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# Impact of Correlations between core configurations for the evaluation of Nuclear Data Uncertainty propagation for Reactivity Coefficients

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## Abstract

*The precise estimation of Pearsons correlation coefficients between core configurations is a fundamental parameter to properly propagate uncertainties, in the so-called re-assimilation and transposition process, from a priori known integral experimental data to a posteriori uncertainty on a target design. In this paper, a traditional adjoint method is used to propagate nuclear data uncertainty on reactivity and reactivity coefficients and estimate their correlations. We show that the estimation of correlation coefficients enables to correctly propagate the whole ND uncertainties on extrapolated configurations. This calculation is made for reactivity at the beginning of life but could be easily extended to other parameters during depletion.*

*Key-words:* uncertainty propagation, nuclear data, transport, reactivity, reactivity coefficients, Pearsons, correlations.

## 1 INTRODUCTION

Sensitivity analysis plays an important role in the field of core physics, as nuclear data Uncertainty propagation and Quantification (UQ) is more and more required in safety calculations of large NPP cores, as well as innovative design relevant of Gen-IV systems. An emerging need also rises for the new generation of very versatile and efficient MTRs, where performances and safety concern both lifetime, and isotope production. A good understanding of biases and uncertainties on reactor core

calculations is essential for assessing safety features and design margins in current and future NPPs, as well as in experimental reactors such as MTRs. In recent years there has been an increasing demand from nuclear industry, safety and regulation for best estimate predictions to be provided with their confidence bounds.

For almost 30 years, nuclear data uncertainty propagation and nuclear data statistical adjustment in fast reactor applications have been widely used to produce “adjusted” sets of multigroup cross sections and to assess the uncertainty on neutronics design parameters. As a consequence, these methods are naturally implemented in calculation tools dedicated to GEN-IV neutron calculations, such as the ERANOS2 code [1] in France.

In this document, we will assess the impact of nuclear data uncertainties on reactivity coefficients at the beginning of life to simplify the problem. The method can be easily extended to depletion calculations and to other local parameters.

To illustrate the performances of the methodology, a Material Testing Reactor benchmark (MTR type) 2D core benchmark has been designed, based on  $U_3Si_2Al$  fuel plate assemblies. The calculation schemes and nuclear data library, as well as nuclear data covariance matrices will be described. A benchmark description will be given, followed by the detailed theoretical analysis of the methods. The last part will detail the results obtained and will give some elements of physical analysis, as well as awaited development perspectives.

## 2 THEORY OF UNCERTAINTY PROPAGATION FOR REACTIVITY COEFFICIENTS

The propagation law of uncertainty comes from a limited development of the calculation code functional, and is known as the sandwich rule. Under a matrix form, it can be written as, for reactivity  $\rho$ :

$$\varepsilon^2(\rho) = [\underline{S}^\rho]^T \underline{\underline{M}} [\underline{S}^\rho] \quad (2.1)$$

Where  $\varepsilon(\rho)$  is the standard deviation of  $\rho$  coming from the nuclear data covariance matrix  $\underline{\underline{M}}$ .  $[\underline{S}^\rho]$  is the sensitivity vector of  $\rho$  to the nuclear data. Knowing  $\underline{\underline{M}}$  from the ND evaluation files, only  $[\underline{S}^\rho]$  needs to be evaluated.

## 2.1 Sensitivity evaluation

The evaluation of  $[\underline{S}^\rho]$  is made using Standard Perturbation Theory [2]. Sensitivities are given by adequate procedures implemented in the APOLLO2 lattice code [3]. The most usual sensitivity value calculated by SPT is the following:

$$\frac{\partial \rho}{\partial \sigma_k} = -\sigma_k \frac{\langle \varphi^\dagger, \left[ \frac{\partial D}{\partial \sigma_k} - \lambda \frac{\partial P}{\partial \sigma_k} \right] \varphi \rangle}{\langle \varphi^\dagger, P \varphi \rangle} \quad (2.2)$$

