

# Local Defect Correction Method coupled with the Zienkiewicz-Zhu a posteriori error estimator in elastostatics Solid Mechanics

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DE LA RECHERCHE À L'INDUSTRIE



# Local Defect Correction Method coupled with the Zienkiewicz-Zhu a posteriori error estimator in elastostatics Solid Mechanics



CMCMM 2015 | HAO LIU (DEN, DEC, SESC) Supervisors: I. RAMIÈRE (DEN, DEC, SESC) AND F. LEBON (LMA, AIX-MARSEILLE UNIVERSITY)

MARCH 22-27, 2015





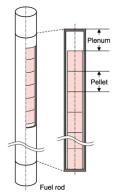
- Industrial context
- Local Defect Correction Method
- A posteriori error estimation in Solid Mechanics
- Numerical results of the combination of LDC and ZZ estimator
- Conclusions and perspectives

# Industrial context





- Pressurized Water Reactors
  - Fuel and Cladding

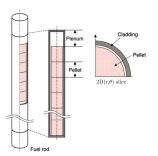






#### Pressurized Water Reactors

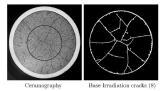
#### Fuel and Cladding



#### During irradiation

#### - Pellet cracking

The pellet cracks and swells and the cladding creeps



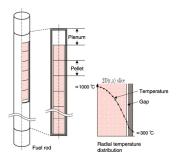
⇒ Discontinuous contacts between pellet and cladding





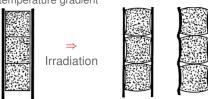
#### Pressurized Water Reactors

Fuel and Cladding



#### During irradiation

 Hourglass shape deformation
 Due to the deforming effects of the high temperature gradient



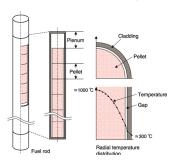
⇒ Contacts occur first in front of the inter-pellet plane





#### Pressurized Water Reactors

#### Fuel and Cladding



#### During irradiation

- Pellet cracking
- Hourglass shape deformation
  - $\Rightarrow$  Localized high stress concentrations on the cladding
  - $\Rightarrow$  Integrity of the cladding (security stake)





■ Simulation in PLEIADES fuel performance software environment





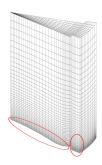
- Simulation in PLEIADES fuel performance software environment
- Representation of 1/32 of the Pellet and the corresponding section of cladding: precise simulations require cells of  $1\mu m$  for a structure of 1cm (unreachable for uniform mesh)







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- Simulation in PLEIADES fuel performance software environment
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 $\hookrightarrow$  Local Mesh Refinement Methods : (h-,p-,r-) adaptive methods or local

multi-grids method





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    - No composite problem





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   → Local multi-grids method
  - Modeling by finite elements
  - No composite problem

# Local Defect Correction Method





- Adaptive Mesh Refinement methods
  - $\equiv$  Local Multigrid methods



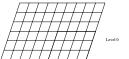


- Adaptive Mesh Refinement methods
  - Local Multigrid methods
    - Inverse multi-grid process :





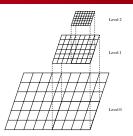
- Adaptive Mesh Refinement methods
  - ≡ Local Multigrid methods
    - Inverse multi-grid process :
      - Global coarse grid







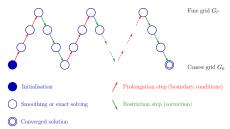
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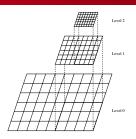






- Adaptive Mesh Refinement methods ≡ Local Multigrid methods
  - Inverse multi-grid process :
    - Global coarse grid
    - Local finer grids generated recursively in zones of interest
  - Prolongation (BC interpolation) and Restriction operators link the grids
    - $\hookrightarrow$  Iterative process ( $\land$ -cycles)





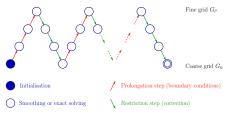


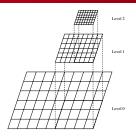


- Adaptive Mesh Refinement methods
   ≡ Local Multigrid methods
  - Inverse multi-grid process :
    - Global coarse grid

