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Challenges in MHD modelling in fusion devices

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Challenges in MHD modelling in fusion devices

Guido Huijsmans

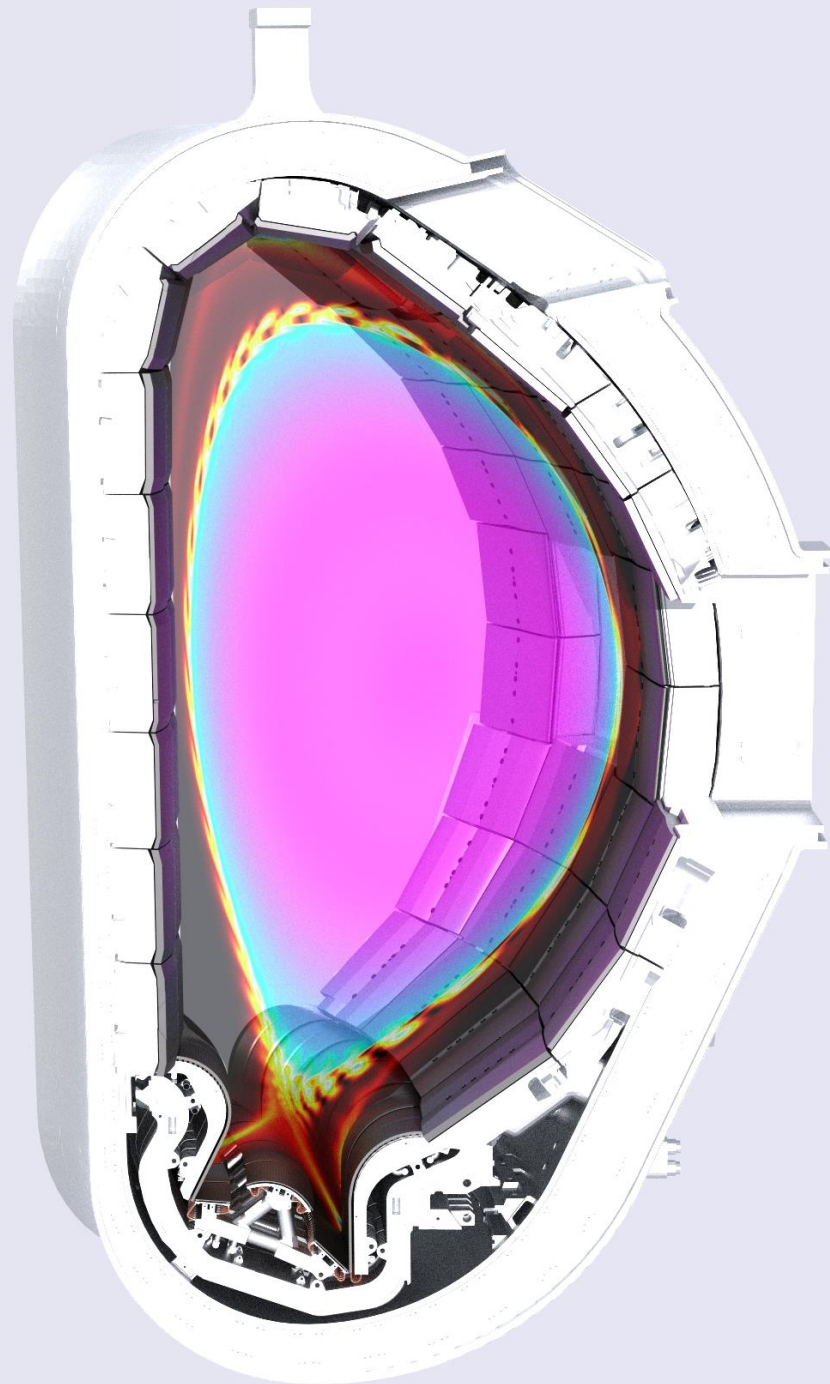


European Commission



IRFM, CEA Cadarache

TU/e *Eindhoven University of
Technology*



Outline

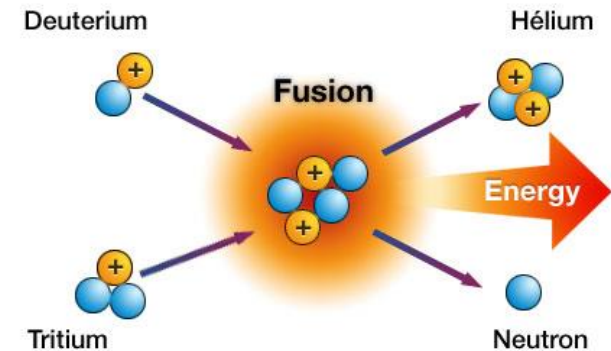
- Fusion
 - Tokamak, ITER
- Magnetohydrodynamic (MHD) Instabilities
 - Edge localised modes (ELMs)
- Nonlinear MHD code JOREK
 - finite elements
 - implicit time evolution
 - parallelisation
- Particles
 - neutrals, impurity transport
- Conclusion

Fusion Energy

- The fusion of light atoms release energy

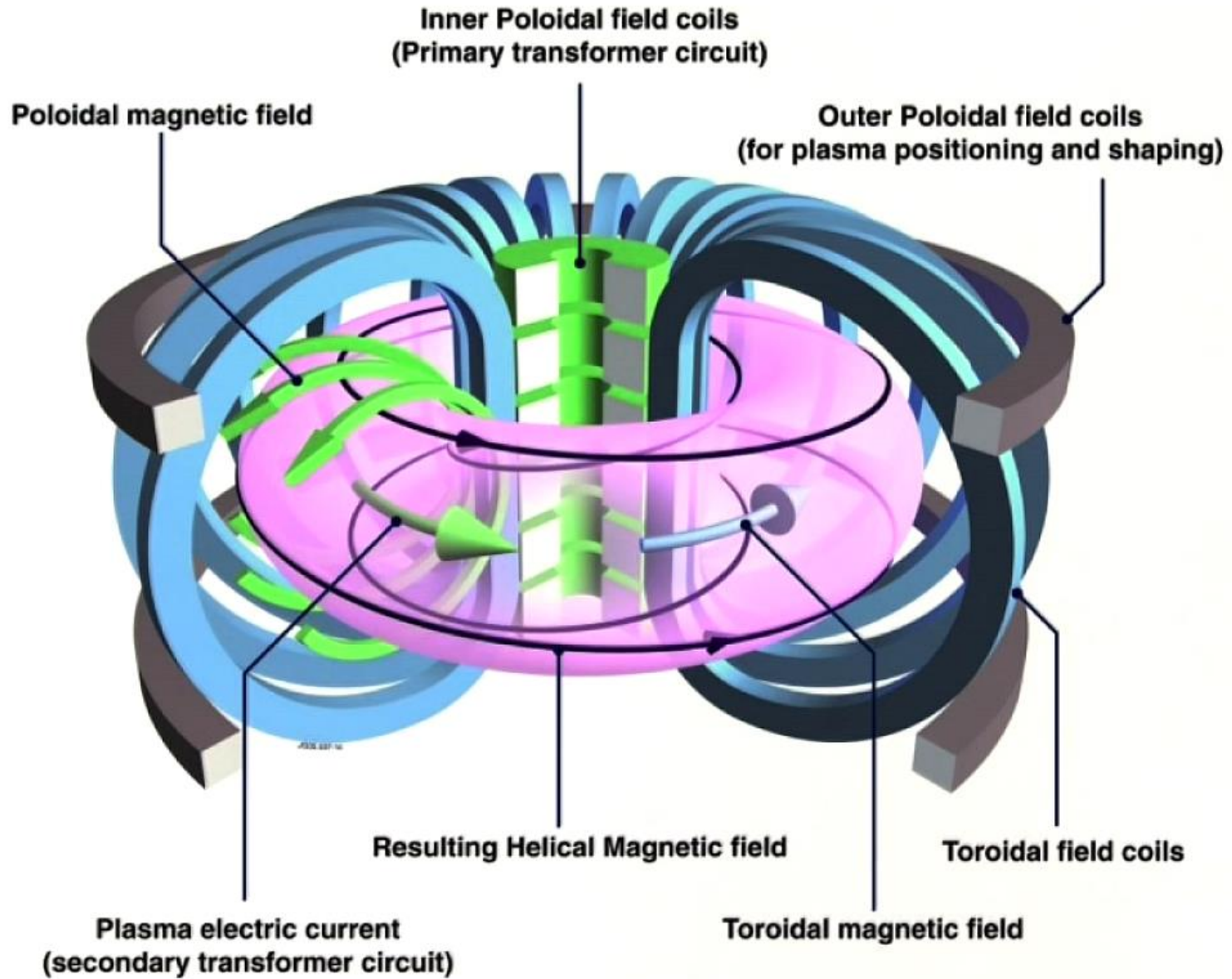


- energy source of the stars
- potential CO₂ free, safe energy source



- The reaction requires very high temperatures (150 M°)
 - gas is ionised and becomes plasma (free ions and electrons)
- The hot plasma can be contained by a magnetic field
 - tokamak is the most advanced approach
 - fusion power record in JET : 16 MW (in 1997)
 - next step: ITER

Tokamak



The ITER Project

- Goals:

- Fusion power amplification factor $Q=10$ for 300-500s
 - Fusion power ~500 MW
- Steady state operation at $Q=5$ (3000s)

