



Direct Forcing methods in Trio_U theory and applications

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Direct Forcing methods in Trio_U: theory and applications

Trio_U Meeting | **MICHEL BELLIARD**

joint work with CLARISSE FOURNIER,
YANNICK GORSSE, CLÉMENT INTROIINI,
BENOIT MATHIEU and BÉATRICE ZOPÉ

CEA/DEN

28/01/2015

Trio_U Meeting
INSTN, Saclay

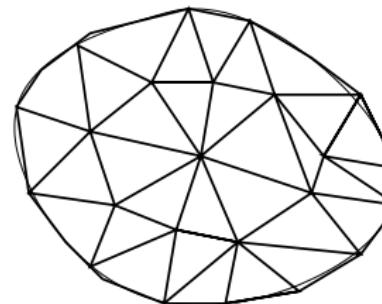
- **Introduction**
- **Theoretical developments**
- **Numerical applications**
- **Conclusions and Perspectives**

Introduction

The Fictitious Domain approach

Complex flow regimes combined with time-varying geometries

- Fluid-Structure Interactions in the nuclear components
 - ★ Flow-induced **vibrations** → possible fuel-rod or U-tube **damages**
 - ★ **Fast** FSI computations with **large deformations**
- Vitrification process for the storage of radioactive wastes
 - ★ Multiphase multicomponent flow at high temperature (gas bubbles, molten glass)
 - ★ Static (vessel structure, measuring apparatus), **moving** (mechanical stirrer) **bodies**



The Fictitious Domain approach

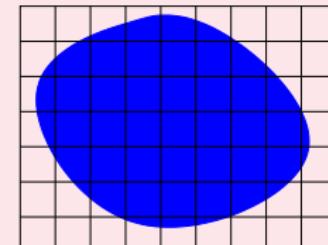
Complex flow regimes combined with time-varying geometries

- Fluid-Structure Interactions in the nuclear components
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Fictitious Domain approach

Pioneer's work ([Saul'ev 1963])

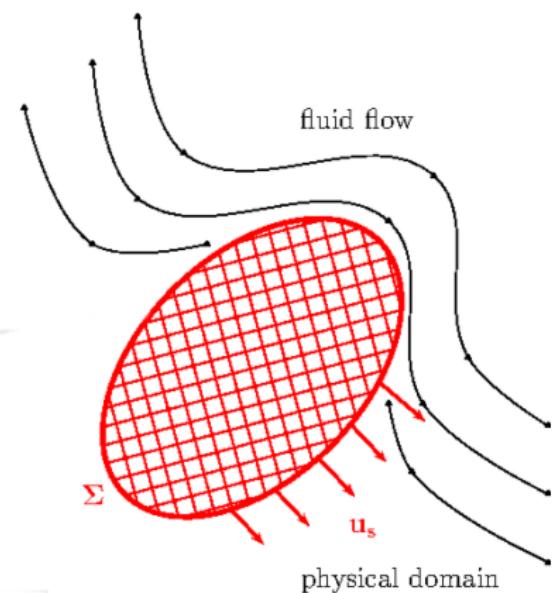
- original domain **embedded** in a geometrically bigger and simply-shaped domain : **fictitious domain**
- computat. domain = fictitious domain
- Cartesian mesh ⇒ fast linear solvers, good numerical properties, easy multiresolution techniques



The Direct Forcing method

Fluid governing equations

$$\begin{cases} \frac{\partial \underline{u}}{\partial t} + \nabla \cdot (\underline{u} \otimes \underline{u}) + \frac{1}{\rho} \nabla p - \nu \nabla^2 \underline{u} = \underline{g} & \text{and} \\ \underline{u} = \underline{u}_D \text{ on } \partial \Omega_F \end{cases}$$



The Direct Forcing method

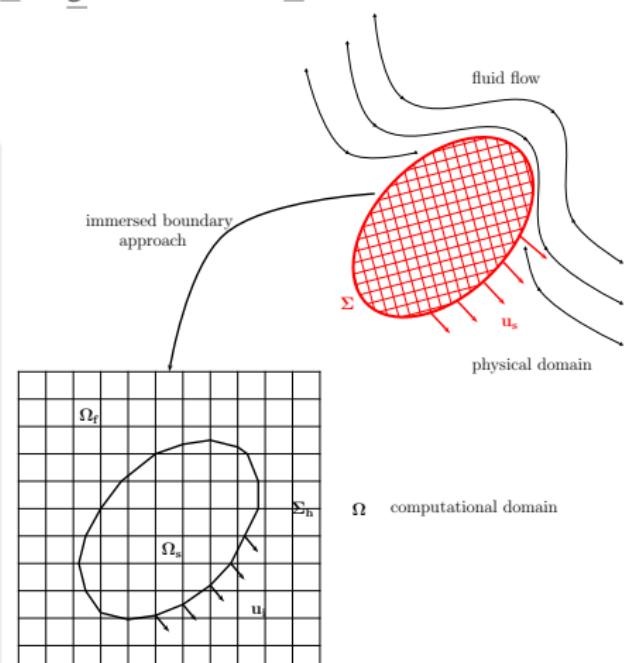
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IBM - DF

$$\begin{aligned} \frac{\underline{u}^* - \underline{u}^n}{\Delta t} + \nabla \cdot (\underline{u}^n \otimes \underline{u}^n) \\ + \frac{1}{\rho} \nabla p^{n+1} - \nu \nabla^2 \underline{u}^n = \underline{g} \text{ in } \Omega \\ \tilde{\underline{u}} = \underline{u}^* + \Delta t \underline{F}_{DF} \end{aligned}$$