Converged solution

- Local finer grids generated recursively in zones of interest
- Prolongation (BC interpolation) and Restriction operators link the grids
  - $\hookrightarrow$  Iterative process ( $\land$ -cycles)





- Work on local fine meshes, Fast resolution on each sub-grid, Generic method: mesh, solver, model, refinement ratio
- Several meshes, Iterative process, Accuracy depending on prolongation and restriction operators





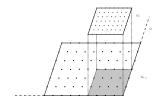
■ Discrete problem on  $G_l$ :  $\mathcal{L}_l(u_l^k) = f_l^k \ \forall l, k$  $\hookrightarrow$  no operator modification

Hao Liu | Copper Mountain, March 22-27, 2015 | PAGE 5/15





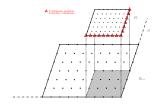
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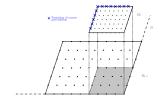
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  - **■** BC of the continuous problem on  $\Gamma_I \cap \Gamma$







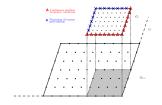
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  - **BC** of the continuous problem on  $\Gamma_I \cap \Gamma$
  - Dirichlet BC on  $\Gamma_l \setminus (\Gamma_l \cap \Gamma)$ :  $u_{\Gamma_l \setminus (\Gamma_l \cap \Gamma)}^k = P_{l-1}^l (u_{l-1}^k)|_{\Gamma_l \setminus (\Gamma_l \cap \Gamma)}$







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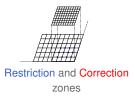


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  - Restriction of the next finer solution at interior nodes :  $\tilde{u}_{l-1}^k(x) = (R_l^{l-1}u_l^k)(x) \quad \forall x \in \blacksquare$

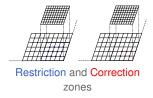




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  - Restriction of the next finer solution at interior nodes :  $\tilde{u}_{l-1}^k(x) = (R_l^{l-1}u_l^k)(x) \quad \forall x \in \blacksquare$
  - Use the corresponding coarse defect as correction :

$$f_{l-1}^k(x) = f_{l-1}^0(x) + \chi_{\blacksquare}(\mathcal{L}_{l-1}(\tilde{u}_{l-1}^k) - f_{l-1}^0)(x)$$

 $\hookrightarrow$  overlaying zone must be large enough







- Discrete problem on  $G_l$ :  $\mathcal{L}_l(u_l^k) = f_l^k \ \forall l, k$  $\hookrightarrow$  no operator modification
- Prolongation step : BC definition on the fine grid G<sub>I</sub>
- Restriction step : voluminous efforts correction on next coarse grid  $G_{l-1}$
- Widely used in Fluid Mechanics, but few studied in Solid Mechanics (Barbié et al, 2014)



#### Local Defect Correction method (Hackbush, 1984)



- Discrete problem on  $G_l$ :  $\mathcal{L}_l(u_l^k) = f_l^k \ \forall l, k$  $\hookrightarrow$  no operator modification
- Prolongation step : BC definition on the fine grid G<sub>I</sub>
- Restriction step : voluminous efforts correction on next coarse grid  $G_{l-1}$
- Widely used in Fluid Mechanics, but few studied in Solid Mechanics (Barbié et al, 2014)
- Depends on zones of interest : either a priori known, or to be detected automatically

# A posteriori error estimation in Solid Mechanics





■ Estimation of the discretization error from a preliminary calculation

→ automatic detection of the zones to be refined





- Estimation of the discretization error from a preliminary calculation → automatic detection of the zones to be refined
- Main error estimations in Solid Mechanics





- Estimation of the discretization error from a preliminary calculation
- Main error estimations in Solid Mechanics
  - Residual type a posteriori error estimators (Babuška and Rheinboldt, 1978)
    - Based on the fact that FE simulation does not verify locally the equilibrium equation
    - Calculation of the interior element residuals and the jumps at the element boundaries
    - Explicit method: post-processing method, easy to implement, dependant on unknown coefficients
    - Implicit method : solve local problems, high computational costs