where  $\varphi^\dagger$  is the adjoint flux,  $\sigma_k$  is the k-th cross-section in the order of the  $\underline{M}$  matrix,  $D$ ,  $P$  and  $\lambda$  are respectively disappearance, production and eigenvalue of the Boltzmann equation and  $\langle \dots \rangle$  represents the dot product on the phase space, defined as follows:

$$\langle \varphi_1, \varphi_2 \rangle = \int_V d^3r \int_0^\infty dE \int_{4\pi} d^2\Omega \varphi_1(r, E, \Omega) \varphi_2(r, E, \Omega)$$

to the calculation of sensitivities to reactivity coefficients is made using the Equivalent Perturbation Theory [4][5]. These reactivity coefficients may be insertion of soluble boron or absorbing material, as well as temperature variation. The derivative of a reactivity coefficient can be expressed as a sum of reactivity derivatives. The sensitivity to a reactivity coefficient is then given by:

$$\frac{\partial \Delta \rho}{\partial \sigma_k} = \frac{\partial \rho_2 - \partial \rho_1}{\partial \sigma_k} \quad (2.3)$$

The  $[\underline{S}^{\rho_2}] - [\underline{S}^{\rho_1}] = [\underline{S}^{\Delta \rho}]$  vector is then built.

## 2.2 Evaluation of the Pearsons correlation coefficients

The Pearson correlation coefficient gives a formal information about the linear relation between two variables  $X_1$  and  $X_2$ . Its variation domain is the interval  $[-1,1]$ . When  $X_1$  and  $X_2$  are strongly positively correlated, the Pearson  $r_{X_1 X_2} \approx 1$ . When they are strongly negatively correlated,  $r_{X_1 X_2} \approx -1$ . This value is close to 0 when the variables are uncorrelated (ie there is no linear relation between  $X_1$  and  $X_2$ ).

The Pearson coefficient can be expressed through the following relations:

$$\begin{aligned}
r_{X_1 X_2} &= \frac{COV_{X_1 X_2}}{\varepsilon(X_1)\varepsilon(X_2)} = \frac{[\underline{S}^{X_1}]^T \underline{\underline{M}} [\underline{S}^{X_2}]}{\sqrt{[\underline{S}^{X_1}]^T \underline{\underline{M}} [\underline{S}^{X_1}] \cdot [\underline{S}^{X_2}]^T \underline{\underline{M}} [\underline{S}^{X_2}]}} \\
&= \frac{\sum_i ((X_{1,i} - \bar{X}_1)(X_{2,i} - \bar{X}_2))}{\sqrt{\sum_i (X_{2,i} - \bar{X}_2)^2 \cdot \sum_i (X_{1,i} - \bar{X}_1)^2}}
\end{aligned} \tag{2.4}$$

where  $\underline{S}^{X_1}$  is the sensitivity of a parameter to  $X_1$ ,  $X_{1,i}$  is a realization of  $X_1$ ,  $\bar{X}_1$  is the average of this realization and  $COV_{X_1 X_2}$  represents the covariance between  $X_1$  and  $X_2$ .

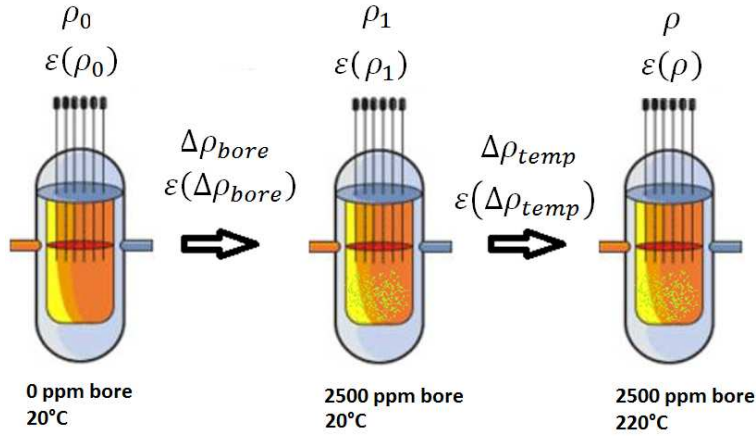
All the Pearson expressions are equivalent. We understand that the knowledge of  $r_{X_1 X_2}$  will be essential to express the covariance, knowing the uncertainties  $\varepsilon(X_1)$  and  $\varepsilon(X_2)$ .

