



HAL
open science

Local Defect Correction Method coupled with the Zienkiewicz-Zhu a *posteriori* error estimator in elastostatics Solid Mechanics

H. Liu, I. Ramiere, Frédéric Lebon

► To cite this version:

H. Liu, I. Ramiere, Frédéric Lebon. Local Defect Correction Method coupled with the Zienkiewicz-Zhu a *posteriori* error estimator in elastostatics Solid Mechanics. CMCMM 2015 - Seventeenth copper mountain conference on multigrid methods, Mar 2015, Copper Mountain, United States. cea-02509248

HAL Id: cea-02509248

<https://cea.hal.science/cea-02509248>

Submitted on 16 Mar 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

DE LA RECHERCHE À L'INDUSTRIE

cea



laboratoire «mécanique» «acoustique»

www.cea.fr

Local Defect Correction Method coupled with the Zienkiewicz-Zhu *a posteriori* error estimator in elastostatics Solid Mechanics

CMCMM 2015 | HAO LIU (DEN, DEC, SESC)
Supervisors: I. RAMIÈRE (DEN, DEC, SESC) AND
F. LEBON (LMA, AIX-MARSEILLE UNIVERSITY)

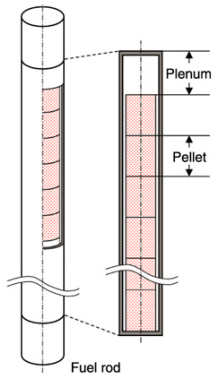
MARCH 22-27, 2015

- **Industrial context**
- **Local Defect Correction Method**
- **A posteriori error estimation in Solid Mechanics**
- **Numerical results of the combination of LDC and ZZ estimator**
- **Conclusions and perspectives**

Industrial context

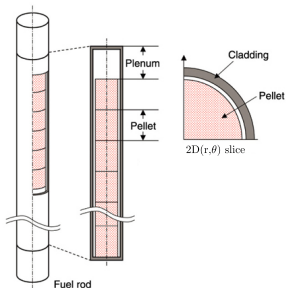
■ Pressurized Water Reactors

■ Fuel and Cladding



■ Pressurized Water Reactors

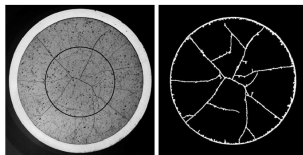
■ Fuel and Cladding



■ During irradiation

■ Pellet cracking

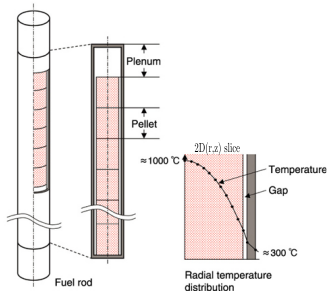
The pellet cracks and swells and the cladding creeps



⇒ **Discontinuous contacts** between pellet and cladding

■ Pressurized Water Reactors

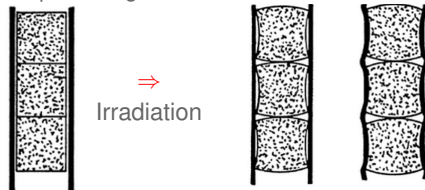
■ Fuel and Cladding



■ During irradiation

■ Hourglass shape deformation

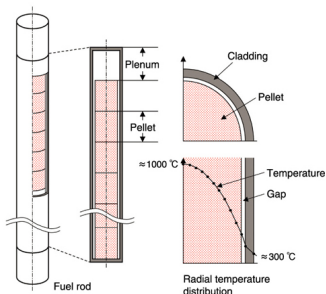
Due to the deforming effects of the high temperature gradient



⇒ **Contacts** occur first in **front of the inter-pellet plane**

■ Pressurized Water Reactors

■ Fuel and Cladding

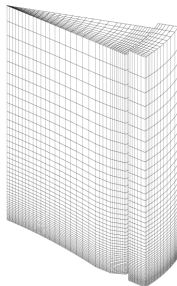


■ During irradiation

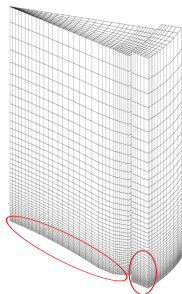
- Pellet cracking
 - Hourglass shape deformation
- ⇒ Localized high stress concentrations on the cladding
- ⇒ Integrity of the cladding (security stake)

