A new Monte Carlo method for neutron noise calculations in the frequency domain

Amélie Rouchon, Andrea Zoia, Richard Sanchez

A NEW MONTE CARLO METHOD FOR NEUTRON NOISE CALCULATIONS

Amélie ROUCHON, Andrea ZOIA & Richard SANCHEZ
CEA-Saclay, DEN, DM2S, SERMA
F-91191 Gif-sur-Yvette, France
amelie.rouchon@cea.fr
andrea.zoia@cea.fr
richard.sanchez@cea.fr

ABSTRACT

A new Monte Carlo algorithm that solves the transport equations for the neutron noise in the frequency domain has been developed. Neutron noise equations, which are obtained by assuming small perturbations of macroscopic cross-sections around a steady state in the neutron field and by subsequently taking the Fourier transform in the frequency domain, are usually solved by analytical techniques or by resorting to diffusion theory. A stochastic approach has been recently proposed in the literature by using particles with complex-valued weights and applying a weight cancellation technique which requires the positive and negative values of the real and imaginary parts of particule weights to be summed up over a sufficiently fine spatial mesh. The new stochastic method presented here does not need any weight cancellation technique and relies on a modified collision operator. In this paper, the two Monte Carlo methods are compared with the usual deterministic methods (diffusion and transport) in the case of a heterogenous one-dimensional rod geometry for several noise frequencies. Our stochastic method is shown to be faster (except at very high frequencies) and easier to implement.

Key Words: **Neutron noise, Monte Carlo, Frequency domain**

1. INTRODUCTION

Traditional neutron noise analysis addresses the description of small time-dependent flux fluctuations induced by small global or local periodic perturbations of the macroscopic cross sections, which may occur in nuclear reactors due to stochastic density fluctuations of the coolant, to vibrations of fuel elements, control rods, or any other structures in the core [1]. Neutron noise techniques are widely used by the nuclear industry for non-invasive general monitoring, control and detection of anomalies in nuclear power plants [2]. They are also applied to the measurement of the properties of the coolant, such as speed and void fraction [3].

In power reactors, ex-core and in-core detectors can be used to monitor neutron noise with the aim of detecting possible anomalies and taking the necessary measures for continuous safe power production.

Noise analysis relies upon the possibility of numerically simulating the behaviour of neutron noise and computing the changes in the neutron field produced by different representative sources of noise in reactor cores.

The general noise equations are obtained by assuming small perturbations around a steady state in the neutron field and by subsequently taking the Fourier transform in the frequency domain. The analysis is performed based on the neutron kinetic equations including the coupling with neutron precursors. The outcome of the Fourier transform analysis is a fixed-source equation for the perturbed neutron field, which can then be solved so as to predict noise measurements at detector locations. For each frequency, the neutron field is a complex function having an intensity and a phase.

Until recently, neutron noise equations have been only solved by analytical techniques [4, 5] and by resorting to diffusion theory [6, 7]. It is therefore necessary to validate them via Monte Carlo simulations. In 2013, a Monte Carlo algorithm was first proposed by Yamamoto in [8] in order to solve the transport equation in neutron noise theory. This algorithm is a cross-over between fixed-source and power iteration methods and uses a weight cancellation technique developed by the same author for neutron leakage-corrected calculations or higher order mode eigenvalue calculations [9–11]. This method gives good results but has some shortcomings, such as the need of introducing a "binning procedure" for the weight cancellation : each fissile region must be divided into a large number of small regions (called bins) where positive and negative weights are summed up and cancelled.

In this paper, we present a new Monte Carlo method that does not need any weight cancellation technique. This method is inspired by a recent technique developed in [12] for alpha eigenvalue calculations. In section 2, the general noise theory will be briefly introduced and the new Monte Carlo method will be presented. In section 3, we will compare our Monte Carlo method and the method proposed in [8] to the deterministic methods (diffusion and transport) in the case of a heterogenous one-dimensional rod geometry. In section 4, we will discuss the figures of merit of each stochastic method and the respective advantages and disadvantages. Finally, conclusions will be presented in section 5.