7 ITER Partners

EU, China, India, Japan,
Korea, Russia, USA

ITER Tokamak

Major radius : 6.2m

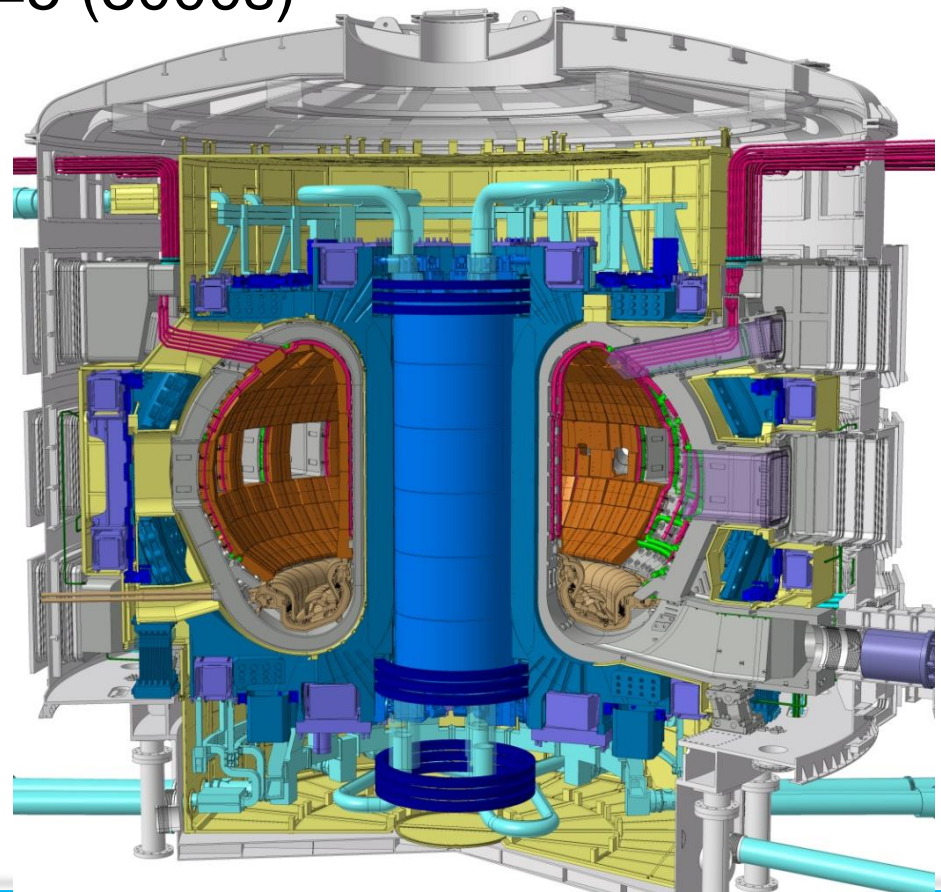
Minor radius : 2.0m

Toroidal field : 5.3T

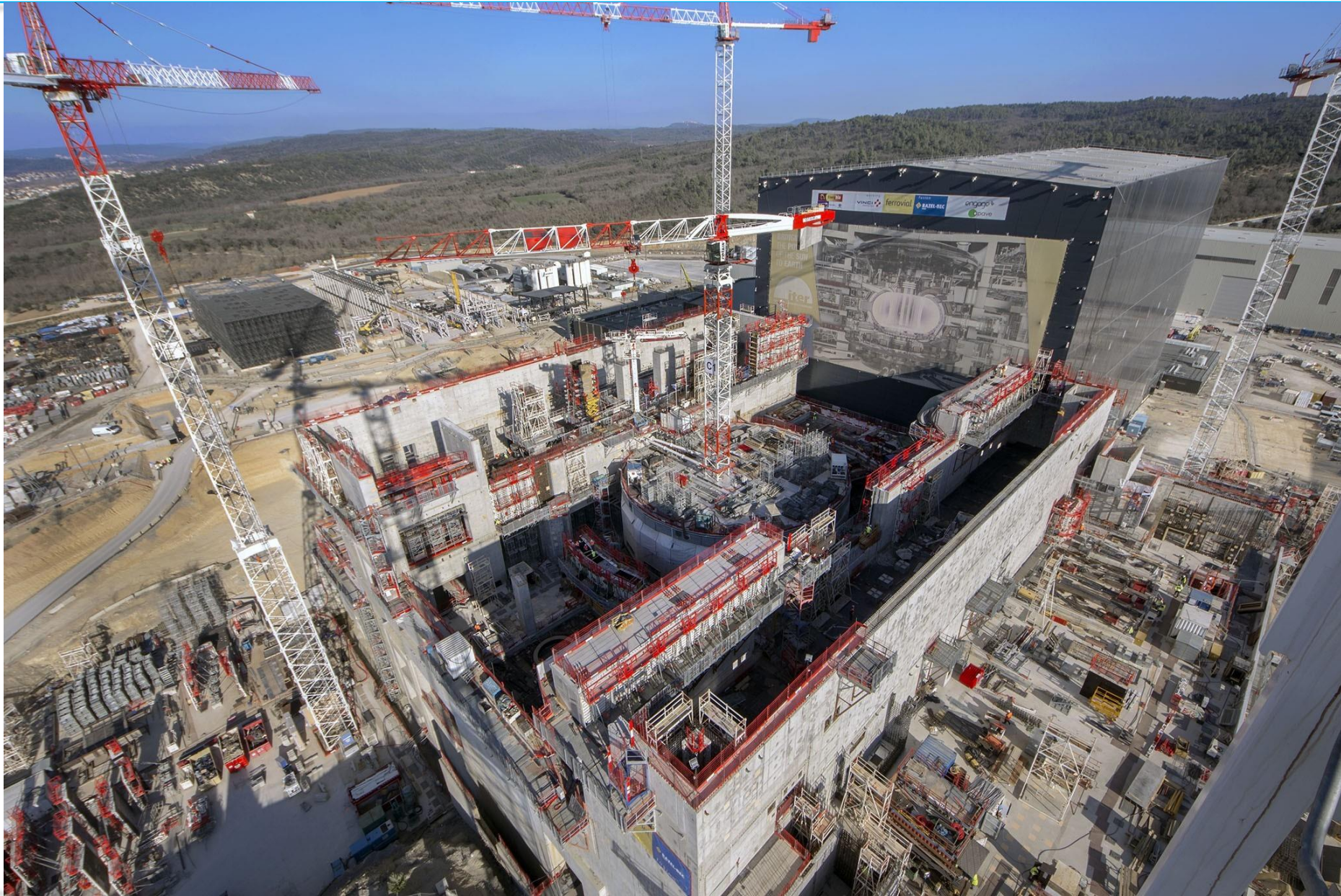
Plasma current : 15 MA

First plasma : end 2025

Fusion power operation: end 2035



ITER Building Site (march 2019)

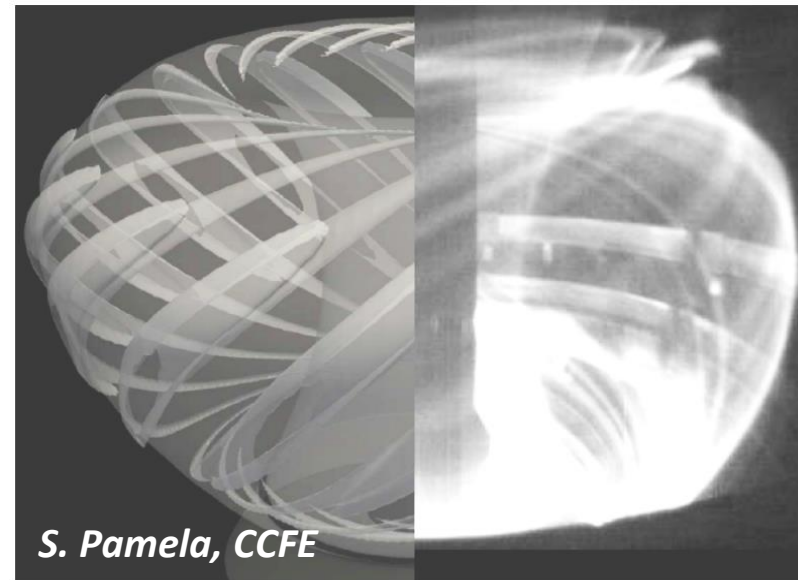


MHD Instabilities

- Large scale magnetic instabilities of the plasma
 - Driven unstable by pressure gradients and plasma current
 - Limit global pressure, current and local pressure, current gradients
 - Typical time scales : 1 millisecond
 - Leading to fast losses of plasma energy and density
 - Large transitory energy fluxes to the plasma facing components
- Disruptions:
 - Global MHD instability leads to fast plasma termination
- Edge Localised Modes (ELMs):
 - Local MHD instability in outer 20% of the plasma leading to repetitive energy and density losses of 1-10% (at 1-100Hz)