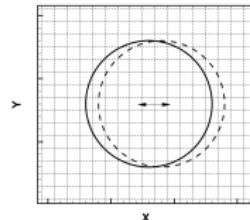
$$\begin{aligned} \underline{F}_{DF} &= \chi_m \frac{\underline{u}_i - \underline{u}^*}{\Delta t} \text{ in } \Omega \\ \chi_m(\underline{x}) &= \begin{cases} 1 & \text{for } \underline{x} \in \Omega_I^m \\ 0 & \text{otherwise.} \end{cases} \end{aligned}$$



A test case from Seo & Mittal, JCP 230 (2011)

$$\begin{aligned}x_c(t) &= x_c(0) + X_0(1 - \cos(2\pi f_0 t)) \\y_c(t) &= y_c(0)\end{aligned}$$

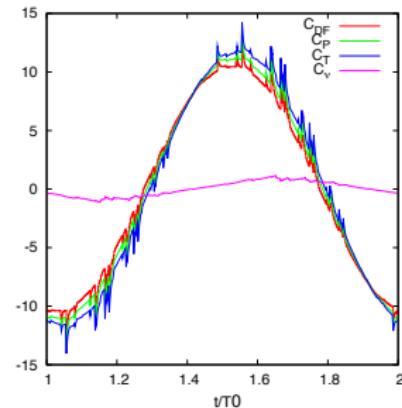
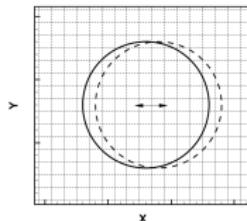
- Fluid at rest
- Drag coefficient $C_T = C_P + C_\nu$
- $C_{DF} = \frac{\iiint_{\Omega_S} \rho \underline{F}_{DF} \cdot \underline{e}_x d\Omega}{\underline{F}_{ref} \cdot \underline{e}_x}$



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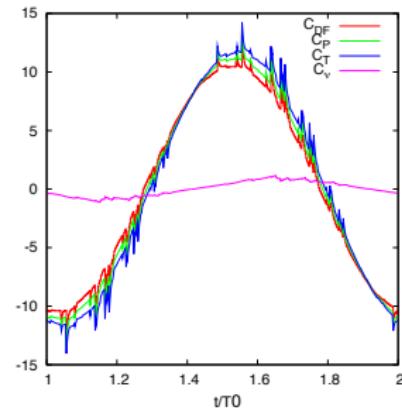
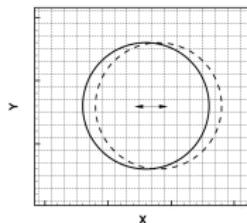


$\Delta x_0 = D/16$ and $\Delta t_0 = 0.002 T_0$

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$\Delta x_0 = D/16$ and $\Delta t_0 = 0.002 T_0$

Forcing term is discontinuous in time . . .

$$\int_{\Omega_{s,h}^{n+1}} \underline{F}_{DF}^{n+1} d\Omega - \int_{\Omega_{s,h}^n} \underline{F}_{DF}^n d\Omega = \mathcal{O}(1) + \mathcal{O}(\Delta x) + \mathcal{O}\left(\frac{\Delta x^{2*}}{\Delta t}\right) \quad * : 3 \text{ for the linear model}$$

Theoretical developments

The Penalized Direct Forcing method

[Belliard, Fournier, *C.R. Acad. Sci. Paris*, 2010] & [Introini *et al.*, *Computers and Fluids*, 2014]

- A combination of the **Direct Forcing** method [Mohd-Yusof, *CTR An. R. Briefs*, 1997] and L^2 -penalty methods [Angot *et al.*, *Num. Math.*, 1999]

The incompressible N-S equations with a penalized forcing term

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u} \otimes \mathbf{u}) + \nabla P - \nu \nabla^2 \mathbf{u} = \mathbf{F} \quad \text{and} \quad \nabla \cdot \mathbf{u} = 0 \quad \text{in } \Omega$$

where $\mathbf{F} = \frac{\alpha \chi_s}{\eta} (\mathbf{u}_s - \mathbf{u})$ with $\alpha > 0$, $0 < \eta \ll 1$ (here, $\eta = 10^{-12}$) and $\chi_s \rightarrow [0, 1]$

- \mathbf{u}_s depending on \mathbf{u}_s and \mathbf{u} following the requested accuracy
- \mathbf{F} is only applied on Cartesian nodes near/inside the immersed boundary ($\chi_s > 0$)

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- A consistent fractional step method [Ikeno & Kajishima, *JCP*, 2007]

(prediction) $\frac{\tilde{\mathbf{u}} - \mathbf{u}^n}{\Delta t} + \nabla_h \cdot (\mathbf{u}^n \otimes \mathbf{u}^n) - \nu \nabla_h^2 \tilde{\mathbf{u}} = \frac{\chi_s}{\eta \Delta t} (\mathbf{u}_i^{n+1} - \tilde{\mathbf{u}}) \quad \text{in } \Omega \text{ with } \alpha = 1/\Delta t$

~ At this stage, IBCs are only satisfied by the provisional velocity $\tilde{\mathbf{u}}$

(correction) $\frac{\mathbf{u}^{n+1} - \tilde{\mathbf{u}}}{\Delta t} + \nabla_h P^{n+1} = \frac{\chi_s}{\eta \Delta t} (\tilde{\mathbf{u}} - \mathbf{u}^{n+1}) \quad \text{in } \Omega$

~ At the end of the projection stage, IBCs are also satisfied by the new velocity \mathbf{u}^{n+1}

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~~~ At this stage, IBCs are only satisfied by the provisional velocity  $\tilde{\mathbf{u}}$