2. MONTE CARLO CALCULATION IN NEUTRON NOISE THEORY

2.1. Neutron noise equation

In this section, we recall the general theory of neutron noise. The generalization of these equations to diffusion theory is straightforward. Note that the zero power noise (fluctuations inherent to the branching process) is neglected in power reactor noise theory. We assume small perturbations of the macroscopic cross-sections around the critical steady state following :

$$B_0(r, \Omega, E)\Psi_0(r, \Omega, E) = 0, \forall r \in D, \tag{1}$$

where $B_0 = \Omega.\nabla + \Sigma_0 - H_0 - P_0$ is the initial steady-state Boltzmann operator with Σ_0 the steady-state total cross-section, H_0 the steady-state scattering operator, P_0 the steady-state production operator, Ψ_0 the initial steady-state angular flux and D the geometric domain. For the steady state, the effective multiplication factor is assumed to be $k = 1$. We impose a temporal perturbation of the cross-sections, which yields the kinetic equation :

$$[\frac{1}{v}\partial_t + B(r, \Omega, E, t)]\Psi(r, \Omega, E, t) = 0, \forall (r, t) \in D \times \mathbb{R}, \tag{2}$$

where v is the neutron velocity and $B = \Omega.\nabla + \Sigma - H - P$ the kinetic Boltzmann operator with Σ the total cross-section, H the scattering operator, P the production operator and Ψ the angular flux. We impose a periodic perturbation of the kinetic operator with a period T_0 :

$$B(r, \Omega, E, t) = B_0(r, \Omega, E) + \delta B(r, \Omega, E, t). \tag{3}$$

This perturbation is supposed to start at $t = -\infty$, so that we can reasonably assume that the asymptotic perturbation regime is attained. As for the kinetic operator B, we split the flux into :

$$\Psi(r, \Omega, E, t) = \Psi_0(r, \Omega, E) + \delta\Psi(r, \Omega, E, t). \tag{4}$$

The term $\delta\Psi$ is called "noise flux". Finally, plugging expressions (3) and (4) into Eq. 2 leads to a kinetic source equation for the noise flux :

$$[\frac{1}{v}\partial_t + B(r, \Omega, E, t)]\delta\Psi(r, \Omega, E, t) = -\delta B(r, \Omega, E, t)\Psi_0(r, \Omega, E). \tag{5}$$

The second order term $\delta B\delta\Psi$ will be neglected, so that we obtain the traditional linearized kinetic equation :

$$[\frac{1}{v}\partial_t + B_0(r, \Omega, E)]\delta\Psi(r, \Omega, E, t) = -\delta B(r, \Omega, E, t)\Psi_0(r, \Omega, E). \tag{6}$$

We want to determine the unique periodic solution of this equation. We apply the Fourier transform and we obtain the traditional noise equation :

$$B_{0,\omega}(r, \Omega, E)\delta\Psi(r, \Omega, E, \omega) = -\delta B(r, \Omega, E, \omega)\Psi_0(r, \Omega, E), \forall \omega \in \mathbb{R}, \tag{7}$$

where $B_{0,\omega} = i\frac{\omega}{v} + \Omega.\nabla + \Sigma_0 - H_0 - P_{0,\omega}$ is a modified (complex) Boltzmann operator. Because of the delayed neutrons, the production operator $P_{0,\omega}$ depends on the frequency. For each frequency, the field $\delta\Psi(r, \Omega, E, \omega)$ has an intensity and a phase and it is therefore a complex function. The real and imaginary components of the noise flux are coupled by two terms: $i\frac{\omega}{v}$ and the production operator $P_{0,\omega}$. The right hand side of Eq. 7 represents a (known) "noise source".

For a system with one precursor group, the critical steady-state Boltzmann equation is :

$$\begin{aligned}(\Omega.\nabla + \Sigma_0(r, E))\,\Psi_0(r, \Omega, E) = &\iint \Sigma_{0,s}(r, \Omega' \to \Omega, E' \to E)\Psi_0(r, \Omega', E')dE'd\Omega' \\ &+ \frac{1}{k}\frac{\chi(E)}{4\pi}\iint \nu(E')\Sigma_{0,f}(r, E')\Psi_0(r, \Omega', E')dE'd\Omega'.\end{aligned} \tag{8}$$

and the noise equation is :

