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# Simultaneous estimation of groundwater recharge and hydrodynamic parameters for groundwater flow modeling

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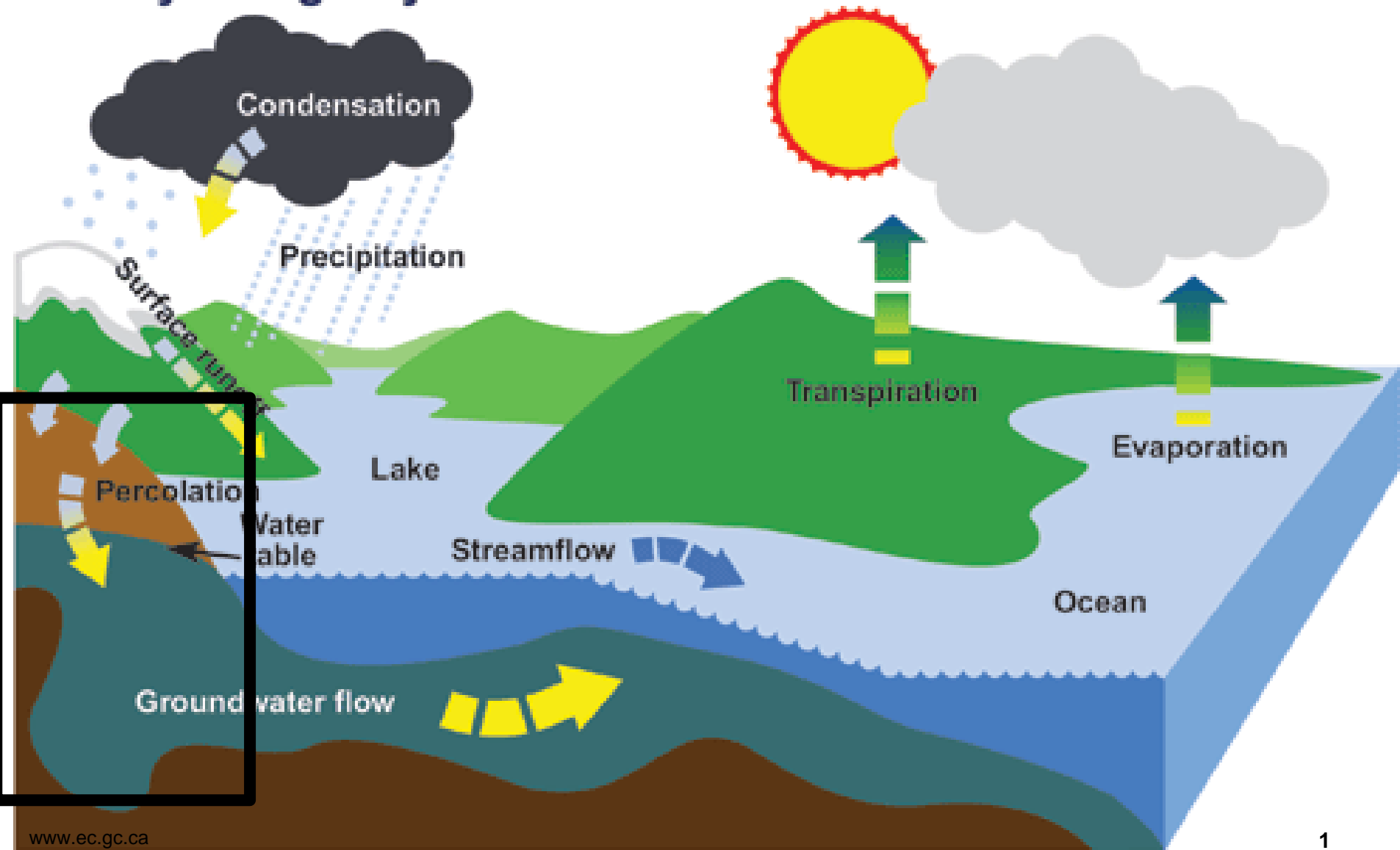
Laboratoire d'Hydrologie et  
de Géochimie de Strasbourg



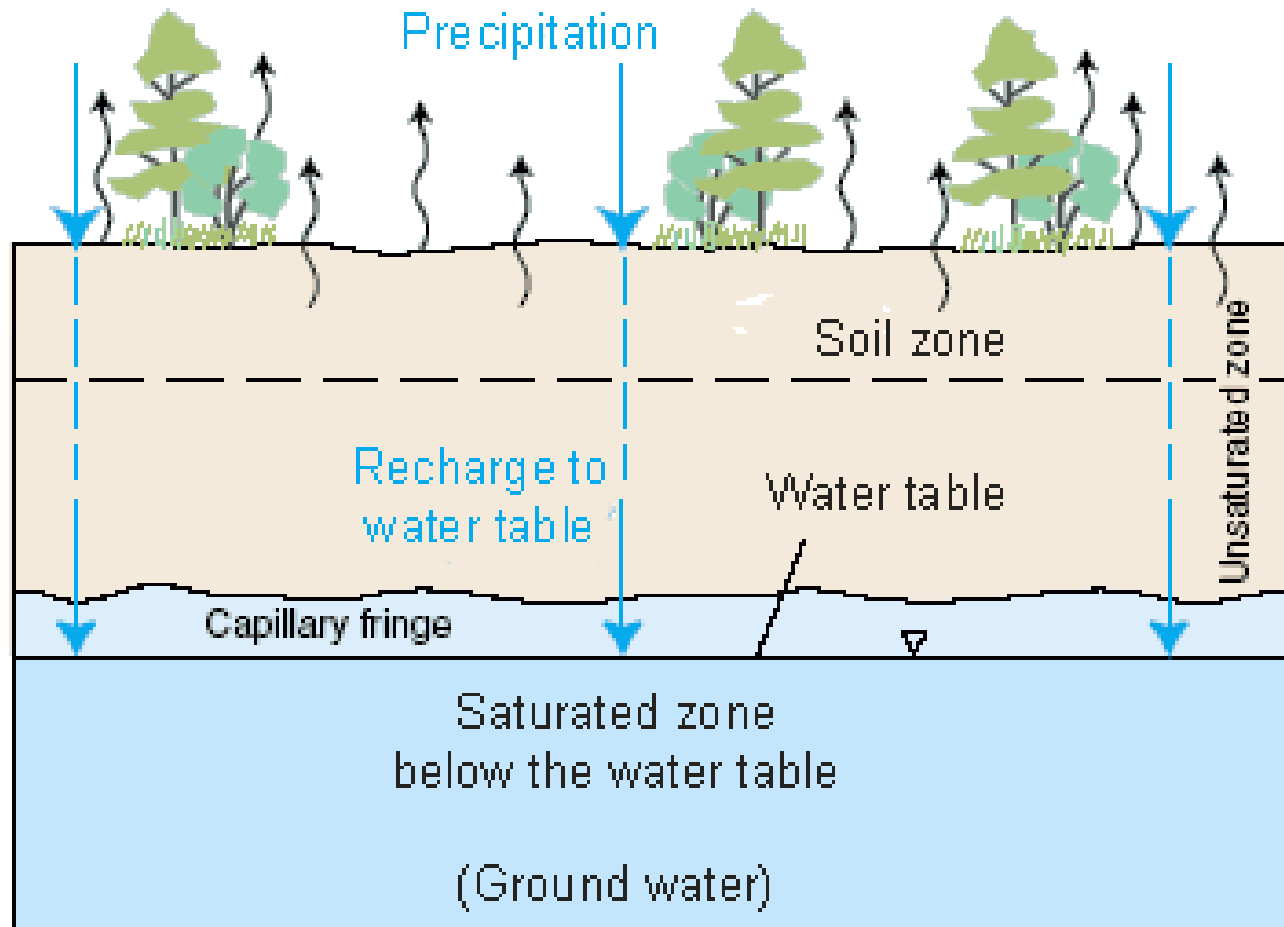
# **SIMULTANEOUS ESTIMATION OF GROUNDWATER RECHARGE AND HYDRODYNAMIC PARAMETERS FOR GROUNDWATER FLOW MODELING**


Fadji HASSANE MAINA (Université de Strasbourg)  
Philippe ACKERER (Université de Strasbourg)  
Olivier BILDSTEIN (CEA Cadarache)

## The hydrologic cycle



- **Groundwater (aquifers):** 3<sup>rd</sup> freshwater reservoir on the planet
- Replenished essentially by precipitation through **groundwater recharge**



- **Groundwater (aquifers):** 3<sup>rd</sup> freshwater reservoir on the planet
- Replenished essentially by precipitation through **groundwater recharge**
- The **main available water resources** for many countries
- Threatened by pollution and over-exploited
- Usually described by models  Simulations of piezometric levels (Hydraulic heads)
- **Physical Models are widely used**

1. **Continuity equation** (mass conservation):  $S \frac{\partial H}{\partial t} + \nabla \cdot q = q_s$



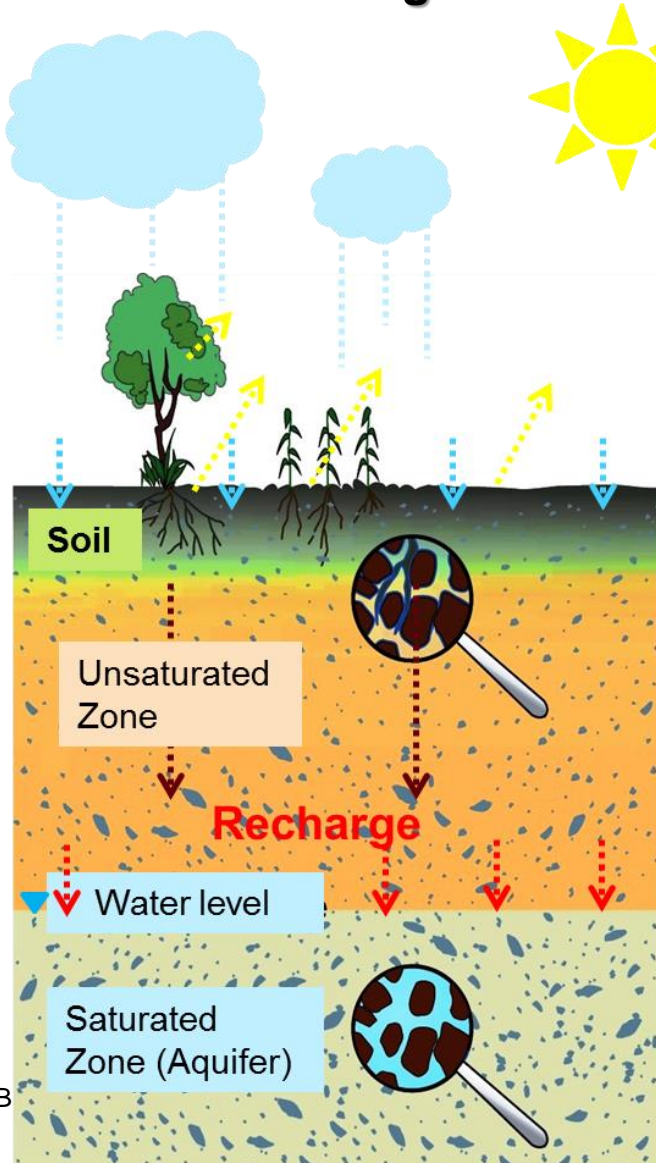
2. **Darcy's law** (computation of fluxes):  $q = -K \nabla H$