(correction)  $\check{\rho} \frac{\mathbf{u}^{n+1} - \tilde{\mathbf{u}}}{\Delta t} = -\nabla_h P^{n+1} \quad \text{in } \Omega \quad \text{where } \check{\rho} = \left(1 + \frac{\chi_s}{\eta}\right) = \begin{cases} 1 & \text{(fluid)} \\ 1 + 1/\eta & \text{(solid)} \end{cases}$

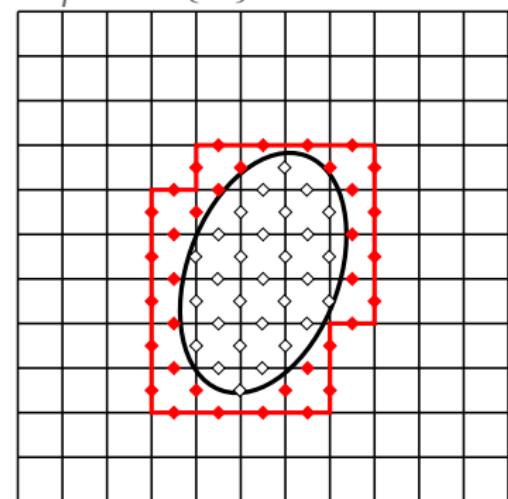
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~~~ Homogeneous Neumann BCs for  $P$  are naturally enforced on obstacles ( $\check{\rho}^{-1} \sim \mathcal{O}(\eta)$ )

## Base scheme

- $\underline{u}_{\text{base}}(\underline{x}) := \underline{u}_S(\underline{x}) \quad \text{if } d(\underline{x}) \geq 0 \quad \diamond$
- $d(\underline{x})$  signed distance to  $\Sigma_h$  ( $> 0$  in  $\Omega_s$ )

$$\Omega_I^{\text{base}} = \{\diamond\}$$



# Interpolation schemes $\underline{u}_i = \underline{u}_m(\underline{u}_S, \underline{u}^*)$

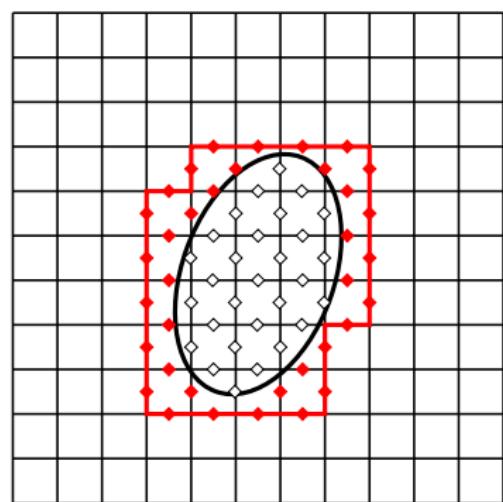
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$$\Omega_I^{\text{linear}} = \{\diamond, \blacklozenge\}$$

## Linear interpolation scheme

$$\underline{u}_{\text{linear}}(\underline{x}) := \begin{cases} \underline{u}_{\text{lin\_int}}(\underline{x}) & \text{if } -\Delta x \leq d(\underline{x}) < 0 \\ \underline{u}_S(\underline{x}) & \text{if } d(\underline{x}) \geq 0 \end{cases} \quad \blacklozenge$$



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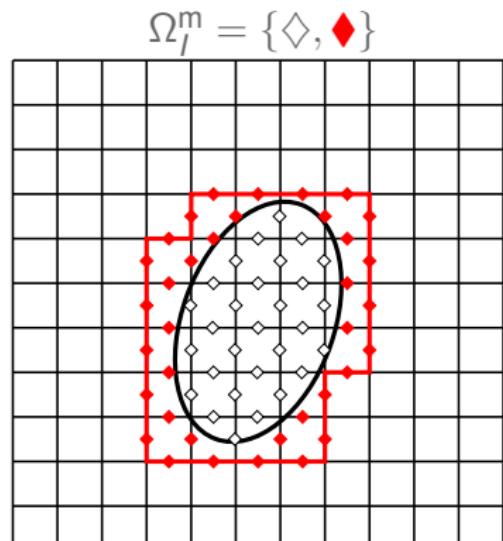
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But possible other choices (higher order schemes, extrapolations, ...)

$$\underline{u}_m(\underline{x}) := \begin{cases} \underline{u}_{\text{ho\_int}}(\underline{x}) & \text{if } -\Delta x \leq d(\underline{x}) < 0 \\ \underline{u}_{\text{ho\_extra}}(\underline{x}) & \text{if } d(\underline{x}) \geq 0 \end{cases} \quad \diamond$$



## In literature ...

- Volume-weighted model : Kajishima & al., *IJ Heat and Fluid Flow 23* (2002)
- Differentially interpolated DF : Chiu & al., *JCP 229* (2010)
- PDE/interpolation hybrid formulation : Luo & al., *Comp. & Fluids 56* (2012)

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Regularization of  $\underline{u}_i$  (Belliard & al. *in prep.*)

$$\begin{cases} \underline{u}_i = \xi_m \underline{u}_m + (1 - \xi_m) \underline{u}^* \\ F_{RG} = \frac{\underline{u}_i - \underline{u}^*}{\Delta t} \end{cases}$$

$0 \leq \xi_m(\underline{x}) \leq 1$  : regular function of  $d(\underline{x})$

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# Regularized Direct Forcing