**Remarks:**

- *The Pearson coefficient allows to analyze sample of bivariate data and not multivariate data,*
- *There is no transitivity relation for the Pearsons, except particular cases [6]*
- *The independence between two variables implies that these variables are not correlated but the reciprocal is wrong. Two variables can have null Pearsons while being dependent.*

**2.3 General theory of uncertainty accumulation**

Let's extend the propagation law to a series of perturbations which are changing the core configuration. Consider the following relation for the reactivity. In the following paragraph, we will use the configuration transformation resumed on Figure 1 as an applicative example.



**Figure 1: Steps of uncertainties accumulations**

We would like to determine the final reactivity  $\rho$  after having added soluble boron in the moderator, followed by a temperature increase, starting from a known reference reactivity state  $\rho_0$ .

We can express the final reactivity state as:

$$\rho = \rho_0 + \underbrace{(\rho_1 - \rho_0)}_{\text{boron addition}} + \underbrace{(\rho_2 - \rho_1)}_{\text{temperature increase}} = \rho_0 + \Delta\rho_{\text{boron}} + \Delta\rho_{\text{temp}} \quad (2.5)$$

The global propagated uncertainty corresponding to this sum ( $\rho$ ) cannot be associated to the quadratic sum of the different uncertainties only, as correlations exist between the three terms of Eqn.2.4. Let's write the uncertainty to  $\rho$  as :

$$\begin{aligned} \varepsilon^2(\rho) = & [\varepsilon^2(\rho_0) + \varepsilon^2(\Delta\rho_{\text{boron}}) + \varepsilon^2(\Delta\rho_{\text{temp}})] \\ & + [2\varepsilon(\rho_0)\varepsilon(\Delta\rho_{\text{boron}})r(\rho_0, \Delta\rho_{\text{boron}}) + 2\varepsilon(\rho_0)\varepsilon(\Delta\rho_{\text{temp}})r(\rho_0, \Delta\rho_{\text{temp}})] \\ & + [2\varepsilon(\Delta\rho_{\text{boron}})\varepsilon(\Delta\rho_{\text{temp}})r(\Delta\rho_{\text{boron}}, \Delta\rho_{\text{temp}})] \end{aligned} \quad (2.6)$$

The first line corresponds to the quadratic sum only. The second line represents the covariances between the initial state  $\rho_0$  and the different reactivity coefficients leading to the final state  $\rho_2$ . The latest line is the covariance between those reactivity coefficients.

Eqn. 2.6 can be written in a more convenient manner in a matrix form:

$$\varepsilon^2(\rho) = \underline{Z}^T \underline{\Omega} \underline{Z} \quad (2.7)$$

Where  $\underline{Z} = [\varepsilon(\rho_0) \quad \varepsilon(\Delta\rho_{\text{boron}}) \quad \varepsilon(\Delta\rho_{\text{temp}})]$

$$\text{And } \underline{\underline{\Omega}} = \begin{bmatrix} 1 & r(\rho_0, \Delta\rho_{boron}) & r(\rho_0, \Delta\rho_{temp}) \\ r(\rho_0, \Delta\rho_{boron}) & 1 & r(\Delta\rho_{boron}, \Delta\rho_{temp}) \\ r(\rho_0, \Delta\rho_{temp}) & r(\Delta\rho_{boron}, \Delta\rho_{temp}) & 1 \end{bmatrix}$$

### 3 RESULTS FOR A “SCHOOL CASE”

#### 3.1 Benchmark description

The 2D benchmark used in the present study is a Material Testing Reactor based on  $U_3Si_2Al$  at 19.95% of  $^{235}U$  fuel. A radial view is reproduced on Figure 2. A single type of assembly has been modelled to build the whole core. For the sake of simplicity, no absorbing material or control element has been included in the benchmark, the goal being only to study the propagation of ND uncertainties as one operating parameter is changed at a time: temperature, or soluble boron.