**Diffusivity equation** (flow equation):  $S \frac{\partial H}{\partial t} - \nabla \cdot [K \nabla H] = q_s$

**Values of H knowing S, K and  $q_s$  → S and K are constant,  $q_s$  includes groundwater recharge**

## Recharge : water transfer from precipitation to groundwater

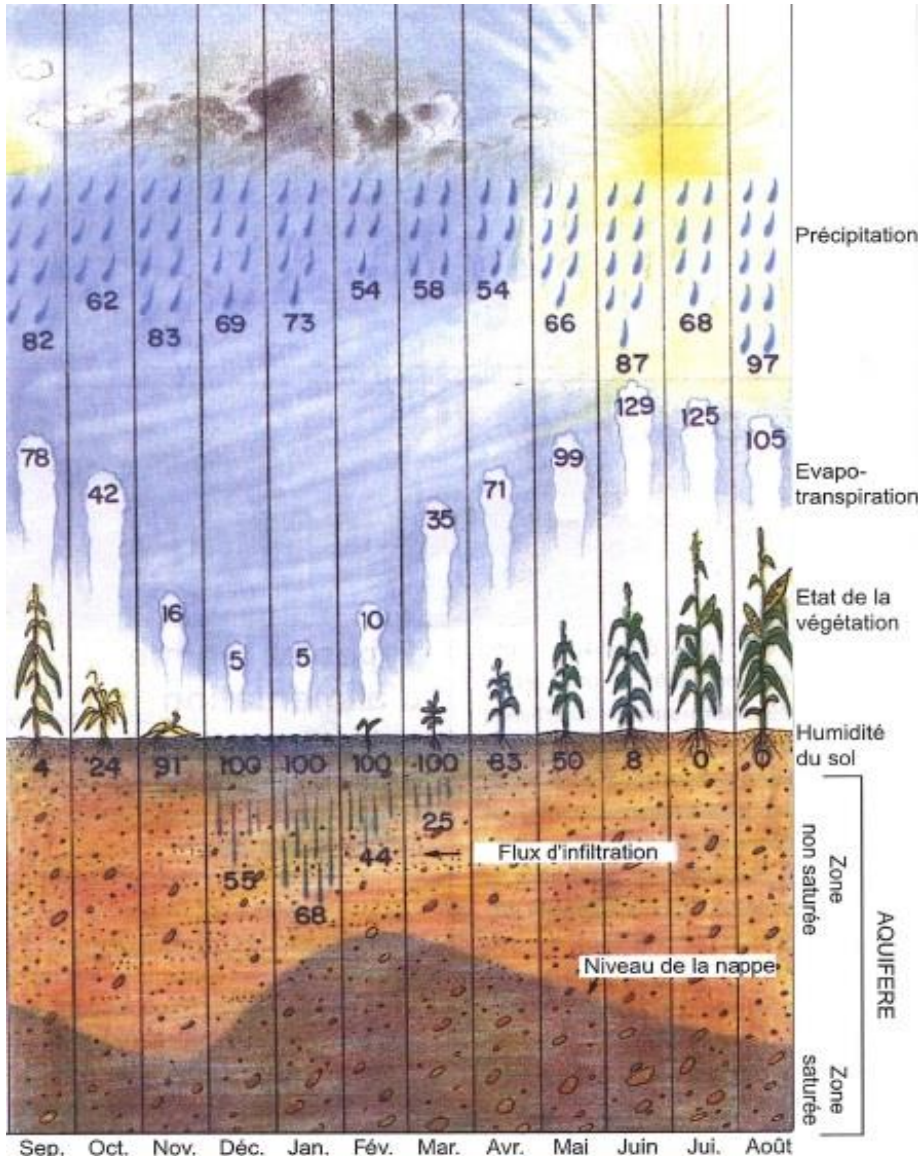


- ☺ Groundwater flow models: **Sophisticated and accurate**
- ☹ Recharge modeling: **Challenge**
  - **Complex hydrological component**

### Groundwater recharge depends on:

- Climatic conditions
- Vegetation
- Soil and root zone
- Unsaturated zone

## Recharge : water transfer from precipitation to groundwater



- ☺ Groundwater flow models: **Sophisticated and accurate**
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### Groundwater recharge depends on:

- Climatic conditions
- Vegetation
- Soil and root zone
- Unsaturated zone

Values can not be directly measured



### Estimation

- Direct methods: lysimeters, TDR
- Empirical methods
- Tracers
- **Mathematical models**

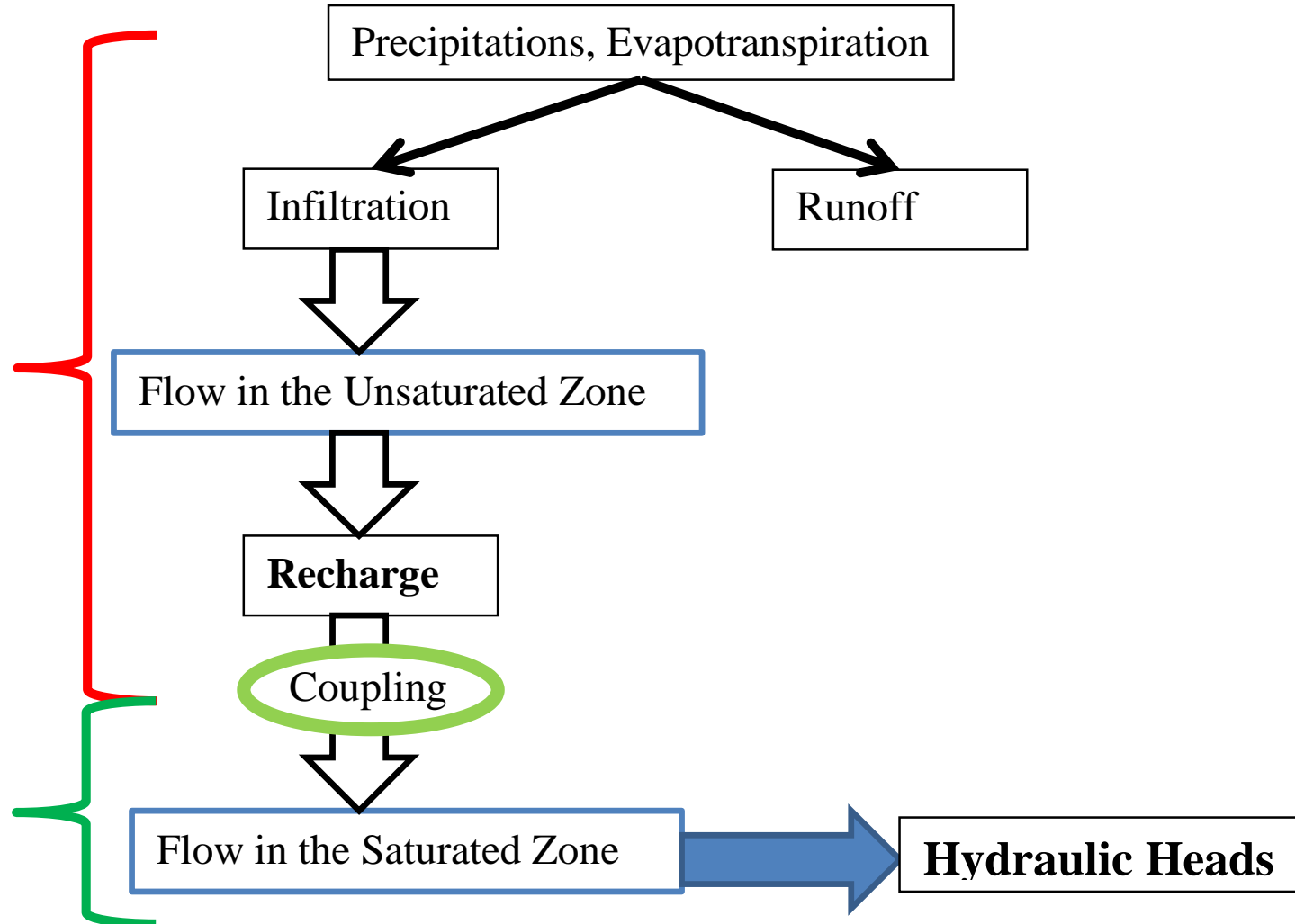
# RECHARGE ESTIMATION: COUPLED MODEL

Simulation of flow in both unsaturated and saturated zones

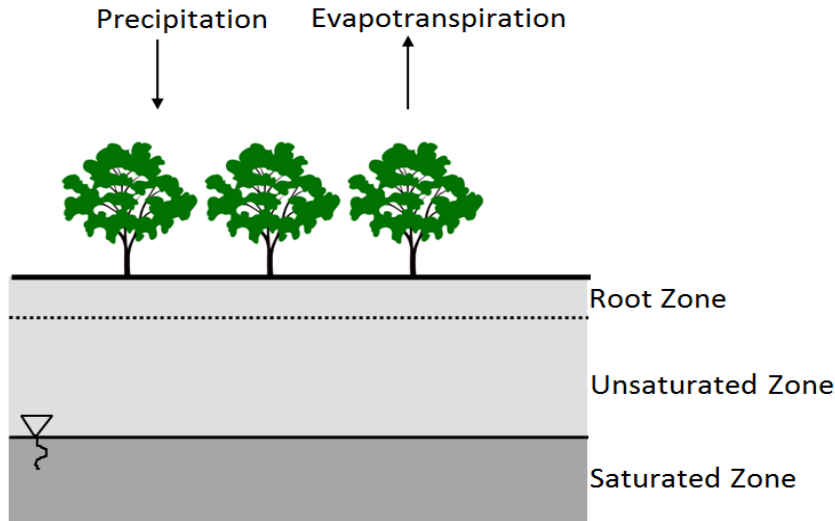
Interactions between  
Atmosphere and Soil  
and Hydrodynamic in  
the Unsaturated Zone :

**Nash**

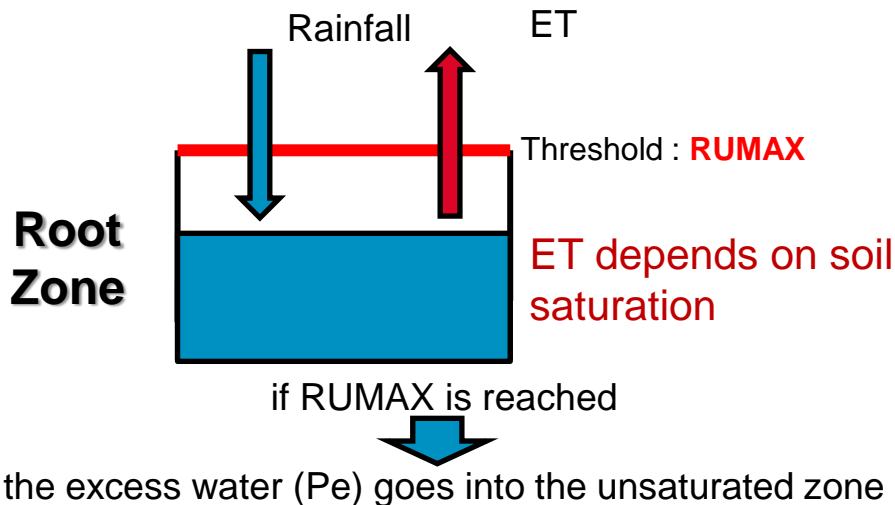
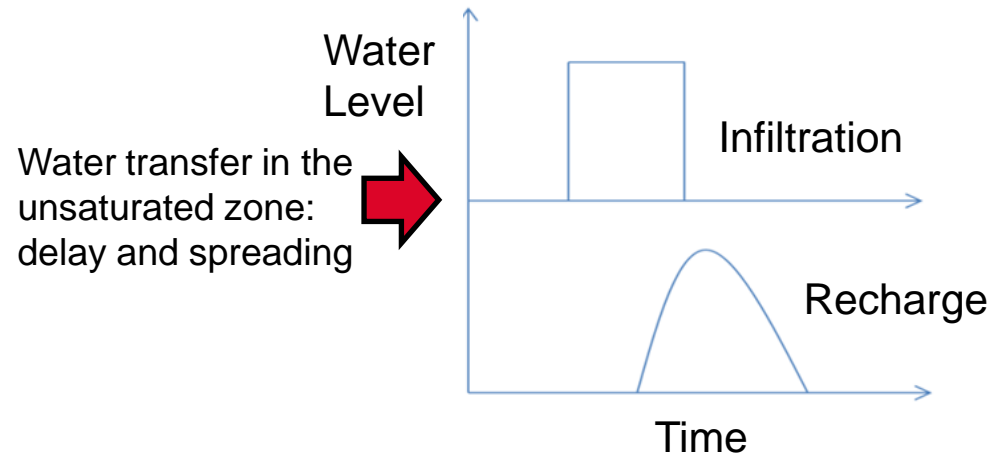
Groundwater Flow:  
**Diffusivity equation**