- Estimation of the discretization error from a preliminary calculation
- Main error estimations in Solid Mechanics
  - Residual type a posteriori error estimators (Babuška and Rheinboldt, 1978)
  - Constitutive relation error estimators (Ladevèze and Leguillon, 1983)
    - Based on the fact that stress field of FE solution is not statically admissible
    - Define local problems to construct a statically admissible solution
    - High computational costs





- Estimation of the discretization error from a preliminary calculation
- Main error estimations in Solid Mechanics
  - Residual type a posteriori error estimators (Babuška and Rheinboldt, 1978)
  - Constitutive relation error estimators (Ladevèze and Leguillon, 1983)
  - Recovery-based error estimators (Zienkiewicz and Zhu, 1987)
    - Based on the fact that the stress field of Lagrange FE method is discontinuous between elements
    - Construction of a smoothed stress field: recover the nodal values of stresses, then construct a continuous stress field using displacement shape functions
    - 2 methods (Projection method, SPR method) based on minimization process
    - Effective, easy to implement, widely used in industrial codes





- Estimation of the discretization error from a preliminary calculation
- Main error estimations in Solid Mechanics
  - Residual type a posteriori error estimators (Babuška and Rheinboldt, 1978)
  - Constitutive relation error estimators (Ladevèze and Leguillon, 1983)
  - Recovery-based error estimators (Zienkiewicz and Zhu, 1987)
    - ⇒ Use the Zienkiewicz and Zhu error estimator



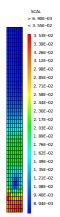


- Construction of a smoothed stress field
  - → Error = difference between the FE and the smoothed stress fields





- Construction of a smoothed stress field
- One classical method to detect the zones of interest
  - Zone to be refined : elements K such that  $e_K > \alpha\%(\max_{L} e_L)$
  - Easy to implement, works well for every kind of estimators and norms
  - α unknown, requirement of a stopping criteria for LDC Method

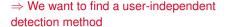


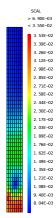
Example of ZZ a posteriori error estimation





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Example of ZZ a posteriori error estimation





Relative error estimator in energy norm :

$$\|e_K\|_E = \left(\frac{\int_K (\sigma^* - \sigma_h) : (\epsilon^* - \epsilon_h) \ dK}{\int_K \sigma^* : \epsilon^* \ dK}\right)^{1/2}$$

with  $\sigma^*, \epsilon^*$  smoothed solutions and  $\epsilon^* = [\mathcal{C}]^{-1} \sigma^*$ ,  $\sigma, \epsilon$  FE solutions





Relative error estimator in energy norm :

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Refine the elements for which the stress error is superior to a threshold :  $\|e_K\|_F > \text{threshold}.$ 





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- Refine the elements for which the stress error is superior to a threshold :  $\|e_K\|_F > \text{threshold}.$
- Global error

$$\|e_{\Omega}\|_{E} = \left(\frac{\int_{\Omega} (\sigma^{*} - \sigma_{h}) : (\epsilon^{*} - \epsilon_{h}) d\Omega}{\int_{\Omega} \sigma^{*} : \epsilon^{*} d\Omega}\right)^{1/2}$$

$$= \left(\frac{\sum \int_{K} (\sigma^{*} - \sigma_{h}) : (\epsilon^{*} - \epsilon_{h}) dK}{\sum \int_{K} \sigma^{*} : \epsilon^{*} dK}\right)^{1/2} < \text{threshold}$$





Error estimator in maximal norm :

$$\|e_K\|_{\infty} = \max_{\text{Gauss point in K}} |\sigma^* - \sigma_h|$$

with  $\sigma^*$  smoothed solutions and  $\sigma$  FE solutions





Error estimator in maximal norm :

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$$||e_K||_{\infty} >$$
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■ Refine the elements for which the stress error is superior to a threshold :

$$||e_K||_{\infty} >$$
threshold.

