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## Entropy and relaxation processes

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# Entropy and relaxation processes

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# Motivation

- Open systems: find constitutive relations that link fluxes to gradients  $\rightarrow$  transport matrix.
- Relaxation processes  $\leftrightarrow$  transport equations.  
Consistency with second principle?
- Principle of minimum entropy production  $\rightarrow$  second principle, relaxation processes, Onsager reciprocal relations. Can be done with quasi-linear theory - not always valid.
- Present status in magnetised plasmas?
- What can be done when this procedure fails?

# Entropy production and Onsager symmetry

- Entropy production  $\dot{S}$  vs “forces”  $\Lambda$  and “fluxes”  $\Gamma$

$$\dot{S} = \Lambda \cdot \Gamma$$

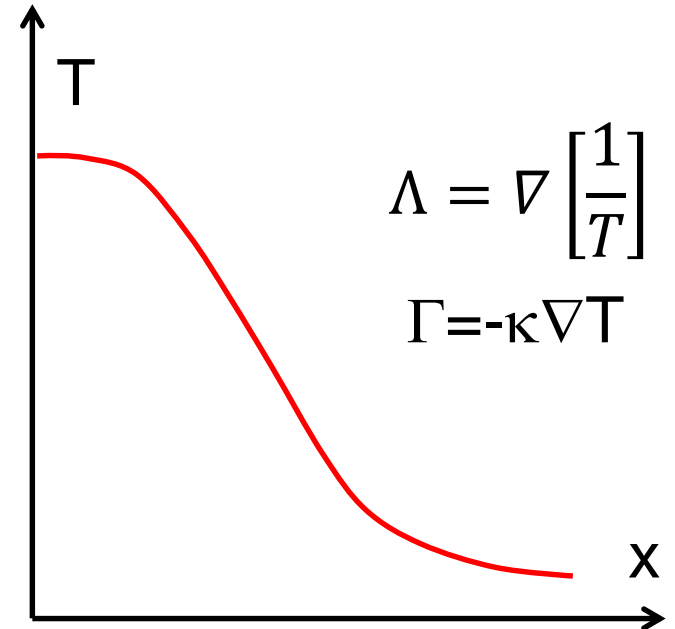
- Linear constitutive relationships

$$\Gamma = \bar{\bar{L}} \cdot \Lambda$$

- $\bar{\bar{L}}$  symmetric  $\mathbf{B} \rightarrow -\mathbf{B}$  Onsager 1931

- Compact form

$$\dot{S} = \Lambda \cdot \bar{\bar{L}} \cdot \Lambda$$



$$\dot{S} = \int dx \kappa \left( \frac{\nabla T}{T} \right)^2$$

- minimum of entropy production Prigogine 47

# Boltzmann-Gibbs statistical mechanics

- Distribution function  $F(\mathbf{x}, \mathbf{v}, t) \rightarrow$   
entropy  $S = - \int d\tau F \ln F$

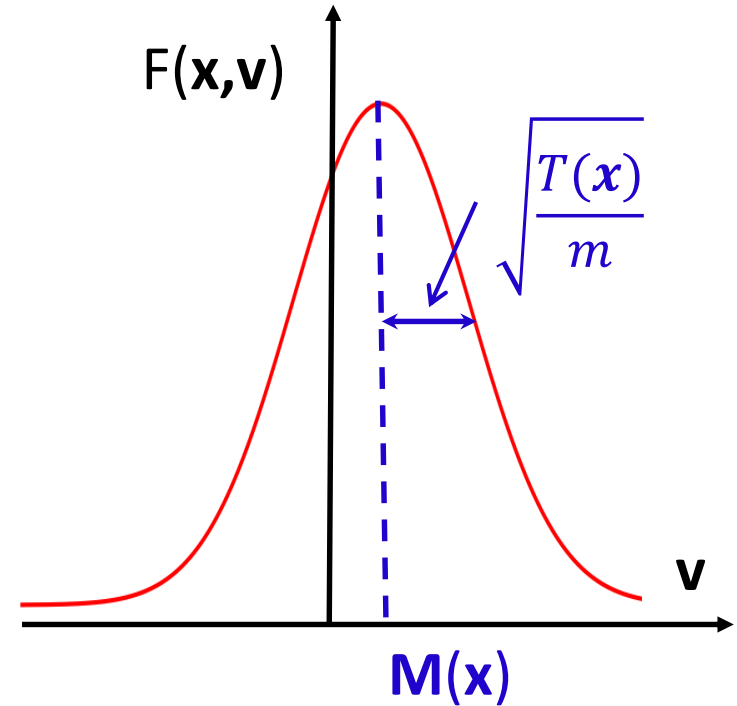
- Maximum of entropy under  
constraints  $\rightarrow$  local Maxwellian