- Simulation in PLEIADES fuel performance software environment

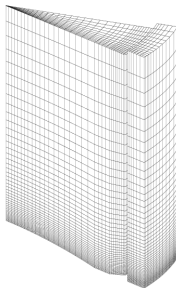
- Simulation in PLEIADES fuel performance software environment
- Representation of 1/32 of the Pellet and the corresponding section of cladding : precise simulations require **cells of $1\mu\text{m}$ for a structure of 1cm** (unreachable for uniform mesh)



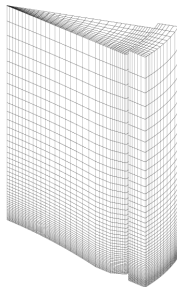
- Simulation in PLEIADES fuel performance software environment
- Representation of 1/32 of the Pellet and the corresponding section of cladding : precise simulations require **cells of $1\mu\text{m}$ for a structure of 1cm (unreachable for uniform mesh)**
- Numerical simulation of several pressure discontinuities



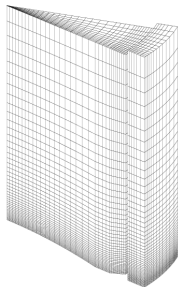
- Simulation in PLEIADES fuel performance software environment
- Representation of 1/32 of the Pellet and the corresponding section of cladding : precise simulations require **cells of $1\mu\text{m}$ for a structure of 1cm (unreachable for uniform mesh)**
- Numerical simulation of several pressure discontinuities
- Acceptable computational times and memory space



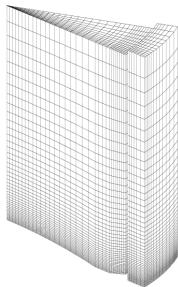
- Simulation in PLEIADES fuel performance software environment
- Representation of 1/32 of the Pellet and the corresponding section of cladding : precise simulations require **cells of $1\mu\text{m}$ for a structure of 1cm (unreachable for uniform mesh)**
- Numerical simulation of several pressure discontinuities
- Acceptable computational times and memory space
↪ **Local Mesh Refinement Methods : (h-,p-,r-) adaptive methods or local multi-grids method**



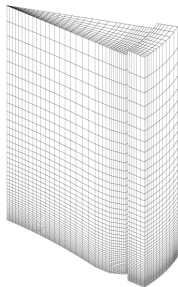
- Simulation in PLEIADES fuel performance software environment
- Representation of 1/32 of the Pellet and the corresponding section of cladding : precise simulations require **cells of $1\mu\text{m}$ for a structure of 1cm (unreachable for uniform mesh)**
- Numerical simulation of several pressure discontinuities
- Acceptable computational times and memory space
 - ↪ **Local Mesh Refinement Methods : (h-,p-,r-) adaptive methods or local multi-grids method**
- Quadrangular linear finite element required around the contact zone (modeling reason)



- Simulation in PLEIADES fuel performance software environment
- Representation of 1/32 of the Pellet and the corresponding section of cladding : precise simulations require **cells of $1\mu\text{m}$ for a structure of 1cm (unreachable for uniform mesh)**
- Numerical simulation of several pressure discontinuities
- Acceptable computational times and memory space
 - ↪ **Local Mesh Refinement Methods : (h-,p-,r-) adaptive methods or local multi-grids method**
- Quadrangular linear finite element required around the contact zone (modeling reason)
- Use the solver as a "black box"



- Simulation in PLEIADES fuel performance software environment
- Representation of 1/32 of the Pellet and the corresponding section of cladding : precise simulations require **cells of $1\mu\text{m}$ for a structure of 1cm (unreachable for uniform mesh)**
- Numerical simulation of several pressure discontinuities
- Acceptable computational times and memory space
 - ↪ **Local Mesh Refinement Methods : (h-,p-,r-) adaptive methods or local multi-grids method**
- Quadrangular linear finite element required around the contact zone (modeling reason)
- Use the solver as a "black box"
- Limited number of degrees of freedom per resolution



