K}\mathbf{J}(\mathbf{x})$$

$\mathbf{J} = Z_0(\mathbf{n} \times \mathbf{H})$  : total surface electric current

$$\mathcal{T}\mathbf{J} = \frac{1}{\kappa^2} \nabla (\mathcal{S} \text{div}_{\Gamma} \mathbf{J}) + \mathcal{S}\mathbf{J}$$

$$\mathcal{K}\mathbf{J} = \nabla \times \mathcal{S}\mathbf{J} \quad \tilde{\mathcal{S}}\lambda(\mathbf{x}) = \int_{\Gamma} G(\mathbf{x}, \mathbf{y}) \lambda(\mathbf{y}) \, ds_{\mathbf{y}}$$

$G$  : out-going Green function of the Helmholtz equation



- $\Omega^{\text{ext}}$  : Exterior domain
  - $\Gamma$  : Scattering surface
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- Jump relations give boundary integral equations

$$\text{(EFIE)} \quad i\kappa \mathcal{T} \mathbf{J} = \mathbf{n} \times (\mathbf{E}^i \times \mathbf{n}) \quad \text{on } \Gamma$$

$$\text{(MFIE)} \quad \frac{1}{2} \mathbf{J} - \mathcal{K} \mathbf{J} = Z_0(\mathbf{n} \times \mathbf{H}^i) \quad \text{on } \Gamma$$

$$\text{(EFIE)} \quad i\kappa T\mathbf{J} = \mathbf{n} \times (\mathbf{E}^i \times \mathbf{n}) \quad \text{on } \Gamma$$

$$\text{(MFIE)} \quad \frac{1}{2}\mathbf{J} - K\mathbf{J} = Z_0(\mathbf{n} \times \mathbf{H}^i) \quad \text{on } \Gamma$$

$$T : H_t^{-\frac{1}{2}}(\text{div}_\Gamma, \Gamma) \longrightarrow H_t^{-\frac{1}{2}}(\text{curl}_\Gamma, \Gamma)$$

$$T\mathbf{J} = \frac{1}{\kappa^2} \nabla_\Gamma (\mathbf{S} \text{div}_\Gamma \mathbf{J}) + \mathbf{S}\mathbf{J}$$

$$K : H_t^{-\frac{1}{2}}(\text{div}_\Gamma, \Gamma) \longrightarrow H_t^{-\frac{1}{2}}(\text{div}_\Gamma, \Gamma)$$

$$K\mathbf{J} = \text{p.v.} [\mathbf{n} \times (\nabla_\Gamma \times \mathbf{S}\mathbf{J})]$$

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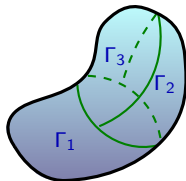
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- $N$  : number of subdomains

$$\bullet \Gamma = \bigcup_{n=1}^N \Gamma_n$$

$$\bullet \gamma_{nm} = \Gamma_n \cap \Gamma_m$$

$$\bullet \gamma_n = \partial\Gamma_n$$



- $\boldsymbol{\tau}_{nm}$  : exterior normal vector to  $\gamma_{nm}$
- $\boldsymbol{\tau}_n$  : exterior normal vector to  $\gamma_n$

$$i\kappa \sum_{m=1}^N T_{nm} \mathbf{J}_m = \mathbf{n} \times (\mathbf{E}^i \times \mathbf{n}) \quad \text{on } \Gamma_n$$

$$T_{nm} : \tilde{\mathbf{H}}_t^{-\frac{1}{2}}(\text{div}_{\Gamma_m}, \Gamma_m) \longrightarrow \tilde{\mathbf{H}}_t^{-\frac{1}{2}}(\text{curl}_{\Gamma_n}, \Gamma_n)$$

$$T_{nm} \mathbf{J}_m = \frac{1}{\kappa^2} \nabla_{\Gamma_n} (S_{nm} \text{div}_{\Gamma_m} \mathbf{J}_m) + S_{nm} \mathbf{J}_m$$

### 1 Restriction to each subdomain

$$\mathbf{J}_m = \mathbf{J}|_{\Gamma_m} \in \tilde{\mathbf{H}}_t^{-\frac{1}{2}}(\text{div}_{\Gamma_m}, \Gamma_m)$$

$$\frac{1}{2} \mathbf{J}_n - \sum_{m=1}^N K_{nm} \mathbf{J}_m = Z_0 (\mathbf{n} \times \mathbf{H}^i) \quad \text{on } \Gamma_n$$