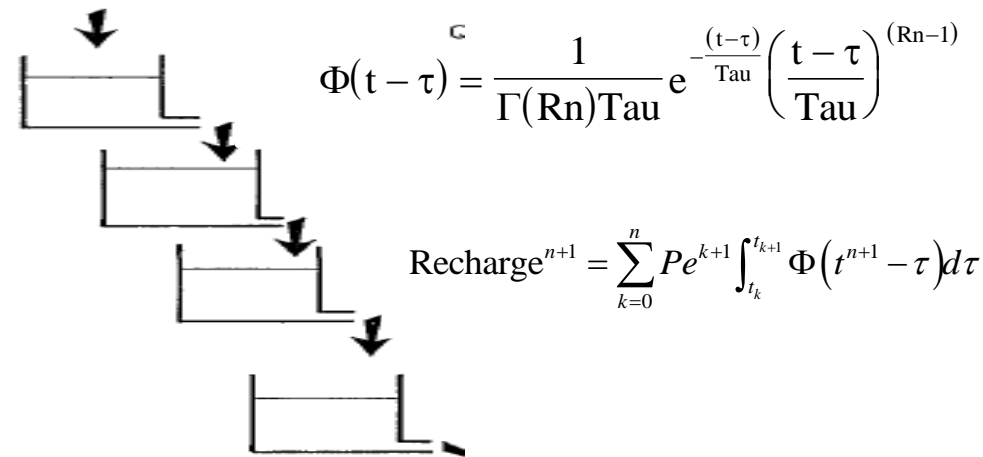




## Hydrodynamic in the unsaturated zone



## Modeling : propagation of a unit pulse signal

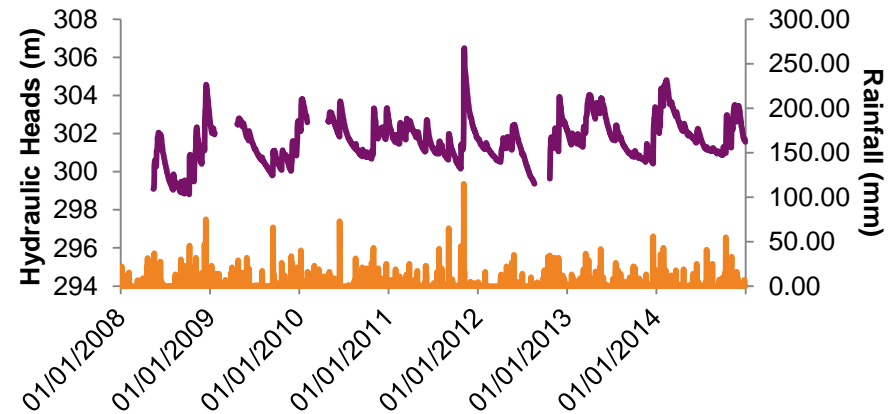
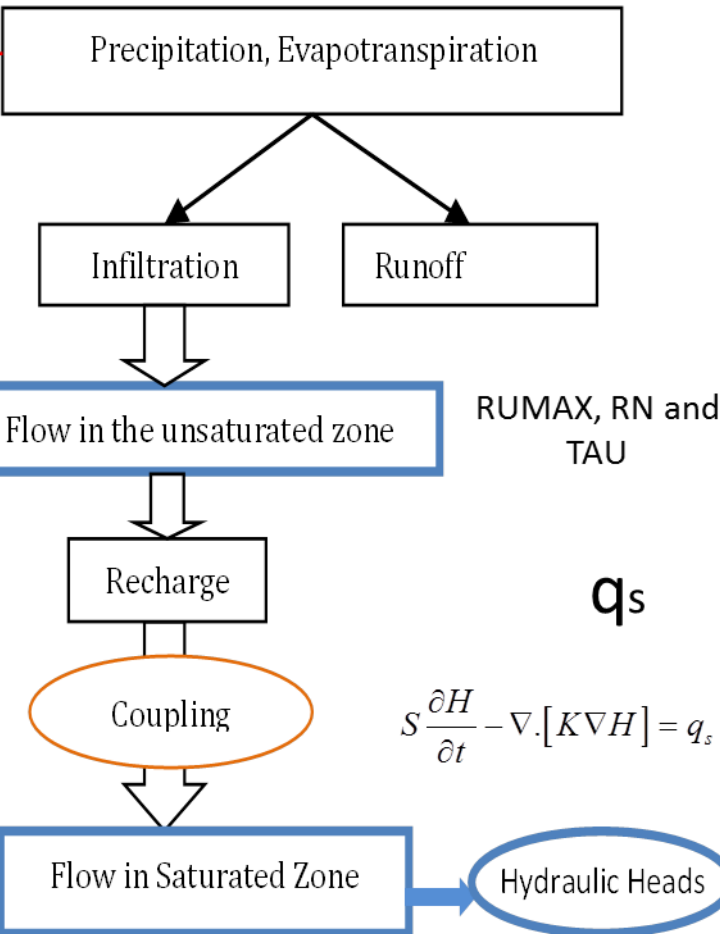


Nash parameters: **RUMAX, RN, TAU**

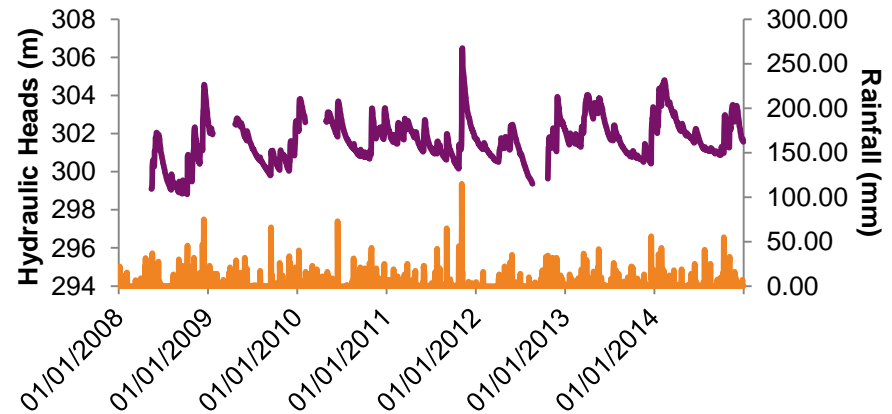
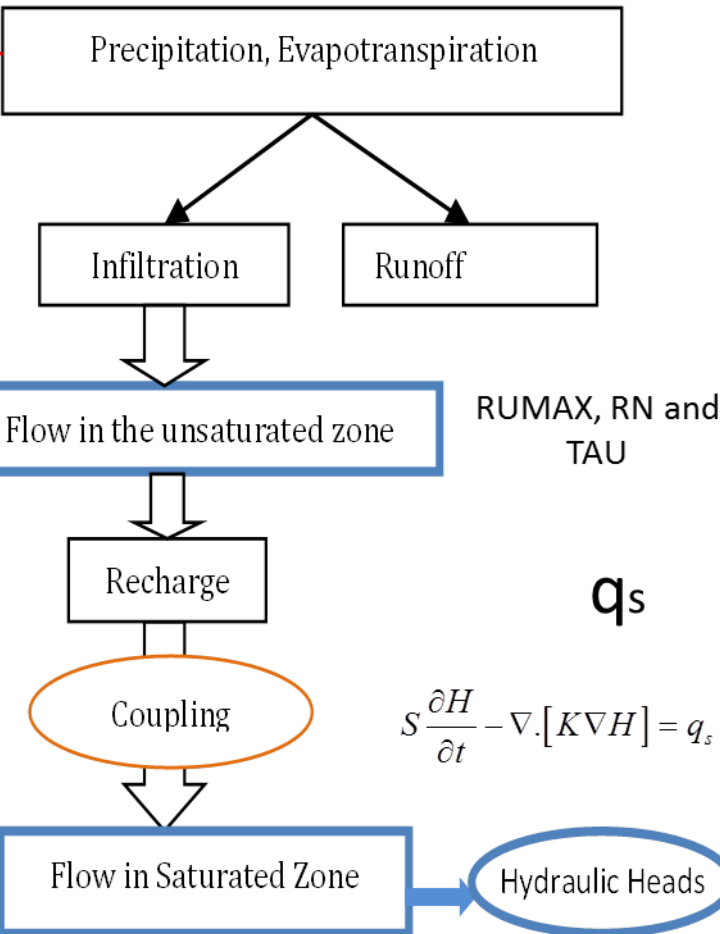
- ☹ Model uses constants called parameters
- ☹ **Aquifers are highly heterogeneous** → Parameters are not known accurately or even **unknown** in natural environments



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- Aquifers are highly heterogeneous → Parameters are not known accurately or even unknown in natural environments**
- Available data: **meteorological and piezometric levels**



- ☹ Model uses constants called parameters
- ☹ **Aquifers are highly heterogeneous** → Parameters are not known accurately or even **unknown** in natural environments
- ☹ Available data: **meteorological and piezometric levels**