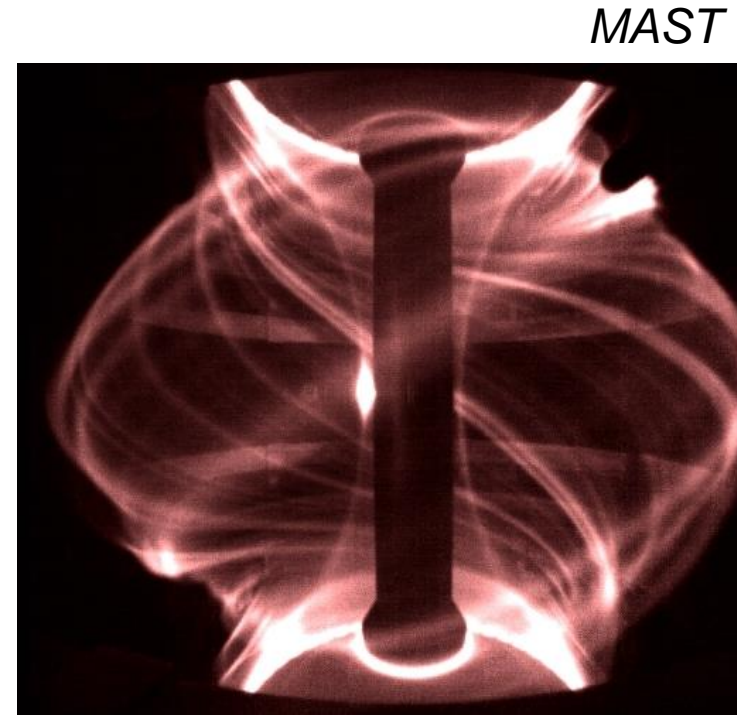
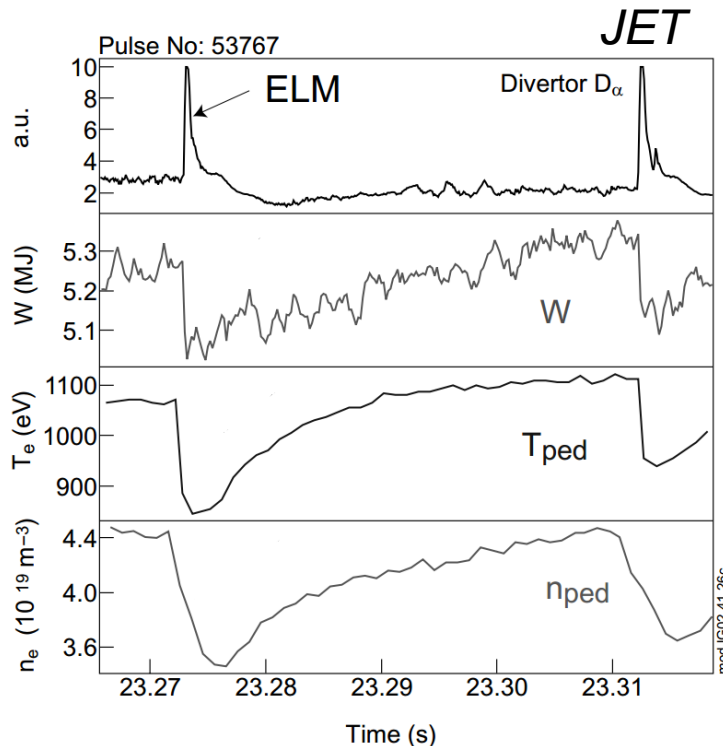
JOREK simulation

MAST



Edge Localised Modes (ELMs)

- H-mode regime: spontaneous stabilisation of turbulence at the plasma edge leads to large pressure gradients in the outer ~5% of the plasma
 - standard operating regime in ITER
- Edge pressure gradient is not limited by transport but by an MHD instability
 - Edge Localised Mode (ELM) removes up to 10% of the plasma energy in several hundred microseconds



[Loarte, PPCF2002]

[Kirk, MAST]

Magneto-Hydro-Dynamics (MHD)

- Theoretical model (H. Alfven, 1942)
 - description of the plasma as a electrically conducting fluid embedded in a time varying magnetic field
 - Combining Navier-Stokes fluid equations with (pre-) Maxwell's equations
 - Conservation of mass, momentum, energy and magnetic flux
 - applications: plasmas, solar physics, astrophysics, dynamos

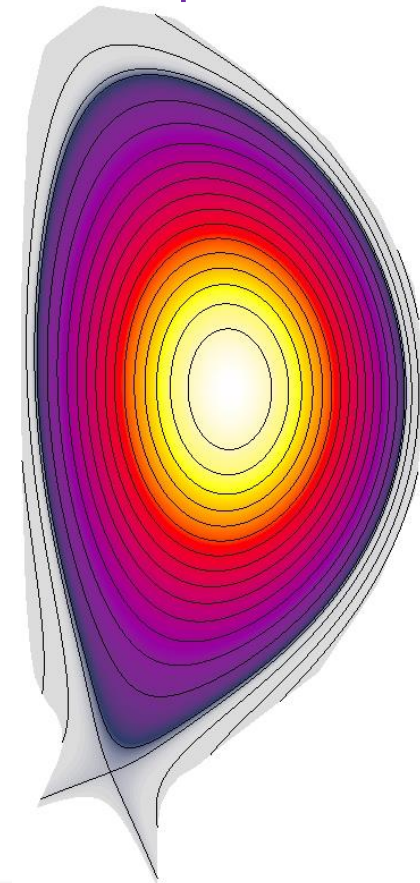
$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{v}) + \nabla \cdot D \nabla \rho + S_\rho$$

$$\rho \frac{\partial \vec{v}}{\partial t} = -\rho \vec{v} \cdot \nabla \vec{v} - \nabla (\rho T) + \vec{J} \times \vec{B}$$

$$\frac{\partial \rho T}{\partial t} = -\vec{v} \cdot \nabla \rho T - \gamma \rho T \nabla \cdot \vec{v} + \nabla \cdot \kappa \nabla T + S_p$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B} - \eta \vec{J})$$

$$\mu_0 \vec{J} = \nabla \times \vec{B}, \quad \nabla \cdot \vec{B} = 0$$



Challenges MHD Simulations

- Time scales:
 - Depending on the application the time scale of interest varies from 1 microsecond to 100's milliseconds
 - Fast waves : 10^{-8} s, Alfvén waves : 10^{-6} s, sound waves : 10^{-4} s
- Length scales:
 - MHD instabilities develop thin current sheets
 - Magnetic Reynolds number $S \sim 10^8 - 10^{10}$ (i.e. normalised resistivity $^{-1}$)
 - Layer width scales as $S^{-1/2}$
- Anisotropy
 - heat conduction is very much larger along a magnetic field line compared to the perpendicular direction
 - Conduction : $K_{//} / K_{\perp} \sim 10^{10}$

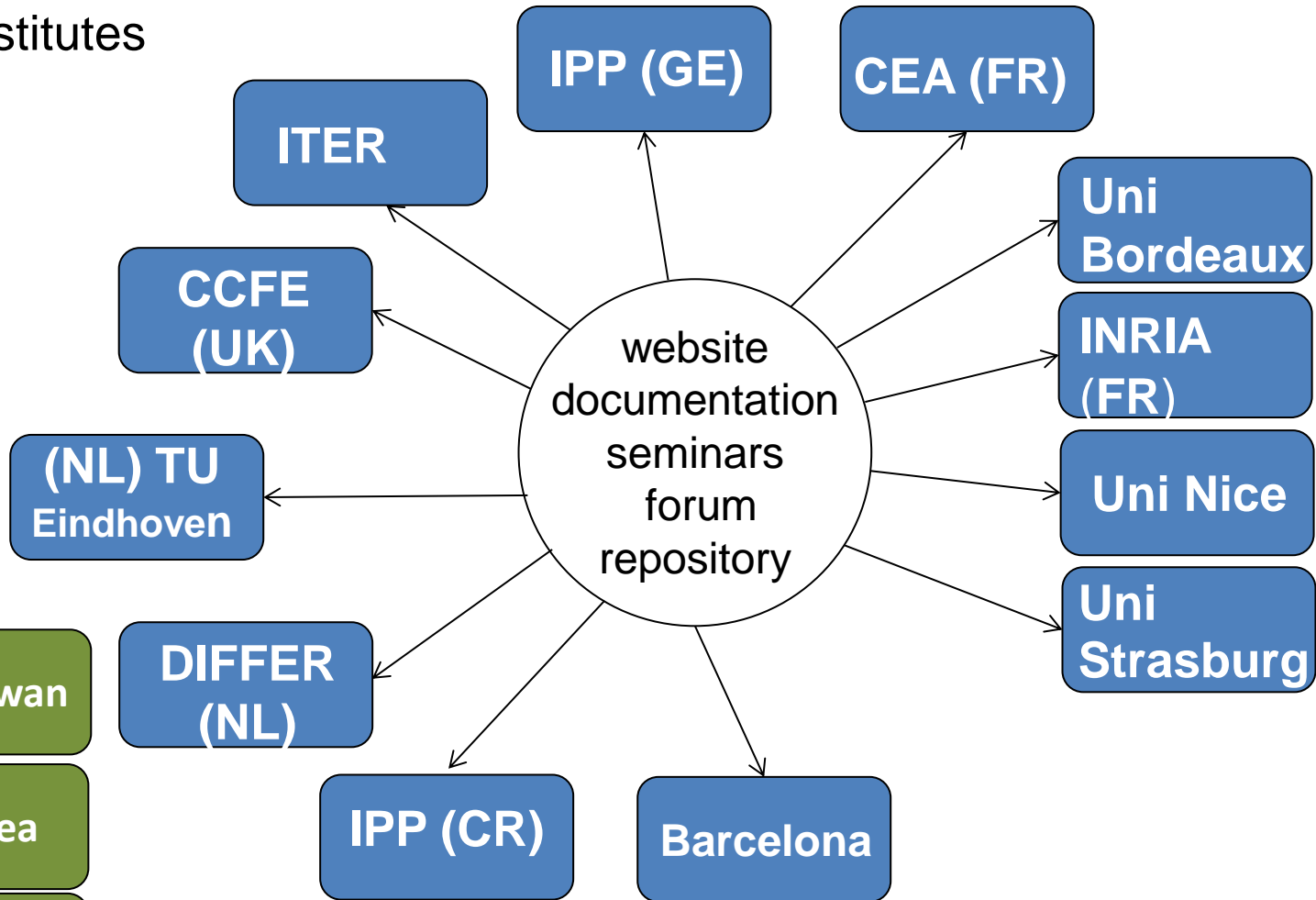
Non-linear MHD code JOREK

- JOREK code solves the (extended) MHD equations in 3D general tokamak geometry
 - Closed and open field lines, plasma wall interaction, conducting structures
 - Main applications: ELMs and disruptions
- now developed with the EU fusion program
 - 14 Fusion Labs + Universities, ~25 persons
 - GIT repository, wiki, decentralised development
- Characteristics
 - Cubic (Bezier) Finite element representation in the poloidal plane
 - Extension of cubic Hermite elements
 - Fourier series in the toroidal direction
 - Fully implicit time evolution
 - MPI/OpenMP parallelisation