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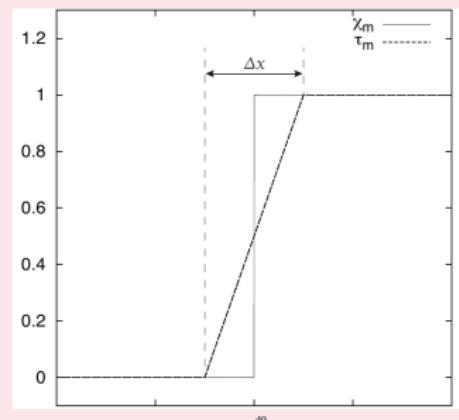
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- An example ...

$$\xi_m(\underline{x}) = \frac{\Delta x + 2[d(\underline{x}) - d_0^m]}{2\Delta x} \equiv \tau_{m(\underline{x})}$$



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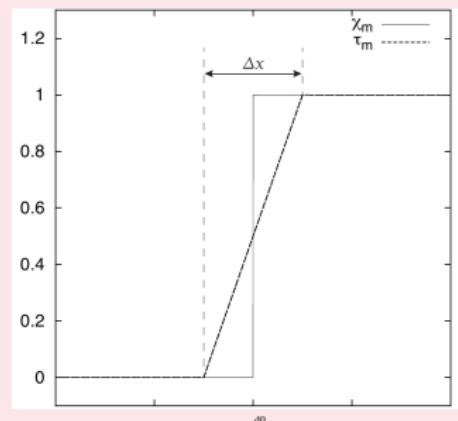
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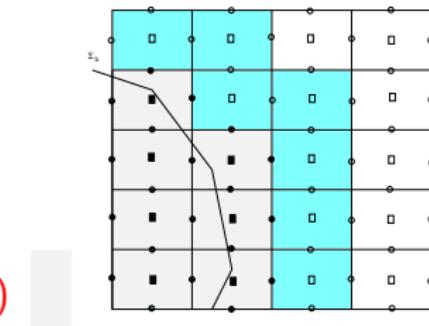
- And also regul. hybrid PDF/DF method ...



# Penalized heat equation

- The heat equation with a penalized forcing term

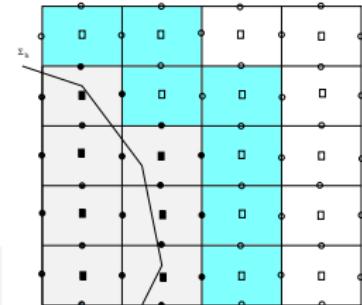
$$\begin{aligned}\partial_t T &= \nabla \left( \frac{\lambda(T)}{\rho_0 C_{P0}} \vec{\nabla} T \right) \\ &+ (1 - \chi_s) \frac{Q}{\rho_0 C_{P0}} \\ &- (1 - \chi_s) \vec{V} \cdot \vec{\nabla} T + \frac{\chi_s}{\eta} (T_i - T)\end{aligned}$$



# Penalized heat equation

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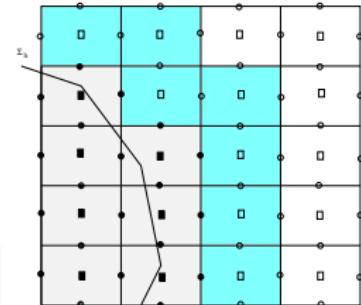
- A fractional semi-implicit time scheme

$$\left\{ \begin{array}{lcl} \frac{T^* - T^n}{\delta t} - \nabla \left( \frac{\lambda(T^n)}{\rho_0 C_{P0}} \vec{\nabla} T^* \right) & = & (1 - \chi_s) \frac{Q}{\rho_0 C_{P0}} - (1 - \chi_s) \vec{V}^n \cdot \vec{\nabla} T^n \\ \frac{T^{n+1} - T^*}{\delta t} - \frac{\chi_s}{\eta} (T_i - T^{n+1}) & = & 0 \end{array} \right.$$

# Penalized heat equation

## ■ The heat equation with a penalized forcing term

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## ■ A fractional semi-implicit time scheme

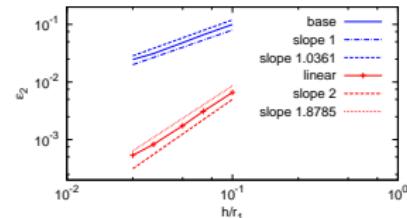
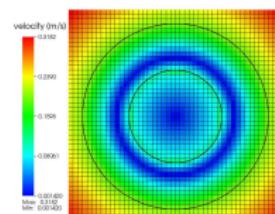
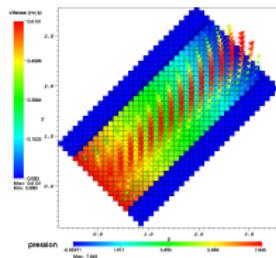
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## ■ Models :

- Base :  $T_i = T_s$
- Weighted :  $T_i = \tau_s T_s + (1 - \tau_s) < T_{neig.} >$  with  $\tau_s|_K = vol_s|_K / vol_K$
- Treatment of fresh cells by an averaging process

# Numerical applications

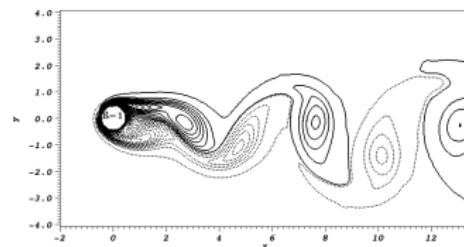
## ■ Poiseuille and Taylor-Couette flows