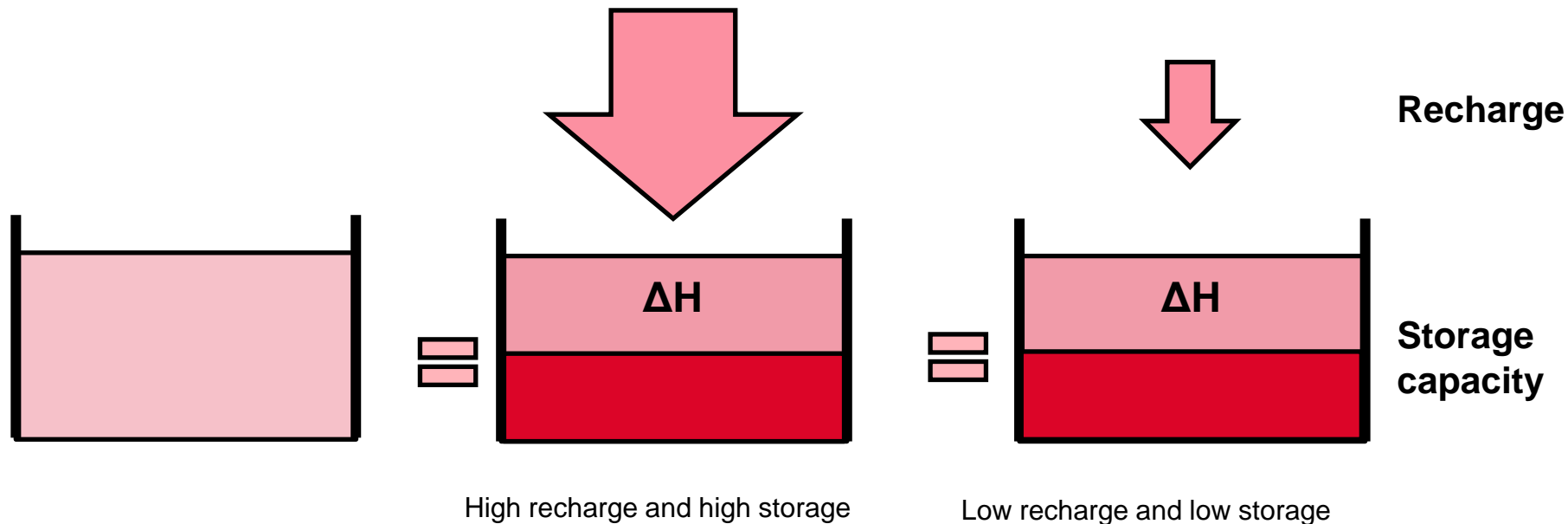


**Recharge is estimated by calibrating piezometric levels**

**Simultaneous estimation of groundwater flow and recharge parameters from hydraulic heads variations**

## Correlation between recharge and storage capacity (S and $q_s$ )

Recharge and storage capacity lead to the same effect



$$q = -K\nabla H \quad \text{Determine } q \text{ et } K \text{ knowing } H !$$

☺ Calibration is performed over time and space

☺ But... recharge is variable over time while the storage capacity is constant

When  $q = 0 \rightarrow 0 = -K\nabla H \rightarrow$  Determination of K

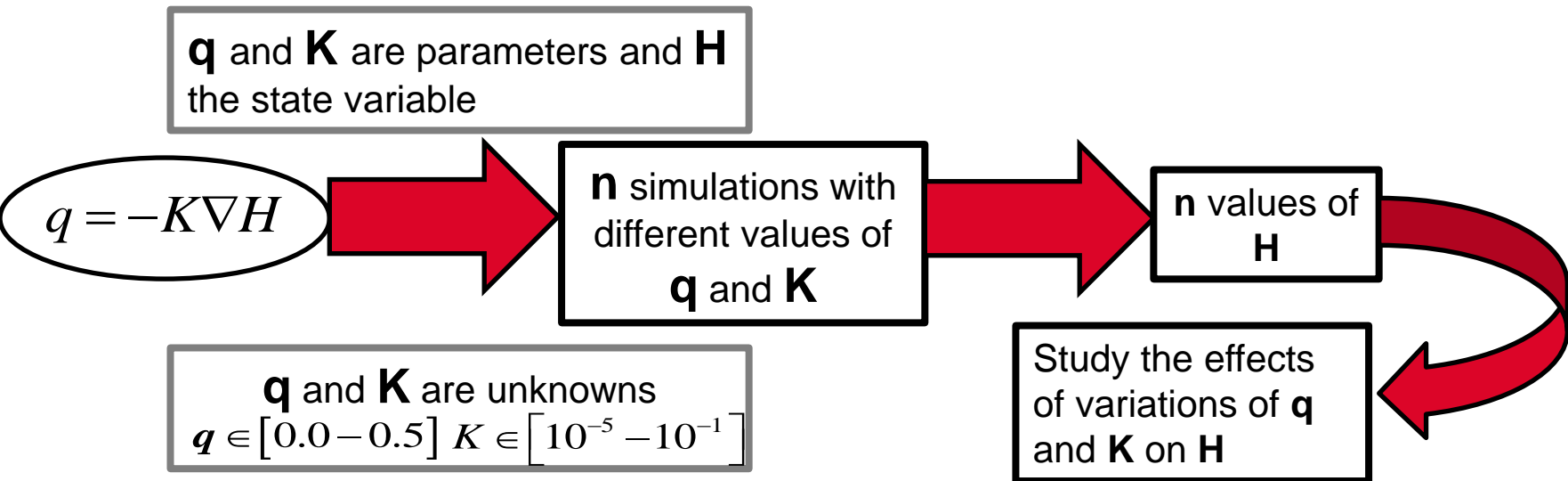
When  $q \neq 0 \rightarrow$  Determination of q knowing K

☺ **Global sensitivity analysis**

**Assess the calibration approach and guide the parameter estimation**

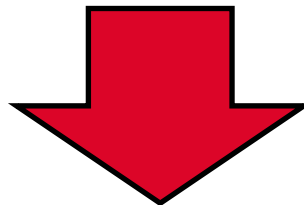
**Uncertain parameters:** variability ranges known → Quantify the effects of parameters uncertainty

**Global sensitivity analysis:** focuses on the output uncertainty over the entire range of values of the input parameters both single and in combination with one another



**Variance based approach:**

Sobol Indices



**Surrogate model: Chaos polynomial expansion**

# SOBOL INDICES : CHAOS POLYNOMIAL EXPANSION


Let us consider a model with  $y$  as output and  $n$  input parameters,  $X$  the parameters vector

➤ **ANOVA decomposition** :  $y = f_0 + \sum_{i=1}^n f_i(x_i) + \sum_{j>1}^n f_{ij}(x_i, x_j) + \dots + f_{1,2,\dots,n}(x_1, x_2, \dots, x_n)$


$f_i$  : contribution of single parameter  $i$  to the output

$f_{ij}$  : contribution of the combined effect of parameters  $i$  and  $j$  to the output

$$H = H_0 + f_S(S) + f_K(K) + f_{RN}(RN) + f_{RN,K}(RN, K) + f_{S,K}(S, K) + \dots$$

**Orthogonality of  $f$**    $V = \sum_{i=1}^n V_i + \sum_{j>1}^n V_{ij} + \dots + V_{1,\dots,n}$

➤ **Sensitivity (Sobol) Indices** :  $S_i = \frac{V_i}{V}$  ;  $S_{ij} = \frac{V_{ij}}{V}$

**Computation of  $f$**   **Polynomial expansion**

➤ **Chaos polynomial expansion**

$$y = \sum_{j=0}^{+\infty} a_j \Psi_j(x_1, \dots, x_n)$$

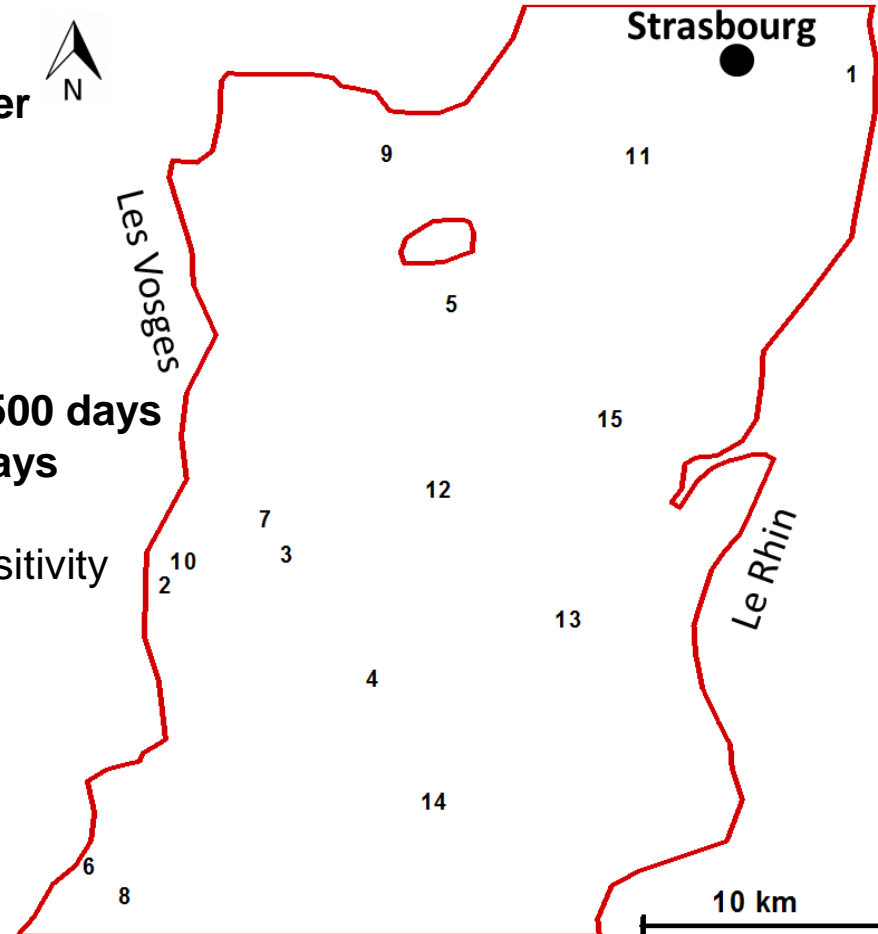
$x$  : variable

$\Psi$  : orthogonal polynomial (Legendre, Hermite)


$a$  : coefficients determined by least squares regression

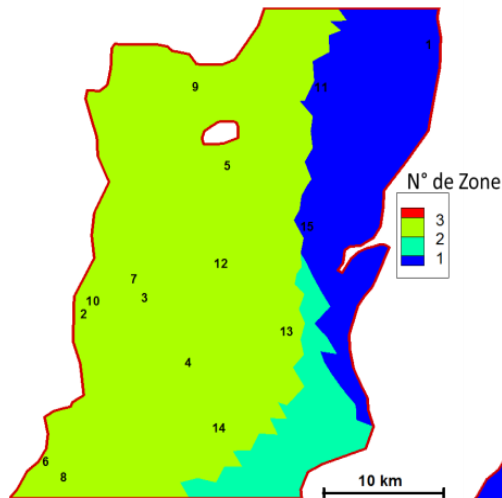
## Field based sensitivity analysis

- **A subdomain of the Upper Rhine alluvial aquifer** widely studied at Université de Strasbourg
- Average rainfall per year: **1100 mm**
- Average of evapotranspiration per year: **900 mm**
- Simulate variations of piezometric levels during **1500 days**
- Sensitivity analysis performed over the last **450 days**
- Investigate spatial and temporal variations of sensitivity indices
- 1500 simulations of Quasi Monte Carlo

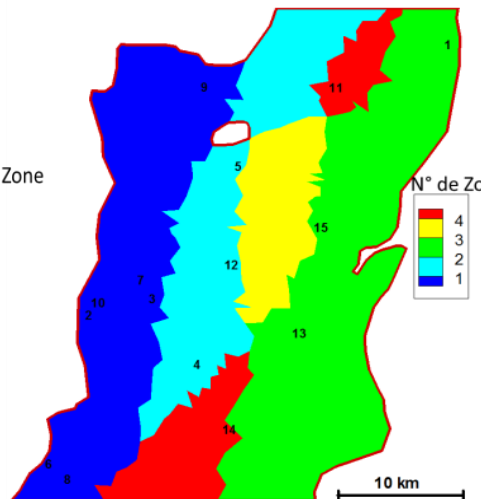




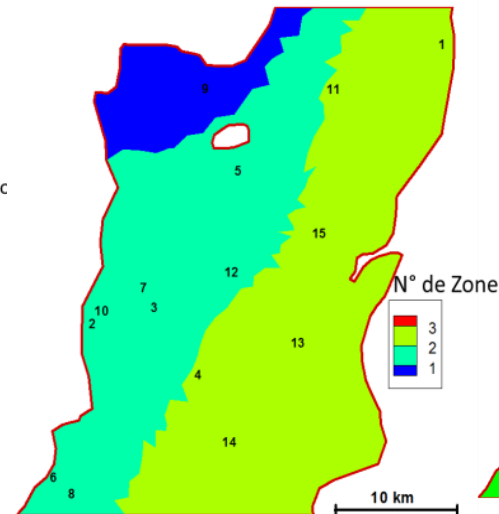
- Nash parameters for recharge: **RUMAX, RN, TAU**
- Aquifer parameters: **K and S**
- Parameters defined by zones (**Zonation**)  20 parameters