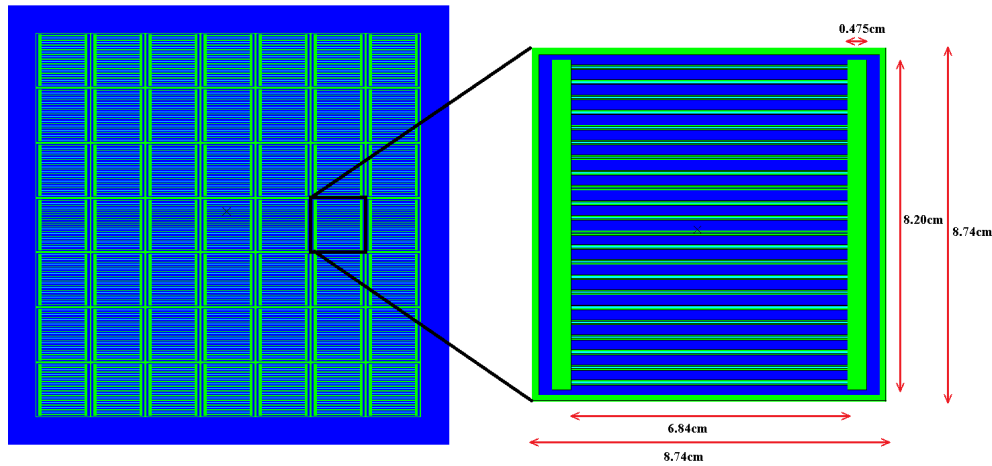


Figure 2: Geometric representation of the benchmark

Each fuel assembly is made of 22 Zircalloy plates (in green) with a thickness of 0.13 cm. Each plate contains a fuel blade of 50 microns thickness. The blue elements represent the surrounding light water (boronless at initial reactivity stage). The 2D geometry is slightly subcritical at last step.

#### 3.2 Calculations tools

The application is made in 15 energy groups with the APOLLO2.8.3 [3] deterministic lattice calculation code on a 2D quarter of core using TDT-MOC (method of characteristics) scheme, described in [7] and ad hoc symmetries.

### **3.3 Nuclear data library and covariance data**

Global uncertainties on core parameters are assessed with the propagation of nuclear data uncertainties only. To obtain reliable covariances associated with JEFF3.1.1 evaluations [8] a nuclear data re-estimation of the major isotopes was performed thanks to selected targeted integral experiments [9]. The CONRAD code is used to produce covariance matrices from marginalization technique [10]. This work led to the emission of a new set of covariance matrices linked to JEFF3.1.1, called the COMAC file (COvariance MATrices Cadarache) [11]. In this covariance file, a particular attention was paid to the re-evaluation of important isotopes  $^{235}\text{U}$  [12],  $^{56}\text{Fe}$  [13],  $^{238}\text{U}$  and  $^{239}\text{Pu}$  [14] meanwhile other evaluations are mainly based on ENDF/B-VII covariance file.

## **4 RESULTS**

In this paragraph, we will first study what happens to reactivity uncertainty when boron is added, or when the core temperature increases. In a second part, the uncertainty on each corresponding reactivity coefficient is calculated, as well as the Pearsons between these different configurations. Finally, we present an example of results obtained with and without taking into account the Pearsons and we give some arguments about the possibility of tabulating these coefficients in the calculation form.

### **4.1 Uncertainties on reactivity**

In this part, the uncertainties are calculated using the SPT (Eqn.(2.2)).