$$\left(\Omega.\nabla + \Sigma_0(r,E) + i\frac{\omega}{v}\right)\delta\Psi(r,\Omega,E,\omega) = \iint \Sigma_{0,s}(r,\Omega'\to\Omega,E'\to E)\delta\Psi(r,\Omega',E',\omega)dE'd\Omega'$$
$$+\frac{1}{k}\frac{1}{4\pi}\left[(1-\beta)\chi_p(E) + \left(\frac{\lambda^2}{\lambda^2+\omega^2} - i\frac{\lambda\omega}{\lambda^2+\omega^2}\right)\beta\chi_d(E)\right]\iint \nu(E')\Sigma_{0,f}(r,E')\delta\Psi(r,\Omega',E',\omega)dE'd\Omega'$$
$$+S(r,\Omega,E,\omega), \tag{9}$$

with i the imaginary unit, $\omega = 2\pi f$ the angular frequency and S the noise source. All other notations are standard. In this paper we shall work with positive frequencies.

2.2. A new Monte Carlo method for the noise equations

To solve the fixed source problem described by Eq. 9, a stochastic method has been provided by Yamamoto in [8] based on the simulation of particles carrying complex weights. Here, we sketch our Monte Carlo method and detail the differences with respect to [8]. Only the modifications with respect to traditional Monte Carlo algorithms for particle transport will be discussed.

In [8], it was proposed to solve the neutron noise equations by Monte Carlo method by introducing particles having complex weights $w(\omega) = \{w_{\Re}(\omega), w_{\Im}(\omega)\}$. The signs of the real and imaginary parts of particle weights can be positive or negative. According to the algorithm in [8], the complex "noise total cross-section" $\Sigma_0 + i\omega/v$ appearing in Eq. 9 is dealt with by modifying the flight length kernel and the particle weights change continuously during each travel. Thus, the track length estimator for the flux for the jth flight path with distance s_j obeys :

$$TL_j = \int_0^{s_j} w_j e^{-\frac{i\omega}{v}r}dr = w_j\frac{v}{\omega}[\sin(\frac{\omega}{v}s_j) + i(\cos(\frac{\omega}{v}s_j) - 1)]. \tag{10}$$

In the present work, we choose instead to deal with the modified total cross-section $\Sigma_0 + i\omega/v$ by changing the collision kernel of Eq. 9. By analogy with the strategy discussed in [12], we add a term $\frac{\eta-i}{\eta}\eta\frac{\omega}{v}$ with $\eta > 0$ to both sides of Eq. 9. Then, the equation becomes :

$$\left(\Omega.\nabla + \Sigma_0(r,E) + \eta\frac{\omega}{v}\right)\delta\Psi(r,\Omega,E,\omega) = \frac{\eta-i}{\eta}\eta\frac{\omega}{v}\delta\Psi(r,\Omega,E,\omega)$$
$$+\iint \Sigma_{0,s}(r,\Omega'\to\Omega,E'\to E)\delta\Psi(r,\Omega',E',\omega)dE'd\Omega'$$
$$+\frac{1}{k}\frac{1}{4\pi}\left[(1-\beta)\chi_p(E) + \left(\frac{\lambda^2}{\lambda^2+\omega^2} - i\frac{\lambda\omega}{\lambda^2+\omega^2}\right)\beta\chi_d(E)\right]\iint \nu(E')\Sigma_{0,f}(r,E')\delta\Psi(r,\Omega',E',\omega)dE'd\Omega'$$
$$+S(r,\Omega,E,\omega), \tag{11}$$

In this case, we work with a real modified total cross-section $\tilde{\Sigma}_0(r,E,\omega) = \Sigma_0(r,E) + \Sigma_\omega(E,\omega) \in \mathbb{R}^+$ where $\Sigma_\omega(E,\omega) = \eta\omega/v$. Hence, flight lengths are sampled as in standard Monte Carlo calculations,

provided that $\tilde{\Sigma}_0$ is used instead of Σ_0. For this algorithm, the track length estimator for the flux calculation is therefore defined as customary, and obeys :

$$TL_j = \int_0^{s_j} w_j dr = w_j s_j. \tag{12}$$

Because of the structure of Eq. 11, the collision operator is now different from that of the regular Boltzmann equation, and we have to treat two types of productions : regular production with probability $\Sigma_{0,f}/\tilde{\Sigma}_0$ and ω-production with probability $\Sigma_\omega/\tilde{\Sigma}_0$. We treat the regular fission as in [8], i.e., the number of fission neutrons is calculated as $\text{Int}(\nu + \epsilon)$ with $\text{Int}(.)$ the integer part and ϵ a uniform random number. Because of the factor appearing in front of the delayed fission production term in Eq. 11, the weight w_d^* of each new delayed particle created by the regular fission is modified by :

