

# Analysis of Vibration-induced Neutron Noise Using One-dimension Noise Diffusion Theory

Amélie ROUCHON, Richard SANCHEZ  
CEA-Saclay, DEN, DM2S, SERMA  
F-91191 Gif-sur-Yvette  
France

Email:amelie.rouchon@cea.fr; richard.sanchez@cea.fr

**Abstract** – *A one-dimension vibration model has been developed in order to simulate a pin vibration of period  $T_0$  in a one-dimensional core and to determine the noise flux generated by this perturbation. We find that this source perturbation excites all multiples of the vibration frequency  $f_0=1/T_0$ . In this work, we analyze a new method aimed to improve traditional linear noise theory. This technique is similar to the traditional linearized noise equations but it uses a different steady-state flux. We compare this method with the exact solution of the non-linearized, fully-coupled noise equations taking into account all the terms neglected in linear theory. The results of the comparisons for a fuel pin vibration in a one-dimensional core are analyzed in four-groups diffusion theory. The temporal reconstruction of the noise flux from its Fourier transform shows that the second harmonic of the noise source is not negligible and should be taken into account, and also that the new method is based on a steady-state flux closer to the steady-state flux of the exact solution compared to the steady-state flux of the traditional method.*

## I. INTRODUCTION

Neutron noise techniques [1] are widely used by the nuclear industry for non-invasive general monitoring, control and detection of anomalies in nuclear power plants. They have also applications to the measurement of the properties of the coolant, such as speed and void fraction. Neutron noise appears as fluctuations of the neutron field induced by stochastic or deterministic changes in the cross-sections. The latter may result from vibrations of fuel elements, control rods or any other mechanical structures in the core, as well as from global or local fluctuations in the flow, density or void fraction of the coolant.

In power reactors, ex-core and in-core detectors can be used to detect neutron noise and pinpoint to the causes behind it so as to take the necessary measures for continuous safe power production. In order to do this, one has to simulate noise calculations and compute the changes in the neutron field produced by different sources representative of the different causes of noise in nuclear cores.

The general noise equations are obtained by assuming small perturbations around a steady state in the neutron field and then by Fourier transform onto the frequency domain. The analysis is conducted with the neutron kinetic

equations including the coupling with neutron precursors. The result is a source equation for the perturbed neutron field which can then be solved to predict noise measurements at detector locations. For each frequency the neutron field has intensity and phase and it is therefore a complex function in the frequency domain.

In this paper, we present the noise flux generated by a fuel pin vibration in a one-dimensional core in multigroup diffusion theory. We find that this source excites all multiples of the vibration frequency  $f_0=1/T_0$ . Because the traditional linearization method of noise theory is not theoretically justified for this type of perturbation [2], in this work we analyze a new method aimed to improve traditional linear noise theory and compare this method to the exact solution. In section II, the general noise theory will be briefly discussed, while the new method will be presented just as the exact solution method. In section III, the vibration model is detailed especially its numerical convergence. In section IV, we discuss and compare the noise flux in the frequency domain and the total flux in the time domain for these different solution methods in case of a fuel pin vibration in a one-dimensional core in four-groups diffusion theory. Finally, section V provides some general conclusions.

## II. NEUTRON NOISE THEORY

In this section, we present the general theory of neutron noise. The extension of these equations to diffusion theory is straightforward. To simplify our notation, we shall omit the velocity variable  $\mathbf{v} = (E, \boldsymbol{\Omega})$ . Note that the zero power noise (fluctuations inherent to the branching process) is always neglected in power reactor noise theory.

### II.A. The traditional noise equation

We assume small perturbations of the macroscopic cross-sections around the initial critical steady state following:

$$B_{0,T}(r)\psi_{0,T}(r) = 0, \forall r \in D. \quad (1)$$

where  $B_{0,T} = \boldsymbol{\Omega} \cdot \nabla + \Sigma_0 - H_0 - P_0$  is the initial steady-state operator with  $\Sigma_0$  the steady-state total cross-section,  $H_0$  the steady-state scattering operator,  $P_0$  the steady-state production operator,  $\psi_{0,T}$  the initial steady-state angular flux and  $D$  the geometric domain. We note  $k_T \approx 1$  the effective multiplication factor. We impose a temporal perturbation of the cross-sections so we need to consider the kinetic equation:

$$\left[ \frac{1}{v} \partial_t + B_T(r, t) \right] \psi_T(r, t) = 0, \forall (r, t) \in D \times \mathbb{R}, \quad (2)$$

where  $v$  is the neutron velocity,  $B_T = \boldsymbol{\Omega} \cdot \nabla + \Sigma - H - P$  the kinetic operator with  $\Sigma$  the total cross-section,  $H$  the scattering operator,  $P$  the production operator (with prompt and delayed fissions) and  $\psi_T$  the angular flux. We impose a periodic perturbation of the kinetic operator with a period  $T_0$ :

$$B_T(r, t) = B_{0,T}(r) + \delta B_T(r, t). \quad (3)$$

This perturbation is supposed to start at  $t = -\infty$  so the transition step is over and we are in an established regime. As for the kinetic operator  $B_T$ , we break down the flux into:

$$\psi_T(r, t) = \psi_{0,T}(r) + \delta\psi_T(r, t). \quad (4)$$

We call ‘‘noise flux’’ the term  $\delta\psi_T$ . Finally, introduction of perturbation expressions (3) and (4) into Eq. (2) leads to a kinetic source equation for the perturbation of the flux:

$$\left[ \frac{1}{v} \partial_t + B_T(r, t) \right] \delta\psi_T(r, t) = -\delta B_T(r, t)\psi_{0,T}(r). \quad (5)$$

Next, we assume that we can neglect the product of any two fluctuating quantities. The second order term

$\delta B_T \delta\psi_T$  is neglected and we obtain the traditional kinetic linearized equation:

$$\left[ \frac{1}{v} \partial_t + B_{0,T}(r) \right] \delta\psi_T(r, t) = -\delta B_T(r, t)\psi_{0,T}(r). \quad (6)$$

We want to find the unique periodic solution of this equation. We apply the Fourier transform and we obtain the traditional noise equation:

$$B_{0,T,\omega}(r)\delta\psi_T(r, \omega) = -\delta B_T(r, \omega)\psi_{0,T}(r), \forall \omega \in \mathbb{R}. \quad (7)$$

where  $B_{0,T,\omega} = \frac{i\omega}{v} + \boldsymbol{\Omega} \cdot \nabla + \Sigma_0 - H_0 - P_{0,\omega}$ . Note that we define the Fourier transform of a function  $f$  by  $f(\omega) = \int_{-\infty}^{+\infty} f(t)e^{-i\omega t} dt$ . Because of the kinetic precursor equation, the production operator  $P_{0,\omega}$  depends on the frequency. Because of the Fourier transform, for each frequency, the neutron field has intensity and phase and it is therefore a complex function in the frequency domain. The real and imaginary equations of the noise flux are coupled by two terms:  $i\omega/v$  and by the production operator  $P_{0,\omega}$ . We call ‘‘noise source’’ the right hand side of Eq. (7).