- Simulation in PLEIADES fuel performance software environment
- Representation of 1/32 of the Pellet and the corresponding section of cladding : precise simulations require **cells of $1\mu\text{m}$ for a structure of 1cm (unreachable for uniform mesh)**
- Numerical simulation of several pressure discontinuities
- Acceptable computational times and memory space
 - ↔ **Local Mesh Refinement Methods : (h-,p-,r-) adaptive methods or local multi-grids method**
- Quadrangular linear finite element required around the contact zone (modeling reason)
- Use the solver as a "black box"
- Limited number of degrees of freedom per resolution
 - ↔ **Local multi-grids method**

- Simulation in PLEIADES fuel performance software environment
- Representation of 1/32 of the Pellet and the corresponding section of cladding : precise simulations require **cells of $1\mu\text{m}$ for a structure of 1cm (unreachable for uniform mesh)**
- Numerical simulation of several pressure discontinuities
- Acceptable computational times and memory space
 - ↔ **Local Mesh Refinement Methods : (h-,p-,r-) adaptive methods or local multi-grids method**
- Quadrangular linear finite element required around the contact zone (modeling reason)
- Use the solver as a "black box"
- Limited number of degrees of freedom per resolution
 - ↔ **Local multi-grids method**
 - Modeling by finite elements

- Simulation in PLEIADES fuel performance software environment
- Representation of 1/32 of the Pellet and the corresponding section of cladding : precise simulations require **cells of $1\mu\text{m}$ for a structure of 1cm (unreachable for uniform mesh)**
- Numerical simulation of several pressure discontinuities
- Acceptable computational times and memory space
 - ↔ **Local Mesh Refinement Methods : (h-,p-,r-) adaptive methods or local multi-grids method**
- Quadrangular linear finite element required around the contact zone (modeling reason)
- Use the solver as a "black box"
- Limited number of degrees of freedom per resolution
 - ↔ **Local multi-grids method**
 - Modeling by finite elements
 - No composite problem

- Simulation in PLEIADES fuel performance software environment
- Representation of 1/32 of the Pellet and the corresponding section of cladding : precise simulations require **cells of $1\mu\text{m}$ for a structure of 1cm (unreachable for uniform mesh)**
- Numerical simulation of several pressure discontinuities
- Acceptable computational times and memory space
 - ↪ **Local Mesh Refinement Methods : (h-,p-,r-) adaptive methods or local multi-grids method**
- Quadrangular linear finite element required around the contact zone (modeling reason)
- Use the solver as a "black box"
- Limited number of degrees of freedom per resolution
 - ↪ **Local multi-grids method**
 - Modeling by finite elements
 - No composite problem
 - ↪ **Local Defect Correction method**

Local Defect Correction Method

- Adaptive Mesh Refinement methods
 - ≡ Local Multigrid methods

■ Adaptive Mesh Refinement methods

≡ Local Multigrid methods

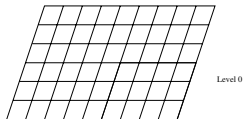
■ Inverse multi-grid process :

- Adaptive Mesh Refinement methods

- ≡ Local Multigrid methods

- Inverse multi-grid process :

- Global coarse grid

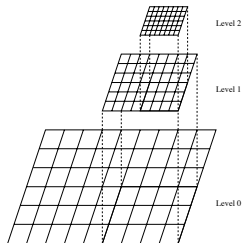


■ Adaptive Mesh Refinement methods

≡ Local Multigrid methods

■ Inverse multi-grid process :

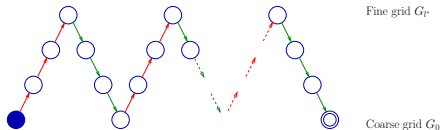
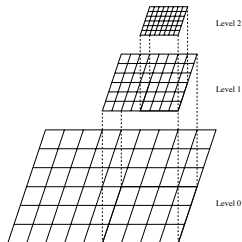
- Global coarse grid
- Local finer grids generated recursively in zones of interest



■ Adaptive Mesh Refinement methods

≡ Local Multigrid methods

- **Inverse multi-grid** process :
 - Global coarse grid
 - Local finer grids generated recursively in zones of interest
- **Prolongation** (BC interpolation) and **Restriction** operators link the grids
- ↪ **Iterative process** (\wedge -cycles)



● Initialisation

○ Smoothing or exact solving

⊙ Converged solution

↗ Prolongation step (boundary conditions)

↘ Restriction step (correction)