$$w_d^* = \frac{w}{k\chi(E)} \left(\frac{\lambda^2}{\lambda^2 + \omega^2} - i \frac{\lambda\omega}{\lambda^2 + \omega^2} \right) \beta\chi_d(E), \tag{13}$$

with w the weight before the fission event. The term representing the ω-fission production consists in a copy of the incident neutron with a new weight w^{**} given by :

$$w^{**} = w \frac{\eta - i}{\eta}. \tag{14}$$

Implicit capture (with forced fission) and Russian roulette can be used as in standard Monte Carlo methods. In the case of implicit capture, after each collision event, the complex weight is modified by $\Sigma_{0,s}/\tilde{\Sigma}_0$. Then, as in [8], the Russian roulette game is applied separately to the absolute value of the real and imaginary parts of the particle weight. The particle is killed only if the real and the imaginary parts are both killed by the Russian roulette. If only the real or the imaginary part is killed, the particle survives. When the real or the imaginary part survives, its value is updated to ± 1 according to its sign before Russian roulette.

2.3. Weight cancellation

At low and high frequencies (more precisely far from the plateau region of the zero-power transfer function of the system), if we use the conventional algorithm for fixed source problems with implicit capture, the Monte Carlo methods described above will lead to the production of a huge number of particles and the calculations will never come to an end. To overcome this problem, the "binning procedure" was proposed in [8] for the weight cancellation. Note that several weight cancellation techniques exist but few of them seem adapted to the noise problem [13]. In order to implement this cancellation technique, for each independent batch, we apply an algorithm very similar to the power iteration method for criticality simulations, i.e., each particle created by fission during a generation is stored and initializes the fission sources of the next generation. Fissile regions are divided into a large number of small spatial bins where complex fission sources with positive and negative weights are stored during each generation and are summed at the end of each generation. After cancellation, the new number n_f of fission sources in each bin is determined by $\text{Int}(\max(\Re(\sum w)), \max(\Im(\sum w)), 1)$

and, for the next generation, fission neutrons are assumed to be uniformly distributed within the bin (this stems from fissions being isotropic in the method proposed in [8]) with an identical new weight equal to $\frac{1}{n_f}\sum w$. The batch ends when $n_f = 0$ for all bins. Thus, the number of fission sources changes from generation to generation until a generation does not produce fission after weight cancellation.

Contrary to [8], we will not use this weight cancellation technique because our ω-fission production depends on the angular distribution of the particles, so that an angular mesh would be also required. We simply use the fission bank, where we store all particles produced by regular and ω-fissions, to initialize the fission sources of the next generation. As in [8], the batch ends when a generation does not produce fission sources, i.e., when the fission bank is empty. We make several independent batches in order to estimate the statistical uncertainties. We will see that it is possible to overcome the problem of particle explosion at low and high frequencies by removing implicit capture and at very hight frequencies by adapting as well the η value (it is necessary to adapt the η factor only at very high frequencies).

3. ANALYSIS AND COMPARISONS

In this section, we compare the two Monte Carlo methods with the deterministic methods (diffusion and transport with isotropic scattering) for a heterogenous one-dimensional rod geometry with 4 energy groups and 6 precursor groups. Note that in a rod geometry, particles can only flight in two directions [14]. Our system is composed of 17 fuel pins of 1.08 cm (the inter-pin size is 0.36 cm so one cell size is 1.44 cm) and we impose vacuum boundaries. The noise source is equal to -1 for the real part of group 4 (the thermal group) in the third pin and zero for other groups. All Monte Carlo calculations were performed with 3 000 independent batches of 10 000 particles. Note that in [8] the author uses only one batch of 1 000 000 particles for his examples so no statistical uncertainties were available. In this paper, we make several independent batches, even with the method of [8], so that we can estimate the statistical uncertainties for both Monte Carlo calculations. The eigenvalue of the steady state is $k = 0.999$.