$$F = \exp\left(-\frac{H}{T} + U(\mathbf{x})\right)$$

- F solution of a kinetic equation

$$\frac{dF}{dt} = C(F)$$

- Collisional transport well  
documented OV's Braginskii 65, Hinton  
76, Balescu 87, Shaing 88  $\rightarrow$  Turbulent  
transport



$$\int d^3 \mathbf{v} F(\mathbf{x}, \mathbf{v}) = N(\mathbf{x})$$

# Quasi-linear theory provides fluxes for a given spectrum of fluctuations

Sheared B field, drift waves, chaotic

Hasegawa-Mima 77, Hasegawa-Wakatani 83

Fluctuations of ExB drift

velocity  $\mathbf{v}_E = \frac{\mathbf{E} \times \mathbf{B}}{B^2}$



Plasma linear response

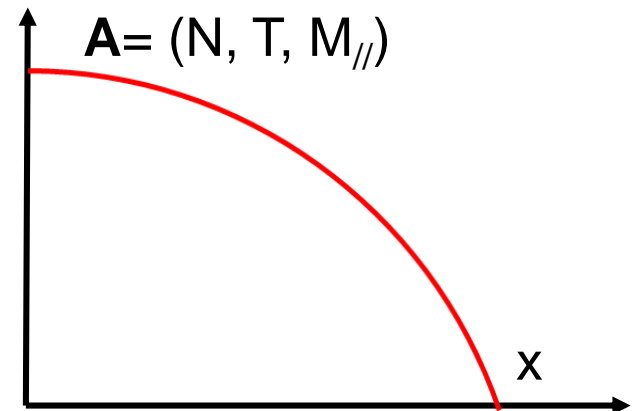
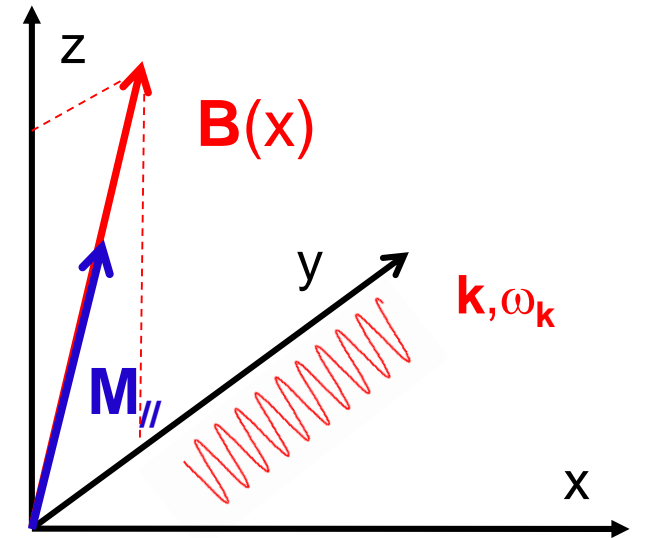
$$F_{\mathbf{k}} = R_{\mathbf{k}}(\Lambda) v_{E\mathbf{k}}$$



Flux  $\Gamma = \sum_{\mathbf{k}} \int d^3\mathbf{v} F_{\mathbf{k}} v_{E\mathbf{k}}^*$

Drummond 62, Vedenov 62, OVs Krommes 02,

Diamond 10



$$\Lambda = \left( \frac{dN}{Ndx}, \frac{dT}{Tdx}, \frac{dM_{||}}{dx} \right)$$

# Entropy production is explicit

- Entropy production rate Horton 80, Itoh 82, Sugama 96, XG 13

$$\dot{S} = \frac{1}{2} \sum_{\mathbf{k}} \int d\tau F D_{\mathbf{k}} \left[ \frac{\partial U}{\partial x} - \frac{eB}{k_y T} (\omega_{\mathbf{k}} - k_{\parallel} M_{\parallel}) \right]^2$$

Diffusion due to ExB  
velocity fluctuations

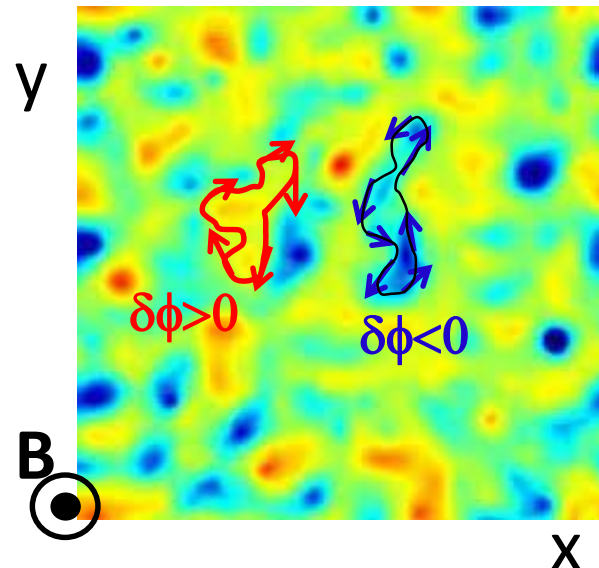
Forces

wave/particle energy  
& momentum  
transfer

$$\Lambda = \left( \frac{dN}{Ndx}, \frac{dT}{Tdx}, \frac{dM_{\parallel}}{dx} \right)$$