- Not widely used in the literature
- More interesting for mechanical engineer because it gives a local information

Numerical results of the combination of LDC and ZZ estimator



#### Test case

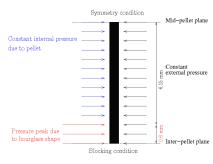


■ Hourglass shape deformation : 2D(r,z) test case





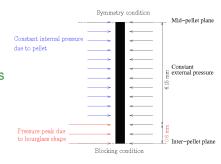
- Hourglass shape deformation : 2D(r,z) test case
- Focus on the elastic response of the cladding



$$E = 100$$
 GPa,  $v = 0.3$ 



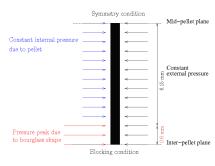
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- Contact with the pellet = discontinuous pressure imposed on the internal radius of the cladding



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 GPa,  $v = 0.3$ 



- Hourglass shape deformation : 2D(r,z) test case
- Focus on the elastic response of the cladding
- Contact with the pellet = discontinuous pressure imposed on the internal radius of the cladding
- LDC coupled with ZZ error estimator
  - Generate sub-grids at the first prolongation step
  - Number of level of sub-grids generated automatically

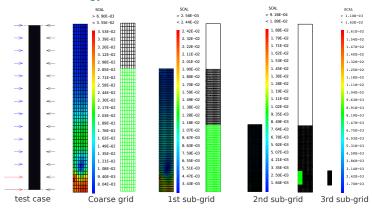


$$E = 100$$
 *GPa*,  $v = 0.3$ 





■ Relative error in energy norm



Example for threshold 1%

- Number of levels stops automatically
- Sub-grids localized around the pressure discontinuity





■ Relative error in energy norm

mesh	threshold				
	5%	2%	1%	0.50%	
hi =328 μm	4.5%	2.04%	0.614%	0.366%	
hi/2	4.20%	1.98%	0.614%	0.366%	
hi/4	1.92%	1.92%	0.527%	0.366%	
hi/8	0.757%	0.757%	0.433%	0.322%	

- Error is calculated between the composite solution and the reference solution which is calculated by a very fine mesh.
- Works well, obtained errors inferior to threshold for different thresholds and different initial coarse meshes.





■ Relative error in energy norm

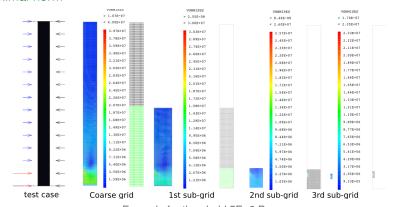
mesh	threshold				
	5%	2%	1%	0.50%	
hi =328 μm	4.5%	2.04%	0.614%	0.366%	
hi/2	4.20%	1.98%	0.614%	0.366%	
hi/4	1.92%	1.92%	0.527%	0.366%	
hi/8	0.757%	0.757%	0.433%	0.322%	

- Error is calculated between the composite solution and the reference solution which is calculated by a very fine mesh.
- Works well, obtained errors inferior to threshold for different thresholds and different initial coarse meshes.
- ullet Error can be sub-estimated by ZZ estimator o security coefficient





#### Maximal norm



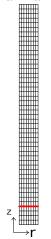
Example for threshold 5E+6 Pa

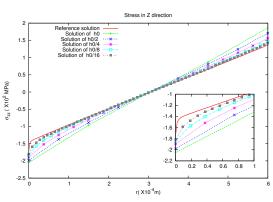
- Generation of sub-levels doesn't stop
- Zone detected more localized





#### Maximal norm





Stress for different meshes

- Stress singularity near the pressure discontinuity
- Never converge with the mesh step
  - $\rightarrow$  Find a method to stop the generation of sub-grids

## Conclusions and perspectives



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#### Conclusions

- Use the solver as a "black box"
- Coupling of ZZ error estimator and LDC method works well.
- Zones of interest are detected automatically
  - Energy norm : automatically stop
  - Maximal norm : generation of sub-grids doesn't stop (singularity)
- Numerical results of the 2D(r,z) test case are promising as zones of interest are more and more localized and the errors decrease.



#### Conclusions and perspectives



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#### Perspectives

- Error in maximal norm
  - $\rightarrow$  Find a user-independent method to stop generating sub-grids
- Nonlinear test case
- Comparison with h-adaptive method (most used space refinement method)

### Thank you

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