3.1. The zero-power transfer function

First, we determine the shape of the zero-power transfer function of our system (see Figure 1). This transfer function G_0 is defined by [1, 15] :

$$G_0(\omega) = \frac{1}{i\omega(\Lambda_{\text{eff}} + \frac{\beta_{\text{eff}}}{\overline{\lambda} + i\omega})}, \tag{15}$$

with Λ_{eff} the mean generation time, β_{eff} the effective yield fraction (weighted by the steady-state adjoint flux) and $\overline{\lambda}$ the averaged decay constant. We can differentiate the low frequency ($<$0.015 Hz), the plateau and the high frequency regions ($>$250 Hz). The boundaries between these three regions depend

on the kinetic parameters : $\omega_l = \overline{\lambda}$ (rad/s) for the low frequency region and $\omega_h = \overline{\lambda} + \beta_{\text{eff}}/\Lambda_{\text{eff}}$ (rad/s) for the high frequency region.

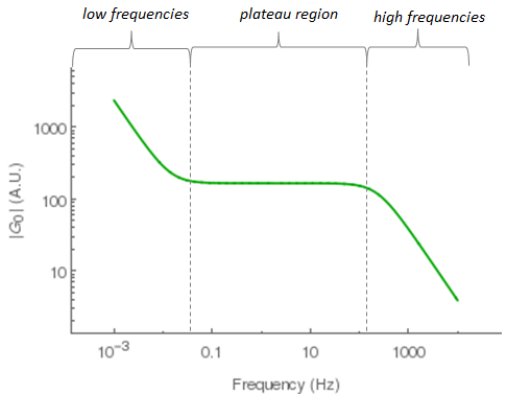


Figure 1. Modulus of the zero-power transfer function of our system.

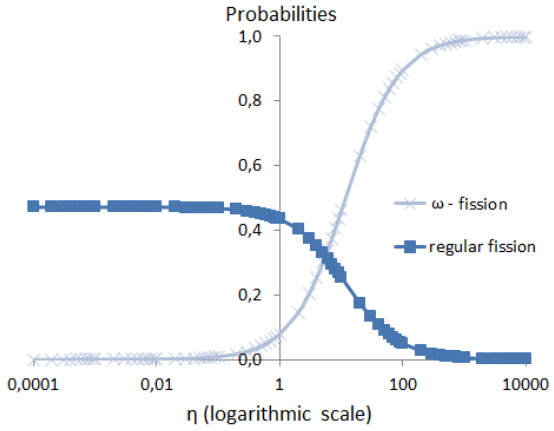
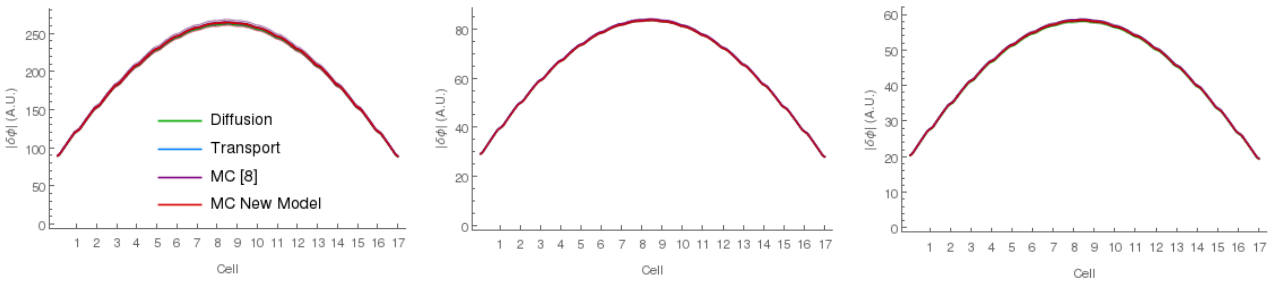


Figure 2. Evolution of the regular fission probability and the ω-fission probability versus η at 2.5 kHz in group 4 and without implicit capture.