### II.B. The ‘new’ steady-state operator

Up to now, we have not made any specific assumptions concerning the form of  $\delta B_T$  excepted that it is the consequence of a small physical perturbation. Contrarily to what is traditionally assumed, we do not assume that the mean value of the perturbed term  $\delta B_T$  is zero. But note that at  $\omega = 0$ , Eq. (7) becomes:

$$B_{0,T}(r)\delta\psi_T(r, \omega = 0) = -\delta B_T(r, \omega = 0)\psi_{0,T}(r). \quad (8)$$

Indeed, we have  $B_{0,T,\omega=0} = B_{0,T}$ . But this equation is solvable if the noise source at  $\omega = 0$  is zero i.e. we must set the mean value of the noise source to zero. To obtain this, we define a new steady-state operator  $B_{0,NF}$ :

$$B_{0,NF}(r) = B_{0,T}(r) + \langle \delta B_T \rangle(r), \quad (9)$$

where  $\langle . \rangle$  is the time-average operator and  $NF$  refers to ‘‘New Flux’’. The kinetic operator  $B_{NF}$  is now broken down into:

$$B_{NF}(r, t) = B_{0,NF}(r) + \delta B_{NF}(r, t), \quad (10)$$

with  $\delta B_{NF}(r, t) = \delta B_T(r, t) - \langle \delta B_T \rangle(r)$ . Obviously, if the perturbation model implies that the mean value of  $\delta B_T$  is zero,  $B_{0,NF} = B_{0,T}$  and we have no problem at  $\omega = 0$ . This is the case with a simple cosine perturbation of the cross-sections but we will see that it is not the case for a vibration perturbation. This new steady-state operator

defines a new steady-state flux, called  $\psi_{0,NF}$ , and a new eigenvalue, called  $k_{NF}$ . We also break down the flux into:

$$\psi_{NF}(r, t) = \psi_{0,NF}(r) + \delta\psi_{NF}(r, t). \quad (11)$$

As for the traditional method, we use the first linear theory and the Fourier transform. Eq. (7) becomes:

$$B_{0,NF,\omega}(r)\delta\psi_{NF}(r, \omega) = -\delta B_{NF}(r, \omega)\psi_{0,NF}(r), \quad (12)$$

with  $\delta\psi_{NF}(r, \omega = 0) = 0$  because we choose to impose  $\langle\psi_{NF}\rangle(r) = \psi_{0,NF}(r)$ .

Until now, we have introduced this new reference steady-state operator  $B_{0,NF}$  in order to solve a mathematical problem but we have to find a physical signification of this new decomposition of the kinetic operator. In power nuclear reactors, a system of regulating rods automatically stabilizes the output power within very narrow limits around the nominal or desired power output. In the case where a perturbation, such as a local vibration, introduces a non-zero amount of reactivity in the steady state of the reactor, the regulating rods act to level the power around the desired output, which amounts to adding reactivity opposite to that introduced by the vibration. The action of the regulating rods effectively cancels the time-averaged reactivity added by the perturbation and has to be accounted for in any sensible modeling of neutron noise. Since an exact modeling of the rods' action is not realistic, a way to implement it is to set the mean value of the perturbation to zero, but this modifies the modeling of the perturbation itself which should be independent of the calculation of the neutron noise. We think that the job can be done with less ambiguity by choosing as reference operator the time-averaged one adjusting thus the effective multiplication factor  $k_{NF}$  to a value slightly different to that for the initial state describes in section II.A.

### II.C. The exact solution of the non-linearized, fully-coupled noise equations

By iterations, we can take into account the term  $\delta B_T \delta\psi_T$  neglected in linear theory, and thus obtain the exact solution of the fully-coupled noise equations. However, we must be careful with the noise equation at  $\omega = 0$ . We want to set the noise source at  $\omega = 0$  to zero as in section II.B. So, we want to find a reference steady-state operator, called  $B_{0,R}$ , and a reference steady-state flux, called  $\psi_{0,R}$ , which permits us to have a nil source at  $\omega = 0$ . Let:

$$B_R(r, t) = \underbrace{[B_{0,T}(r) + \alpha(r)]}_{B_{0,R}(r)} + \underbrace{[\delta B_T(r, t) - \alpha(r)]}_{\delta B_R(r, t)}, \quad (14)$$

$$\psi_R(r, t) = \psi_{0,R}(r) + \delta\psi_R(r, t), \quad (15)$$

with  $B_{0,R}(r)\psi_{0,R}(r) = 0$  the new steady state (with the eigenvalue  $k_R$ ) and  $\alpha$  a function of  $r$ . We have:

$$\left[\frac{1}{v}\partial_t + B_{0,R}(r)\right]\delta\psi_R(r, t) = -\delta B_R(r, t)\psi_R(r, t). \quad (16)$$

We apply Fourier transform and we obtain:

$$B_{0,R,\omega}(r)\delta\psi_R(r, \omega) = -\frac{1}{2\pi}[\delta B_R * \psi_R](r, \omega), \quad (17)$$

with  $*$  the convolution operator. Hence, at  $\omega = 0$  we have:

$$\begin{aligned} B_{0,R}(r)\delta\psi_R(r, \omega = 0) &= -\frac{1}{2\pi}[\delta B_R * \psi_R](r, \omega = 0) \\ &= -\left[\frac{1}{2\pi}[\delta B_T * \psi_R](r, \omega = 0) - \alpha(r)\psi(r, \omega = 0)\right]. \end{aligned} \quad (18)$$

We want to cancel out the noise source at  $\omega = 0$  and we find that for

$$\alpha(r) = \frac{\frac{1}{2\pi}[\delta B_T * \psi_R](r, \omega = 0)}{\psi_R(r, \omega = 0)} = \frac{\langle\delta B_T * \psi_R\rangle(r)}{\langle\psi_R\rangle(r)}, \quad (19)$$

the noise source vanishes. Thus, we have a new steady-state operator  $B_{0,R}(r) = B_{0,T}(r) + \frac{\langle\delta B_T * \psi_R\rangle(r)}{\langle\psi_R\rangle(r)}$  which permits us to have no noise source at  $\omega = 0$  and, as in section II.B, to have  $\delta\psi_R(r, \omega = 0) = 0$  so that  $\langle\psi_R\rangle(r) = \psi_{0,R}(r)$ .

## III. VIBRATION-GENERATED NOISE SOURCE

In this section, we will detail the principal steps of the one-dimension vibration model. Here we do not use the Taylor expansion of the noise source [2], [3] and we find that this source excites all multiples of the vibration frequency  $f_0 = 1/T_0$  as it was earlier pointed out [4]. We also comment on the numerical convergence of this model.

### III.A. A multi-frequency noise source

We will describe the variations of all cross-sections when a fuel pin vibrates with an angular frequency  $\omega_0 = 2\pi/T_0$  and with a maximal amplitude  $\Delta_L$ . We define four perturbed regions in our model which are depicted in Fig. 1(a).

