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A. Rouchon, A. Zoia, R. Sanchez

PHYSOR 2016 - Unifying Theory and Experiments in the 21st Century
Sun Valley Resort, Sun Valley, Idaho, USA, May 1 - 5, 2016, on CD-ROM (2016)

A NEW MONTE CARLO METHOD FOR NEUTRON NOISE CALCULATIONS

EXTENDED SUMMARY

Amélie ROUCHON, Andrea ZOIA & Richard SANCHEZ
CEA-Saclay, DEN, DM2S, SERMA
F-91191 Gif-sur-Yvette, France
amelie.rouchon@cea.fr
andrea.zoia@cea.fr
richard.sanchez@cea.fr

Key Words: **Neutron noise, Monte Carlo, Frequency domain**

1. INTRODUCTION

Traditional neutron noise analysis [1] addresses the description of small time-dependent flux fluctuations caused by small global or local periodic perturbations of the macroscopic cross sections, which may occur in nuclear reactors due to stochastic density fluctuations of the coolant or to vibrations of fuel elements, control rods or any other structures in the reactor core. Neutron noise techniques are widely used by the nuclear industry for non-invasive general monitoring, control and detection of anomalies in nuclear power plants. They are also applied to the measurement of the properties of the coolant, such as speed and void fraction.

Until recently, neutron noise equations have been only solved by analytical methods (see [2] or [3]) or by diffusion theory (see [4],[5] or [6]). As all deterministic methods, it is important to validate them thanks to Monte Carlo simulations. In 2013, an original stochastic method was proposed by Yamamoto in [7] in order to solve the transport equation in neutron noise theory thanks to a Monte Carlo algorithm. This algorithm is similar to the power iteration method and uses a weight cancellation technique developed by the same author into neutron leakage-corrected calculations or higher order mode eigenvalue calculations (see [8], [9] and [10]). This method gives good results but has some disadvantages especially the use of the ”binning procedure” for the weight cancellation: each fissile region must be divided into a large number of small regions (called bins) where positive and negative weights are cancelled.

In this paper, we present a new Monte Carlo method that does not use any weight cancellation technique. This new method has been inspired by a recent technique developed in [11] for alpha eigenvalue calculations. In section 2, the general noise theory will be briefly introduced and the new Monte Carlo method will be presented. In section 3, we compare our Monte Carlo method and the method proposed in [7] to the deterministic methods (diffusion and transport) in the case of a heterogenous one-dimensional system for several frequencies. In section 4, we discuss the Figure Of Merit of each stochastic method and the respective advantages and disadvantages. Finally, section 5 provides some general conclusions.

2. MONTE CARLO CALCULATION IN NEUTRON NOISE THEORY

2.1. Neutron noise equation

The general noise equations are obtained by assuming small periodic perturbations (which allows for a linear theory) around a steady state in the neutron field and then by Fourier transform in the frequency domain. The analysis is conducted with the neutron kinetic equations including the coupling with neutron precursors. The result is a fixed-source equation for the perturbed neutron field, which can then be solved so as to predict noise measurements at detector locations. For each frequency, the neutron field has an intensity and a phase and it is therefore a complex function in the frequency domain.

In case of a one-dimensional system in P_0 approximation with only one precursor group, the critical steady-state equation for each group g is :

$$(\Omega.\nabla + \Sigma_0^g(r))\,\Psi_{0,g}(r,\Omega) = \frac{1}{2}\sum_{g'}\Sigma_{0,s0}^{g'\to g}(r)\Phi_{0,g'}(r) + \frac{1}{2}\chi^g\sum_{g'}\frac{\nu^{g'}\Sigma_{0,f}^{g'}(r)}{k}\Phi_{0,g'}(r). \tag{1}$$