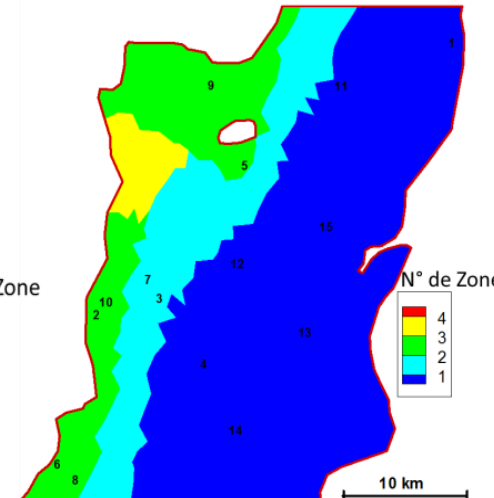
RUMAX




RN and TAU



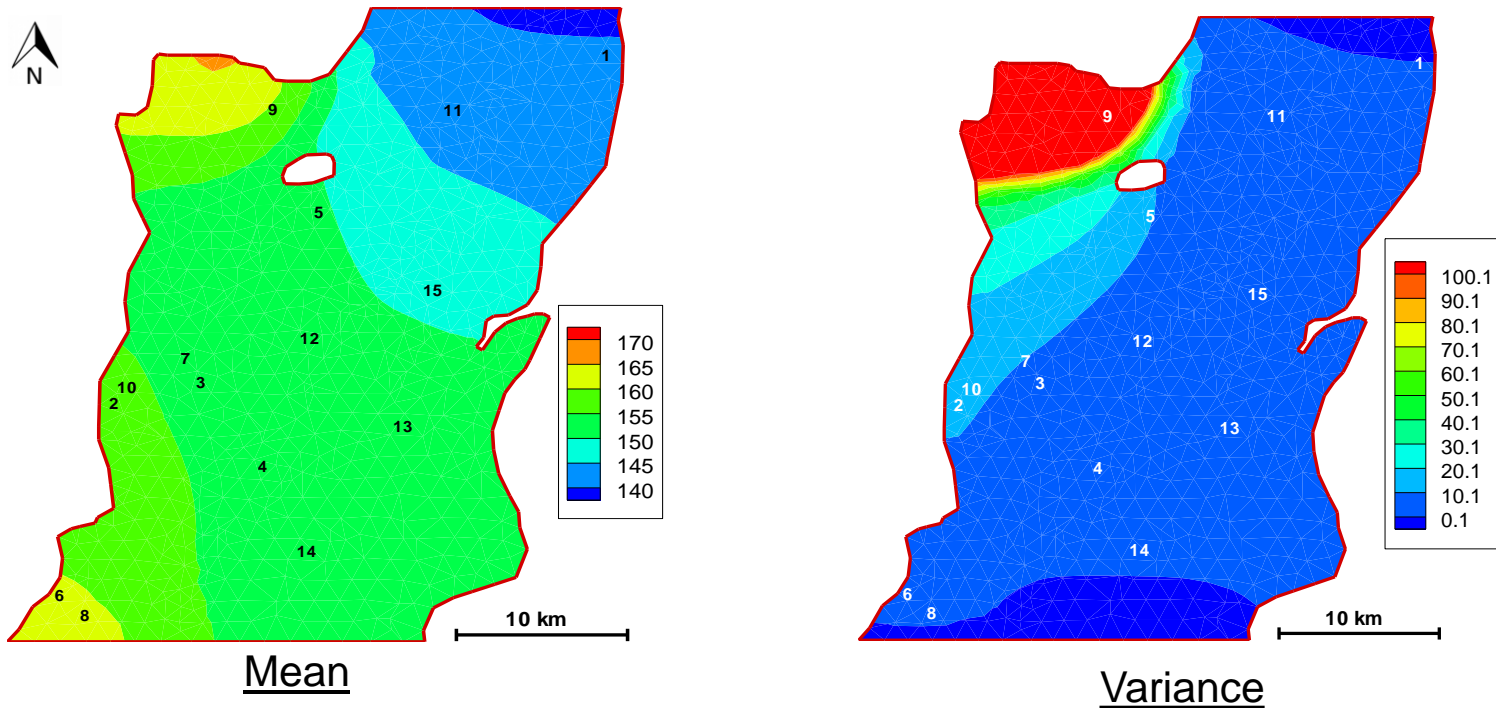
Permeability



Storage capacity

- Nash parameters for recharge: RUMAX, RN, TAU
- Aquifer parameters: **K** and **S**
- Parameters defined by zones (**Zonation**)  20 parameters

## Mean and Variance of the hydraulic head

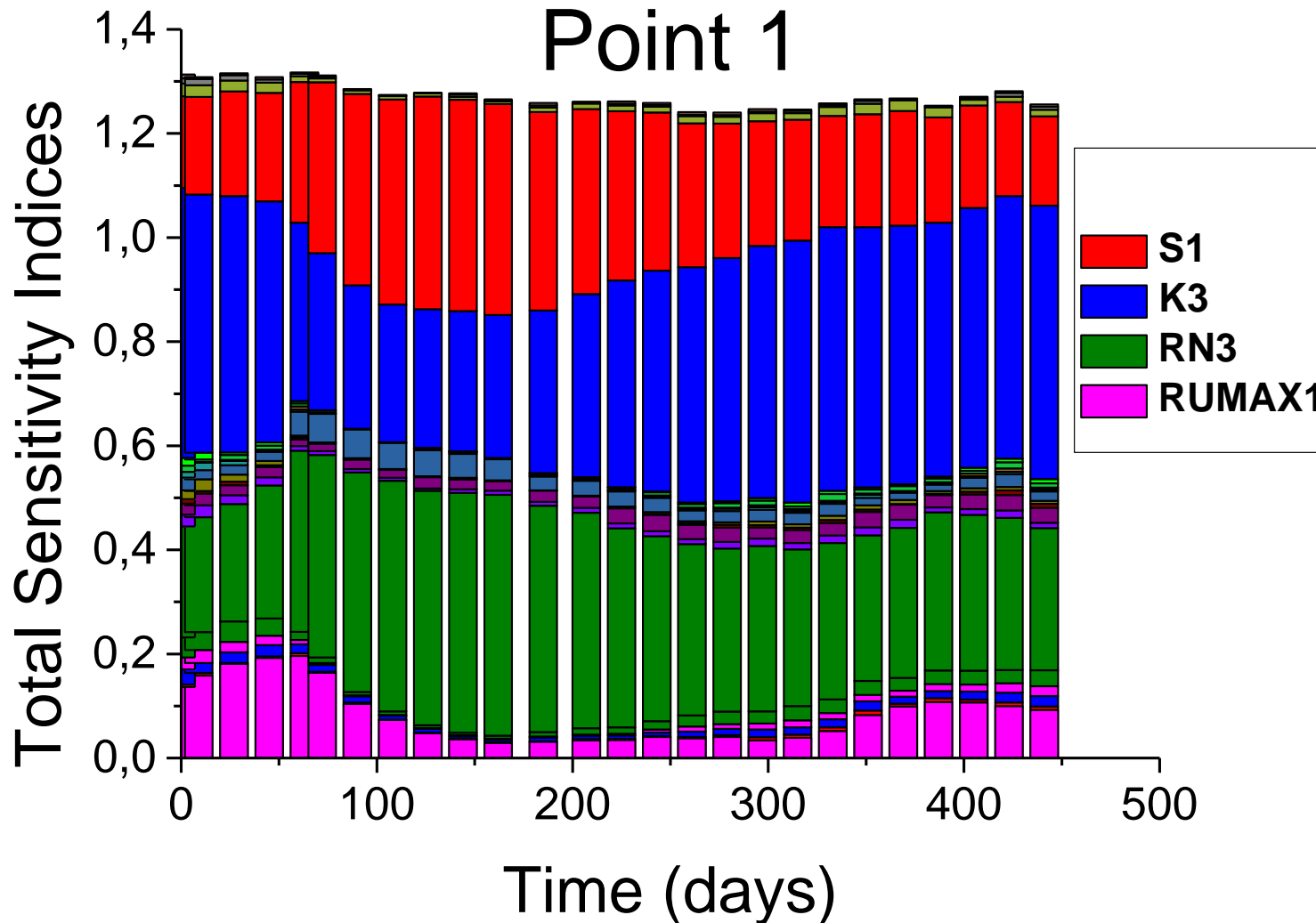


- Hydraulic heads range between 140 and 165 m
- Variance ranges between 0 and 100: the imposed boundaries have the lowest variance

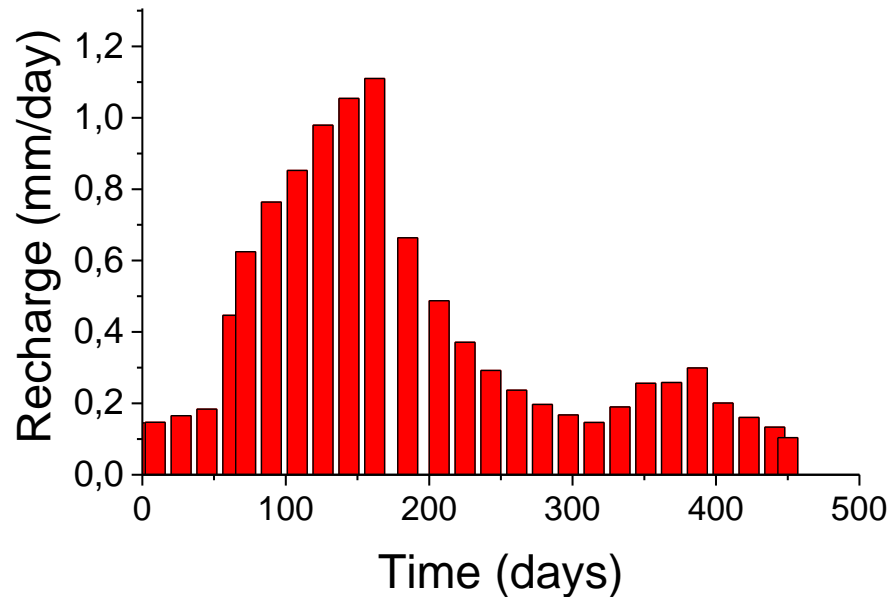
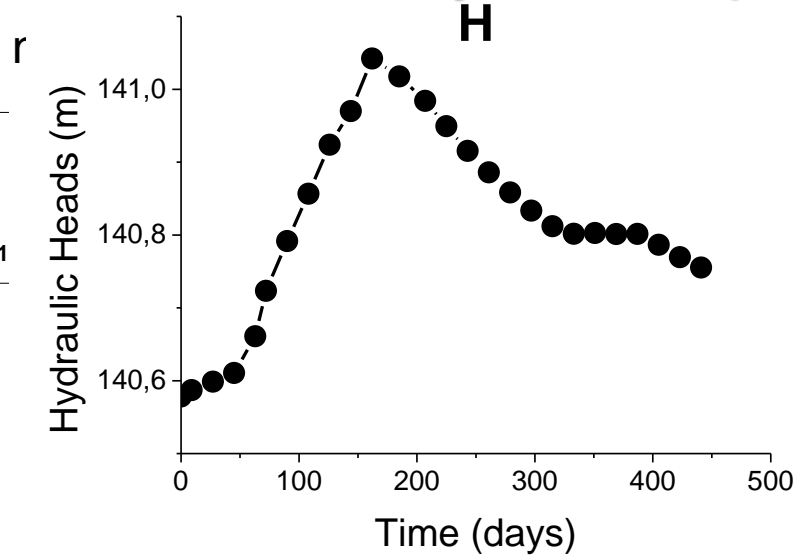
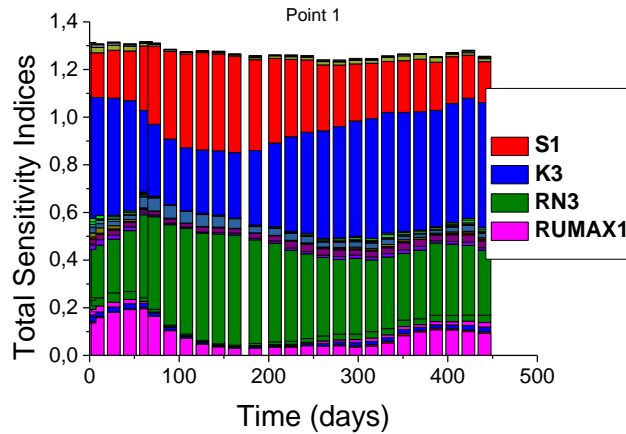
# GLOBAL SENSITIVITY ANALYSIS: NASH MODEL