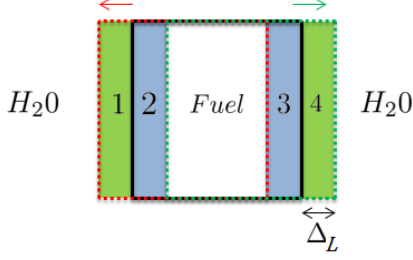


Fig. 1(a) - The four regions affected by the vibration

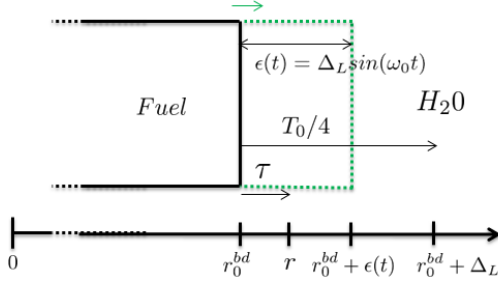


Fig. 1(b) - Zoom in on the fourth region

Fig. 1 - Fuel pin vibration scheme

We will develop our calculation for the cross-sections of region 4 (see Fig. 1(b)). Let  $r \in [r_0^{bd}, r_0^{bd} + \Delta_L]$  with  $r_0^{bd}$  the steady-state position of the right boundary of the vibrated pin and  $\epsilon(t) = \Delta_L \sin(\omega_0 t)$  for all  $t \in \mathbb{R}$ . We have:

$$\delta\Sigma(r, t) = \Delta_\Sigma H(r_0^{bd} + \epsilon(t) - r), \quad (20)$$

where:

- $\delta\Sigma(r, t)$  is the perturbed cross-section term in  $r$  at  $t$  ( $\Sigma(r, t) = \Sigma_0(r) + \delta\Sigma(r, t)$ ),
- $\Delta_\Sigma$  is the amplitude of the cross-section perturbation (in our case  $\Delta_\Sigma = \Sigma_{0, fuel} - \Sigma_{0, H_2O}$ ),
- $H$  is the Heaviside function.

We take  $\Delta_L$  to be a small fraction of the pin size. For  $r \in [r_0^{bd}, r_0^{bd} + \Delta_L]$ ,  $H(r_0^{bd} + \epsilon(t) - r) = 1$  in a continuous interval  $[\tau(r), \frac{T_0}{2} - \tau(r)]$ . The Fourier transform of the periodic function  $\delta\Sigma$  is:

$$\begin{aligned} \delta\Sigma(r, \omega) &= 2\pi\Delta_\Sigma \left( \frac{1}{2} - 2\frac{\tau(r)}{T_0} \right) \delta(\omega) \\ &\quad - 2\Delta_\Sigma \sum_{p=-\infty, p \neq 0}^{p=+\infty} \frac{\sin(2p\omega_0\tau(r))}{2p} \delta(\omega - 2p\omega_0) \\ &\quad - 2i\Delta_\Sigma \sum_{p=-\infty}^{p=+\infty} \frac{\cos((2p+1)p\omega_0\tau(r))}{2p+1} \delta(\omega - (2p+1)\omega_0), \end{aligned} \quad (21)$$

with  $\tau(r) = \sin^{-1}\left(\frac{r-r_0^{bd}}{\Delta_L}\right)/\omega_0$  and  $\delta$  the Kronecker symbol.

Now, we introduce a meshing in our geometry. We use a simple diamond scheme for our numerical solution so the flux is space-independent on each mesh. For simplicity we use the same mesh size  $\Delta_{\text{mesh}}$  in the four regions with  $\Delta_L = q\Delta_{\text{mesh}}$  (with  $q \in \mathbb{N}^*$ ). We integrate the noise source  $\delta\Sigma(r, \omega)\psi_0(r)$  on each mesh included in  $[r_0^{bd}, r_0^{bd} + \Delta_L]$ . Finally, we derive simple relations between the perturbed cross-sections of these four regions (the lower indexes 1,2,3 and 4 refer to the four regions):

$$\begin{aligned} \delta\Sigma_2(\omega) &= -\delta\Sigma_4(\omega), \\ \delta\Sigma_1(\omega) &= \delta\Sigma_4(\omega)e^{-i\frac{\pi}{\omega_0}\omega}, \\ \delta\Sigma_3(\omega) &= -\delta\Sigma_4(\omega)e^{-i\frac{\pi}{\omega_0}\omega}. \end{aligned} \quad (22)$$

In particular, for  $\omega = (2p+1)\omega_0$  ( $p \in \mathbb{Z}$ ) we have:

$$\delta\Sigma_1(\omega) = \delta\Sigma_2(\omega) = -\delta\Sigma_3(\omega) = -\delta\Sigma_4(\omega), \quad (23)$$

and for  $\omega = 2p\omega_0$  we have:

$$\delta\Sigma_1(\omega) = \delta\Sigma_4(\omega) = -\delta\Sigma_3(\omega) = -\delta\Sigma_2(\omega). \quad (24)$$

These results confirm the vibration-generated noise source excites all multiples of the vibration frequency  $f_0$ . We remark that the mean value of each perturbed cross-section term (proportional to  $\delta\Sigma(r, \omega = 0)$ ) is not zero as we mentioned it previously.

For the exact solution described in section II.C, we choose to impose a convolution centered in  $\delta\psi_R(r, \omega = 0)$  i.e., for all  $m \in \mathbb{Z}^*$ :

$$[\delta B_R * \delta\psi_R](r, m\omega_0) = \sum_{n=-\infty}^{n=+\infty} \delta B_R(r, (m-n)\omega_0) \delta\psi_R(r, n\omega_0). \quad (25)$$

If we choose a convolution amplitude of  $N$ , we have to solve a system of  $2N+1$  equations with  $2N+1$  unknowns ( $2N$  in reality because we impose  $\delta\psi_R(r, \omega = 0) = 0$ ). Note that this scheme converges when one increases the convolution amplitude.

### III.B. Numerical convergence of the vibration model

One of the most important points about the vibration model presented in the previous section III.A is that we have to choose a numerical meshing refined enough in order to correctly represent the sources for all frequencies of interest. For example, if we use only one mesh in each region, we obtain (the lower index 4 refers to the fourth region):

$$\begin{aligned} \delta\Sigma_4(r, t) &= \Delta_\Sigma \left[ -\frac{i\pi}{2} (\delta(\omega - \omega_0) - \delta(\omega + \omega_0)) \right. \\ &\quad \left. - 2 \sum_{p=-\infty}^{p=+\infty} \frac{\delta(\omega - 2p\omega_0)}{4p^2 - 1} \right], \end{aligned} \quad (26)$$

and we lose all the odd frequencies so this meshing is not refined enough within a single mesh point. In the same way, if we choose to unify regions 3 and 4, we have:

$$\delta\Sigma_{3-4}(r, t) = -\frac{i\pi}{2} \Delta\Sigma(\delta(\omega-\omega_0) - \delta(\omega+\omega_0)), \quad (27)$$

and this result is the Fourier transform of a simple sine oscillation. An infinity of frequencies is lost and only the first harmonic outlives this poorly refined meshing. So the numerical meshing must be, at a minimal level, greater than one mesh per region.