## ■ Rotating cylinder ( $\mathfrak{R} = 100$ , $D/h \sim 50$ )

[Stojković *et al.*, Phys. Fluids, 02]<sup>1</sup>  
 [Kang *et al.*, Phys. Fluids, 99]<sup>2</sup>

| Present results | Ref.       |              |
|-----------------|------------|--------------|
|                 | [1]        | [2]          |
| $C_d$           | 1.12       | 1.1080       |
| $C_d'$          | $\pm 0.11$ | $\pm 0.0986$ |
| $C_l$           | 2.51       | 2.504        |
| $C_l'$          | $\pm 0.37$ | $\pm 0.3616$ |
| St              | 0.165      | 0.1658       |
|                 | 0.1650     | 0.1650       |



(solid/dotted lines : neg./pos. contours)

# About the SFO damping

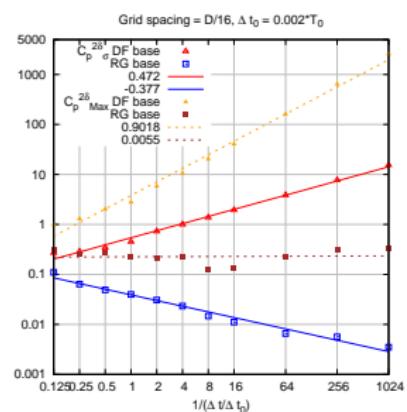
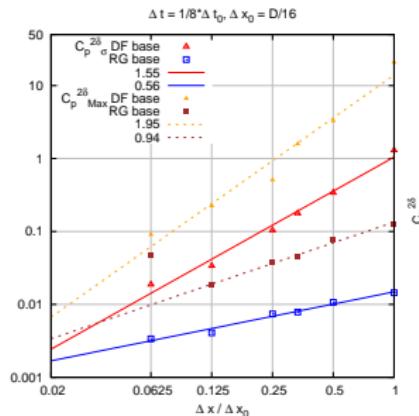
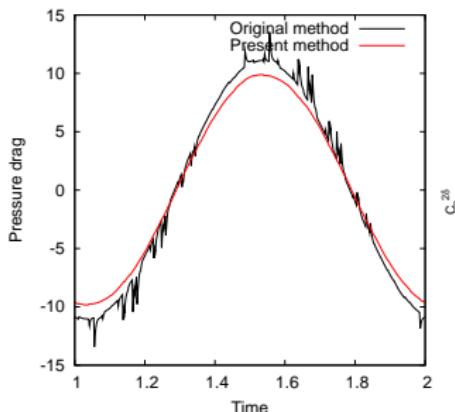
Analysis for the base model . . .

$$\begin{aligned} \int_{\Omega_{s,h}^{n+1}} \underline{F}_{RDF}^{n+1} d\Omega - \int_{\Omega_{s,h}^n} \underline{F}_{RDF}^n d\Omega &= \underbrace{\mathcal{O}(1) + \mathcal{O}(\Delta t) + \mathcal{O}(\Delta x)}_{K \in \Omega_{s,h}^{n+1} \cap \Omega_{s,h}^n} + \underbrace{\mathcal{O}(\Delta t) + \mathcal{O}(\Delta x)}_{K \in DC} + \underbrace{\mathcal{O}(\Delta t)}_{K \in FC} \\ &= \mathcal{O}(1) + \mathcal{O}(\Delta t) + \mathcal{O}(\Delta x^*) \quad * : 2 \text{ for the linear model} \end{aligned}$$

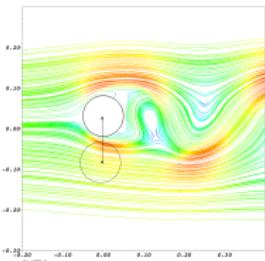
# About the SFO damping

Analysis for the base model . . .

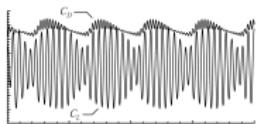
$$\begin{aligned} \int_{\Omega_{s,h}^{n+1}} F_{RDF}^{n+1} d\Omega - \int_{\Omega_{s,h}^n} F_{RDF}^n d\Omega &= \underbrace{\mathcal{O}(1) + \mathcal{O}(\Delta t) + \mathcal{O}(\Delta x)}_{K \in \Omega_{s,h}^{n+1} \cap \Omega_{s,h}^n} + \underbrace{\mathcal{O}(\Delta t) + \mathcal{O}(\Delta x)}_{K \in DC} + \underbrace{\mathcal{O}(\Delta t)}_{K \in FC} \\ &= \mathcal{O}(1) + \mathcal{O}(\Delta t) + \mathcal{O}(\Delta x^*) \quad * : 2 \text{ for the linear model} \end{aligned}$$



# An oscillating cylinder in cross-flow ( $R_e = 185$ , Guilmineau & Queutey, *JFS* 16, 2002)



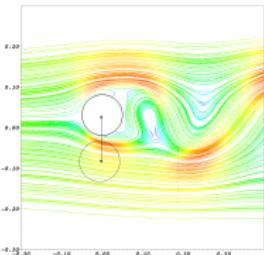
( $D/\Delta x = 25$ )



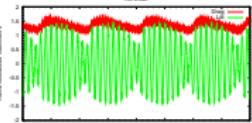
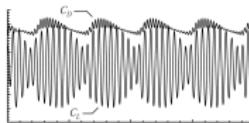
# An oscillating cylinder in cross-flow ( $R_e = 185$ , Guilmineau & Queutey, JFS 16, 2002)



( $D/\Delta x = 12.5$ )



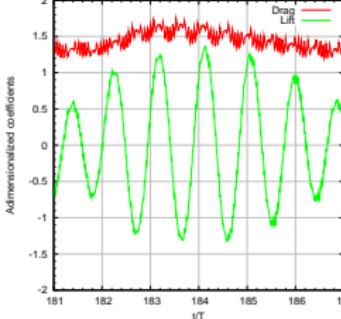
( $D/\Delta x = 25$ )