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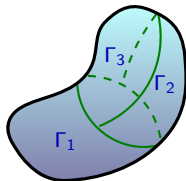
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$$\mathbf{J}_m = \mathbf{J}|_{\Gamma_m} \in \tilde{H}_t^{-\frac{1}{2}}(\text{div}_{\Gamma_m}, \Gamma_m)$$

### 2 Obtention of variational formulations

- ▶ Multiplication by a test-function
- ▶ Integration on partial surfaces
- ▶ Integration by parts (EFIE only)

$$\langle \nabla_{\Gamma} (S_{nm} \text{div}_{\Gamma} \mathbf{J}_m), \mathbf{v}_n \rangle_{\Gamma_n} =$$

$$- \langle S_{nm} \text{div}_{\Gamma} \mathbf{J}_m, \text{div}_{\Gamma} \mathbf{v}_n \rangle_{\Gamma_n} + \langle S_{nm} \text{div}_{\Gamma} \mathbf{J}_m, \mathbf{v}_n \cdot \boldsymbol{\tau}_n \rangle_{\gamma_n}$$

- ▶ which requires **more regularity** ( $\varepsilon > 0$ )

$$\text{div}_{\Gamma} \mathbf{J}_m \in H^{-\frac{1}{2}+\varepsilon}(\Gamma_m) \quad \text{and} \quad \mathbf{v}_n \in H_t^{\frac{1}{2}-\varepsilon}(\text{div}_{\Gamma}, \Gamma_n)$$

$$\frac{1}{2} \mathbf{J}_n - \sum_{m=1}^N K_{nm} \mathbf{J}_m = Z_0 (\mathbf{n} \times \mathbf{H}^i) \quad \text{on } \Gamma_n$$

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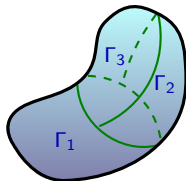
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### 2 Obtention of variational formulations

- ▶ Multiplication by a test-function
- ▶ Integration on partial surfaces
- ▶ Integration by parts (EFIE only)

### 3 Summation over all the subdomains

$$\sum_{n=1}^N \langle S_{nm} \text{div}_{\Gamma} \mathbf{J}_m, \mathbf{v}_n \cdot \boldsymbol{\tau}_n \rangle_{\gamma_n} =$$

$$\sum_{\gamma_{nm}} \langle S_{nm} \text{div}_{\Gamma} \mathbf{J}_m, [\mathbf{v}]_{\gamma_{nm}} \rangle_{\gamma_{nm}}$$

$$[\mathbf{v}]_{\gamma_{nm}} = \boldsymbol{\tau}_{nm} \cdot \mathbf{v}_m + \boldsymbol{\tau}_{mn} \cdot \mathbf{v}_n : \text{jump across } \gamma_{nm}$$

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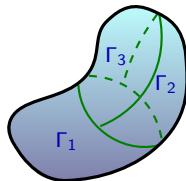
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- $\boldsymbol{\tau}_{nm}$  : exterior normal vector to  $\gamma_{nm}$

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Find  $\mathbf{J} \in \bigoplus_{m=1}^N \mathbf{H}_t^{-\frac{1}{2}+\varepsilon}(\text{div}_{\Gamma_m}, \Gamma_m)$  with  $\varepsilon > 0$  such that

$$a_{\Gamma}(\mathbf{J}, \mathbf{v}) + a_{\gamma}^{\pm}(\mathbf{J}, \mathbf{v}) + p_{\gamma}^*(\mathbf{J}, \mathbf{v}) = \ell(\mathbf{v}) \quad \text{for any } \mathbf{v} \in \bigoplus_{n=1}^N \mathbf{H}_t^{\frac{1}{2}-\varepsilon}(\text{div}_{\Gamma_n}, \Gamma_n)$$

$$a_{\Gamma}(\mathbf{J}, \mathbf{v}) = \sum_{n=1}^N \sum_{m=1}^N \left\{ \frac{1}{i\kappa} \langle S_{nm} \text{div}_{\Gamma_m} \mathbf{J}_m, \text{div}_{\Gamma_n} \mathbf{v}_n \rangle_{\Gamma_n} + i\kappa \langle S_{nm} \mathbf{J}_m, \mathbf{v}_n \rangle_{\Gamma_n} \right\} \quad (\text{continuous EFIE})$$

$$a_{\gamma}^{\pm}(\mathbf{J}, \mathbf{v}) = -\frac{1}{i\kappa} \sum_{m=1}^N \sum_{\gamma_{nm}} \langle S_{nm} \text{div}_{\Gamma_m} \mathbf{J}_m, [\mathbf{v}]_{\gamma_{nm}} \rangle_{\gamma_{nm}} + \tilde{a}_{\gamma}^{\pm}(\mathbf{J}, \mathbf{v}) \quad (\text{symmetrization})$$

$$p_{\gamma}^*(\mathbf{J}, \mathbf{v}) \text{ is function of } [\mathbf{J}]_{\gamma_{nm}} \text{ and } [\mathbf{v}]_{\gamma_{nm}} \quad (\text{penalization})$$