JOREK Network (2019)

- Decentralised development

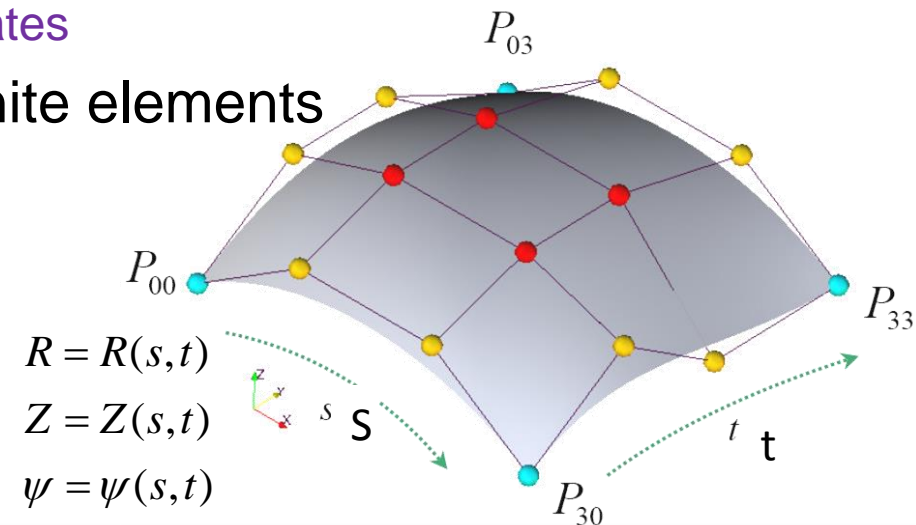
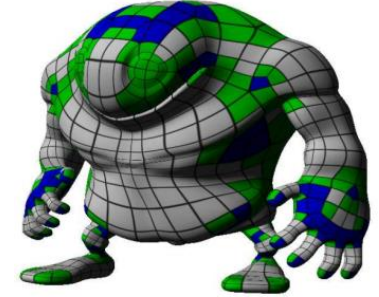
-14 institutes



Eurofusion enabling research project (M. Hoelzl)

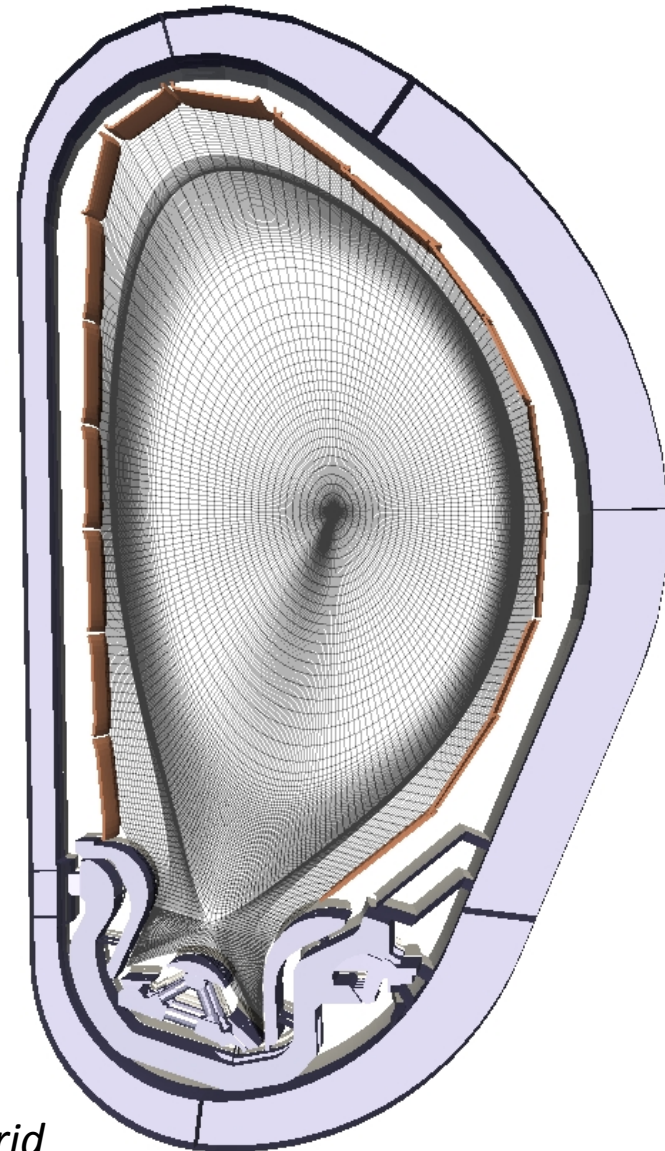
Bezier Finite Elements

- Bezier patches are widely used in CAD, computer graphics
 - to describe 2D surface in 3D physical space
 - parametrised in local coordinates (s,t)
 - $X(s,t)$, $Y(s,t)$, $Z(s,t)$
 - Cubic bezier patch defined by 16 control points
 - here the height represents a physics variable
 - (R,Z) space and variables described by Bezier patches :
Isoparametric representation: $R(s,t)$, $Z(s,t)$, $\psi(s,t)$
 - values and derivatives continuous in space (C^1)
 - but not continuous in local (s,t) coordinates
 - generalisation of cubic Hermite finite elements
 - Allows local refinement



JOREK Grids

- Elements aligned to magnetic geometry
 - the curved boundaries of the Bezier patches can be accurately aligned with the magnetic geometry
 - alignment and higher order (cubic) elements allow an accurate description of the anisotropy of the heat conduction ($K_{//} / K_{\perp} = 10^{10}$)
- Aligned grids can be extended (non-aligned) up to the plasma facing components
 - essential for an accurate description of the plasma-wall interaction

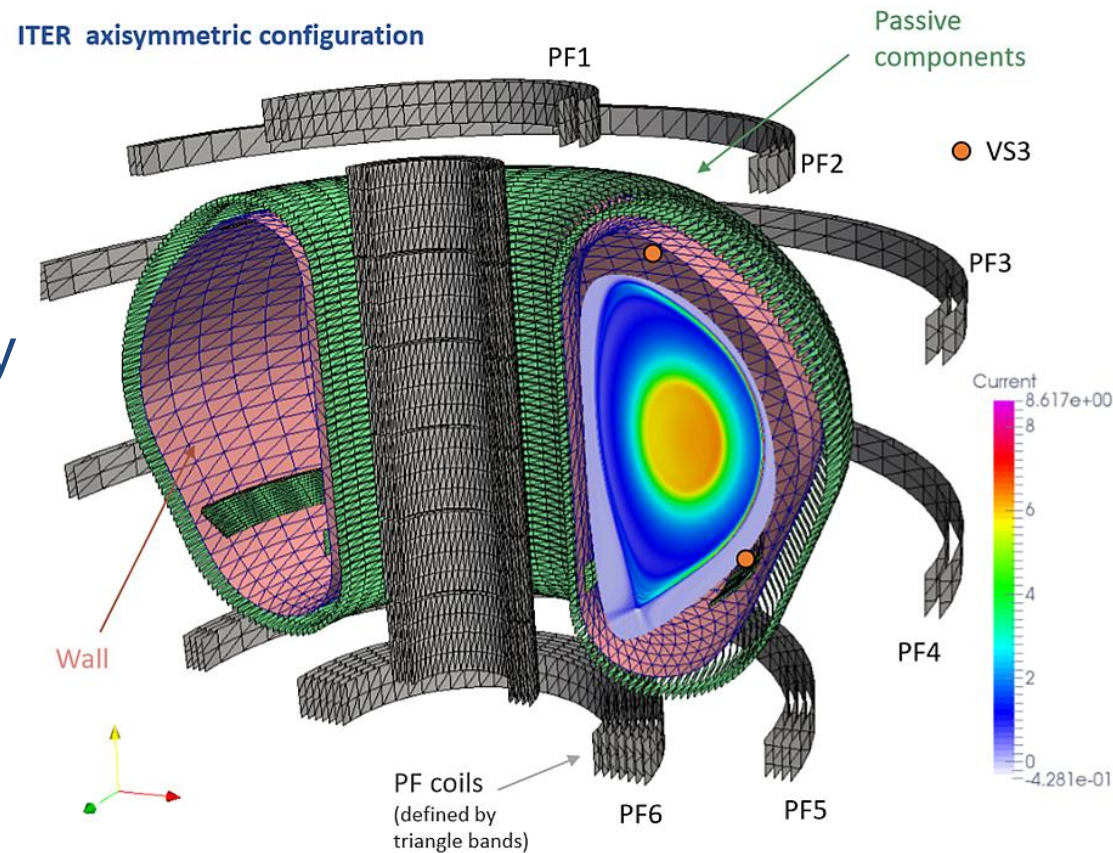


example ITER grid

Boundary Conditions

- The evolution of magnetic fields and currents in coils and conducting structures can be solved independently from the main plasma
 - Solved (once) using the code **STARWALL** (boundary element method)

- Provides boundary conditions for the magnetic fields at the JOREK domain boundary
 - $B_{\text{normal}} / B_{\text{tan}} = M$



Implicit time evolution

- The time scales of interest can vary from the Alfvén time (microsecond) to slow growing resistive instabilities (10s of seconds in ITER)
- Explicit schemes would be limited by the fast wave frequency (full MHD) or by Alfvén waves (reduced MHD)
 - Not practical for most MHD tokamak applications
- Implicit scheme allows large time steps resolving the phenomena of interest
 - Linearised Crank-Nicholson or BDF (Gear's) schemes