$$\mathbf{V}_E = \frac{\mathbf{E} \times \mathbf{B}}{B^2}$$

$$D_{\mathbf{k}} = 2\pi |v_{E\mathbf{k}}|^2 R(\omega_{\mathbf{k}} - k_{\parallel} v_{\parallel})$$



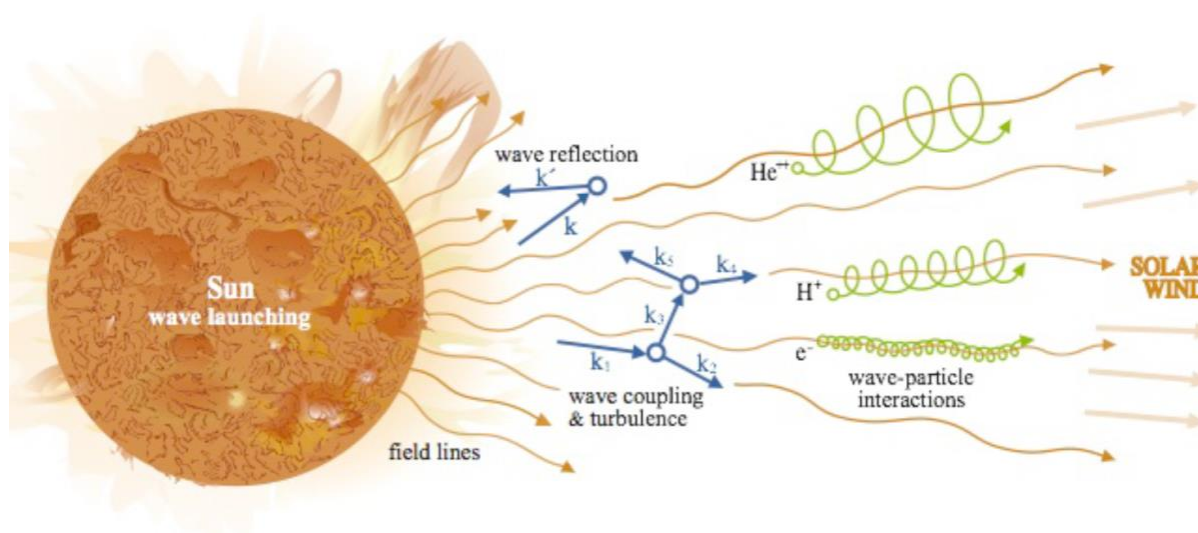
Numata 06

# Transport equations bear a puzzling shape

- Evolution of thermodynamical variables  $\mathbf{A}=(N,T,M_{//})$

$$\frac{\partial A}{\partial t} + \nabla \cdot \mathbf{\Gamma} = \Sigma \quad \mathbf{\Gamma} = \bar{\bar{\mathbf{L}}} \cdot \mathbf{\Lambda} + \mathbf{\Gamma}_{\text{res}}$$

- Transport matrix  $\bar{\bar{\mathbf{L}}}$  Onsager symmetric. However :
  - “Residual” momentum and energy fluxes  $\mathbf{\Gamma}_{\text{res}}$
  - Sources  $\Sigma =$  turbulent heating and acceleration Rudakov 71, Ott 72