Next, we check the convergence of the vibration model when we increase the refinement of the meshing in the four perturbed regions. To do this, we will compare six different meshings in a one-dimensional core in two-groups (fast and thermal) with vacuum boundaries. The core is composed by 17 assemblies themselves composed by 17 fuel cells. We choose to perturb the central fuel pin of this symmetric geometry with a vibration frequency  $f_0 = 1$  Hz and a particularly small amplitude of 0.018 cm (the fuel pin size is 1.08 cm and the inter-pin size is 0.36 cm so one cell size is 1.44 cm). Here we simply use the traditional linearization method of section II.A in diffusion theory and we only show the results for the modulus of the fast noise flux. Figure 2 illustrates the results.

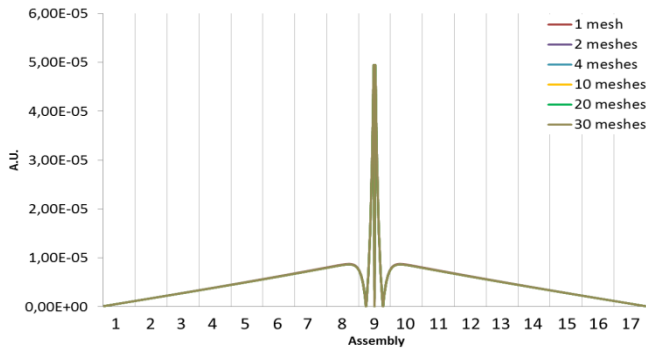


Fig. 2 (a) - Moduli of the fast noise flux at  $f_0$

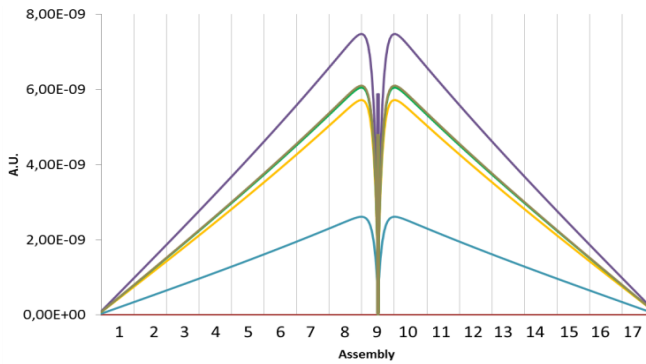


Fig. 2(b) - Moduli of the fast noise flux at  $7f_0$

Fig. 2 - Convergence of the vibration model in function of the mesh refinement

We find that the vibration model converges if we increase the refinement of the meshing. We find the same convergence with the thermal noise flux. Note that the higher the harmonic order is the more the mesh has to be refined.

#### IV. ANALYSIS OF A FUEL PIN VIBRATION IN A ONE-DIMENSIONAL CORE

In this section, we present the results of a fuel pin vibration in the one-dimension diffusion theory with four-groups (and six precursors groups). Note that the fluctuations of the diffusion coefficients are disregarded. We also neglect the term  $\frac{1}{v^g} \partial_t J^g$  (with  $J^g$  the scalar current of group  $g$ ). Groups 1 and 2 are fast groups and groups 3 and 4 are thermal groups. We work with the same core of section III.B. but we choose to perturb the central fuel pin of the third assembly with an amplitude of 0.09 cm (with 20 meshes per perturbed region). We use also a frequency  $f_0 = 1$  Hz, a common frequency for a fuel pin vibration. The fuel pin moves to the right side during the interval  $[0, \frac{T_0}{2}]$  and to the left side during the interval  $[\frac{T_0}{2}, T_0]$ .

Firstly, we quickly present the global and local components of the noise flux in order to understand more easily the following results. After that, we compare the noise fluxes in the frequency domain and the total fluxes in the time domain between three solution methods:

- the traditional method noted  $\Phi_T$  with the steady-state flux  $\Phi_{0,T}$  (section II.A, notation  $T$  for “Traditional”);
- the method with  $B_{0,NF}$  noted  $\Phi_{NF}$  with the steady-state flux  $\Phi_{0,NF}$  (section II.B, notation  $NF$  for “New Flux”);
- the exact solution (in diffusion theory) noted  $\Phi_R$  with the steady-state flux  $\Phi_{0,R}$  (section II.C, we choose a large convolution amplitude of 15, notation  $R$  for “Reference”);

To simplify the analysis, we present the results for groups 1 and 4. Group 2 results (resp. 3) are equivalents to group 1 results (resp. 4). For all calculations, the maximal relative errors of the noise flux and the fission sources between two iterations are  $10^{-6}$ .

#### IV.A. General proprieties of the global and local components

The neutron noise in a nuclear reactor can be separated into two components: a global and a local components [5], [6]. The local component is rapidly changing along the axis and is of a high frequency nature while the global component is slowly varying in space and is confined to low frequencies. The local component corresponds to a rapid relaxation in space and so exists only in the vicinity of the perturbation, and the global component follows a much lower relaxation and impacts the entire core. In the time domain, the noise flux is always composed by these two components but the balance between them can be different in function of the energy group, the vibration frequency or the core size. Here we have some important proprieties of these components:

- the global component dominates in low frequencies while the local in high frequencies;
- the global component dominates in small or tightly coupled core while the local in large or few coupled core;
- the local component affects more the thermal noise flux than the fast noise flux;
- the global component can be described by the point reactor model in small core, not in large core;
- the global component should be more affected by the high order terms of the noise flux than the local component.

#### IV.B. The noise flux harmonics in the frequency domain

Figure 3 compares the moduli of harmonics 1, 2, 3 and 4 of the exact noise flux in groups 1 and 4. We use 20 meshes per perturbed region so we have a good convergence of the noise source for all these harmonics. We remark that harmonics 1 and 2 dominate and that the contribution of harmonics 3 and 4 (and therefore of the higher-order harmonics) can be neglected. We have obtained similar results with the other methods. Thus, the more important harmonics of the noise source are at  $f_0$  and  $2f_0$ .