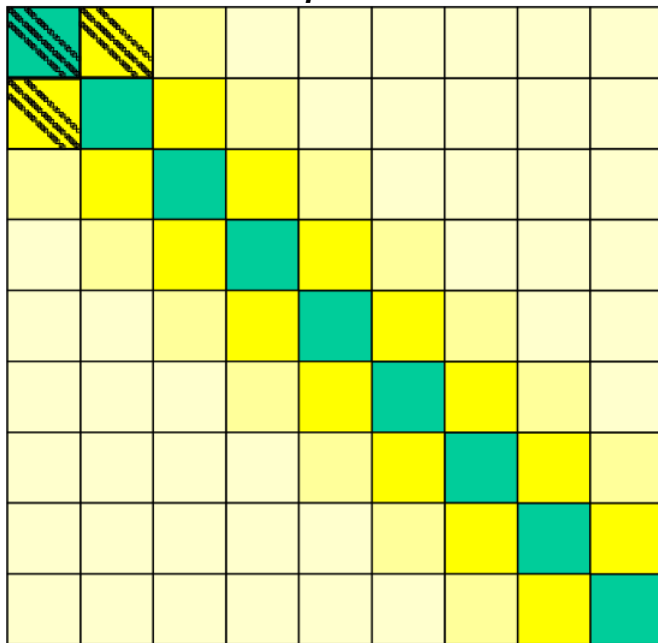
$$\frac{\partial A(\vec{y})}{\partial t} = B(\vec{y}) \quad \Rightarrow \quad \left(\frac{\partial A(\vec{y}_n)}{\partial y} - \frac{1}{2} \delta t \frac{\partial B(\vec{y}_n)}{\partial y} \right) \delta \vec{y} = B(\vec{y}_n) \delta t$$

- Implicit schemes yields large matrix problem at each time step
 - 5×10^4 nodes, 8 variables, 4 dof per node, 15 Fourier modes: $N = 5 \times 10^7$
 - Challenging for large scale parallelisation

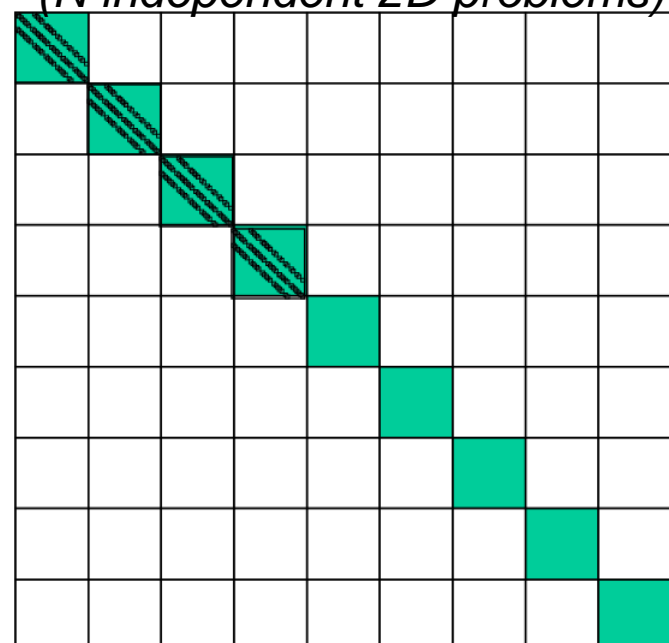
Preconditioned GMRES

- Matrix problem ($\sim 10^7$ unknowns) solved using preconditioned GMRES
- standard preconditioners not satisfactory for MHD
 - condition number MHD matrices very large
- use physics-based preconditioner
 - ignore non-linear coupling of N toroidal Fourier modes in preconditioning matrix
 - N independent, sparse, blocks solved using **PaStiX** library

full matrix 3D problem

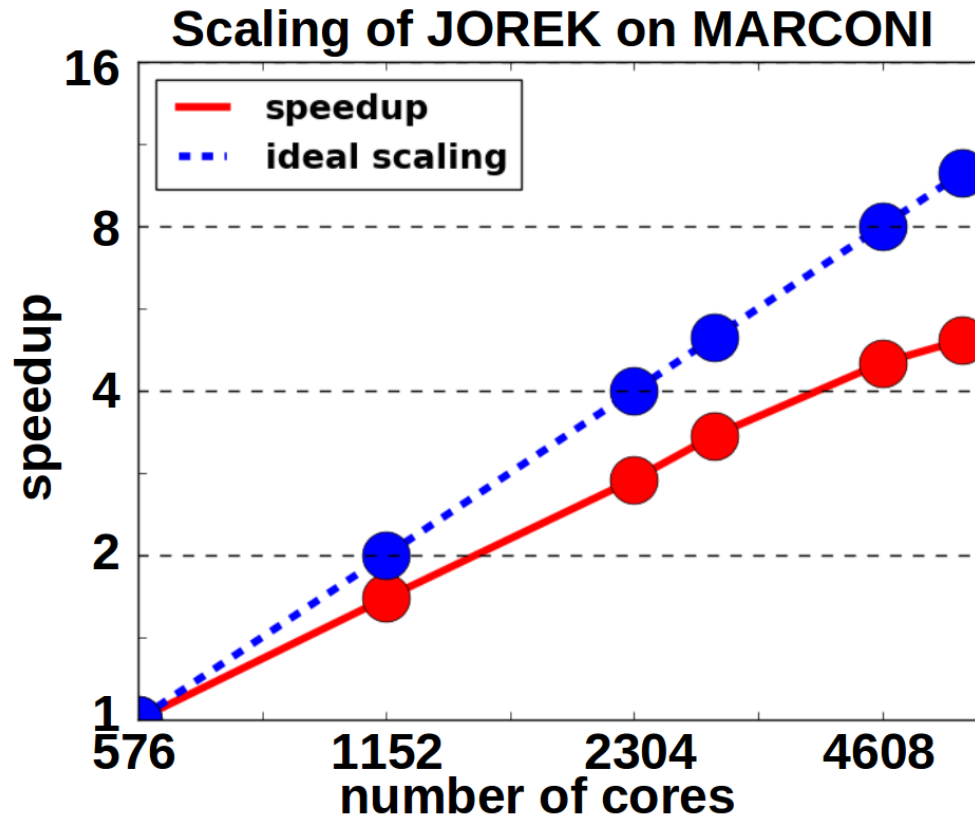


*preconditioning matrix
(N independent 2D problems)*



JOREK Parallelisation

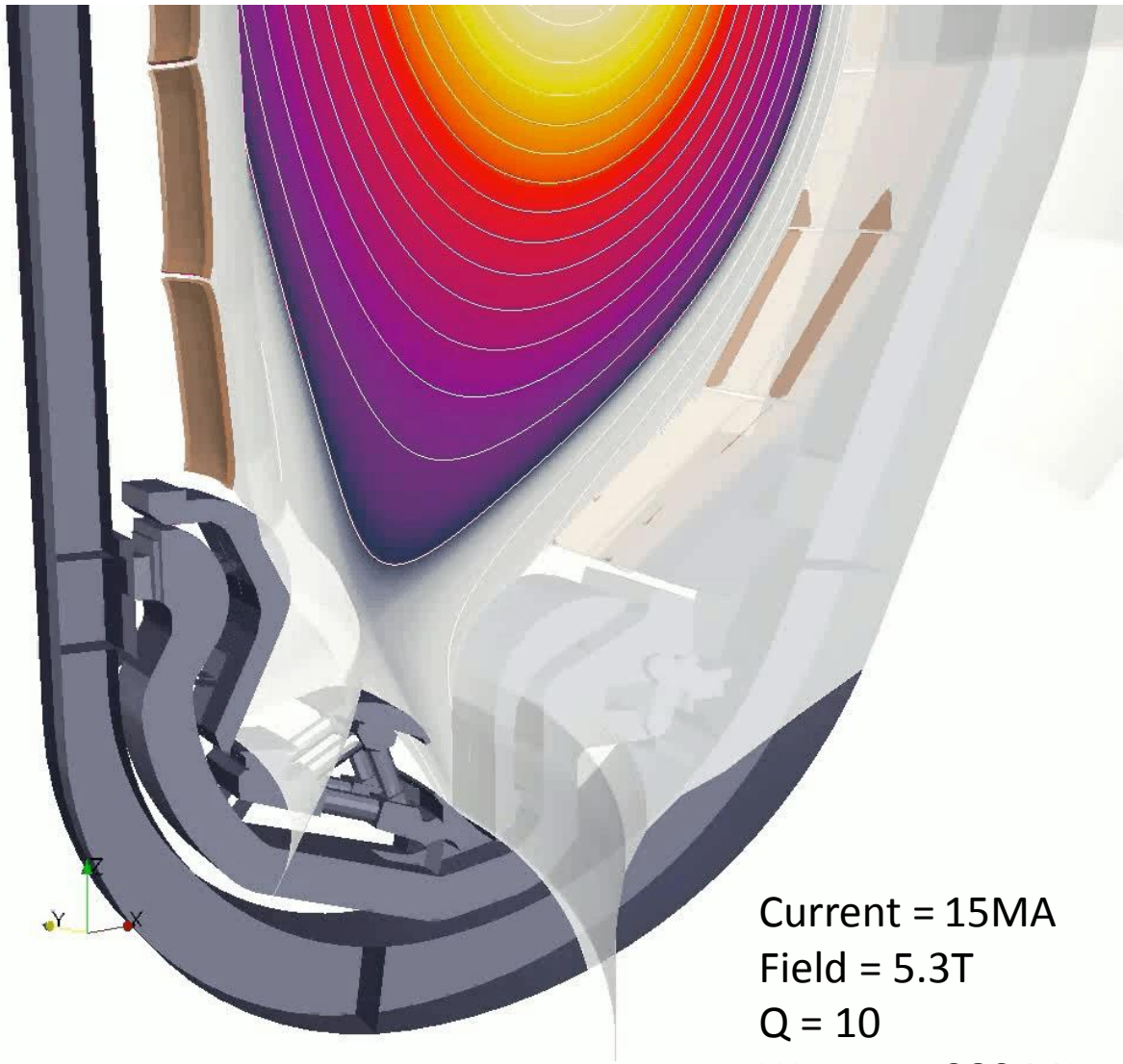
- Parallelisation : MPI+openMP,
 - # CPU : mostly due to memory requirements
- 3 main parts:
 - Matrix construction, factorisation preconditioner and GMRES solve