We perturb all cross-sections and we decompose the angular flux into $\Psi_g(r,\Omega,t) = \Psi_{0,g}(r,\Omega) + \delta\Psi_g(r,\Omega,t)$. Thus, in this case, the detailed noise equation is :

$$\begin{aligned}\left(\Omega.\nabla + \Sigma_0^g(r) + i\frac{\omega}{v^g}\right)\delta\Psi_g(r,\Omega,\omega) &= \frac{1}{2}\sum_{g'}\Sigma_{0,s0}^{g'\to g}(r)\delta\Phi_{g'}(r,\omega)\\ &+ \frac{1}{2}\left[(1-\beta)\chi_p^g + \frac{\lambda^2\beta}{\lambda^2+\omega^2}\chi_d^g - i\frac{\lambda\beta\omega}{\lambda^2+\omega^2}\chi_d^g\right]\sum_{g'}\frac{\nu^{g'}\Sigma_{0,f}^{g'}(r)}{k}\delta\Phi_{g'}(r,\omega)\\ &+ S_g(r,\Omega,\omega),\end{aligned} \tag{2}$$

with i the imaginary number, $\delta\Phi$ the scalar noise flux, $\omega = 2\pi f$ the angular frequency and S the noise source. All other notations are standard. In all this paper, we work with positive frequencies.

2.2. A new Monte Carlo method for solving the noise equations

A stochastic method for solving Eq. 2 has been provided by Yamamoto in [7]. Here, we only detail our new Monte Carlo method and explain the differences with respect to [7]. We also only detail the differences with the classical Monte Carlo calculations. For exemple, in noise calculations, the scattering is treated exactly like in classical calculations so we do not detail this step.

A way to solve the neutron noise equations by Monte Carlo method is to work with particles having complex weights [7] $w(\omega) = \{w_{\Re}(\omega), w_{\Im}(\omega)\}$. The signs of the real and imaginary parts of particle weights can be positive or negative.

In [7], particle flights are based on the "noise total cross-section" $\Sigma_0^g + i\frac{\omega}{v^g}$ appearing in Eq. 2, so that the particle weights change continuously during each travel. Thus, the track length estimator calculation is more cumbersome than the classical one. In the present work, we choose to add the term $\frac{\eta - i}{\eta}\eta\frac{\omega}{v^g}$ with $\eta \in \mathbb{R}^+\backslash\{0\}$ to both sides of Eq. 2. So, the equation becomes :

$$\left(\Omega.\nabla + \Sigma_0^g(r) + \eta\frac{\omega}{v^g}\right)\delta\Psi_g(r,\Omega,\omega) = \frac{\eta - i}{\eta}\eta\frac{\omega}{v^g}\delta\Psi_g(r,\Omega,\omega) + (...). \tag{3}$$

In this case, we work with a real total cross-section $\tilde{\Sigma}_0^g(\omega) = \Sigma_0^g + \Sigma_{f,spe}^g(\omega) \in \mathbb{R}^+$ where $\Sigma_{f,spe}^g(\omega) = \eta\frac{\omega}{v^g}$. So, flight paths are sampled as in classical Monte Carlo calculations, provided that $\tilde{\Sigma}_0^g$ is used instead of Σ_0^g. The track length estimator calculation is therefore trivial.

Because of the structure of Eq. 3, we have to treat two types of fission : the "normal" fission with the probability $\Sigma_{0,f}^{g'}/\tilde{\Sigma}_0^{g'}(\omega)$ and the "special" fission with the probability $\Sigma_{f,spe}^{g'}/\tilde{\Sigma}_0^{g'}(\omega)$. We treat the "normal" fission as in Yamamoto's method i.e. the number of "normal" fission sources is calculated as Int$(\nu^{g'} + \epsilon)$ with Int$(.)$ the integer part and ϵ a random number. Moreover, the weight w^* of each new particle created by the "normal" fission is modified by :

$$w^* = \frac{w}{k\chi_{\text{total}}^g}[(1-\beta)\chi_p^g + \frac{\lambda^2\beta}{\lambda^2+\omega^2}\chi_d^g - i\frac{\lambda\beta\omega}{\lambda^2+\omega^2}\chi_d^g] \tag{4}$$