Bale 2012



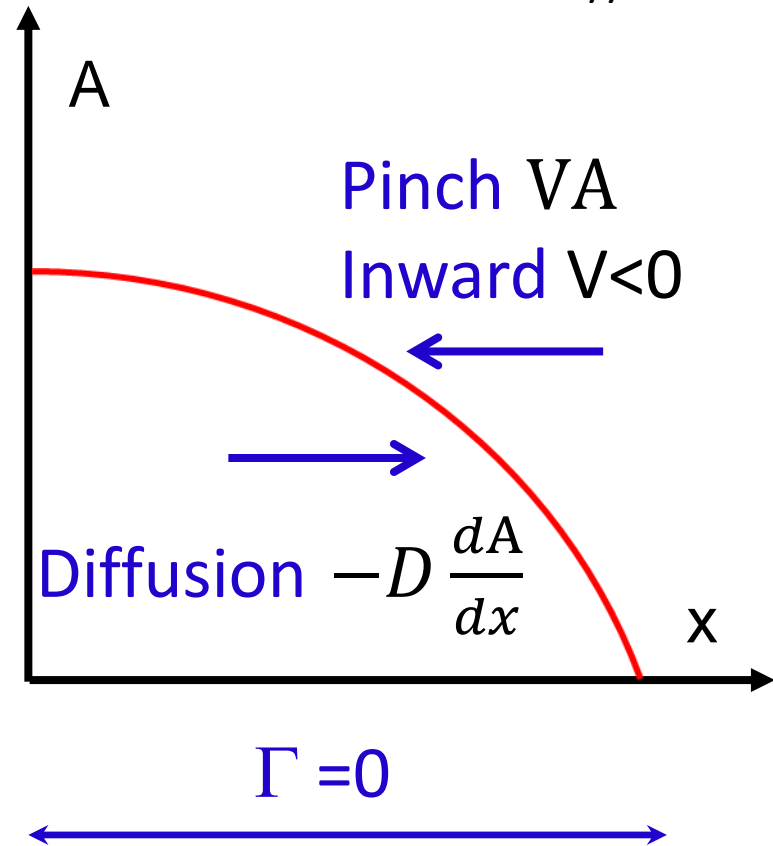
# Fluxes possess pinch and residual components

- Diffusion/convection flux structure

$$\Gamma = -D \frac{dA}{dx} + VA + \Gamma_{res}$$

$$\mathbf{A} = (N, T, M_{//})$$

- Diffusion = average of  $\sum_k D_k$ .
- Pinch velocity due forces other than  $\frac{dA}{dx}$ .
- Residual momentum and heat fluxes  $\neq 0 \rightarrow$  requires symmetry breaking  $\langle k_{//} k_y \rangle \neq 0$ .



# Plasma sped up and heated by turbulence

- Source terms Itoh 88, Hinton 06, Lu Wang 13, XG 13

$$\Sigma_{M\parallel} = \langle NeE_{\parallel} \rangle$$

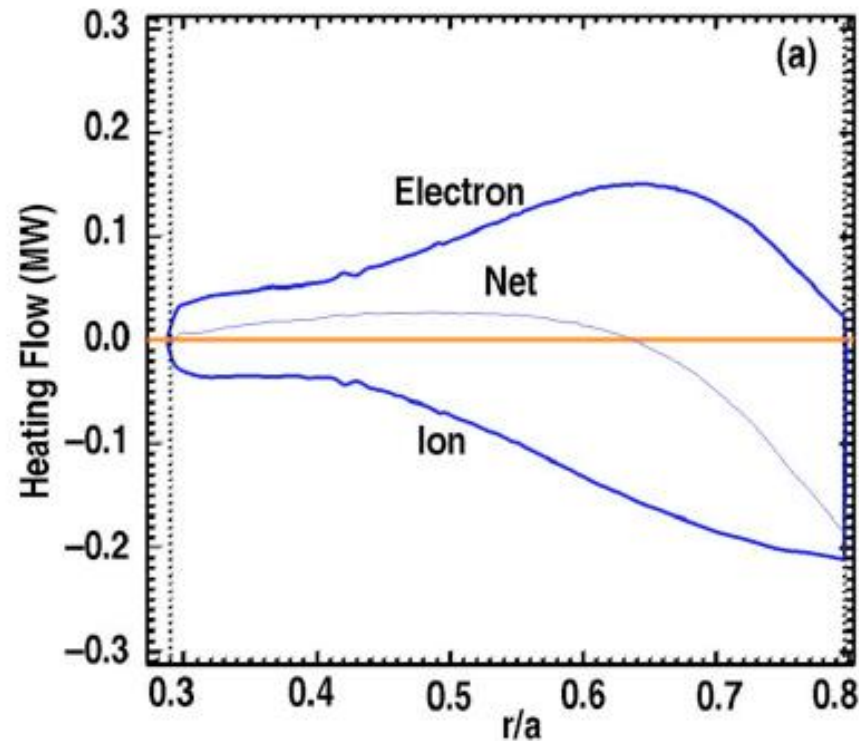
$$\Sigma_T = \langle NeV \cdot E \rangle$$