Strong scaling
(fixed problem size)

ITER ELM Simulation

- Ballooning instability
 - $n=10$
- Losses:
 - Energy loss : 1%
 - Density loss : 2%
 - ELM depth : $\sim 12\text{cm}$
 - Duration 250 micro sec

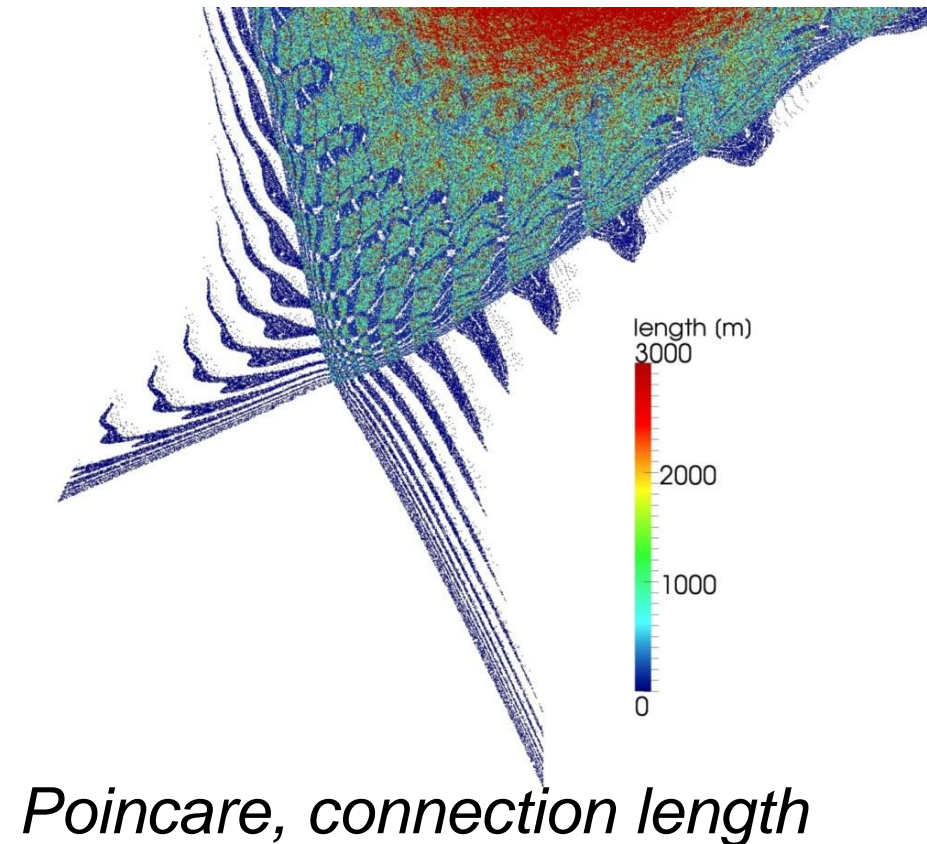
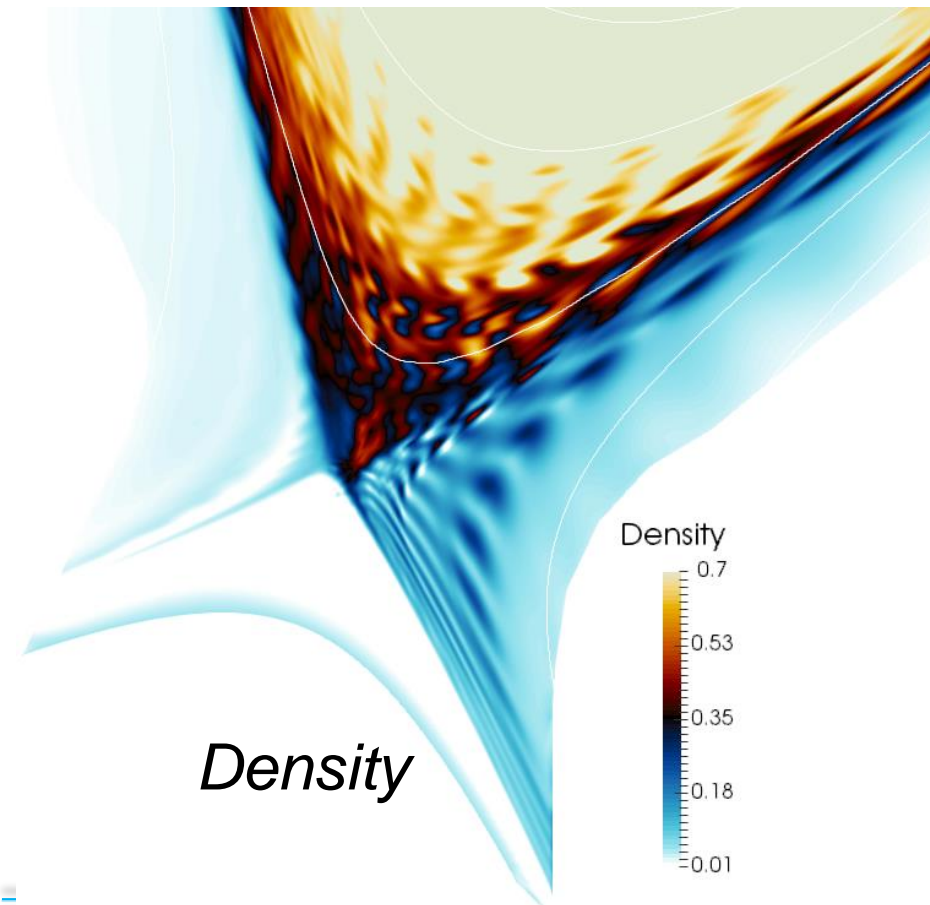


Plasma temperature (0-20keV)
Heat flux to divertor (0-3 GW/m²)

Current = 15MA
Field = 5.3T
 $Q = 10$
 $W_{\text{thermal}} = 380 \text{ MJ}$

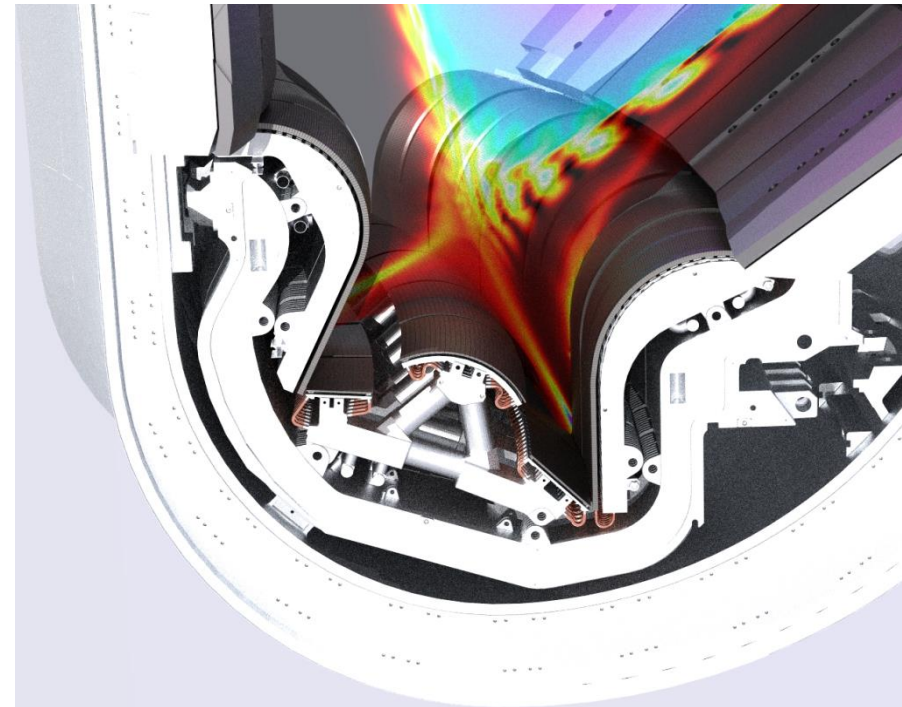
ELM Energy Losses

- Energy loss during ELMs is due to two mechanisms:
 - Ejection of filaments : convective energy loss
 - Formation of magnetic tangles : conduction parallel to perturbed field lines
 - **conductive – convective ELMs**