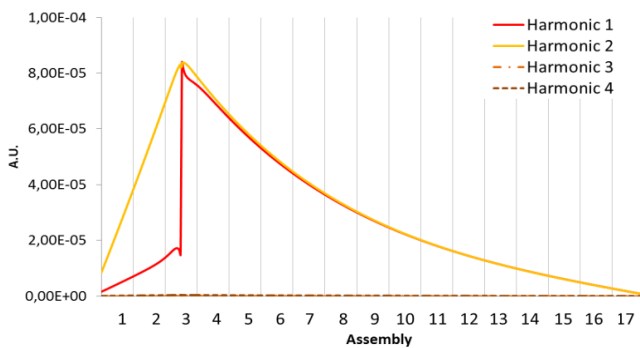


Fig. 3(a) - Moduli in group 1

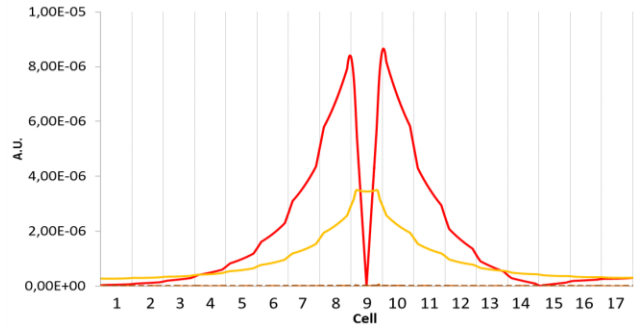


Fig. 3(b) - Moduli in group 4 (zoom in on the third assembly)

Fig. 3 - Comparison between the moduli of harmonics 1, 2, 3 and 4 of the exact noise flux in groups 1 and 4

This observation can be explained by the fact that, in each one of the four regions (see Fig. 1(a)),  $\delta\Sigma(r, \omega)$  does not change of sign for  $\omega_0$  and  $2\omega_0$  whereas it changes of sign for all others harmonics (see Fig.4). Thus, for harmonics greater than two, the noise source changes of sign within each perturbed region and, as shown in Fig. 3, the final effect is a very small noise flux. Consequently, we will only detail the two first components of the noise flux.

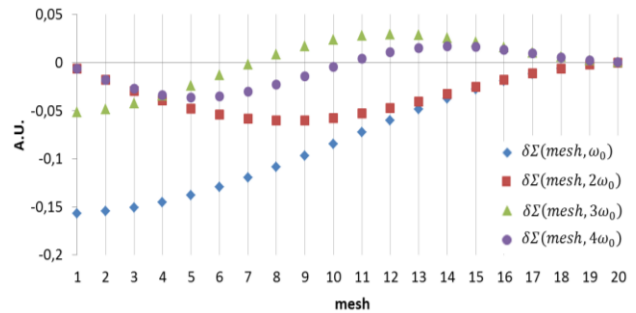


Fig. 4 -  $\delta\Sigma$  in function of mesh and frequency for region 4 with 20 meshes.

Figures 5 and 6 compare the moduli and the phases of the first and second harmonics of the noise flux in groups 1 and 4 for the three methods. Note that we have zoom in on the third assembly for the moduli of group 4. At  $f_0$ , the three methods are very close to each other while at  $2f_0$  we observe some significant differences between them.

Let us begin by analyzing the modulus at  $f_0$ . For group 1, we have a global component, which has a slow variation in space, and a local component in the vicinity of the vibration in the third assembly. Its phase is constant in the entire core (around  $-\pi/2$ ). For group 4, the local component strongly dominates in the vicinity of the vibration with several rough phase changes. Its phase is constant far from the perturbation (around  $-\pi/2$ ). Compared to group 1, the global component far from the vibration seems to be negligible and crushed by the amplitude of the local

component. At  $2f_0$ , we can observe similar behaviors but with a phase around 0 for  $\Phi_T$  and  $\Phi_{NF}$  and around  $+\pi$  for  $\Phi_R$ .

We can conclude that in the vicinity of the perturbation the global and local component for  $f_0$  and  $2f_0$  are visible for the fast group while only the local component is visible for the thermal one. Far from the perturbation, only the global component exists for all groups but its amplitude is greater for the fast group than for the thermal group. Moreover, we have a phase shift of  $\pm\pi/2$  depending on the method between these two harmonics.

Next, we compare the three methods. At  $f_0$ ,  $\Phi_{NF}$  is closer to the exact solution than  $\Phi_T$  while at  $2f_0$   $\Phi_T$  is closer to the exact solution than  $\Phi_{NF}$ . But as the first harmonic is more important than the second harmonic (not always but especially in group 4), we have a balance between these

differences and we cannot say that  $\Phi_T$  is closer to the exact solution than  $\Phi_{NF}$ , or conversely.

The phase shift of  $\pi$  between  $\Phi_T$  or  $\Phi_{NF}$  and the exact solution can be explained by the importance of the convolution term  $[\delta B_R * \delta \Phi_R](r, 2\omega_0)$  especially the term  $\delta B_R(\omega_0)\delta \Phi_R(\omega_0)$  which is the more important term of the convolution.

We note that the second harmonic affects more the fast group than the thermal one and so affects more the global than the local component. We also verify that the global component cannot be described by the point reactor model in a large core and that the local component affects more the thermal noise flux than the fast noise flux.

Let us precise that some of our observations about the global and local components and the first and second harmonics are in agreement with [7] but in our case the second harmonic is not negligible.

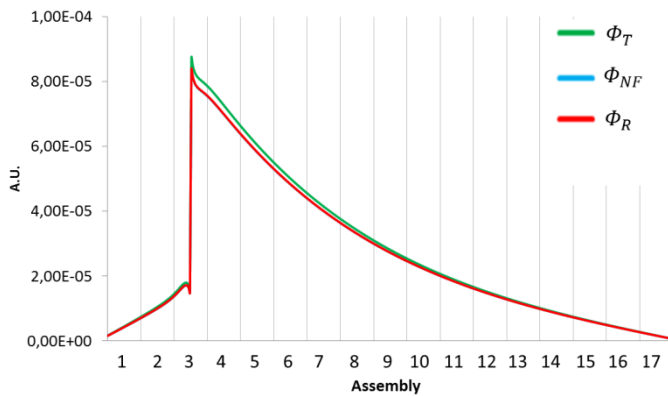


Fig. 5(a) – Moduli at  $f_0$  (blue and red curves are superposed)