■ Adaptive Mesh Refinement methods

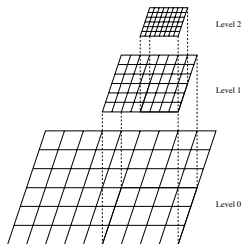
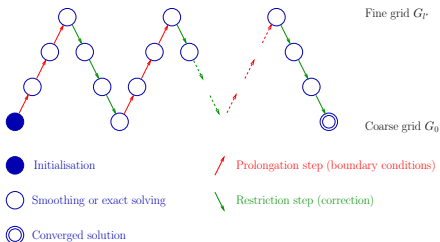
≡ Local Multigrid methods

■ Inverse multi-grid process :

- Global coarse grid
- Local finer grids generated recursively in zones of interest

■ Prolongation (BC interpolation) and Restriction operators link the grids

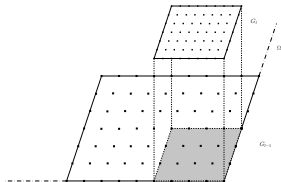
↔ Iterative process (\wedge -cycles)



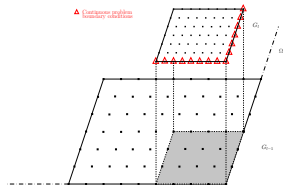
- 😊 Work on local fine meshes, Fast resolution on each sub-grid, Generic method : mesh, solver, model, refinement ratio
- ☹ Several meshes, Iterative process, Accuracy depending on prolongation and restriction operators

- Discrete problem on $G_l : \mathcal{L}_l(u_l^k) = f_l^k \forall l, k$
↪ no operator modification

- Discrete problem on $G_l : \mathcal{L}_l(u_l^k) = f_l^k \forall l, k$
 ↪ no operator modification
- Prolongation step : BC definition on the fine grid G_l

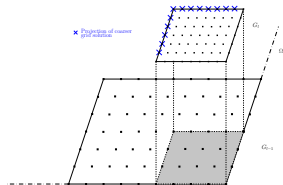


- Discrete problem on $G_l : \mathcal{L}_l(u_l^k) = f_l^k \forall l, k$
 ↪ no operator modification
- Prolongation step : BC definition on the fine grid G_l
 - BC of the continuous problem on $\Gamma_l \cap \Gamma$



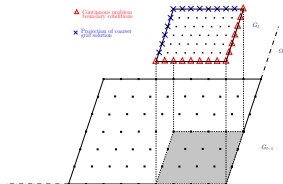
- Discrete problem on $G_l : \mathcal{L}_l(u_l^k) = f_l^k \forall l, k$
 ↪ no operator modification
- Prolongation step : BC definition on the fine grid G_l
 - BC of the **continuous problem** on $\Gamma_l \cap \Gamma$
 - **Dirichlet BC** on $\Gamma_l \setminus (\Gamma_l \cap \Gamma)$:

$$u_{\Gamma_l \setminus (\Gamma_l \cap \Gamma)}^k = P_{l-1}^l(u_{l-1}^k)|_{\Gamma_l \setminus (\Gamma_l \cap \Gamma)}$$



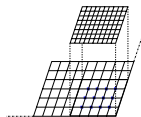
- Discrete problem on $G_l : \mathcal{L}_l(u_l^k) = f_l^k \forall l, k$
 ↪ no operator modification
- Prolongation step : BC definition on the fine grid G_l
 - BC of the **continuous problem** on $\Gamma_l \cap \Gamma$
 - **Dirichlet BC** on $\Gamma_l \setminus (\Gamma_l \cap \Gamma)$:

$$u_{\Gamma_l \setminus (\Gamma_l \cap \Gamma)}^k = P_{l-1}^l(u_{l-1}^k)|_{\Gamma_l \setminus (\Gamma_l \cap \Gamma)}$$



- Discrete problem on $G_l : \mathcal{L}_l(u_l^k) = f_l^k \forall l, k$
↪ no operator modification
- Prolongation step : BC definition on the fine grid G_l
- Restriction step : voluminous efforts correction on next coarse grid G_{l-1}

- Discrete problem on $G_l : \mathcal{L}_l(u_l^k) = f_l^k \quad \forall l, k$
 ↪ no operator modification
- Prolongation step : BC definition on the fine grid G_l
- Restriction step : voluminous efforts correction on next coarse grid G_{l-1}
 - Restriction of the next finer solution at interior nodes : $\tilde{u}_{l-1}^k(x) = (R_l^{l-1} u_l^k)(x) \quad \forall x \in \blacksquare$