3.2. Comparison with deterministic methods

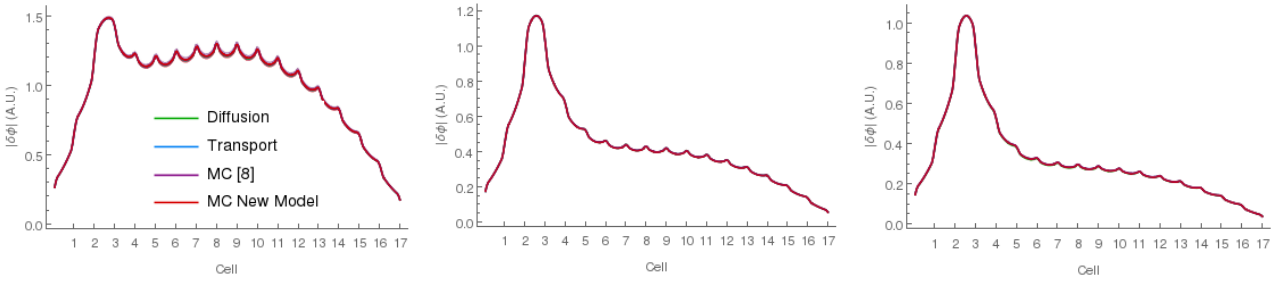
Figures 3 and 4 illustrate, respectively, the moduli of the noise flux for groups 1 and 4 at 0.01 Hz (low frequency), 1 Hz (in the plateau region) and 2.5 kHz (very high frequency). For our method, we activate implicite capture only at 1 Hz. Moreover, $\eta = 100$ at 2.5 kHz and $\eta = 1$ at 0.01 Hz and 1 Hz. For the method in [8], we always use the "binning procedure" (with 302 bins) and implicit capture.



(a) 0.01 Hz (b) 1 Hz (c) 2.5 kHz

Figure 3. Moduli of the fast noise flux (group 1) at 0.01 Hz, 1 Hz and 2.5 kHz. Monte Carlo results (track length estimator) are plotted with the 3σ error bars (barely visible).

As shown in Figs 3 and 4, these two Monte Carlo methods give good results for the moduli of the noise fluxes and allow validating deterministic calculations.



(a) 0.01 Hz (b) 1 Hz (c) 2.5 kHz

Figure 4. Moduli of the thermal noise flux (group 4) at 0.01 Hz, 1 Hz and 2.5 kHz. Monte Carlo results (track length estimator) are plotted with the 3σ error bars (barely visible).

3.3. Influence of implicit capture and of the η factor

As described previously, we have to remove implicit capture in order to perform our method at low and high frequencies. Note that the method proposed in [8] without the "binning procedure" works and gives also good results provided that we remove implicit capture at low and high frequencies. So the problem of the explosion of particle number outside the plateau region is due to implicit capture. Let us note that we have tested implicit capture followed by Russian roulette but, regardless of the value of the minimum weight of this latter (by default 0.25 in our simulations), we were not able to perform calculations at low and high frequencies. Thus, the most straight forward choice is to remove implicit capture.

Unfortunately, at very high frequencies ($\gg \omega_h$), removing implicit capture in our method is necessary but not sufficient. We also have to adjust the η value. Figure 2 presents the evolution of the regular fission probability and the ω-fission probability versus η at 2.5 kHz in group 4 without implicit capture (and so without forced fission). Knowing that $\nu_{\text{group 4}}^{\text{regular}} \approx 3$ and $\nu_{\text{group 4}}^{\omega\text{-fission}} = 1$, we notice that with a large η value we can decrease the total number of fission sources. In our case, we note that a η equal to 100 is sufficient to perform all our calculations at very high frequencies. Similar results have been obtained for several other systems (not presented here).

4. DISCUSSION

The figures of merit of the Monte Carlo calculations are reported in Tab. I : our method shows better performances than [8] except at very high frequencies. The method proposed in this paper uses a real total cross-section and not a complex one; hence, since we do not have to bias weights during each travel, the flight path and the track length estimator calculations are numerically less expensive than in [8]. Moreover, our method does not suffer from the explosion of the CPU time at low frequencies, contrary to [8] (see Fig. 10 of [8]). Nevertheless, our method is not faster at very high frequencies because, via the η factor, we artificially increase the ω-fission probability which becomes close to 1.

Thus, for each system, we have to find the best η factor which allows performing calculations at very high frequencies but without significantly deteriorate performances. However, we note that only the region between 0.01 Hz and 100 Hz are physical relevant and experimentally viable [3]. Thus, in the frequency region of interest, our method shows better performances and it is not necessary to adapt the η factor.

We can summarize the advantages and disadvantages of both Monte Carlo methods in four points : implicit capture can be used for all frequencies with the method proposed in [8] and only in the plateau region with our method; a weight cancellation technique is essential at low and high frequencies if implicit capture is activated with the method in [8], whereas our method does not need any weight cancellation technique; except at very high frequencies, our method is faster than the method developed in [8]; finally, our method is easier to implement because all modifications concern only the production term in the collision operator.