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$p_{\gamma}^*(\mathbf{J}, \mathbf{v})$  is function of  $[\mathbf{J}]_{\gamma_{nm}}$  and  $[\mathbf{v}]_{\gamma_{nm}}$  (penalization)

$$(S) \quad \tilde{a}_{\gamma}^{-}(\mathbf{J}, \mathbf{v}) = -\frac{1}{i\kappa} \sum_{n=1}^N \sum_{\gamma_{mn}} \langle S_{mn} \text{div}_{\Gamma_n} \mathbf{v}_n, [\mathbf{J}]_{\gamma_{mn}} \rangle_{\gamma_{mn}}$$

$$(AS) \quad \tilde{a}_{\gamma}^{+}(\mathbf{J}, \mathbf{v}) = +\frac{1}{i\kappa} \sum_{n=1}^N \sum_{\gamma_{mn}} \langle S_{mn} \text{div}_{\Gamma_n} \mathbf{v}_n, [\mathbf{J}]_{\gamma_{mn}} \rangle_{\gamma_{mn}}$$

► Requires more regularity :  $\varepsilon = \frac{1}{2}$

$$\mathbf{J} \text{ and } \mathbf{v} \in \bigoplus_{n=1}^N \mathbf{L}_t^2(\text{div}_{\Gamma_n}, \Gamma_n)$$

Find  $\mathbf{J} \in \bigoplus_{m=1}^N L_t^2(\text{div}_{\Gamma_m}, \Gamma_m)$  such that

$$a_{\Gamma}(\mathbf{J}, \mathbf{v}) + a_{\gamma}^*(\mathbf{J}, \mathbf{v}) + p_{\gamma}^*(\mathbf{J}, \mathbf{v}) = \ell(\mathbf{v}) \quad \text{for any } \mathbf{v} \in \bigoplus_{n=1}^N L_t^2(\text{div}_{\Gamma_n}, \Gamma_n)$$

- Empirical choice coming from Peng, Hiptmair and Shao (2016) :  $L^2(\gamma)$ -inner product

$$(L^2) \quad p_{\gamma}^0(\mathbf{J}, \mathbf{v}) = \frac{\beta}{\kappa} \sum_{\gamma_{nm}} \langle [\mathbf{J}]_{\gamma_{nm}}, [\mathbf{v}]_{\gamma_{nm}} \rangle_{\gamma_{nm}}$$

where  $\beta$  have to be without dimension

- ▶ requires **more regularity** (one more time!) to make sense :  $\mathbf{J}$  and  $\mathbf{v} \in \bigoplus_{n=1}^N H_t^{\frac{1}{2}}(\text{div}_{\Gamma_n}, \Gamma_n)$

- A new penalization :  $H^{-\frac{1}{2}}(\gamma)$ -inner product (positive definite bilinear form)

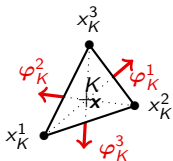
$$(H^{-\frac{1}{2}}) \quad p_{\gamma}^{-\frac{1}{2}}(\mathbf{J}, \mathbf{v}) = \beta \sum_{\gamma_{nm}} \langle S_{\gamma_{nm}}[\mathbf{J}]_{\gamma_{nm}}, [\mathbf{v}]_{\gamma_{nm}} \rangle_{\gamma_{nm}}$$

where  $\tilde{S}_{\gamma} \lambda(\mathbf{x}) = \frac{1}{2\pi} \int_{\gamma} K_0(\kappa|\mathbf{x} - \mathbf{y}|) \lambda(\mathbf{y}) d\sigma_{\mathbf{y}}$  and  $\beta$  have to be without dimension

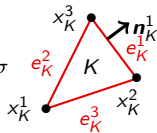
- Discretization space : Restrictions of Raviart-Thomas boundary elements of smallest degree

$$\mathbf{J} \approx \mathbf{J}^h \in \mathcal{V}^h \quad \Longleftrightarrow \quad \mathbf{J}^h = \sum_{m=1}^N \mathbf{J}_m^h \quad \mathbf{J}_m^h \in \mathcal{V}_m^h \subset L_t^2(\operatorname{div}_{\Gamma_m}, \Gamma_m)$$

where  $\mathbf{J}_m^h(\mathbf{x}) = \sum_{K \in \Gamma_m^h} J_{K,1}^{h,m} \varphi_K^1(\mathbf{x}) + J_{K,2}^{h,m} \varphi_K^2(\mathbf{x}) + J_{K,3}^{h,m} \varphi_K^3(\mathbf{x}) \quad \mathbf{x} \in \Gamma_m^h$  (triangulation of  $\Gamma_m$ )