## Temporal variation of sensitivity indices at point 1

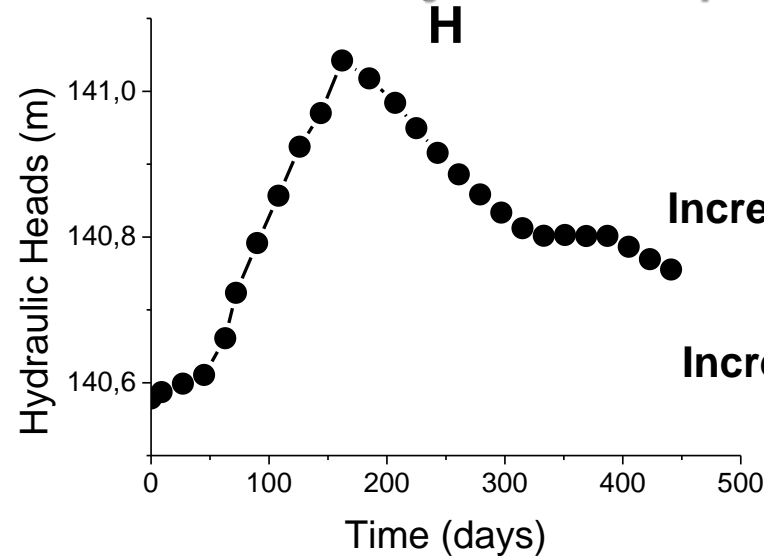
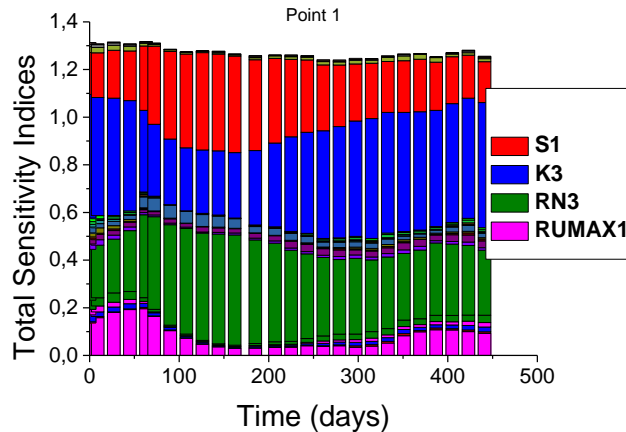
Influential parameters: S, K, RUMAX, RN



## Temporal variation of sensitivity indices at point 1



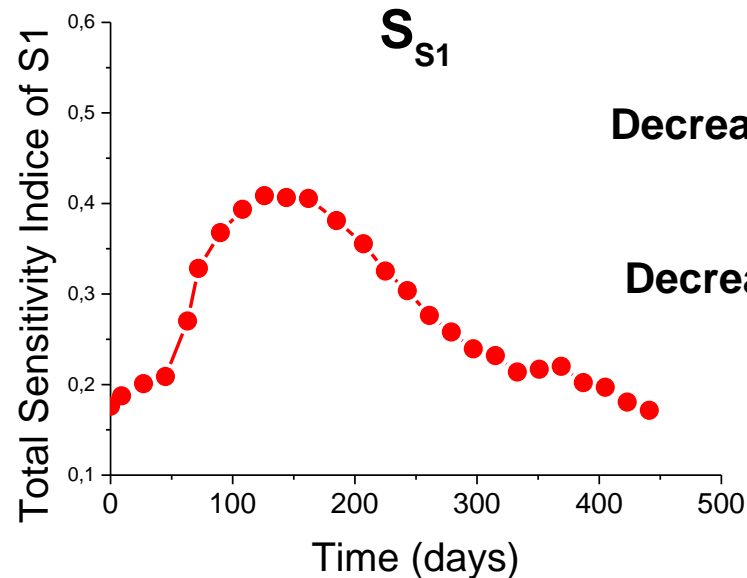
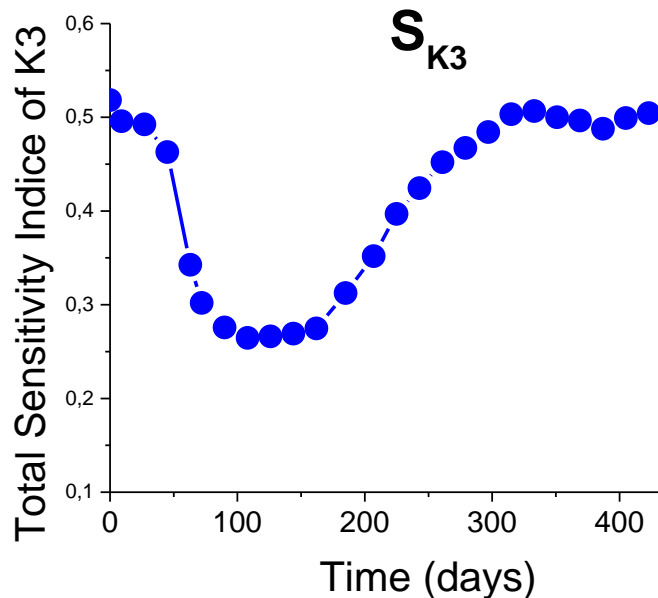
## Temporal variation of sensitivity indices at point 1



Increase of piezometric level



Increase of the influence of storage capacity

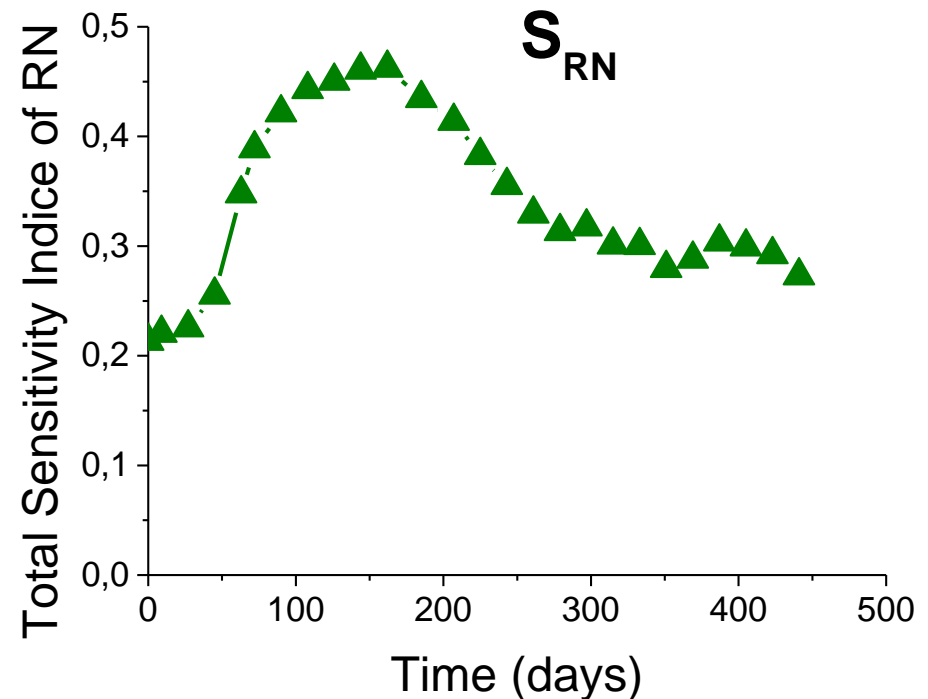
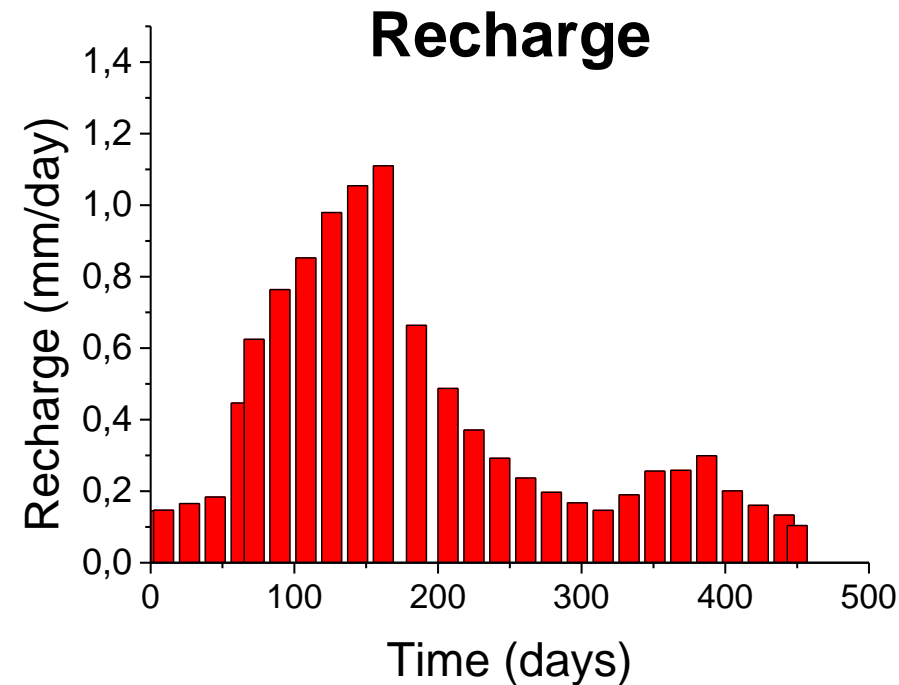