Table I. Figures of merit of the Monte Carlo calculations

	0.01 Hz	1 Hz	2.5 kHz
Method in [8]	1.00	3.21	9.45
New method	2.78	7.88	7.69

5. CONCLUSIONS

In this paper we have presented a new Monte Carlo method that solves neutron noise equations in the frequency domain. Contrary to the method developed in [8], our method does not need any weight cancellation technique (instead, we remove implicit capture at low and high frequencies) and it is based on a real total cross-section and a modified collision kernel. We compared the two Monte Carlo methods in a heterogeneous one-dimensional rod geometry with the deterministic methods for several frequencies. The comparisons showed that, except at very high frequencies, our Monte Carlo method is faster than the method developed in [8]. Our method is also easier to implement because no weight cancellation technique is used and all complex operators and modifications with respect to standard Monte Carlo codes concern only the production term in the collision operator.

Future work will concern the optimisation of the algorithm proposed in this paper in view of a possible implementation in the reference Monte Carlo code TRIPOLI-4® developed at CEA [16].

ACKNOWLEDGMENT

We gratefully acknowledge AREVA and EDF for their long term partnership and their support.

REFERENCES

[1] Pázsit I. and Demazière C. *Noise Techniques in Nuclear Systems*. Handbook of Nuclear Engineering, Ed. D. G. Cacuci,**3**, 2, Springer Verlag (2010).

[2] Fry D. N., March-Leuba J., and Sweeney F. J. "Use of neutron noise of diagnosis of in-vessel anomalies in light-water reactors." *ORNL/TM-8774* (1984).

[3] Kosály G. "Noise investigations in boiling-water and pressurized-water reactors." *Progress in Nuclear Energy*, **5**: pp. 145–199 (1980).

[4] Pázsit I. and Analytis G. Th. "Theoretical investigation of the neutron noise diagnostics of two-dimensional control rod vibrations in a PWR." *Annals of Nuclear Energy*, **7**: pp. 171–183 (1980).

[5] Jonsson A. *et al.* "Analytical inverstigation of the properties of the neutron noise induced by vibrationg absorber and fuel rods." *Kerntechnik*, **77**: pp. 371–380 (2012).

[6] Demazière C. "CORE SIM: A multi-purpose neutronic tool for research and education." *Annals of Nuclear Energy*, **38**: pp. 2698–2718 (2011).

[7] Malmir H., Vosoughi N., and Zahedinejad E. "Development of a 2-D 2-group neutron noise simulator for hexagonal geometries." *Annals of Nuclear Energy*, **37**: pp. 1089–1100 (2010).

[8] Yamamoto T. "Monte carlo method with complex-valued weights for frequency domain analyses of neutron noise." *Annals of Nuclear Energy*, **58**: pp. 72–79 (2013).

[9] —. "Convergence of the second eigenfunction in monte-carlo power iteration." *Annals of Nuclear Energy*, **36**: pp. 7–14 (2009).

[10] —. "Monte carlo algorithm for buckling search and neutron leakage-corrected calculations." *Annals of Nuclear Energy*, **47**: pp. 14–20 (2012).

[11] —. "Monte carlo method with complex weights for neutron leakage-corrected calculations and anisotropic diffusion coefficient generations." *Annals of Nuclear Energy*, **50**: pp. 141–149 (2012).

[12] Zoia A., Brun E., and Malvagi F. "Alpha eigenvalue calculations with TRIPOLI-4®." *Annals of Nuclear Energy*, **63**: pp. 276–284 (2014).

[13] Booth T.E. and Gubernatis J.E. "Exact regional monte carlo weight cancellation for second eigenfunction calculations." *Nuclear Science and Engineering*, **165**: pp. 129–153 (2010).

[14] Wing G.M. *An introduction to transport theory*. John Wiley & Sons (1962).

[15] Duderstadt J.J. and Hamilton L.J. *Nuclear Reactor Analysis*. John Wiley & Sons (1976).

[16] Brun E. *et al.* "TRIPOLI-4®, CEA, EDF and AREVA reference Monte Carlo code." *Annals of Nuclear Energy*, **82**: pp. 151–160 (2015).