Stand. linear DF

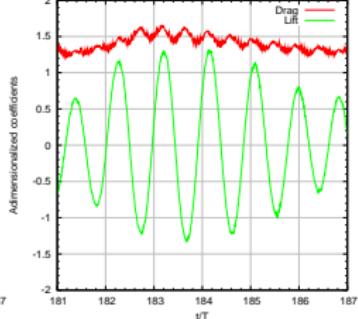
Drag - Lift

Adimensionnalized coefficients

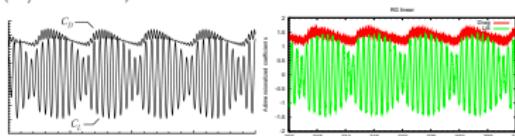
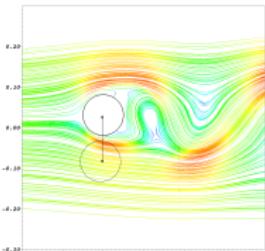


RG linear DF

Drag - Lift

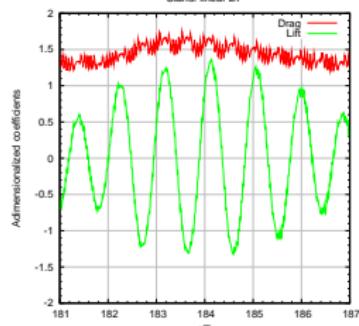


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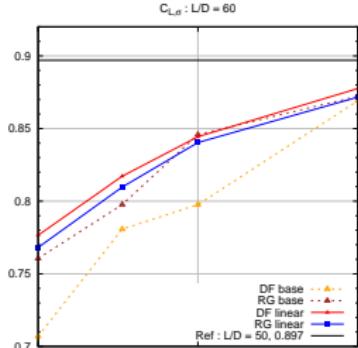
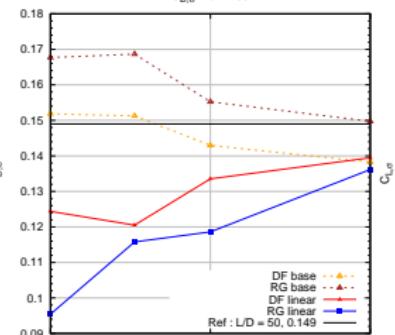
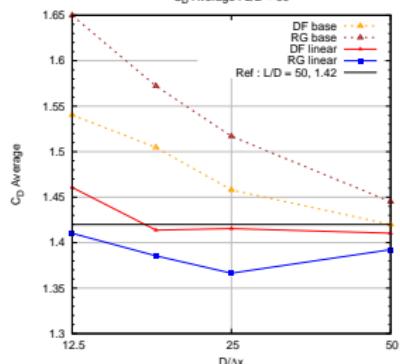
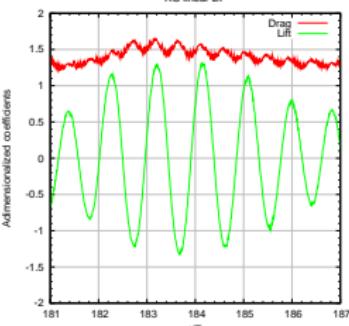


$(D/\Delta x = 12.5)$

Stand. linear DF

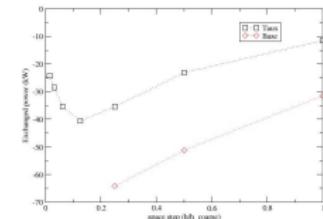


RG linear DF



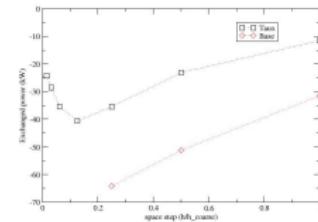
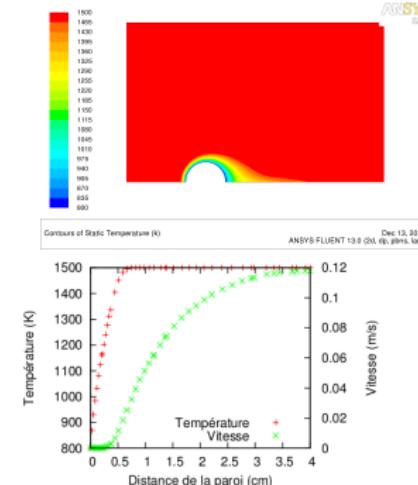
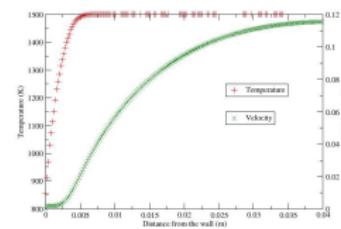
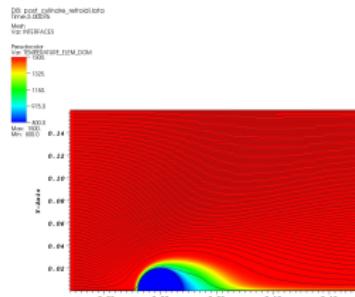
# 2D molten glass flow over a cold cylinder

- $\mu(T) \in [1; 10^5]$ ,  $\Lambda(T)$  and Boussinesq approx.
- Fluent benchmark simulation  
(boundary layer : AMR ; 14,000 faces)
- Mesh convergence study ( $2,400 \rightarrow 10$  M cells)