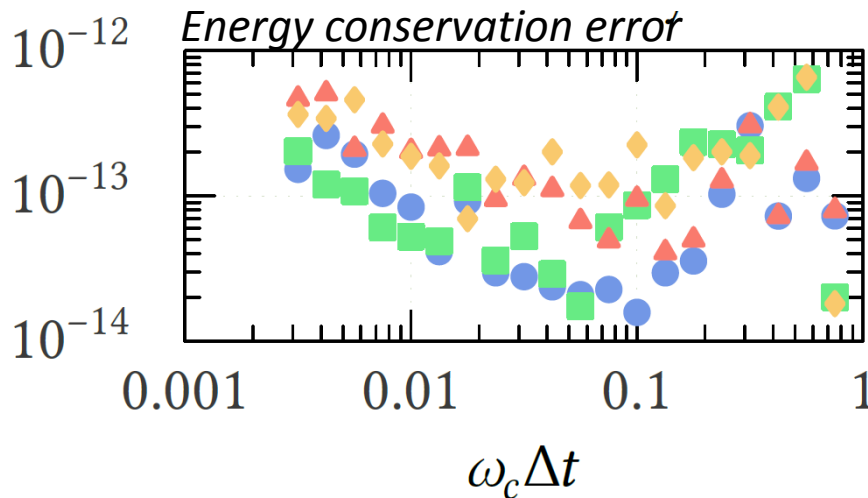
Plasma-Wall Interaction

- To describe the consequences of the MHD instabilities (ELMs, disruptions), the plasma-wall interaction needs to be accurately modeled:
 - **MHD model extended with “divertor” boundary conditions:**
 - Plasma flows into the wall at Mach one (or larger)
 - Relation between conductive and convective energy fluxes: $K \nabla_{\parallel} T = c n v_{\parallel} T$
 - **Discrete particles** to model
 - recycling of plasma into neutrals,
 - ionisation, recombination, charge exchange
 - Impurities : sputtering and spreading due to MHD instabilities

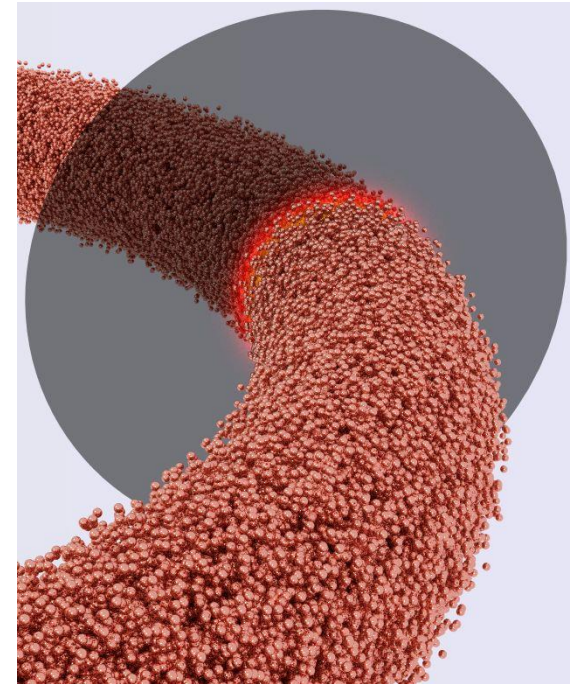


Particles

- Particles are followed in the time evolving magnetic and electric field of the MHD fluid
 - full orbit using well-known Boris method (with correction toroidal geometry)
 - particles are followed in real space (R,Z,φ) and finite element space (s,t,φ)
 - charge states are evolving in time (Ionisation, recombination, CX)
 - Particle-background collisions with binary collision model
 - sputtering model for impurity and neutral sources



Energy conservation independent of time step Δt ,
momentum conservation $\sim \Delta t^2$



Kinetic Neutrals

- Coupling of JOREK MHD fluid and particle

- Particle contributions become sources in MHD equations

- Conservative form:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = S_\rho$$

$$\frac{\partial (\rho \vec{v})}{\partial t} + \nabla \cdot (\rho \vec{v} \vec{v}) + \nabla \cdot \bar{\bar{P}} - \vec{J} \times \vec{B} = \vec{S}_v$$

$$\frac{\partial}{\partial t} \left(\frac{1}{2} \rho v^2 + \frac{3}{2} \rho T \right) + \nabla \cdot \left(\frac{1}{2} \rho v^2 \vec{v} + \frac{3}{2} \rho T \vec{v} + \vec{v} \cdot \bar{\bar{P}} \right) = S_E$$

- JOREK form

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = S_\rho$$

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho (\nabla \cdot \vec{v}) \vec{v} + \nabla \cdot \bar{\bar{P}} = \vec{S}_v - \vec{v} S_\rho$$

$$\frac{\partial \left(\frac{3}{2} \rho T \right)}{\partial t} + \vec{v} \cdot \nabla \left(\frac{3}{2} \rho T \right) + \frac{5}{2} \rho T (\nabla \cdot \vec{v}) = S_E - v S_v + \frac{1}{2} v^2 S_\rho$$

[D. van Vugt, S. Franssen]

Sources, Moments

- The sources are given by:
$$S_\rho(\vec{x}) = \int mf(\vec{x}, \vec{v}) d\vec{v}$$
$$S_v(\vec{x}) = \int m\vec{v}f(\vec{x}, \vec{v}) d\vec{v}$$
$$S_E(\vec{x}) = \int \frac{1}{2}mv^2 f(\vec{x}, \vec{v}) d\vec{v}$$

- Express sources in JOREK finite element representation to obtain a continuous function from a list of discrete particles

$$\tilde{S}_\rho(\vec{x}) = \sum_{ijk} p_{\rho,ijk} H_{ij}(s, t) H_{\varphi,k}(\varphi)$$

- Projection using weak form (system of equations to be solved)

$$\int v^*(\vec{x}) \tilde{S}_\rho(\vec{x}) d\vec{x} = \int v^* S_\rho(\vec{x}) d\vec{x}$$

$$\int v^*(s, t, \varphi) \sum_{ijk} p_{\rho,ijk} H_{ij}(s, t) H_{\varphi,k}(\varphi) d\vec{x} = \int v^* mf(\vec{x}, \vec{v}) d\vec{v} d\vec{x}$$

$$\int v^*(s, t, \varphi) \sum_{ijk} p_{\rho,ijk} H_{ij}(s, t) H_{\varphi,k}(\varphi) d\vec{x} = \sum_n v^* m w_n \delta(\vec{x} - \vec{x}_n)$$

Moments: smoothing

- Some smoothing is required
 - depending on number of particles
- Modified projection equation:

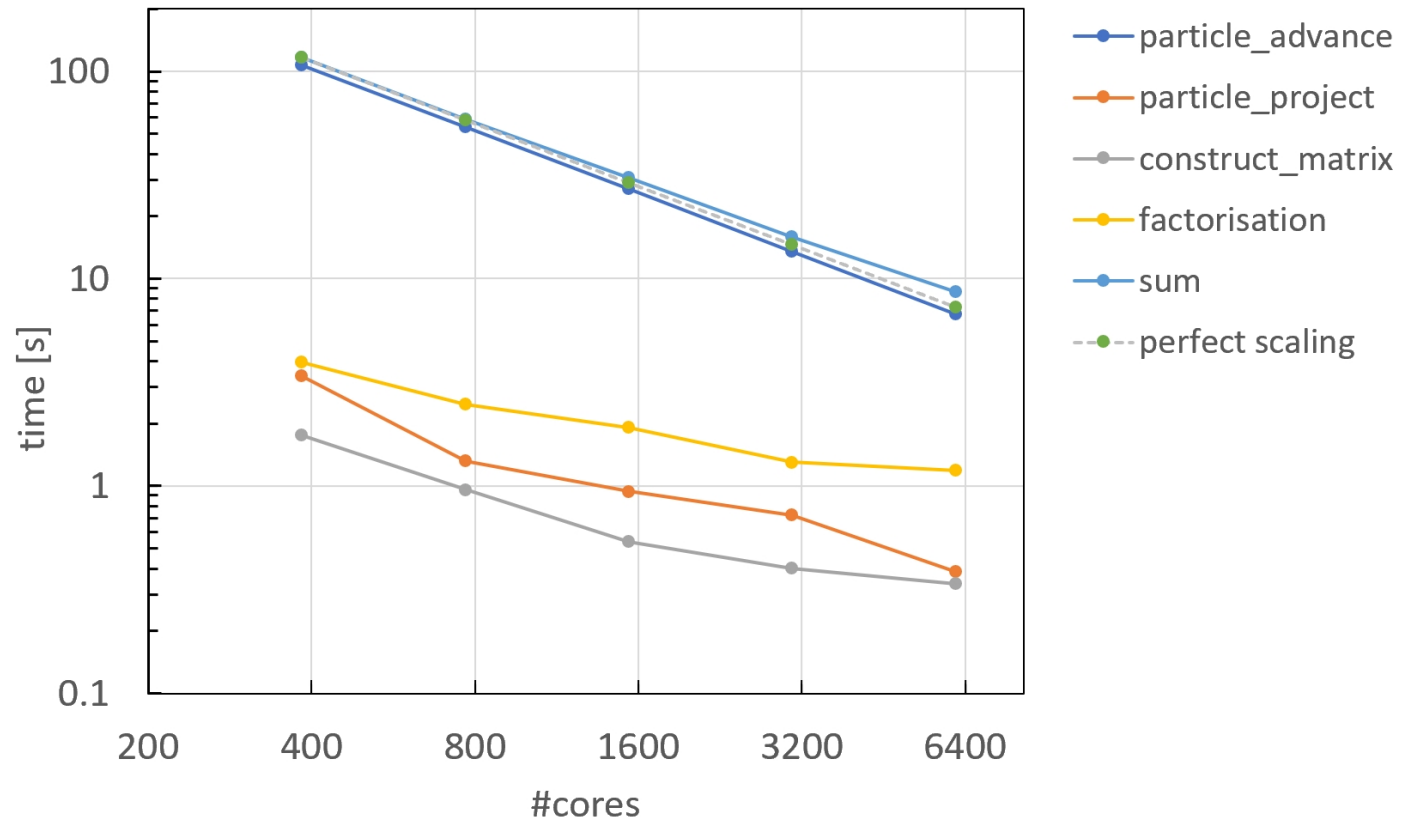
$$\int v^*(\vec{x})(1 - \lambda \nabla^2 - \zeta \nabla^4) \tilde{S}_\rho(\vec{x}) d\vec{x} = \int v^* S_\rho(\vec{x}) d\vec{x}$$

$$\int (v^* \tilde{S}_\rho + \lambda \nabla v \cdot \nabla \tilde{S}_\rho + \zeta \nabla^2 v^* \nabla^2 \tilde{S}_\rho) d\vec{x} = \int v^* S_\rho(\vec{x}) d\vec{x}$$