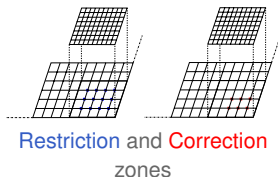
Restriction and Correction zones

- Discrete problem on $G_l : \mathcal{L}_l(u_l^k) = f_l^k \forall l, k$
 ↪ no operator modification
- Prolongation step : BC definition on the fine grid G_l
- Restriction step : voluminous efforts correction on next coarse grid G_{l-1}

- Restriction of the next finer solution at interior nodes : $\tilde{u}_{l-1}^k(x) = (R_l^{l-1} u_l^k)(x) \quad \forall x \in \blacksquare$
- Use the corresponding coarse defect as correction :

$$f_{l-1}^k(x) = f_{l-1}^0(x) + \chi_{\blacksquare}(\mathcal{L}_{l-1}(\tilde{u}_{l-1}^k) - f_{l-1}^0)(x)$$

↪ overlaying zone must be large enough



- Discrete problem on $G_l : \mathcal{L}_l(u_l^k) = f_l^k \forall l, k$
↪ no operator modification
- Prolongation step : BC definition on the fine grid G_l
- Restriction step : voluminous efforts correction on next coarse grid G_{l-1}
- Widely used in Fluid Mechanics, but few studied in Solid Mechanics (Barbié et al, 2014)

- Discrete problem on $G_l : \mathcal{L}_l(u_l^k) = f_l^k \forall l, k$
↪ no operator modification
- Prolongation step : BC definition on the fine grid G_l
- Restriction step : voluminous efforts correction on next coarse grid G_{l-1}
- Widely used in Fluid Mechanics, but few studied in Solid Mechanics (Barbié et al, 2014)
- Depends on zones of interest : either a priori known, or to be detected automatically

A posteriori error estimation in Solid Mechanics

- Estimation of the discretization error from a preliminary calculation
↳ automatic detection of the zones to be refined

- Estimation of the discretization error from a preliminary calculation
↳ automatic detection of the zones to be refined
- Main error estimations in Solid Mechanics

- Estimation of the discretization error from a preliminary calculation
 - ↪ automatic detection of the zones to be refined

- Main error estimations in Solid Mechanics
 - Residual type a posteriori error estimators (Babuška and Rheinboldt, 1978)
 - Based on the fact that FE simulation does not verify locally the equilibrium equation
 - Calculation of the interior element residuals and the jumps at the element boundaries
 - Explicit method : post-processing method, easy to implement, dependant on unknown coefficients
 - Implicit method : solve local problems, high computational costs

- Estimation of the discretization error from a preliminary calculation
 - ↪ automatic detection of the zones to be refined

- Main error estimations in Solid Mechanics
 - Residual type a posteriori error estimators (Babuška and Rheinboldt, 1978)
 - Constitutive relation error estimators (Ladevèze and Leguillon, 1983)
 - Based on the fact that stress field of FE solution is not statically admissible
 - Define local problems to construct a statically admissible solution
 - High computational costs

- Estimation of the discretization error from a preliminary calculation
 - ↪ automatic detection of the zones to be refined

- Main error estimations in Solid Mechanics
 - Residual type a posteriori error estimators (Babuška and Rheinboldt, 1978)
 - Constitutive relation error estimators (Ladevèze and Leguillon, 1983)
 - Recovery-based error estimators (Zienkiewicz and Zhu, 1987)
 - Based on the fact that the stress field of Lagrange FE method is discontinuous between elements
 - Construction of a smoothed stress field : recover the nodal values of stresses, then construct a continuous stress field using displacement shape functions
 - 2 methods (Projection method, SPR method) based on minimization process
 - Effective, easy to implement, widely used in industrial codes

- Estimation of the discretization error from a preliminary calculation
 - ↪ automatic detection of the zones to be refined

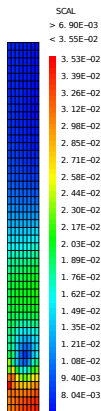
 - Main error estimations in Solid Mechanics
 - Residual type a posteriori error estimators (Babuška and Rheinboldt, 1978)
 - Constitutive relation error estimators (Ladevèze and Leguillon, 1983)
 - Recovery-based error estimators (Zienkiewicz and Zhu, 1987)
- ⇒ Use the Zienkiewicz and Zhu error estimator