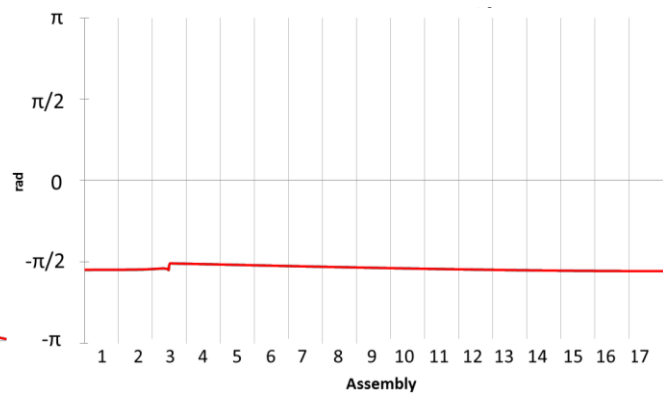


Fig. 5(b) – Phases at  $f_0$

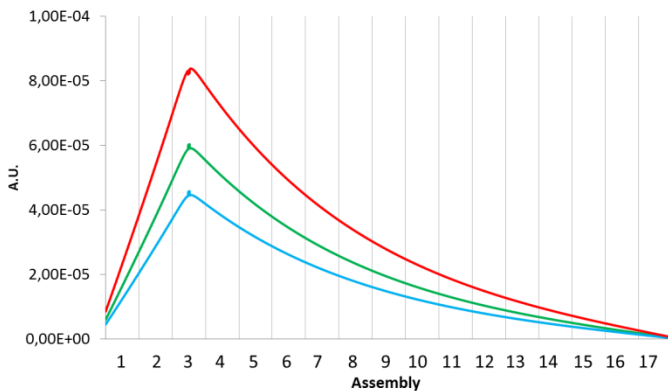


Fig. 5(c) – Moduli at  $2f_0$

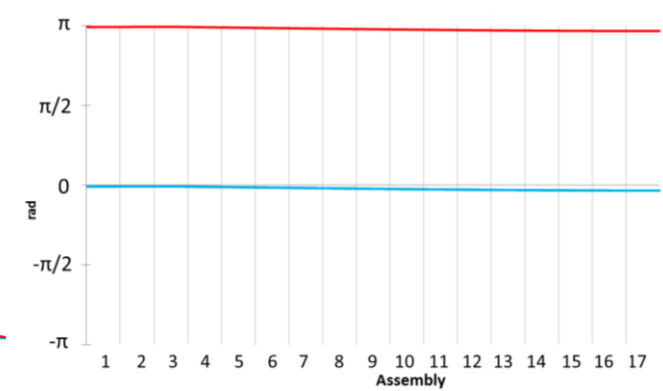


Fig. 5(d) – Phases at  $2f_0$  (green and blue curves are superposed)

Fig. 5 - Moduli and phases of the noise flux in group 1 at  $f_0$  and  $2f_0$



#### IV.C. The total flux in the time domain

The total flux in the time domain is here defined by the steady-state flux and the first and second harmonics of the noise flux:

$$\Phi(r, t) \approx \Phi_0(r) + \frac{|\delta\phi|(r, \omega_0)}{\pi} \cos(\omega_0 t + \varphi(r, \omega_0)) + \frac{|\delta\phi|(r, 2\omega_0)}{\pi} \cos(2\omega_0 t + \varphi(r, 2\omega_0)), \quad (28)$$

where  $\varphi$  is the scalar flux,  $|\delta\phi|$  is the modulus of the noise scalar flux and  $\varphi$  its phase.

The eigenvalues of the three steady states are  $k_T=1.000016$ ,  $k_{NF}=0.9999347$  and  $k_R=0.9999401$ . These eigenvalues are very close to each other. Figure 7 presents the steady-state fluxes in groups 1 and 4. We remark that the steady-state operator  $B_{0,NF}$  seems to be a good approximation of the exact steady-state operator.

Figures 8 and 9 present the total fluxes of groups 1 and 4 in the time domain at  $t = T_0/4$  i.e. when the fuel pin vibration hits its maximal amplitude at the right side.

Figures 8(a) and 9(a) show that, at this time and at the right side of the perturbation, the fast flux is greater than the steady-state fast flux and that the thermal flux is smaller than the steady-state thermal flux. We also remark that, as expected, the global component is more dominating in the fast group than in the thermal group while the local component dominates more in the thermal group than in the fast group. Moreover, Fig. 9(b) points out the rapid relaxation length of the local component.

Figures 10 and 11 show the temporal variations of the noise flux in a position  $r_a$  close to the perturbation (see Fig. 8(a)) and a position  $r_b$  far from the perturbation (close to the core center).

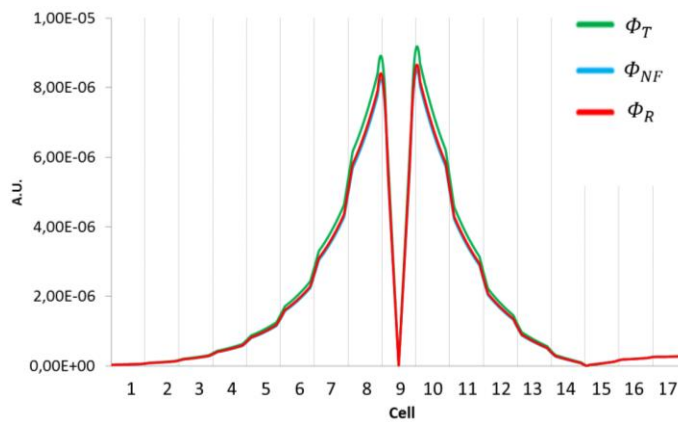


Fig. 6(a) – Moduli at  $f_0$  (zoom in on the third assembly)

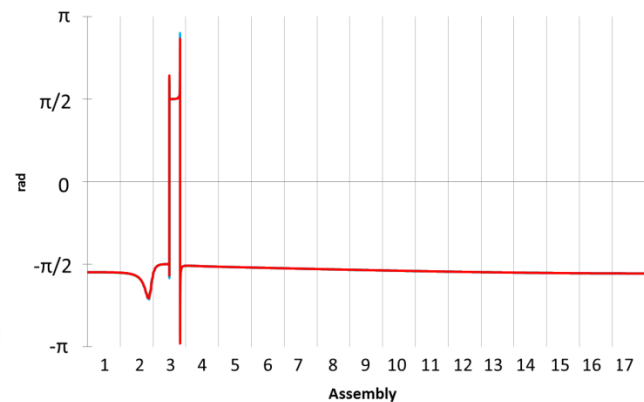


Fig. 6(b) – Phases at  $f_0$

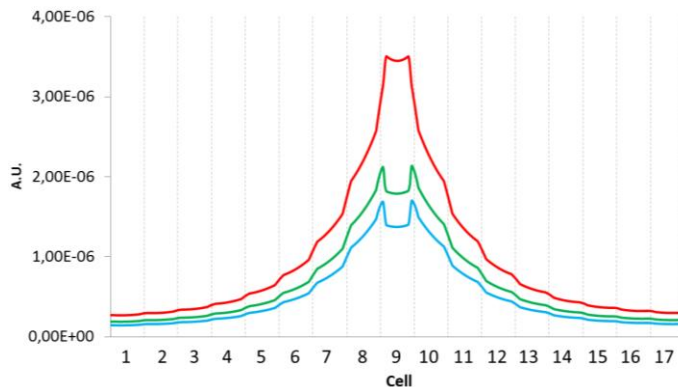


Fig. 6(c) – Moduli at  $2f_0$  (zoom in on the third assembly)

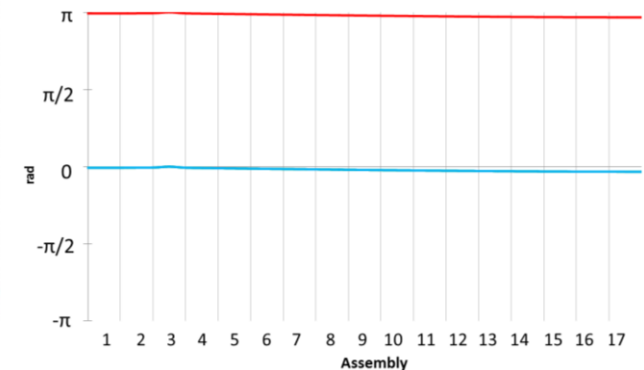


Fig. 6(d) – Phases at  $2f_0$  (green and blue curves are superposed)

Fig. 6 – Moduli and phases of the noise flux in group 4 at  $f_0$  and  $2f_0$



For the position  $r_a$  close to the vibration, in group 1, the second harmonic is clearly not negligible compared to the first harmonic. In group 4, the second harmonic is less visible. Moreover, we note the differences between  $\Phi_{NF}$  and the exact solution  $\Phi_R$  are more visible in group 1 than in group 4. But we saw that, close to the vibration, the global and local component are visible for the fast group 1 while only the local component dominates for the thermal group 4. So this observation is coherent with the fact that the global component should be more affected by the high order terms of the noise flux than the local component. It is also coherent with the fact that the second harmonic of the

noise flux impacts more the global than the local component.