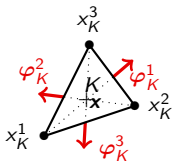
with  $\varphi_K^i(\mathbf{x}) = \frac{1}{2|K|} (\mathbf{x} - x_K^i)$  and  $J_{K,i}^{h,m} = \int_{e_K^i} \mathbf{J}_m^h \cdot \mathbf{n}_K^i d\sigma$



- Discretization space : Restrictions of Raviart-Thomas boundary elements of smallest degree

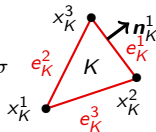
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Find  $\mathbf{J}^h \in \mathcal{V}^h$  such that for any  $\mathbf{v}^h \in \mathcal{V}^h$

(DG-EFIE)  $a_{\Gamma}^h(\mathbf{J}^h, \mathbf{v}^h) + a_{\gamma}^{*,h}(\mathbf{J}^h, \mathbf{v}^h) + p_{\gamma}^{*,h}(\mathbf{J}^h, \mathbf{v}^h) = \ell^h(\mathbf{v}^h)$

(DG-MFIE)  $b_{\Gamma}^h(\mathbf{J}^h, \mathbf{v}^h) = \tilde{\ell}^h(\mathbf{v}^h)$

(from continuous MFIE)

(DG-CFIE)  $\alpha$  (DG-EFIE) +  $(1 - \alpha)$  (DG-MFIE)

(with  $0 < \alpha < 1$ )

- Use of **GMRes solver** (from CERFACS) with **block-diagonal Jacobi preconditioning**
  - ▶ Diagonal blocks correspond to matrices for individual subdomain
    - Flexibility in choosing subdomain solvers ( $LL^t$ , LU, Block low-rank, H-matrix, . . .)
  - ▶ Off-diagonal blocks correspond to interactions between subdomains
    - Could be sped-up by compression techniques

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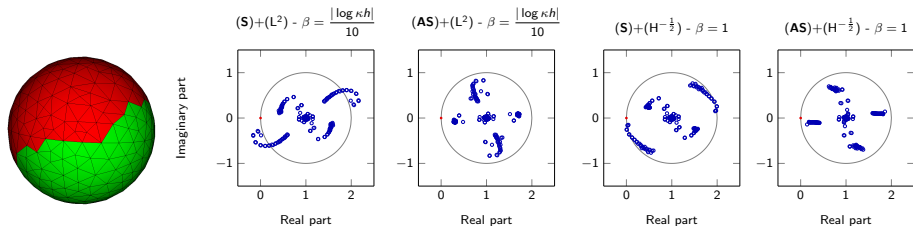
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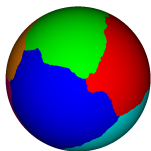
- Eigenvalue distribution : (**DG-CFIE**) simulation at 95 MHz with CFIE parameter  $\alpha = 0.5$



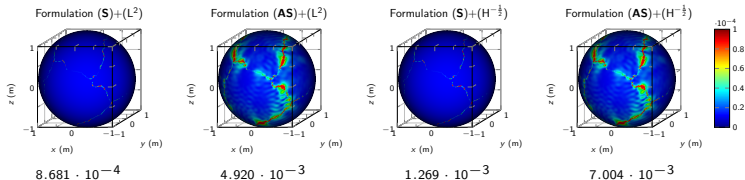


- 1 Discontinuous Galerkin-based surface domain decomposition method
  - Model problem
  - Discontinuous formulation
  - Discrete formulation
  - Iterative procedure
  
- 2 Numerical results
  - Accuracy of solutions
  - Convergence of the iterative solver
  - Computational costs
  
- 3 Conclusion and perspectives

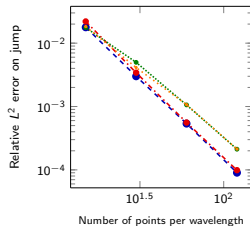
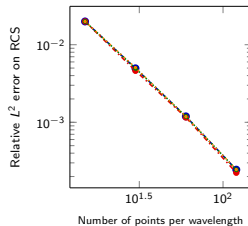
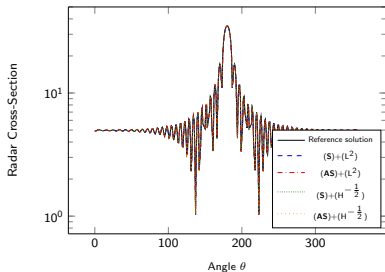
- Difference modulus between electric currents : **(DG-CFIE)** at 1.52 GHz -  $h = \frac{\lambda}{15}$