- Construction of a **smoothed stress field**
↔ **Error = difference** between the FE and the smoothed stress fields

- Construction of a **smoothed stress field**
 - ↪ **Error = difference** between the FE and the smoothed stress fields

- One classical method to detect the **zones of interest**
 - **Zone to be refined** : elements K such that

$$e_K > \alpha \% (\max_L e_L)$$
 - 😊 Easy to implement, **works well for every kind of estimators and norms**
 - ☹ **α unknown**, requirement of a **stopping criteria** for LDC Method



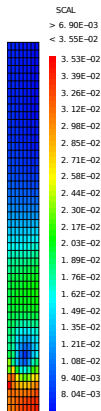
Example of ZZ a posteriori error estimation

- Construction of a **smoothed stress field**
 ↪ **Error = difference** between the FE and the smoothed stress fields

- One classical method to detect the **zones of interest**
 - **Zone to be refined** : elements K such that

$$e_K > \alpha\%(\max_L e_L)$$
 - 😊 Easy to implement, **works well for every kind of estimators and norms**
 - ☹ **α unknown**, requirement of a **stopping criteria** for LDC Method

⇒ We want to find a user-independent detection method



Example of ZZ a posteriori error estimation

■ Relative error estimator in energy norm :

$$\|e_K\|_E = \left(\frac{\int_K (\sigma^* - \sigma_h) : (\varepsilon^* - \varepsilon_h) dK}{\int_K \sigma^* : \varepsilon^* dK} \right)^{1/2}$$

with σ^*, ε^* smoothed solutions and $\varepsilon^* = [C]^{-1} \sigma^*$, σ, ε FE solutions

- Relative error estimator in energy norm :

$$\|e_K\|_E = \left(\frac{\int_K (\sigma^* - \sigma_h) : (\varepsilon^* - \varepsilon_h) dK}{\int_K \sigma^* : \varepsilon^* dK} \right)^{1/2}$$

with σ^*, ε^* smoothed solutions and $\varepsilon^* = [C]^{-1} \sigma^*$, σ, ε FE solutions

- Refine the elements for which the stress error is superior to a threshold :

$$\|e_K\|_E > \text{threshold.}$$

- Relative error estimator in energy norm :

$$\|e_K\|_E = \left(\frac{\int_K (\boldsymbol{\sigma}^* - \boldsymbol{\sigma}_h) : (\boldsymbol{\varepsilon}^* - \boldsymbol{\varepsilon}_h) dK}{\int_K \boldsymbol{\sigma}^* : \boldsymbol{\varepsilon}^* dK} \right)^{1/2}$$

with $\boldsymbol{\sigma}^*, \boldsymbol{\varepsilon}^*$ smoothed solutions and $\boldsymbol{\varepsilon}^* = [C]^{-1} \boldsymbol{\sigma}^*$, $\boldsymbol{\sigma}, \boldsymbol{\varepsilon}$ FE solutions

- Refine the elements for which the stress error is superior to a threshold :

$$\|e_K\|_E > \text{threshold.}$$

- Global error

$$\begin{aligned} \|e_\Omega\|_E &= \left(\frac{\int_\Omega (\boldsymbol{\sigma}^* - \boldsymbol{\sigma}_h) : (\boldsymbol{\varepsilon}^* - \boldsymbol{\varepsilon}_h) d\Omega}{\int_\Omega \boldsymbol{\sigma}^* : \boldsymbol{\varepsilon}^* d\Omega} \right)^{1/2} \\ &= \left(\frac{\sum \int_K (\boldsymbol{\sigma}^* - \boldsymbol{\sigma}_h) : (\boldsymbol{\varepsilon}^* - \boldsymbol{\varepsilon}_h) dK}{\sum \int_K \boldsymbol{\sigma}^* : \boldsymbol{\varepsilon}^* dK} \right)^{1/2} < \text{threshold} \end{aligned}$$