Decrease of piezometric level



Decrease of the influence of permeability

## Temporal variation of sensitivity indices at point 1

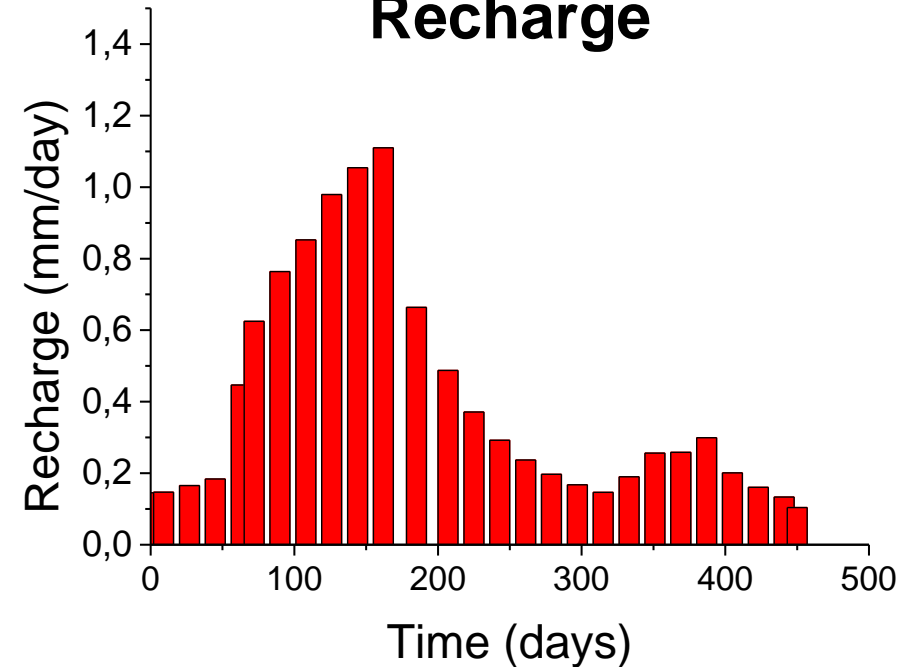


**RN:** water transfer in the unsaturated zone  
(delay and spreading)

**RUMAX:** threshold parameter

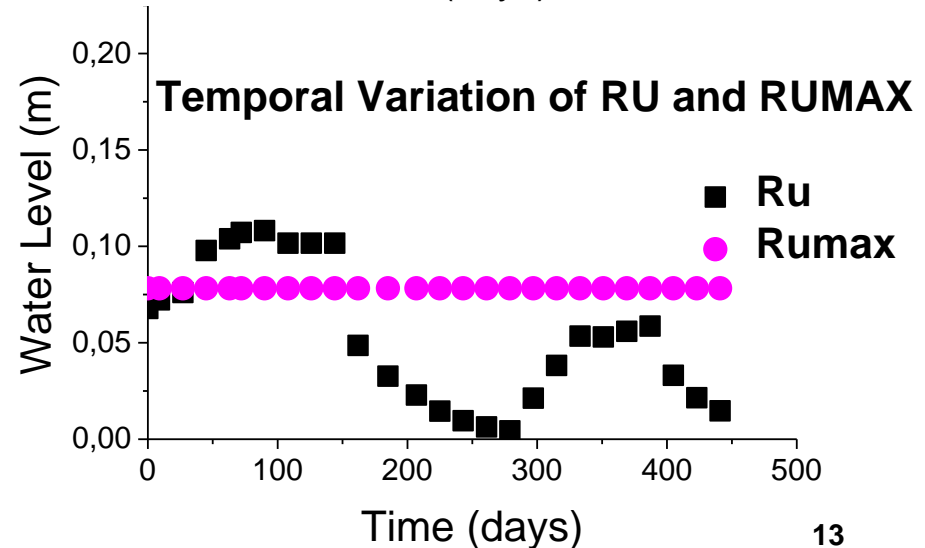
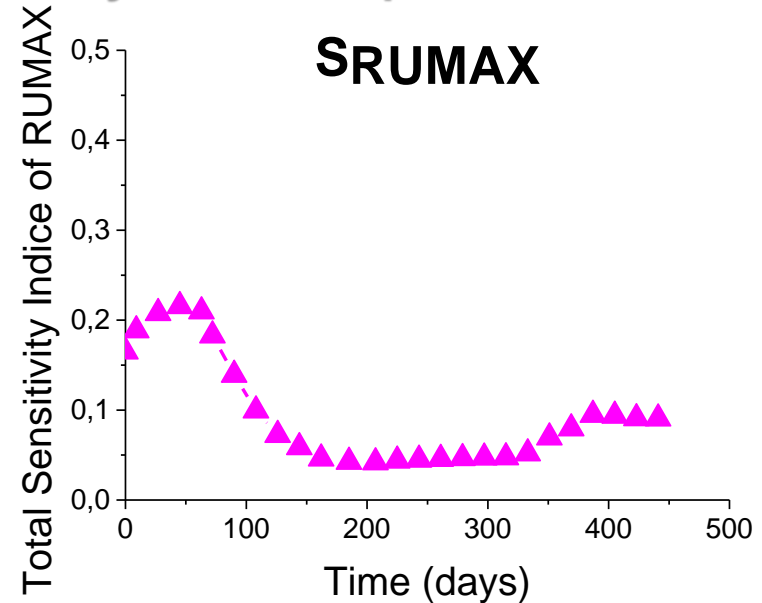
## Temporal variation of sensitivity indices at point 1

### Recharge

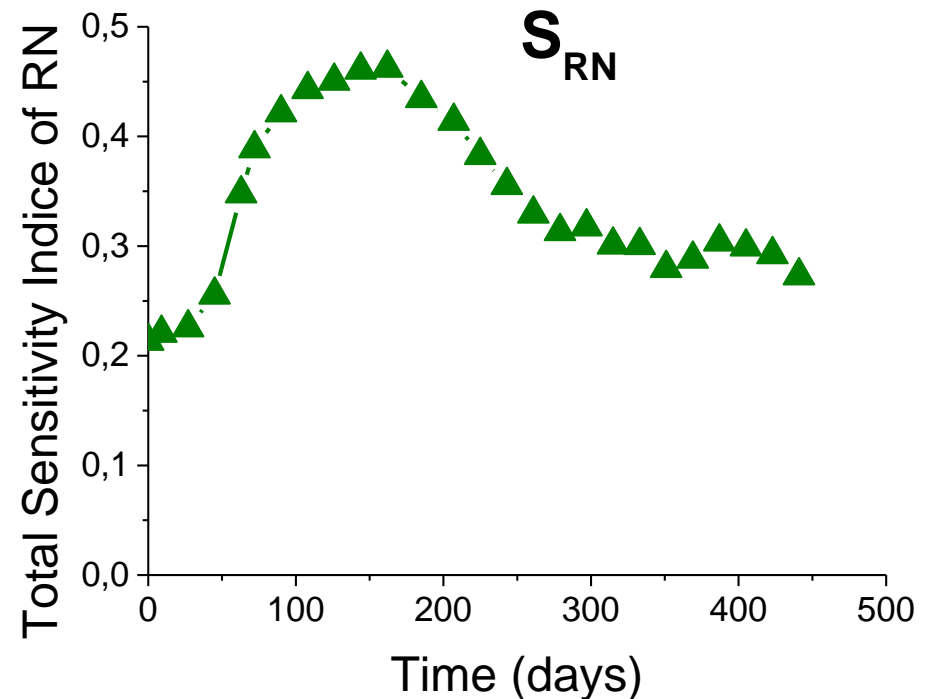
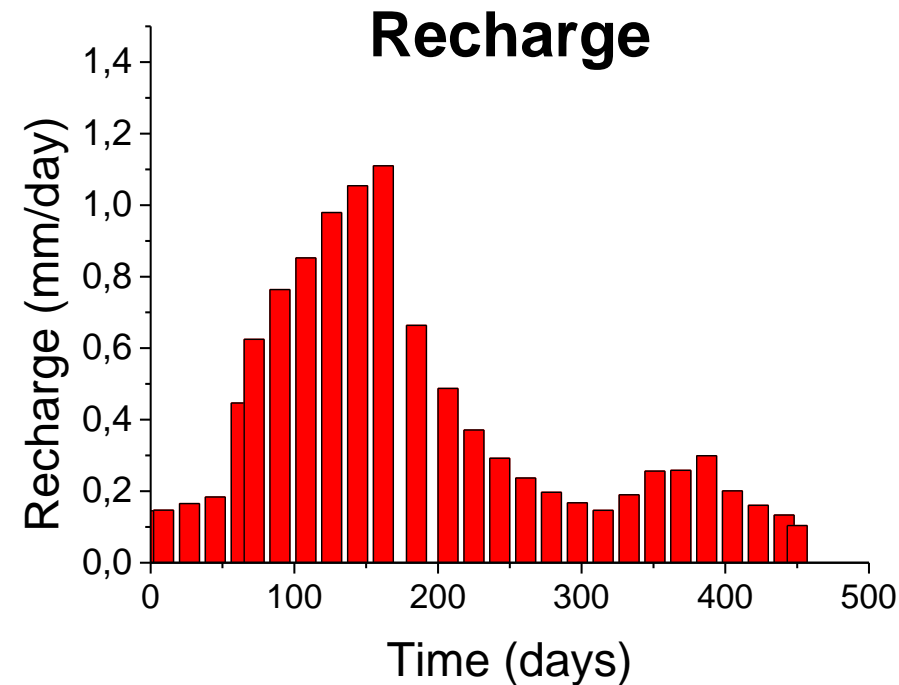


**RN:** water transfer in the unsaturated zone (delay and spreading)  
**RUMAX:** threshold parameter

### SRUMAX



## Temporal variation of sensitivity indices at point 1

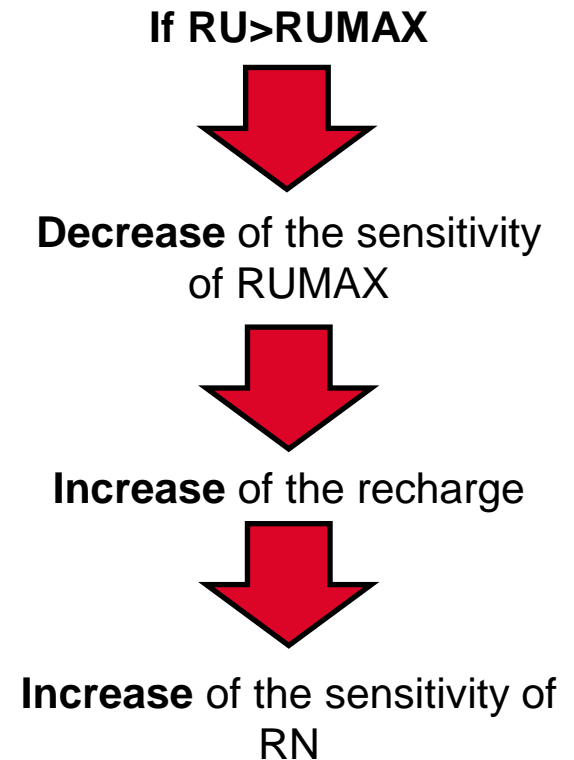
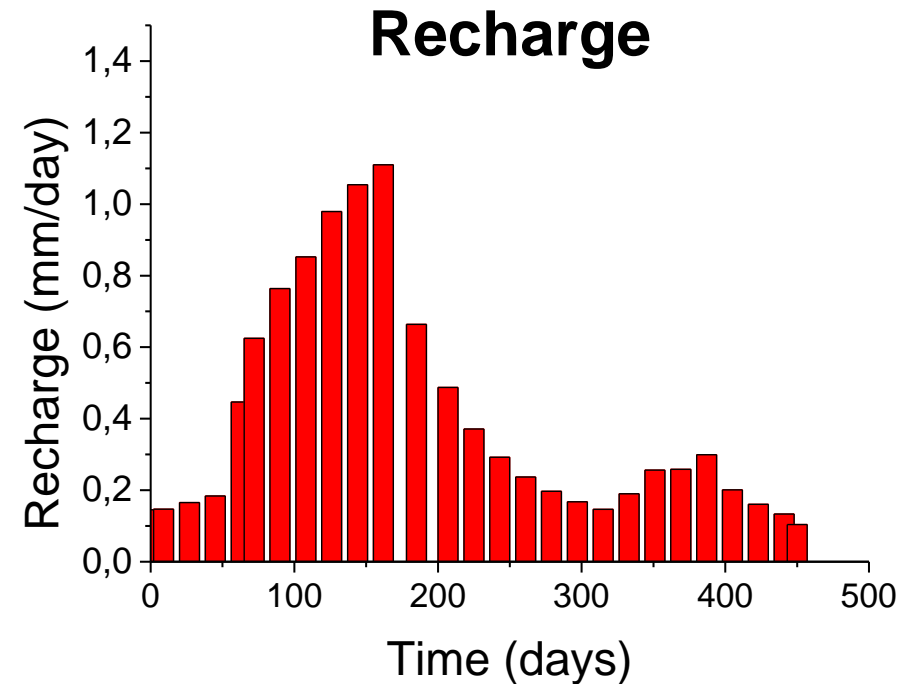