The calculated uncertainties on initial state reactivity (largely supercritical) give a result of 350 pcm at  $1\sigma$  (first column of Tab.1 and Tab.2). The main contributors are fission of  $^{235}\text{U}$ , and scattering of  $\text{H}_2\text{O}$  and  $^{27}\text{Al}$ . In Tab.1, the soluble boron concentrations increased stepwisefrom 0 to 2800 ppm (parts per million  $10^{-6}$ ). We observe an increase of the whole uncertainties except for  $^{27}\text{Al}$  which remains almost constant on the whole boron range. The uncertainty increase is a linear function of the boron concentration, essentially due to the spectrum hardening caused by thermal absorption. The sensitivity profiles moves to higher energies, where associated uncertainties in both  $^{235}\text{U}$  fission, and  $^{238}\text{U}$  resonant capture, are also higher. At 2800 ppm, the reactivity uncertainty gets the value of 460 pcm at  $1\sigma$ . For  $\text{H}_2\text{O}$  (in fact bounded hydrogen in  $\text{H}_2\text{O}$ ), we see, in the interval [0-600] ppm slight decrease of the uncertainties, followed by an increase after 600 ppm. However, the trend remains non-significant.



Temp. 20°C						
ppmB	0	300	600	2000	2500	2800
U235	268	280	291	344	363	374
U238	60	62	65	78	83	86
H2O	180	177	177	182	185	186
Al27	121	121	121	121	121	121
B10	0	14	28	90	112	124
Tot. Unc.	350	358	368	425	447	460

**Table 1: Reactivity uncertainty as a function of the soluble boron concentration (pcm at 1 $\sigma$ )**

Tab.2 shows the variations of reactivity uncertainties when the core temperature is modified. No particular crystalline effect is taken into account for the Doppler resonant treatment. Moreover, all materials are increased to the same temperature, and no additional temperature gradient is modeled in the fuel. Uncertainty modifications are much lower compared to the boron effect. Going from 20°C to 250°C, the reactivity uncertainty grows from 350 to 363 pcm at 1 $\sigma$ , which is totally negligible. For the uranium isotopes, we observe a decrease of their propagated uncertainties between 20 and 200 °C as for the other isotopes, the uncertainties are increased as the temperature rises..

Boron 0 ppm						
T °C	20	100	150	180	220	250
U235	268	260	262	264	267	270
U238	60	58	61	63	66	69
H2O	180	190	190	190	191	192
Al27	121	124	126	127	130	132
B10	0	0	0	0	0	0
Tot. Unc.	350	350	353	355	359	363

**Table 2: Reactivity uncertainty as a function of the core temperature (pcm at 1 $\sigma$ )**

Tab.3 presents results for simultaneous boron and temperature modifications. We concentrated on 2 temperatures. At 150°C, the boron produces a slightly more important uncertainty on the reactivity than at 220°C.

	T 220°C				T 150°C		
ppmB	100	600	2500	ppmB	100	600	2500
U235	283	300	363	U235	277	295	363
U238	73	78	96	U238	67	72	90
H2O	177	177	187	H2O	177	176	185
Al27	127	127	129	Al27	123	124	124
B10	4	26	99	B10	5	27	106
Tot. Unc.	365	380	450	Tot. Unc.	357	373	448

**Table 3: Reactivity uncertainty for simultaneous variations of boron concentration and core temperature (pcm at 1 $\sigma$ )**

To resume, when the temperature decreases with an increase of the boron amount, the reactivity uncertainty coming from boron increases but the reactivity uncertainty coming from other isotopes decreases. It follows a light decrease of the total reactivity uncertainty. It is light because, temperature impact on reactivity uncertainty is light, according to the Tab.3.