■ Error estimator in maximal norm :

$$\|e_K\|_\infty = \max_{\text{Gauss point in } K} |\sigma^* - \sigma_h|$$

with σ^* smoothed solutions and σ FE solutions

■ Error estimator in maximal norm :

$$\|e_K\|_\infty = \max_{\text{Gauss point in } K} |\sigma^* - \sigma_h|$$

with σ^* smoothed solutions and σ FE solutions

■ Refine the elements for which the stress error is superior to a threshold :

$$\|e_K\|_\infty > \text{threshold.}$$

- Error estimator in maximal norm :

$$\|e_K\|_\infty = \max_{\text{Gauss point in } K} |\sigma^* - \sigma_h|$$

with σ^* smoothed solutions and σ FE solutions

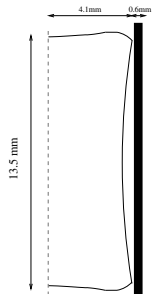
- Refine the elements for which the stress error is superior to a threshold :

$$\|e_K\|_\infty > \text{threshold.}$$

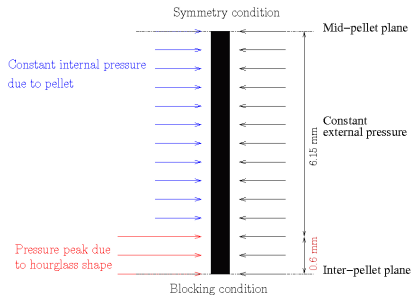
- Not widely used in the literature
- More interesting for mechanical engineer because it gives a local information

**Numerical results of
the combination of LDC
and ZZ estimator**

- Hourglass shape deformation :
2D(r, z) test case

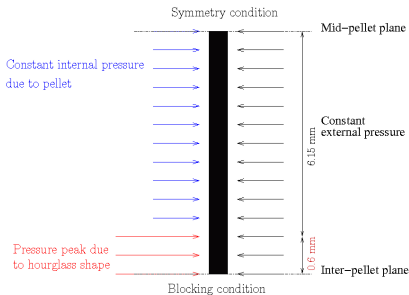


- Hourglass shape deformation :
 $2D(r, z)$ test case
- Focus on the elastic response of the cladding



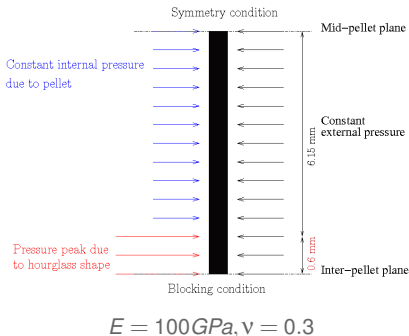
$$E = 100 \text{ GPa}, \nu = 0.3$$

- Hourglass shape deformation : $2D(r, z)$ test case
- Focus on the elastic response of the cladding
- Contact with the pellet = discontinuous pressure imposed on the internal radius of the cladding

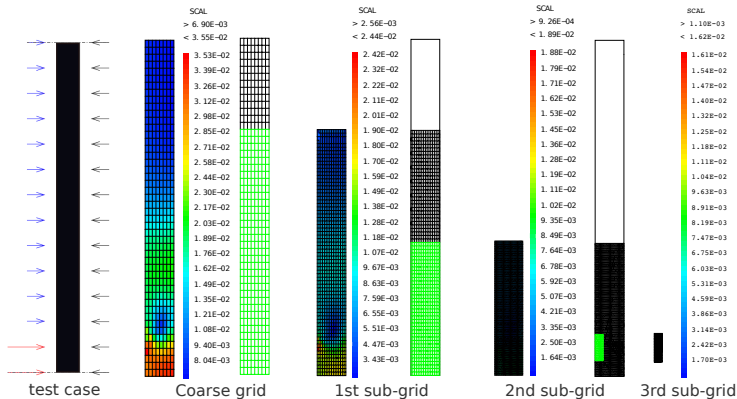


$$E = 100 \text{ GPa}, \nu = 0.3$$

- Hourglass shape deformation :
 $2D(r, z)$ test case
- Focus on the elastic response of the cladding
- Contact with the pellet = discontinuous pressure imposed on the internal radius of the cladding
- LDC coupled with ZZ error estimator
 - Generate sub-grids at the first prolongation step
 - Number of level of sub-grids generated automatically



Relative error in energy norm



Example for threshold 1%

- Number of levels stops automatically
- Sub-grids localized around the pressure discontinuity