**RN:** water transfer in the unsaturated zone  
(delay and spreading)

**RUMAX:** threshold parameter



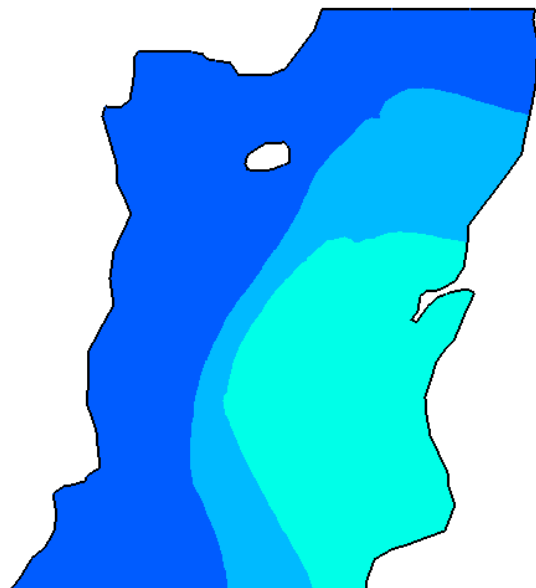
## Temporal variation of sensitivity indices at point 1



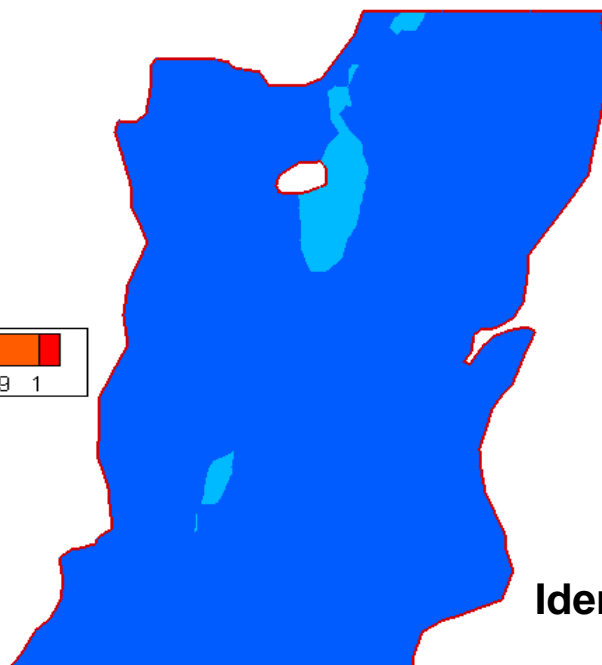
**$R_N$** : water transfer in the unsaturated zone (delay and spreading)

**$R_{UMAX}$** : threshold parameter

## Sensitivity maps over time



**S1**

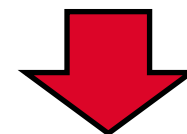


**RUMAX3**

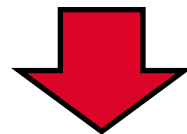
**RUMAX3**: sensitive from the **1<sup>st</sup>** to **185<sup>th</sup>** day  
**S1**: influential between the **185<sup>th</sup>** and the **346<sup>th</sup>** day



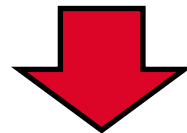
Weak or even negligible correlations between the 185<sup>th</sup> and the 346<sup>th</sup> day



Identification of S



During the first days



Identification of RUMAX and RN knowing S

- Determination of influential parameters

**Nash:** RUMAX and RN

**Groundwater flow:** K and S

- Temporal variation of parameter sensitivity

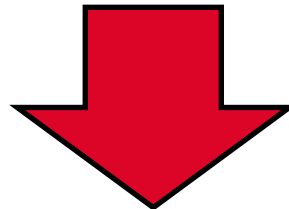
**Nash:** saturation of the 1<sup>st</sup> reservoir (RUMAX) and recharge (RN)

**Permeability:** decrease of the hydraulic heads

**Storage capacity:** increase of the hydraulic heads

- Weak interactions between aquifer and recharge parameters

$$q = -K\nabla H \quad K \text{ constant, } q \text{ variable} \rightarrow q=0, \text{ détermination of } K$$



**Confirmation of the feasibility of simultaneous estimation of recharge and groundwater flow parameters**

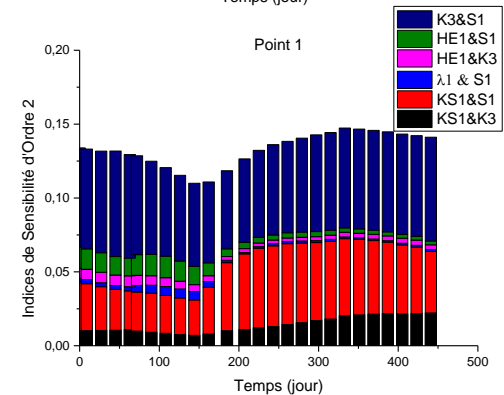
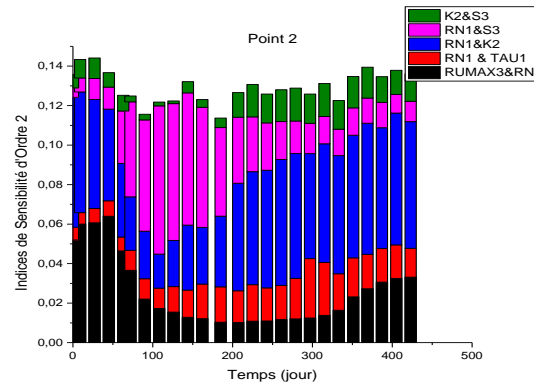
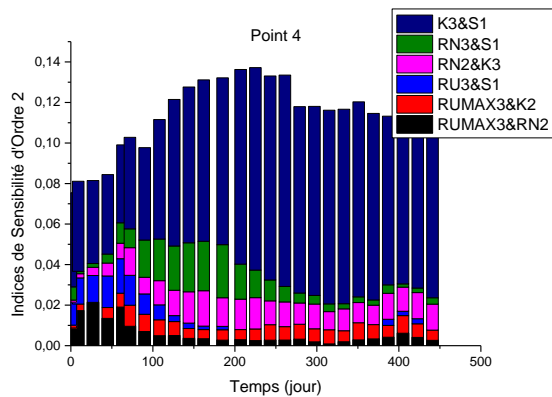
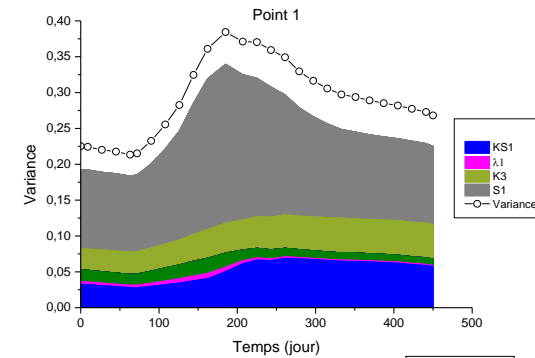
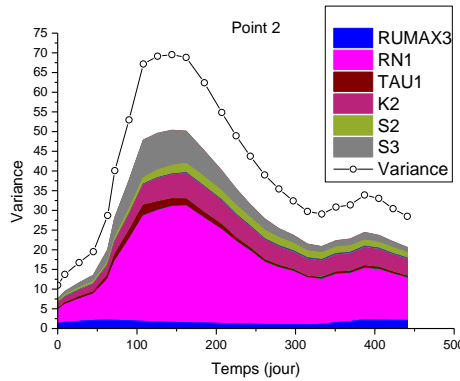
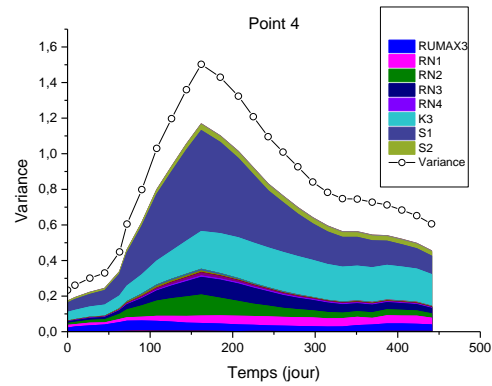
A high-speed photograph of a water splash, showing a central droplet rising from a pool of water, surrounded by smaller droplets and ripples. The background is a soft, out-of-focus blue.

*Thank you for  
your attention*

## Parameters Interactions

### Nash model

### Richards model



Diffusivity equation (Transient state with recharge )

$$S \frac{\partial H}{\partial t} - \nabla \cdot [K \nabla H] = q_s$$

Steady state without recharge

$$-\nabla \cdot [K \nabla H] = 0$$

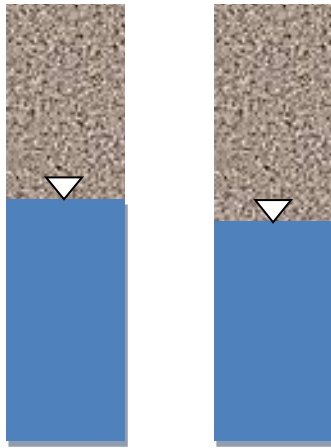
Steady state with recharge

$$-\nabla \cdot [K \nabla H] = q_s$$

Transient state without recharge

$$S \frac{\partial H}{\partial t} - \nabla \cdot [K \nabla H] = 0$$

## Assumption of constant hydraulic head over decade



$$q_i^n = -K \frac{\partial}{\partial z} (h - z) = K \left( h_{i+\frac{1}{2}} \right) \left( \frac{h_i^n - h_{i+1}^n}{\Delta z_i} + 1 \right)$$

If the hydraulic heads decreases, the imposed pressure decreases, and the flux calculated is positive and strong

If the hydraulic heads increases, the imposed pressure increases, and the flux calculated is negative

# RESOLUTION OF RICHARDS EQUATION