Waltz 11

- Charge conservation  $\rightarrow$  global conservation

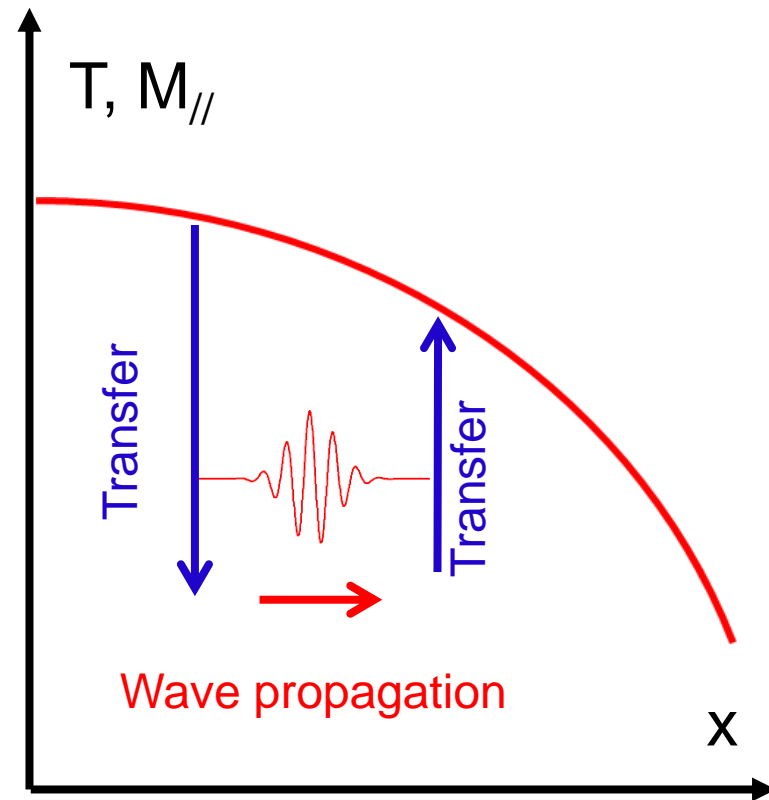
$$\sum_{\text{species}} \text{sources} = 0$$

- Momentum and energy transfer between species Zhao 12



# Residual fluxes and sources are Onsager symmetric

- Define new forces  $\frac{1}{T}, \frac{M_{\parallel}}{T} \rightarrow$   
**transport matrix is symmetric**  
Horton 80, Itoh 82, Sugama 96
- Total field + particle momentum/  
energy is conserved  $\rightarrow$  extended  
thermodynamics Boozer 92,  
Krommes 93, Watanabe 06, XG 12
- Sources = fluxes of **momentum/  
energy carried by waves** Diamond  
08



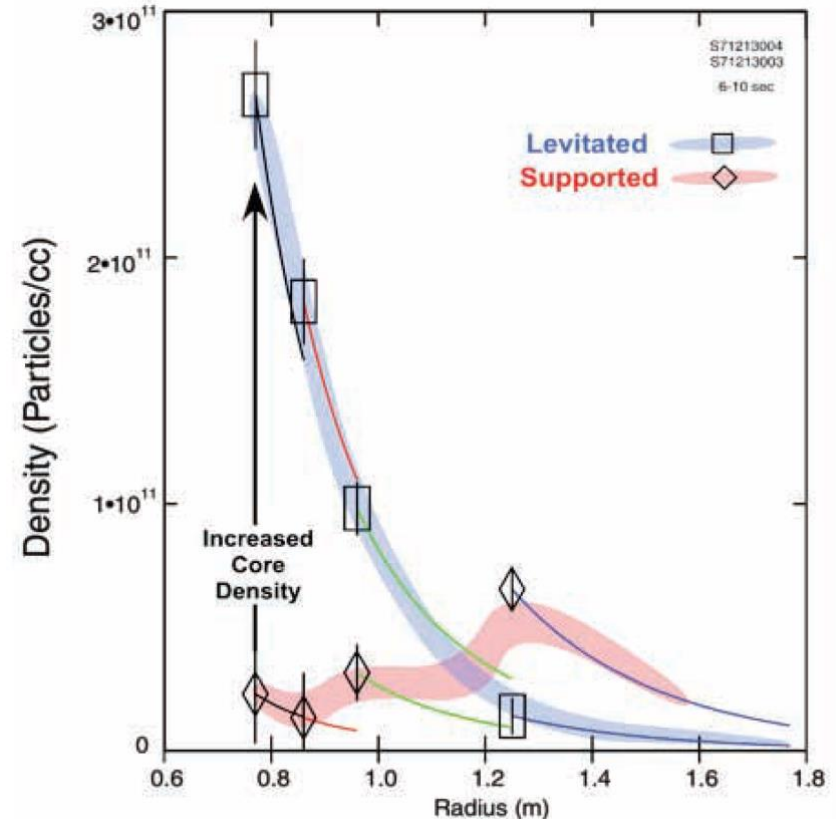
# Magnetic drift contributes to pinch velocities

- Resonance  $\omega = k_{\parallel} v_{\parallel} \rightarrow \omega = k_{\parallel} v_{\parallel} + \mathbf{k} \cdot \mathbf{v}_D \rightarrow$  introduces magnetic drift  $\rightarrow$  pinch velocities proportional to  $\nabla B/B$ .