For the position  $r_b$  far from the perturbation where the global component dominates for all groups, the balance between harmonic 1 and 2 is the same for groups 1 and 4 but the amplitudes of the total flux variations are really different. In group 1, the maximal amplitude is around  $2.10^{-5}$  while in group 4 is around  $7.10^{-8}$ . As we saw previously, the amplitude of the global component is clearly more important for the fast group than for the thermal group.

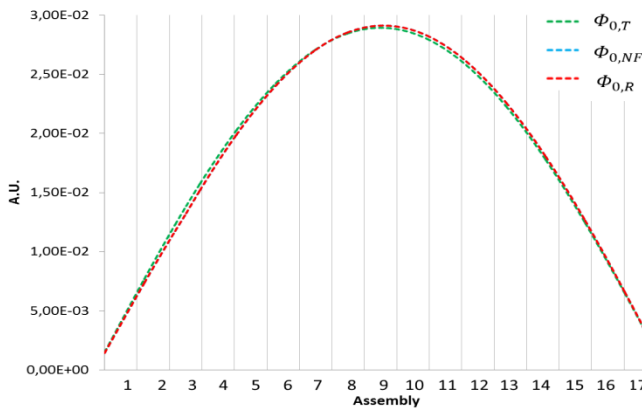


Fig. 7(a) – Group 1

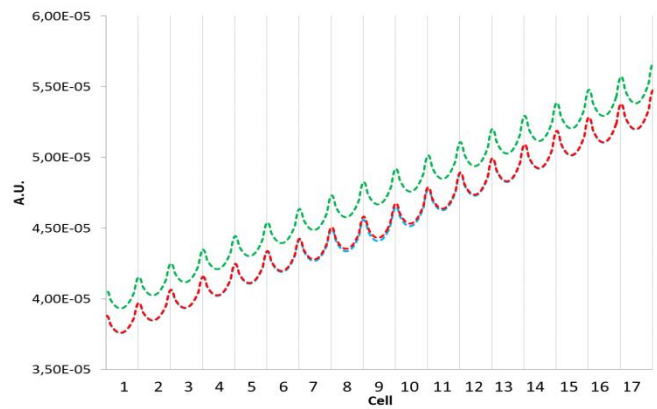


Fig. 7(b) – Group 4 (zoom in on the third assembly)

Fig.7 – Steady-state fluxes in groups 1 and 4 (blue and green curves are superposed)

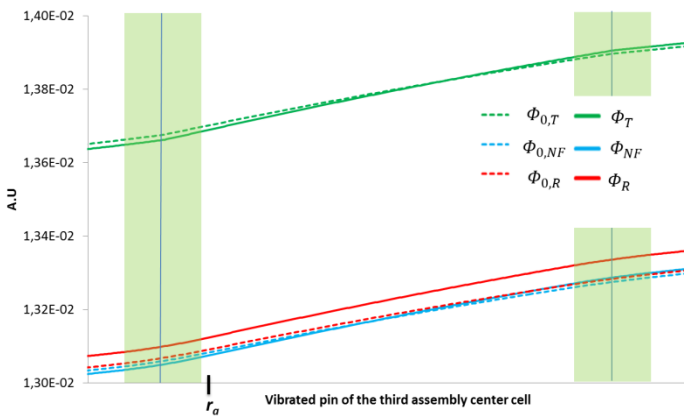


Fig. 8(a) - The vibrated fuel pin (green regions delimit the perturbed regions)

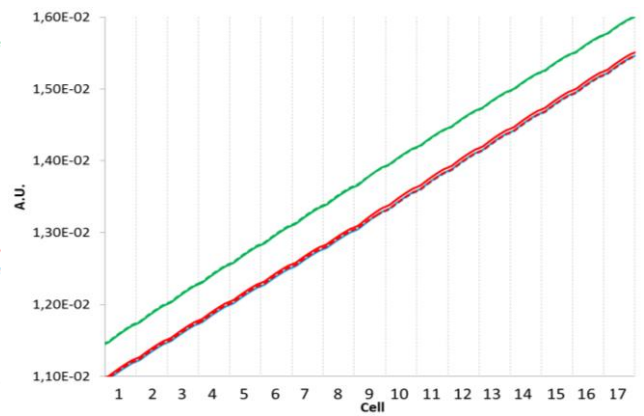


Fig. 8(b) - The third assembly

Fig. 8 - Total fluxes in group 1 at  $t = \frac{T_0}{4}$

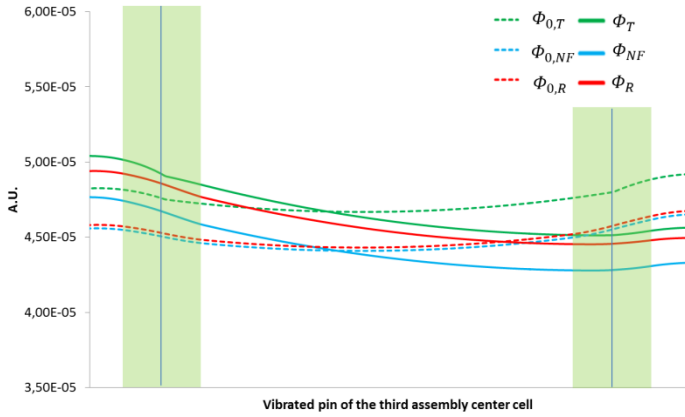


Fig. 9(a) - The vibrated fuel pin (green regions delimit the perturbed regions)