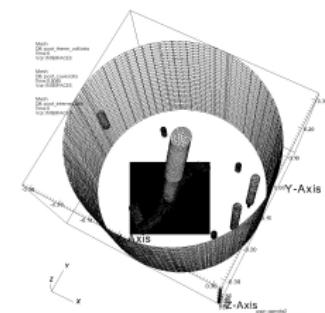
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Hydraulic validations of flows induced by a stirrer

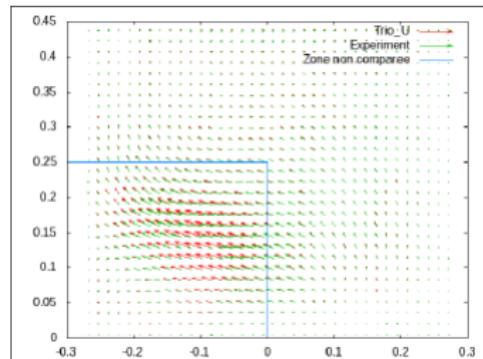
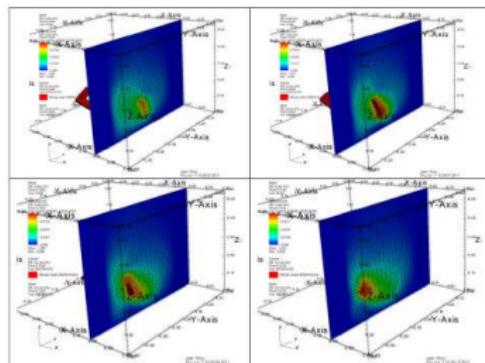
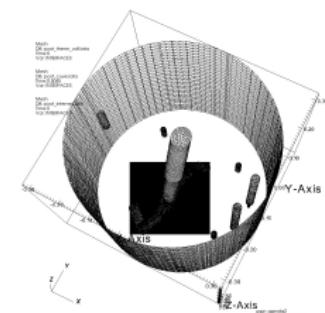
- Experimental device filled with viscous oil ( $\mathcal{R} \approx 100$ ) - PIV -
  - IBC Lagrangian meshes from CAD
  - Regular Eulerian meshes



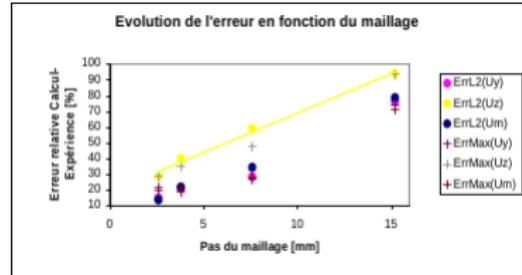
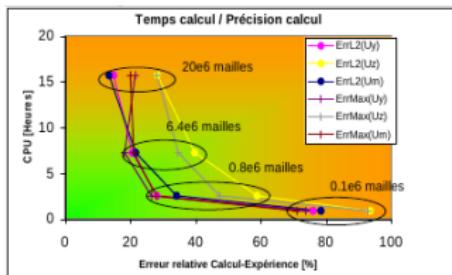
# cea Cold crucible simulations

## Hydraulic validations of flows induced by a stirrer

- Experimental device filled with viscous oil ( $\mathfrak{R} \approx 100$ ) - PIV -
- IBC Lagrangian meshes from CAD
- Regular Eulerian meshes
- Velocity-profile comparisons on a cut-plane and lines
- Fluent body-fitted sliding-mesh simulations (1 M cells with AMR)

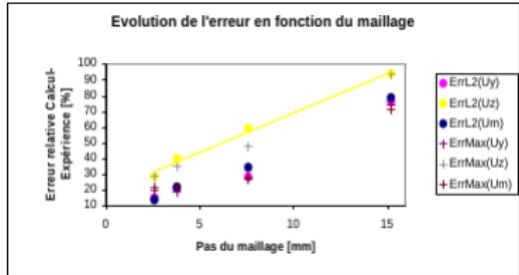
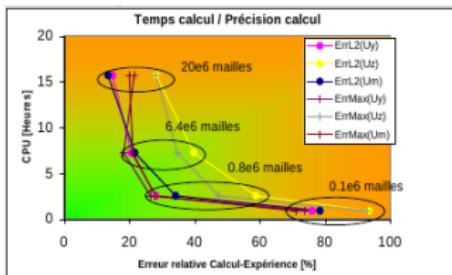


## ■ Base model : mesh convergence study ( $0.05 \rightarrow 20$ M cells ; $2 \rightarrow 400$ CPUs)



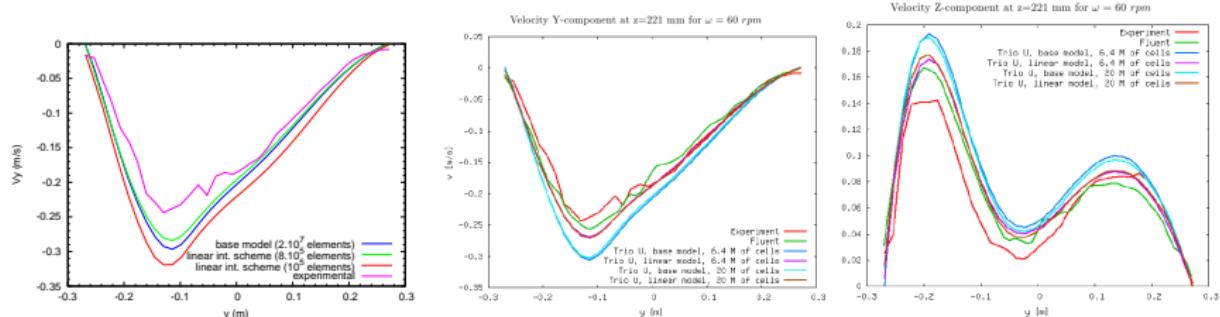
- Experimental rate of convergence about 0.6