- Related to Lagrangian invariants  $\rightarrow$  compressibility  $\nabla \cdot \mathbf{v}_E \neq 0$  Yankov 94, Isichenko 95&97, Baker 01, XG 04, Gürcan 10

$$\frac{d}{dt} \left( \frac{N}{B^2} \right) = 0$$

Boxer 10 - LDX



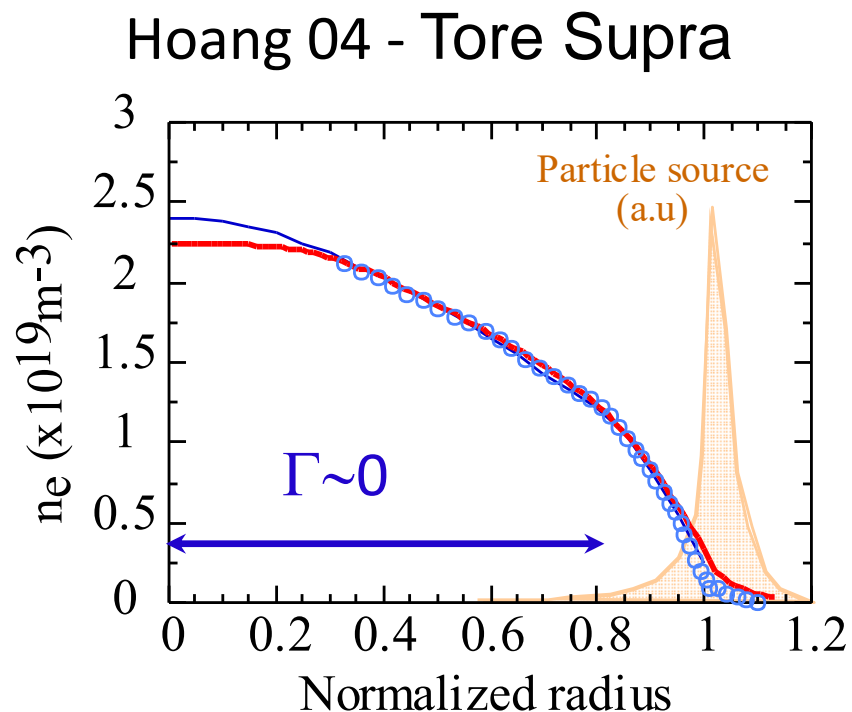
# Turbulent pinch theory successfully tested in tokamaks

- Particle flux  $\Gamma = -D \frac{dN}{dx} + VN$
- Pinch velocity XG 04, Angioni 04 & 06, Camenen 09

$$V = V_T \frac{dT}{T dx} + V_{M\parallel} \frac{dM_{\parallel}}{dx} + V_B \frac{dB}{B dx}$$

Thermo diffusion
Roto diffusion
compression

- Onsager symmetry  $\rightarrow$  **thermal pinch** Luce 92, Itoh 96, Mantica 05, Lu Wang 11



# Turbulent pinch theory successfully tested in tokamaks

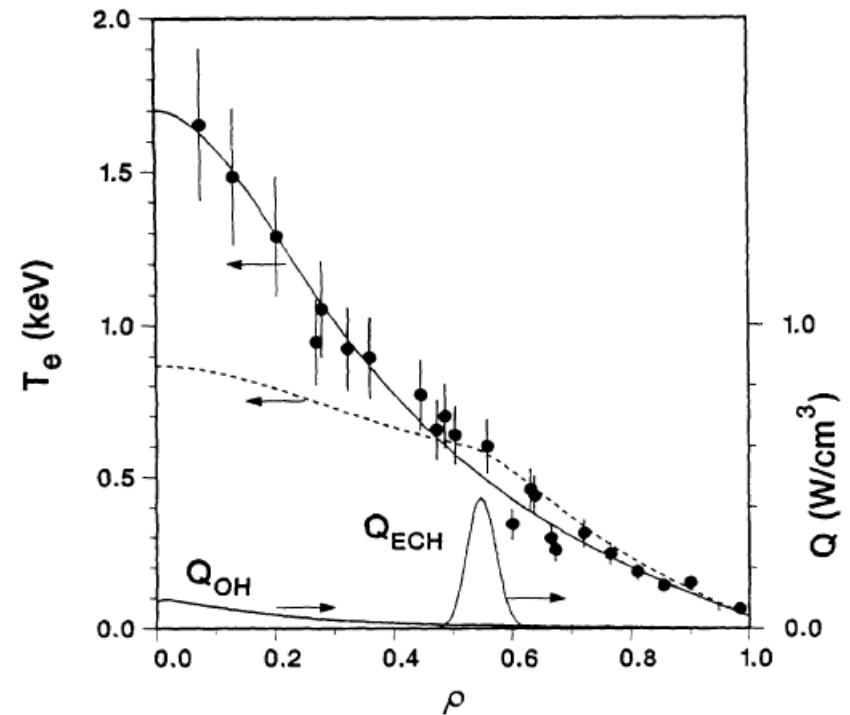
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$$V = V_T \frac{dT}{T dx} + V_{M\parallel} \frac{dM_{\parallel}}{dx} + V_B \frac{dB}{B dx}$$