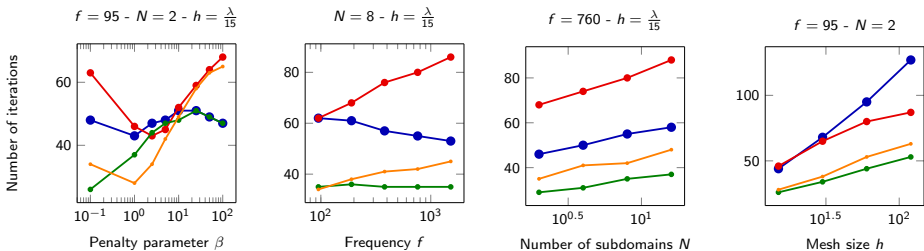
Maximal errors :



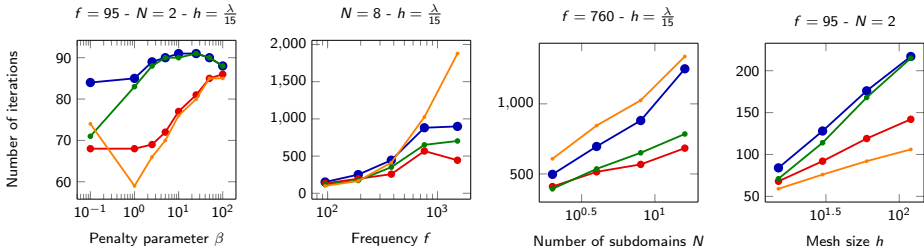
- Comparison of associated radar cross-sections and convergence (95 MHz - 2 subdomains)

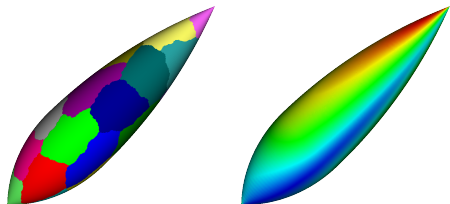


- (DG-CFIE) with  $\alpha = 0.5$     • (S)+(L<sup>2</sup>)    • (AS)+(L<sup>2</sup>)    • (S)+(H<sup>-1/2</sup>)    • (AS)+(H<sup>-1/2</sup>)



- (DG-EFIE)





- Dimensions :  $2\lambda \times 2\lambda \times 6\lambda$
- Number of degrees of freedom : 273 312
- Number of subdomains : 24
- (DG-CFIE) simulation with  $\alpha = 0.9$

- Comparison between CFIE+LU, CFIE+GMRes and DG-CFIE+GMRes

Formulation	Solver	Number of CPU	Factorization (in CPU·H)	Memory (in GB/MPI process)
CFIE	LU	3 328	5 018	11.43
DG-CFIE	GMRes	2 560	45	18.81

Formulation	Solver	Relative residual	Number of iterations	Convergence (in CPU·H)
CFIE	GMRes	$10^{-3}$	202	5 697
DG-CFIE	GMRes	$10^{-3}$	49	1 382
DG-CFIE	GMRes	$10^{-6}$	149	4 203

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## Conclusion

- Development and numerical analysis of a discontinuous Galerkin surface domain decomposition method for electromagnetic scattering by non-penetrable objects
  - ▶ Comparison of **symmetric** and **anti-symmetric** formulations
    - Symmetric formulation involves **better accuracy** on currents and **preserves symmetry in EFIE**
    - Anti-symmetric formulation involves **better conditioning number** in CFIE formulation
  - ▶ Comparison of  $L^2$  and  $H^{-\frac{1}{2}}$  interior penalty terms
    - $L^2$  penalization is robust with respect to the **frequency** but **parameter  $\beta$  have to be calibrated**
    - $H^{-\frac{1}{2}}$  penalization is more robust with respect to the **discretization size**
- Comparison with a boundary element method
  - ▶ Computational costs
    - Using a direct solver : **large gain** in factorization time
    - Using a preconditioned iterative solver : **faster convergence** for EFIE formulation
  - ▶ Accuracy : Jump **does not pollute** radar cross-sections

## Perspectives

- Integration of **hp-refinement**
  - ▶ **Non-conformal** meshes
  - ▶ **High-order** boundary elements
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- Extension to **dielectric** scatterers

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Thank you for your attention !