with w the weight before fission event, $\chi_{\text{total}}^g = (1-\beta)\chi_p^g + \beta\chi_d^g$ and g the fission energy group (sampled with probability $\chi_{\text{total}}^g/\sum_{g'}\chi_{\text{total}}^{g'}$). The "special" fission is a copy of the incident neutron with a new biased weight w^{**} given by :

$$w^{**} = w\frac{\eta - i}{\eta} \tag{5}$$

Implicit capture and Russian roulette can be also used. In the case of implicit capture, after each collision event, the complex weight is modified by $\Sigma_{0,s0}^{g'}/\tilde{\Sigma}_0^{g'}(\omega)$. Then, as in Yamamoto's method, the Russian roulette game is applied separately to the absolute value of the real and imaginary parts. The particle is killed only if the real and the imaginary parts are killed by the Russian roulette at the same time. If only the real or the imaginary part is killed, the particle survives. When the real or the imaginary part survives, its value is updated to ± 1 according to its sign before Russian roulette.

2.3. Weight cancellation

At low and high frequencies (more precisely outside the plateau region of the zero-power transfer function of the system), the Monte Carlo methods described above possibly lead to the production of a huge number of particles and the calculations are not normally terminated. To overcome this problem, Yamamoto's method uses the "binning procedure" for the weight cancellation. With this technique, fissile regions are divided into a large number of small spatial bins where fission sources with positive and negative weights are summed. After cancellation, the new number n_f of fission sources in each bin is equal to Int$(\max(\Re(\sum w)), \max(\Im(\sum w)), 1)$ and, for the next generation, they are distributed uniformly within the bin (this is allowed because all fissions in the method proposed in [7] are isotropic) with a unique new weight equal to $\frac{1}{n_f}\sum w$.

Contrary to Yamamoto's method, we will not use this weight cancellation technique because our "special" fission depends on the angular distribution of the particles, so that an angular mesh would be also required. Nevertheless, we will see that it is possible to overcome the problem of particle explosion at low and high frequencies by removing the implicit capture and adapting the η value.

3. ANALYSIS AND COMPARISONS

In this section, we compare the two Monte Carlo methods with the deterministic methods (diffusion and transport) in case of a heterogenous one-dimensional system with 4 energy groups and 6 precursor groups. Our system is composed of 17 fuel pins of 1.08 cm (the inter-pin size is 0.36 cm so one cell size is 1.44 cm) and we impose vacuum boundaries. The noise source is equal to -1 for the real part of group 4 (the thermal group) in the entire third pin and zero for other groups. All Monte Carlo calculations were performed with 3 000 independent batches of 10 000 particles. Note that in [7] the author uses only one batch of 1 000 000 particles so no statistical uncertainties are available. In this paper, we make several independent batches even with Yamamoto's method, so that we can estimate the statistical uncertainties for all Monte Carlo calculations.

Tab. I details the calculation parameters for the new Monte Carlo method. For Yamamoto's method, we always use the "binning procedure" (with 302 bins) and the implicit capture. Figure 1 illustrates the results of the track length estimator for the group 4 in 0.01 Hz (low frequency), 1 Hz (in the plateau region) and 10 000 Hz (high frequency).

Table I. Calculation parameters for the new Monte Carlo method

	0.01 Hz	1 Hz	10 000 Hz
Implicit capture	no	yes	no
η value	1	1	100

Thus, these two Monte Carlo methods give good results for the moduli of the noise fluxes and allow validating deterministic calculations as shown in Fig. 1(c).