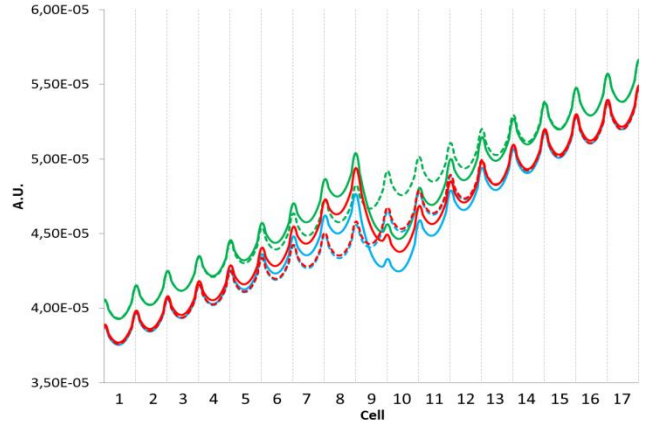


Fig. 9(b) - The third assembly

Fig. 9 - Total fluxes in group 4 at  $t = \frac{T_0}{4}$

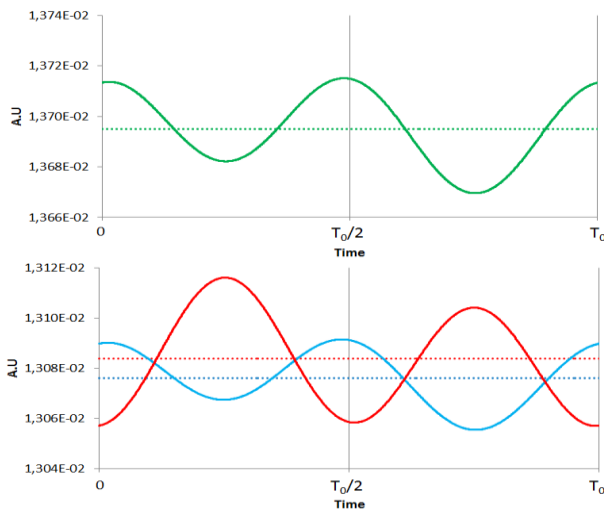


Fig. 10(a) - Total fluxes at  $r_a$

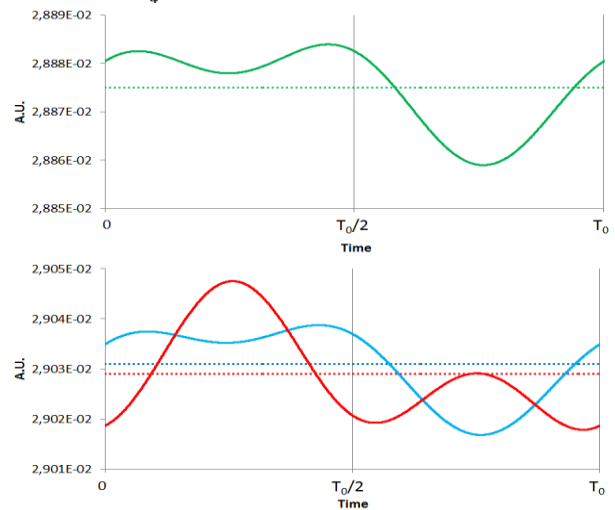


Fig. 10(b) - Total fluxes at  $r_b$

Fig. 10 - Total fluxes in group 1 at  $r_a$  and  $r_b$  in function of time (same legend of previous figures)

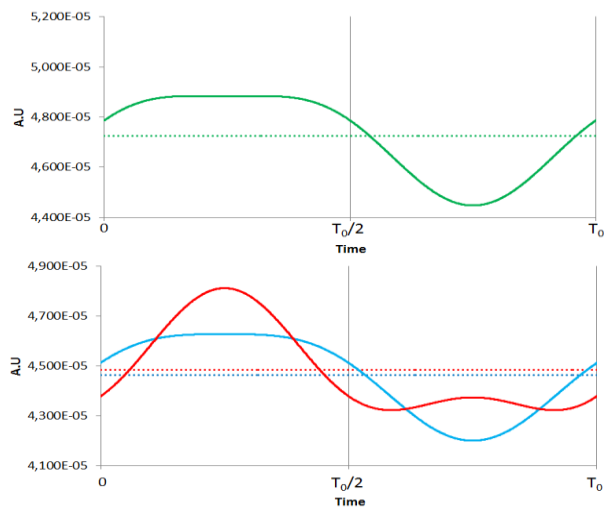


Fig. 11(a) - Total fluxes at  $r_a$

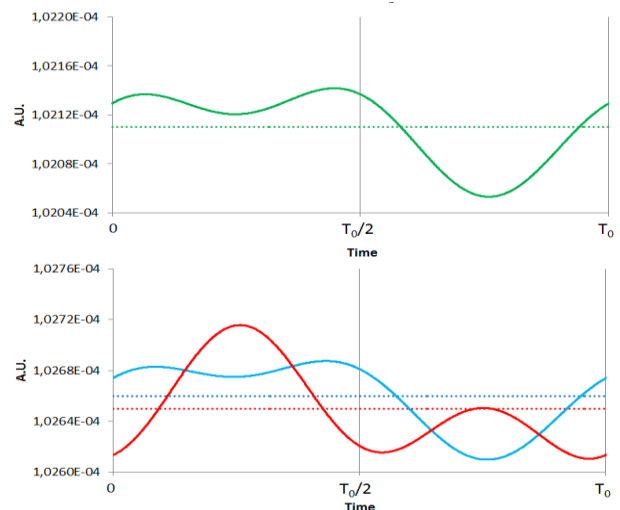


Fig. 11(b) - Total fluxes at  $r_b$

Fig. 11 - Total fluxes in group 4 at  $r_a$  and  $r_b$  in function of time (same legend of previous figures)

## V. CONCLUSIONS

A one-dimension vibration model has been developed. This source perturbation excites all multiples of the vibration frequency  $f_0$ . The more important harmonics of this noise source are at  $f_0$  and  $2f_0$ . We noted that the modulus of this second harmonic is more important and closer to the first harmonic for the global than the local component of the noise flux and for fast groups than for thermal groups.

We introduced a new method in order to improve the traditional linearization method, which is not theoretically justified in case of vibration perturbation. Because in a vibration perturbation the mean values of the perturbed cross-sections are not zero, in this new method we use a new steady-state operator which is the time-average of the kinetic operator and not the initial critical operator as in the traditional method.

We compared this new method with the traditional method and with the exact solution of the non-linearized, fully-coupled noise equations in diffusion theory and for four energy groups. We concluded that the second harmonic of the noise flux is not negligible, should be taken into account and affects especially the global component of the noise flux. We also noted that the differences between all methods are more important at  $2f_0$  than  $f_0$ . Moreover, the new method is based on a steady-state flux closer to the steady-state flux of the exact solution compared to the traditional steady-state flux.

We also verified that the global (resp. local) component of the noise flux dominates in fast (resp. thermal) group, that it cannot be described by the point reactor model in a large system and more especially that it is more affected by the high order terms of the noise flux than the local component.

Thanks to many other calculations not presented here with different vibration amplitudes and different system sizes, we observed that the modulus of the second harmonic is more important and closer to the first harmonic for a large system than for a small system and for a large vibration amplitude than for a small vibration amplitude. Moreover, the high order terms are logically more important in case of a large vibration amplitude compared to a small vibration amplitude and they are also more important for a small system than for a large system.

In this paper, we use the classical assumptions of noise diffusion theory i.e. the fluctuations of the diffusion coefficients and the term  $\frac{1}{v\beta} \partial_t J^g$  are neglected. Future work will investigate these assumptions and will compare transport and diffusion results in case of fuel pin vibration in two-dimensional systems.

Moreover, a new method, which should improve traditional linear noise theory, will be presented. This new technique respects also the linear theory but improves the noise source term by using a more physically realistic steady-state flux.

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