↑
↑
↑

Thermo diffusion
Roto diffusion
compression

Luce 95 – DIII-D



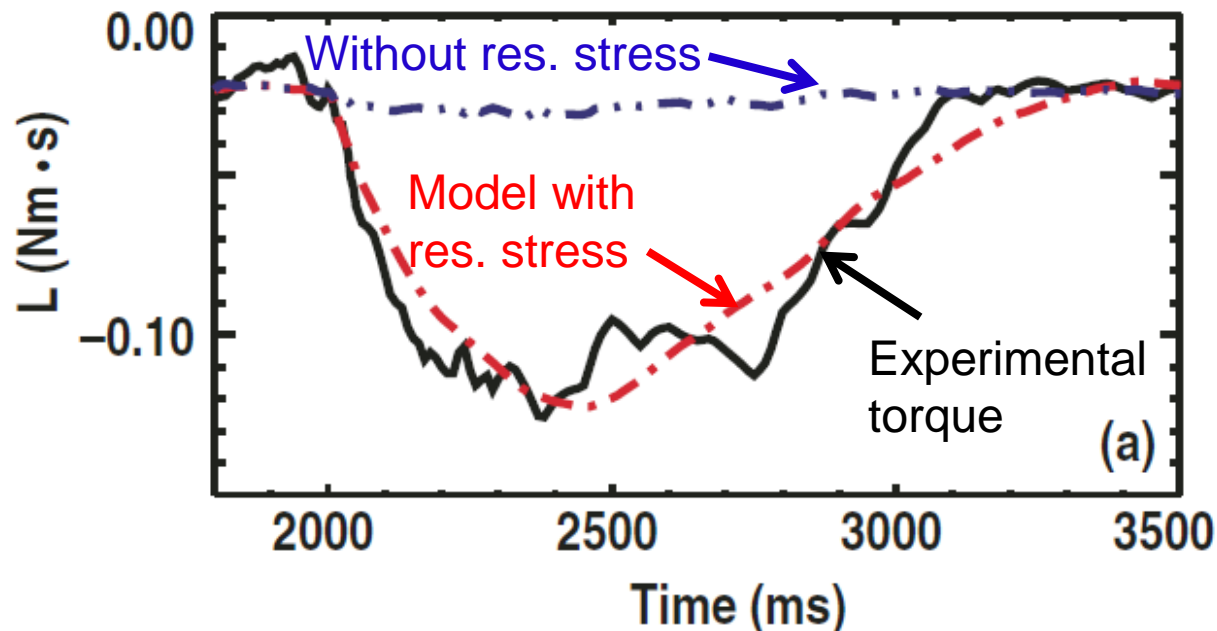
- Onsager symmetry → thermal pinch Luce 92, Itoh 96, Mantica 05, Lu Wang 11

# Momentum flux has both pinch and residual components

- Plasma spin-up in tokamaks without external torque.

- Pinch and residual stress  $\Gamma = -D \frac{dM_{\parallel}}{dx} + VM_{\parallel} + \Gamma_{res}$

OVs Diamond 09, Peeters 11, Ida 13, Tynan 19 CD-I9 entropy production Kosuga 10



Solomon  
09 - DIII-D

# Thermodynamics of non local transport is an open issue

- Non local transport Van Milligen

04, Del-Castillo-Negrete 05, Dif-Pradalier 10, OV Ida 15

$$\Gamma(x) = - \int dx' \kappa(x - x') \nabla N(x')$$

- Pinch effect with single force

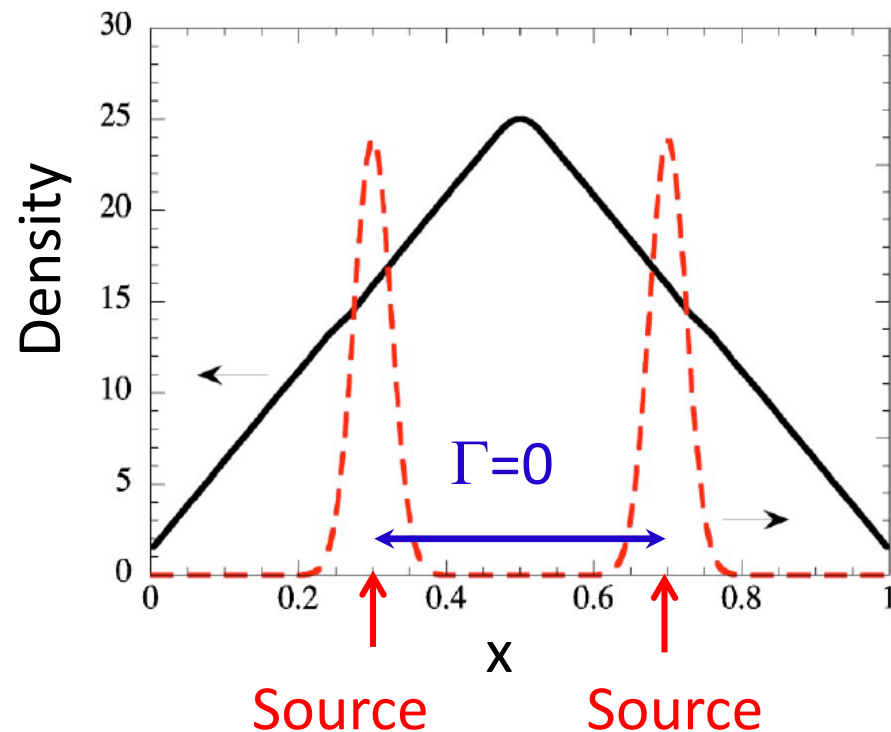
Del-Castillo-Negrete 05, Bouzat 05.