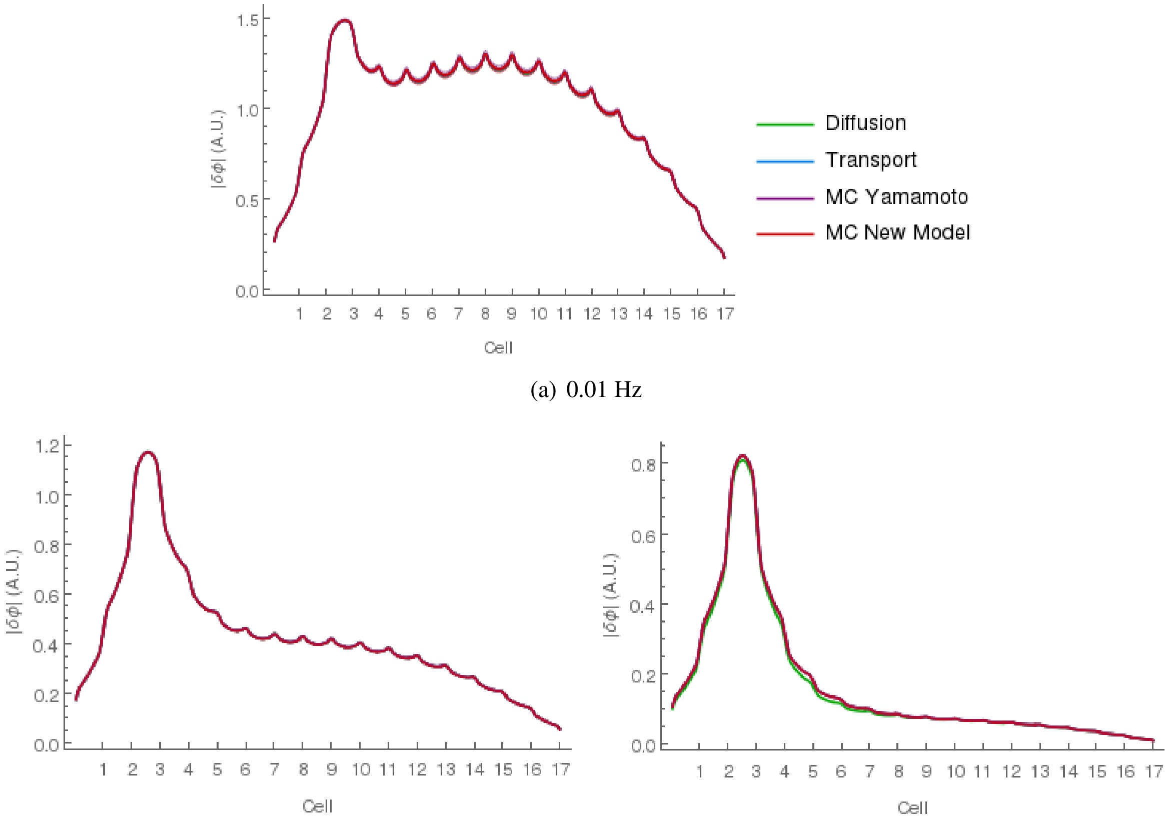
4. DISCUSSION

The Figures Of Merit of the Monte Carlo calculations are reported in Tab. II: our new method shows better performances with respect to [7]. Moreover, we do not have any explosion of the CPU time at low frequencies, contrary to [7] (see Fig. 10 of [7]).

Table II. Figures Of Merit of the Monte Carlo calculations

	0.01 Hz	1 Hz	10 000 Hz
Yamamoto's method	1.10E-06	4.40E-05	4.96E-05
New method	7.73E-06	2.30E-04	2.46E-04

Furthermore, if we separately analyse the real and the imaginary parts of the noise flux, we note that, at high frequencies, the track length estimator of [7] is not robust because of the sinusoidal components $cos(\omega/v^g * l)$ and $sin(\omega/v^g * l)$ of this estimator. Our method does not suffer from this issue, and the track length and collision estimators are always coherent.



(a) 0.01 Hz

(b) 1 Hz

(c) 10 000 Hz

Figure 1. Moduli of the thermal noise flux (group 4) in 0.01 Hz, 1 Hz and 10 000 Hz. Monte Carlo results are plotted with the 3σ error bars (barely visible).

5. CONCLUSION

In this paper we have presented a new Monte Carlo method that solves neutron noise equations. This new method does not use any weight cancellation techniques, is faster than the method developed in [7] and is more robust at high frequencies. Nevertheless, we have to remove the implicit capture at low and high frequencies. Future work will investigate new weight cancellation techniques based on the combing method in order to allow us keeping implicit capture for all frequencies and not only in the plateau region.

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