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Calculation of beta spectra The BetaShape program

- > Motivations
- > Assumptions of the calculations
- Comparison with other programs
- Parameters that influence the spectrum shape
- > Comparison with the measured ²⁴¹Pu spectrum
- Conclusions and further developments

CEA Saclay / LNHB : *Xavier Mougeot*, M.-M. Bé, V. Chisté, C. Dulieu, M. Loidl JINR Dubna / DLNP : V. Gorozhankin



Users ask for a precise knowledge of **beta spectrum shapes** coupled with **well established uncertainties**

Ionizing radiation metrology

Activity measurements by LSC : better knowledge \Rightarrow better uncertainties

Nuclear industry, medical care

Residual power in nuclear reactor, doses delivered to patient's cells, ...

Past measurements : ~ 10 published form factors since 1976 ! Exhaustive bibliographical compilation by V. Gorozhankin up to 2003

⇒ Not enough experimental data to test theoretical beta spectrum shapes, especially for high forbidding orders and at low energy

Theoretical studies stopped in the 1980's

Almost no theorist still works on beta spectra

V. Gorozhankin, Construction of the beta spectra on the basis of nuclear experimental decay data, LSC 2010, P–68 poster sessions (2010)





Remarks:

- In this presentation only the first step of this work, the calculations part, is introduced.
- The experimental set up is in progress...



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Dirac's equation with Coulomb potential

$$\begin{bmatrix} i\vec{\alpha}\cdot\vec{\nabla}-\beta m\end{bmatrix}\Psi(\vec{r},t) = \begin{bmatrix} i\frac{\partial}{\partial t}-V(\vec{r})\end{bmatrix}\Psi(\vec{r},t) \longrightarrow \begin{cases} \frac{d}{dr}g_{\kappa}(r)+\frac{\kappa+1}{r}g_{\kappa}(r)-[E_{tot}+m-V(r)]f_{\kappa}(r)=0\\ \frac{d}{dr}f_{\kappa}(r)-\frac{\kappa-1}{r}f_{\kappa}(r)+[E_{tot}-m-V(r)]g_{\kappa}(r)=0 \end{cases}$$

c .

<u>Solving</u>

- analytically (Rose): fast calculations, point source. Ex: RadList program
- numerically (Behrens, Dzhelepov): more real source, slowness → tables. Ex: Beta Spectrum

Assumptions

- $m_{\text{nucleus}} = \infty$ and $m_{\nu} = 0 \rightarrow \text{simplification of 3-body decay}$
- spherical symmetry and $\lambda_{C}(e^{-}) >> R \rightarrow$ plane waves approximation
- Coulomb field: Fermi function → models for charge, potential, screening from electron cloud

Classification





- Radiative corrections (virtual photons, internal bremsstrahlung)
- Spectrum normalized by its integral

R.H. Good Jr, *Effect of atomic electron screening on the shape of forbidden beta spectra*, Physical Review 94, 931 (1954)
N.B. Gove, M.J. Martin, *Log-f tables for beta decay*, Nuclear Data Tables 10, 205 (1971)
H. Behrens, W. Bühring, *Electron radial wave functions and nuclear beta decay*, Oxford Science Publications (1982)



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⁹⁹Tc (9/2⁺,gs) → ⁹⁹Ru (5/2⁺,gs) β⁻ 2nd non-unique forbidden transition P_β = 99.99855 (30) % Q_β = 293.8 (14) keV T_{1/2} = 211.5 (11) ·10³ a



Nucleus	Finite nuclear size correction	Screening correction	Radiative corrections \geq 1% if
²⁴¹ Pu	~ 0.03 %	~ 4.7 %	$E \ge 53 \% E_{max}$
²⁰³ Hg	~ 0.16 %	~ 4.5 %	$E \ge 58 \% E_{max}$
¹⁷⁶ Lu	~ 0.22 %	~ 4 %	$E \ge 72 \% E_{max}$
¹³³ Xe	~ 0.007 %	~ 3 %	$E \ge 75 \% E_{max}$
¹²⁹	~ 0.44 %	~ 17 %	$E \ge 78 \% E_{max}$







Discrepancies at low energy \Rightarrow Exchange effect ?

M. Loidl et al., First measurement of the beta spectrum of ²⁴¹Pu with a cryogenic detector, Applied Radiation and Isotopes 68 (2010) 1460



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List The BetaShape program

- Analytical calculations of beta spectrum shapes
- Coulomb corrections which depend on the forbidding order of the transition
- Other corrections : finite nuclear size, screening, radiative
- C++ class: easy implementation in Monte-Carlo simulations or nuclear database programs
 - Atomic corrections: exchange effect at low energy
 - Screening correction: derive a smooth function from experimental spectra
 - Parity change: specific treatment of the non-unique forbidden transitions

Form factors

Not enough experiments to validate them, especially for high forbidding orders

- \Rightarrow Do not trust blindly a given form factor
- \Rightarrow For each published form factor, each experiment must be analyzed carefully

Experiments

- Measurements with a silicon detector (setup in progress)
 - Shapes: to test and to validate theoretical predictions
 - Models: to establish associated uncertainties