- Second principle  $\rightarrow$  Tsallis

entropy Tsallis 88, Anderson 18

Onsager symmetry ?

Van Milligen 04





# Violent relaxation theory predict coherent states

- Fast relaxation of a system with long range interactions. Entropy

Lynden-Bell 67

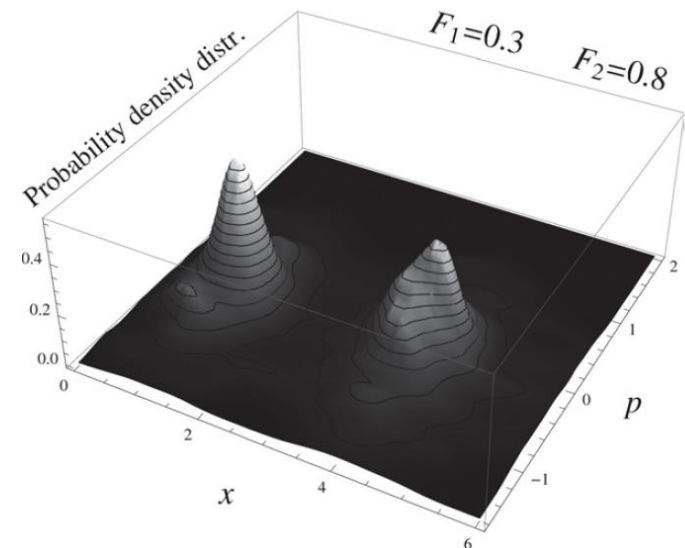
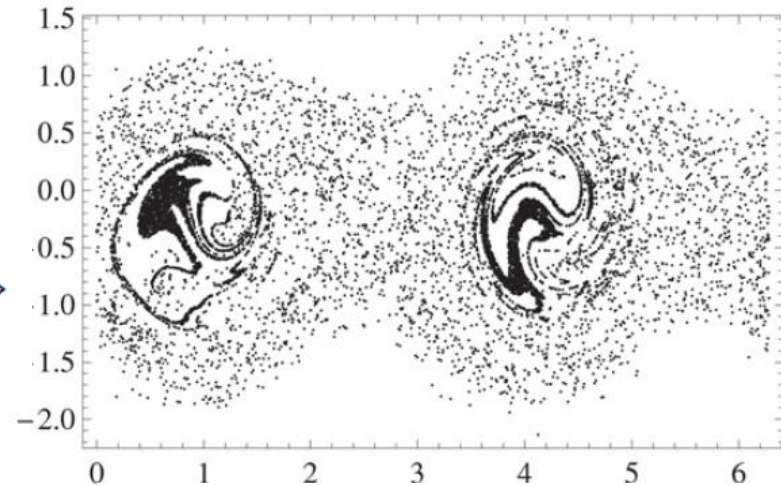
$$S = - \int d\tau \left\{ \frac{F}{F_0} \ln \frac{F}{F_0} + \left( 1 - \frac{F}{F_0} \right) \ln \left( 1 - \frac{F}{F_0} \right) \right\}$$

- Maximum entropy with conservation constraints  $\rightarrow$  Quasi-Stationary States Robert 92, Antoniazzi

08, Chavanis 06, Carlevaro 13

- Relaxation? Maximum entropy production principle? Martyushev 06

Carlevaro 13



# Conclusions

- Minimum of entropy production principle coupled to quasi-linear theory predicts fluxes vs forces in turbulent magnetised plasmas.
- Predicts pinches, residual contributions to energy and momentum fluxes, turbulent heating and acceleration.
- Onsager symmetry respected under conditions.
- May fail, typically in systems with long range interactions, with memory effects. Tsallis and Lynden Bell statistics offer alternatives.