#### **4.2 Uncertainties on reactivity coefficients**

Uncertainties of reactivity coefficients are calculated using EPT (Eqn.(2.3)). In Tab.4, we fixed the temperature and made boron variations. The  $\Delta\rho$  line is the value of the reactivity coefficient and the Tot. Unc. Line corresponds to its uncertainty. We see that the reactivity coefficient uncertainty, for low boron adds, is more important at high temperature but is almost the same for the highest boron concentration (2500 ppm). The propagated value rises to 177pcmat 220°C for 169 pcm at 20°C. For both temperatures, the total uncertainty value is a linear function of  $\Delta\rho$  (Pearson > 0.999). But the function coefficients are not the same for both temperatures. This mean it is possible to predict the value of the uncertainty, knowing the  $\Delta\rho$  for boron amount in the interval [0-2500] ppm. Moreover, we remark that the relative uncertainty of this reactivity coefficient is constant.

$\Delta p_{\text{boron}}$	T 20°C			$\Delta p_{\text{boron}}$	T 220°C		
	ppmB	0->100	0->600		0->2500	ppmB	0->100
U235	4	29	116	U235	36	52	125
U238	1	6	28	U238	25	29	47
H2O	2	13	38	H2O	25	34	57
Al27	0	3	13	Al27	11	14	24
B10	5	28	112	B10	4	26	99
Tot. Unc.	7	43	169	Tot. Unc.	52	75	177
$\Delta \rho$	-1141	-6658	-26493	$\Delta \rho$	-1008	-6012	-23877

**Table 4: Reactivity coefficients uncertainties, on the left, at 20°C, on the right at 220°C for boron amount variations (pcm at 1 $\sigma$ )**

For the temperature coefficients, the trend is different. It seems the relative uncertainty of the reactivity coefficient is constant for low boron amount but not when there is a lot of boron in the moderator. The uncertainties remain weak for temperature coefficients despite the important  $\Delta \rho$  when the boron amount is weak. We remark than for all the cases, the uncertainties coming from the different isotopes remain close. But the uncertainties coming from boron, obviously, change.

$\Delta p_{\text{temp}}$	B100ppm		$\Delta p_{\text{temp}}$	B2500ppm	
	ppmB	20->150		20->220	ppmB
U235	5	11	U235	4	10
U238	6	12	U238	7	13
H2O	10	23	H2O	11	24
Al27	5	11	Al27	6	14
B10	0	0	B10	5	12
Unc. Tot	14	30	Unc. Tot	16	34
$\Delta \rho$	-1359	-2866	$\Delta \rho$	-163	-383

**Table 5: Reactivity coefficients uncertainties, on the left, at 100 ppm of boron, on the right at 2500 ppm of boron for core temperature variations (pcm at 1 $\sigma$ )**

These reactivity coefficients uncertainties will be use in the following to calculate the uncertainty of the core with different configurations.

#### 4.3 Pearsons calculation

The Pearson correlation coefficients are the last parameters to be calculated in order to properly propagate uncertainties for a particular configuration. This coefficient, describing the linear relation between two parameters is calculated from the second equality of Eqn.(2.4). The obtained values are tabulated for some configurations in Tab.6. The symbol

“->” represents the modified value used to calculate the  $\Delta\rho$ . Two kinds of information are tabulated in the Tab.6. The one mentioned in blue, is the simple correlation between the initial reactivity and the reactivity coefficient  $r(\rho_0, \Delta\rho)$ . The second information, mentioned in red, corresponds to a correlation between two reactivity coefficients  $r(\Delta\rho_1, \Delta\rho_2)$ .

For the first one, we observe that the Pearson's correlation follows the same behavior than the boron concentration. However the reverse trend is observed for the temperature: the Pearson decreases as the temperature rises.

The red values have completely different trends. The Pearson's increase when the boron content increases for a temperature change from 20 to 150°C, and is inverted if the range of temperature variation goes from 20°C to 220°C. However, if the correlation coefficients are relatively high for the boron concentrations, they remain low to very low for other quantities.