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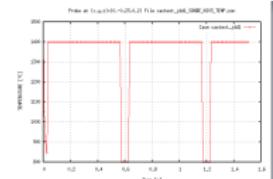
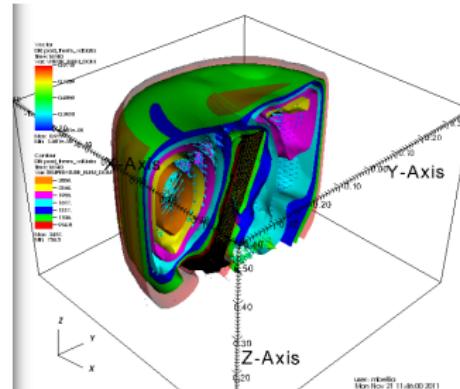
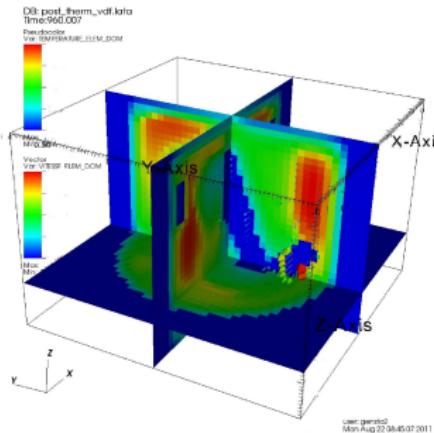
- Experimental rate of convergence about 0.6

## ■ Linear vs base model : cost ↓ for a given accuracy (1 M cells is quasi-optimal)



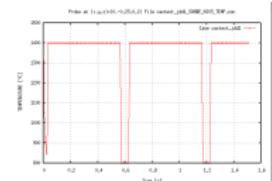
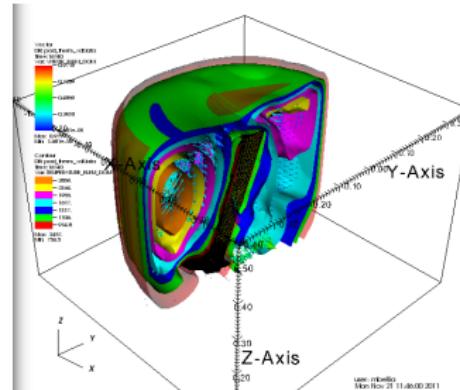
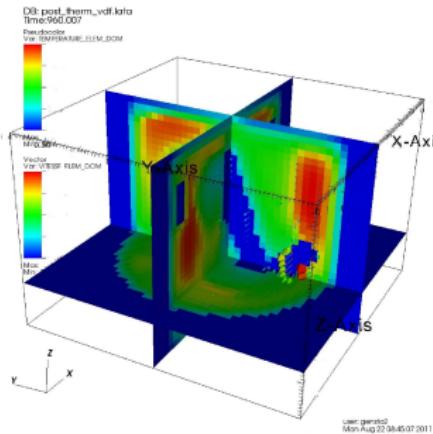
- Experimental rate of convergence about 1.0 for coarse meshes (but, exp. errors...)
- Trio\_U results compare well with experiment and Fluent ones

■ Thermal-hydraulic simulations;  $\mu(T) \in [1; 10^5]$ ,  $\Lambda(T)$  and Boussinesq approx.,  $\mathfrak{R} \approx 100$

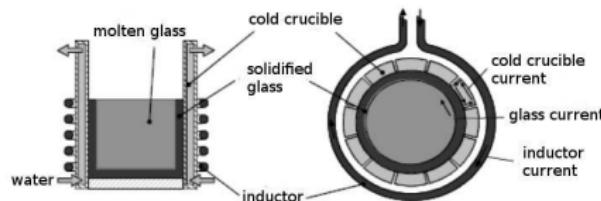


# Cold crucible simulations

■ Thermal-hydraulic simulations ;  $\mu(T) \in [1; 10^5]$ ,  $\Lambda(T)$  and Boussinesq approx.,  $\mathfrak{R} \approx 100$



■ Hydraulic bubbling simulations



# **Conclusions - Perspectives**

# Conclusions and perspectives

- Development of a **Penalized Direct Forcing** method
  - Finite Volume approximation with a staggered arrangement of variables
    - ~~ To solve the Navier-Stokes equations on Cartesian grid with penalized forcing term taking into account immersed boundary conditions
    - ~~ To account for the IBCs in the prediction and correction equations
- Development of an original **robust interpolation scheme**
  - ~~ To reconstruct accurately the velocity field near the immersed boundary
- SFO analysis & development of **regularized** DF and PDF methods
- CFD assessment on Academical test cases ( $\mathcal{O}(h^2)$  in  $L^2$ - and  $L^\infty$ -norms)
- Cutting off the SFO without altering physical phenomena nor the space convergence orders
- Numerical **simulations** of the **cold crucible** process
  - ~~ Hydraulic assessment vs experiment and code results
  - ~~ Thermal-hydraulic assessment vs code benchmark
  - ~~ Hydraulic bubbling simulations with IBCs give satisfactory results

- ⇒ Increase of the validation basis
- ⇒ Extension of our Penalized Direct Forcing method
  - Energy equation : linear interpolation for the temperature
  - Including AMR ...
  - Increasing the Reynolds number ...
- ⇒ FSI applications for single-phase and two-phase fluids
- ⇒ Cold crucible process simulations
  - Bubbling : convergence study and assessment vs experiment

# Thank you for your attention ...