■ Relative error in energy norm

mesh	threshold			
	5%	2%	1%	0.50%
hi = 328 μm	4.5%	2.04%	0.614%	0.366%
hi/2	4.20%	1.98%	0.614%	0.366%
hi/4	1.92%	1.92%	0.527%	0.366%
hi/8	0.757%	0.757%	0.433%	0.322%

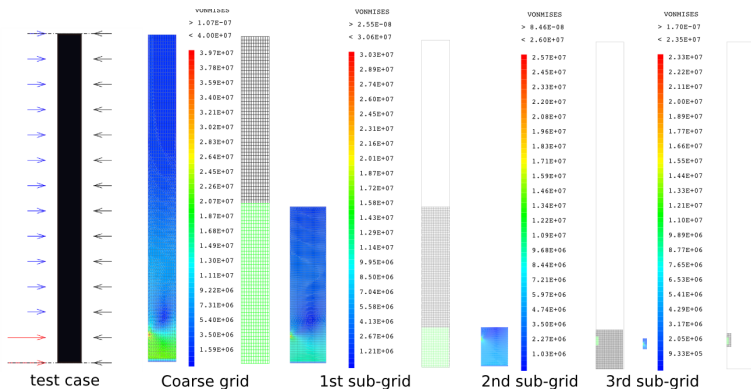
- Error is calculated between the composite solution and the reference solution which is calculated by a very fine mesh.
- Works well, obtained errors inferior to threshold for different thresholds and different initial coarse meshes.

■ Relative error in energy norm

mesh	threshold			
	5%	2%	1%	0.50%
hi =328 μm	4.5%	2.04%	0.614%	0.366%
hi/2	4.20%	1.98%	0.614%	0.366%
hi/4	1.92%	1.92%	0.527%	0.366%
hi/8	0.757%	0.757%	0.433%	0.322%

- Error is calculated between the composite solution and the reference solution which is calculated by a very fine mesh.
- Works well, obtained errors inferior to threshold for different thresholds and different initial coarse meshes.
- Error can be sub-estimated by ZZ estimator \rightarrow security coefficient

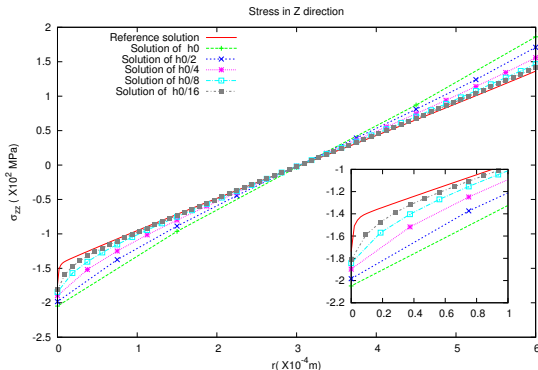
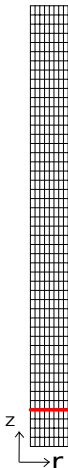
Maximal norm



Example for threshold $5E+6$ Pa

- Generation of sub-levels **doesn't stop**
- Zone detected **more localized**

Maximal norm



Stress for different meshes

Stress singularity near the pressure discontinuity

Never converge with the mesh step

→ Find a method to stop the generation of sub-grids

Conclusions and perspectives

■ Conclusions

- Use the solver as a "black box"
- Coupling of ZZ error estimator and LDC method works well.
- Zones of interest are detected automatically
 - Energy norm : automatically stop
 - Maximal norm : generation of sub-grids doesn't stop (singularity)
- Numerical results of the 2D(r,z) test case are promising as zones of interest are more and more localized and the errors decrease.

■ Conclusions

- Use the solver as a "black box"
- Coupling of ZZ error estimator and LDC method works well.
- Zones of interest are detected automatically
 - Energy norm : automatically stop
 - Maximal norm : generation of sub-grids doesn't stop (singularity)
- Numerical results of the 2D(r,z) test case are promising as zones of interest are more and more localized and the errors decrease.

■ Perspectives

- Error in maximal norm
 - Find a user-independent method to stop generating sub-grids
- Nonlinear test case
- Comparison with h-adaptive method (most used space refinement method)

Thank you

This work has been achieved in the framework of the PLEIADES project, financially supported by CEA (Commissariat à l'Énergie Atomique et aux Énergies Alternatives), EDF (Électricité de France) and AREVA.