- LHS matrix is calculated and factorised once (using MUMPS)
- **RHS is collected at every particle step**
 - Important for conservation properties
- Projection (i.e. MUMPS solve) at every JOREK fluid time step

Parallelisation Scaling

- Good parallel scaling as long as particles dominate total time
 - 4×10^7 particles, 1600 elements, 8 toroidal harmonics
 - Marconi, Skylake (1 mpi, 48 threads per node)
 - efficiency 84% at 6000 cores

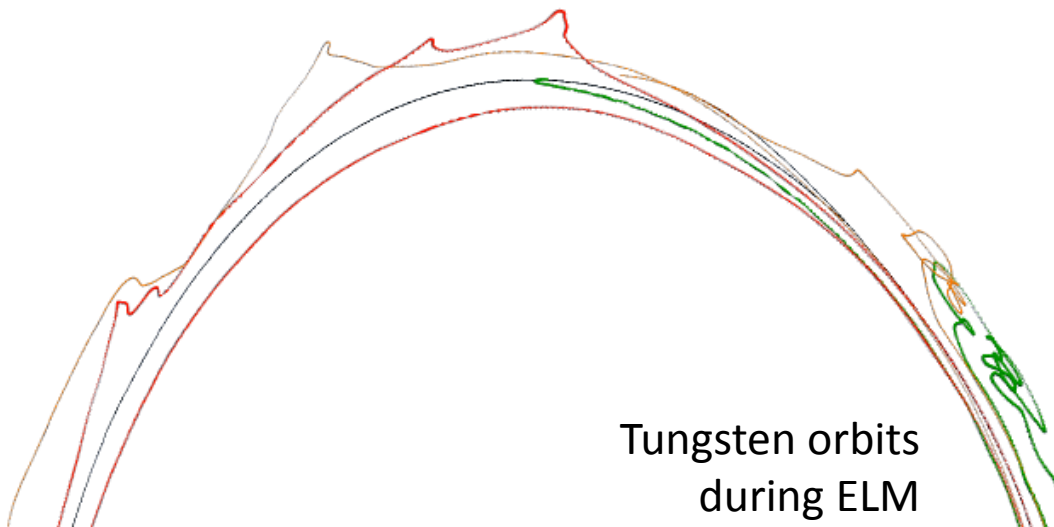
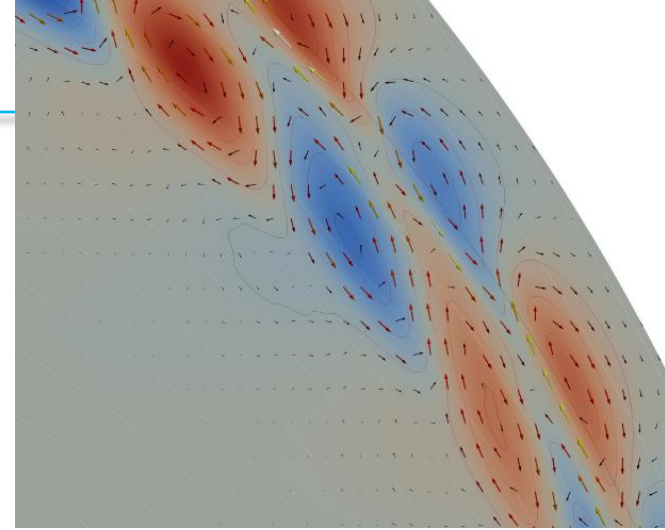


Tungsten (W) transport and ELMs

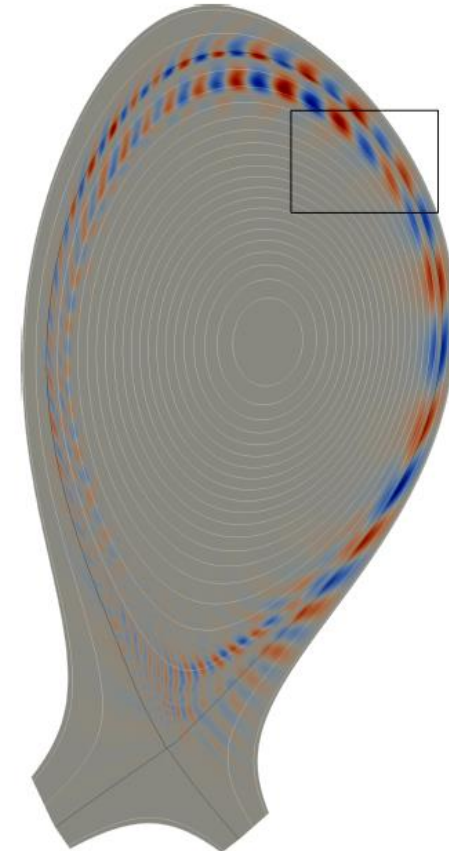
- Modern tokamaks (and ITER) have Tungsten divertors
 - high melting temperature
 - low retention of Tritium
- Even low concentrations of W lead to large radiation losses
 - leading to disruptions
- Main source of W is due to sputtering of the divertor due to large energy deposition due to ELMs
 - i.e. produces W in the divertor, outside main plasma
- However, ELMs also expel Tungsten from the main plasma
 - at least in current experiments
- JOREK MHD + W-particles simulations (D. van Vugt, TUE)
 - Study the ELM induced W transport
 - W production due to sputtering

ELM induced W Transport

- 3D vortices of the ELM instability at the edge of the plasma
 - associated electric field perturbs W orbits and causes radial transport
 - magnetic field perturbation unimportant
- Transport is due 3D to interchange motion

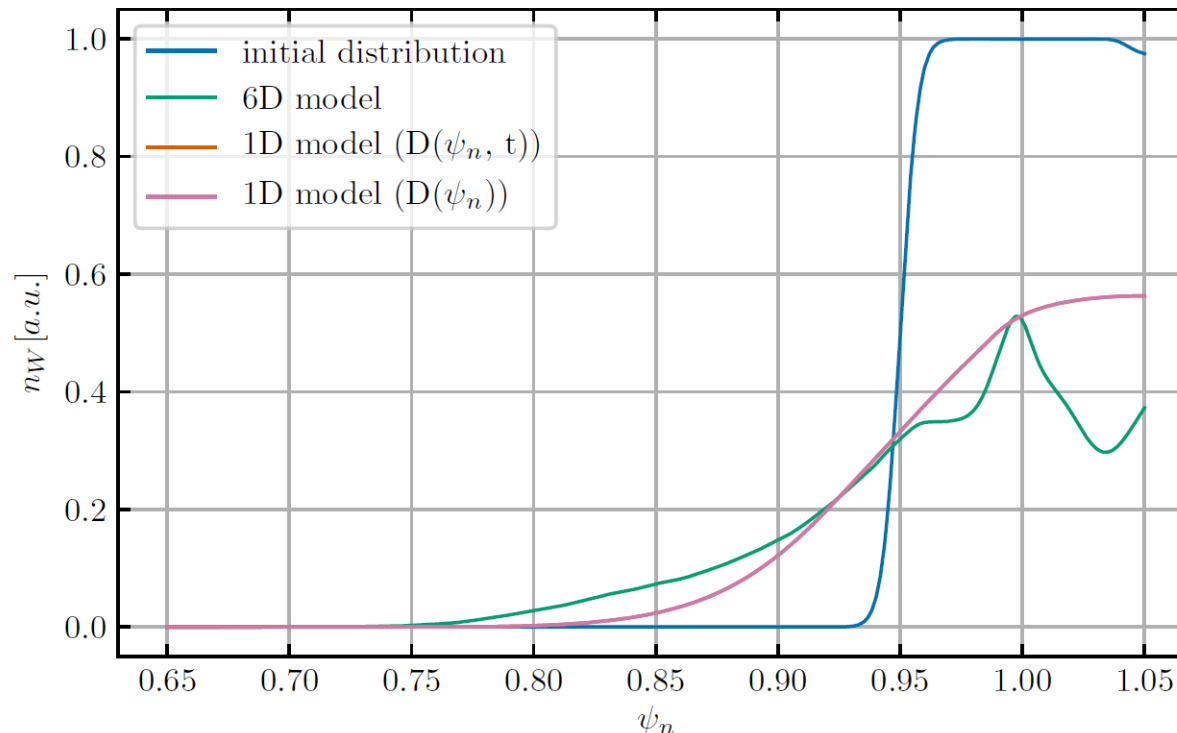


Tungsten orbits
during ELM



ELM induced W transport

- Effective ELM induced W radial transport (in 1D) is diffusive
 - In the case of W accumulation inside the plasma ELMs expel Tungsten
 - As in current experiments
 - In ITER, ELMs are **predicted to move Tungsten into the plasma**
 - Due to inter-ELM profiles of Tungsten mainly outside the main plasma
 - Opposite to current experiments



Summary

- MHD instabilities are one of the main concerns for ITER operation
 - ELMS, disruptions
 - MHD instabilities must be controlled in ITER
- Nonlinear MHD code JOREK
 - simulation of MHD and its control in current and future fusion experiments
 - MHD+particle model for plasma wall interaction, fast particles
- HPC requirements
 - fully implicit scheme due to large range of time scales
 - Large sparse matrices \Rightarrow memory requirements, parallelisation inefficiency beyond several 1000 cores (fluid model)
 - MHD+particle model scales much better
- Next: extending particle model
 - verification and validation: towards a predictive capability
 - fast α -particles, massive material injection