Constant Boron \ Constant Temp			$r(\rho_0, \Delta\rho_{\text{boron}})$			
			0->100	0->600	0->2500	
			0,31860	0,36453	0,41094	
$r(\rho_0, \Delta\rho_{\text{temp}})$	20->150	0,01097	0,02524	0,03172	0,04740	$r(\Delta\rho_{\text{temp}}, \Delta\rho_{\text{boron}})$
	20->220	0,07628	0,10986	0,09382	0,06187	
			$r(\Delta\rho_{\text{temp}}, \Delta\rho_{\text{boron}})$			

**Table 6: Pearson's calculated between reactivity coefficients or reference reactivity and reactivity coefficients**

These correlation coefficients will be used in the next part to calculate the final uncertainty after changing the temperature and the boron amount in the core.

#### 4.4 Example of uncertainty accumulation with non-zero correlations

In this part, we will consider an example and show the importance of the correlations term to calculate the uncertainty. We will show that some simplifications can be done in the correlation matrix.

We consider the following simple case: suppose the reactivity uncertainty for a case without boron and at 20°C (noted  $\varepsilon(\rho_0)$ ) to be known, as well as the uncertainty of the boron insertion  $\varepsilon(\Delta\rho_{boron})$ , the Pearson correlation between  $\varepsilon(\rho_0)$  and  $\varepsilon(\Delta\rho_{boron})$ , written  $r(\rho_0, \rho_{\Delta\rho_{bore}})$  or the Pearson correlation between  $\varepsilon(\rho_0)$  and the final case with boron  $\varepsilon(\rho_0 + \Delta\rho_{boron})$ , written  $r(\rho_0, \rho_1)$ .

We want to calculate the uncertainty of the final case  $\varepsilon(\rho_1)$ .

Two possibilities can be used, given by the uncertainty propagation law, isolating the quantity of interest:

$$\begin{aligned}\varepsilon(\rho_1) &= \pm\sqrt{[\varepsilon(\rho_0)]^2 r(\rho_0, \rho_1) - [\varepsilon(\rho_0)] + [\varepsilon(\Delta\rho_{boron})]^2 + [\varepsilon(\rho_0)]r(\rho_0, \rho_1)} \\ &= \pm\sqrt{[\varepsilon(\rho_0)]^2 + [\varepsilon(\Delta\rho_{boron})]^2 + 2[\varepsilon(\rho_0)][\varepsilon(\Delta\rho_{boron})]r(\rho_0, \rho_{\Delta\rho_{bore}})}\end{aligned}$$

A numerical application can be performed, considering a boron injection of 2500 ppm., then, using the second equation:

$$\varepsilon(\rho_1) = \pm\sqrt{[350]^2 + [169]^2 + 2[350][169] * 0.41094} = 447 \text{ pcm}$$

It corresponds to the value calculated in the Tab.1. Performing the application without the correlation term would give:

$$\varepsilon(\rho_1) = \pm\sqrt{[350]^2 + [169]^2} = 389 \text{ pcm}$$

The calculated uncertainty without correlation would be 389 pcm instead of 447 pcm. This represents an error of 13% on the reactivity uncertainty estimation.

Let's try to generalize the process for different reactivity coefficients and different core configurations, as presented on Figure 1.

The final calculated reactivity is given by:

$$\rho = \rho_0 + \Delta\rho_{bore} + \Delta\rho_{temp} + \Delta\rho_{croix} = 29264 + (-26493) + (-383) = 2388 \text{ pcm}$$

Using the different tables previously presented, the correlation matrix and the uncertainty vector can be built from Eqn. 2.7:

$$\left\{ \begin{array}{l} \underline{Z} = [350 \quad 169 \quad 34] \\ \underline{\underline{\Omega}} = \begin{bmatrix} 1 & 0.41094 & 0.07628 \\ 0.41094 & 1 & 0.06187 \\ 0.07628 & 0.06187 & 1 \end{bmatrix} \end{array} \right.$$

Then we get  $\varepsilon(\rho) = 450$  pcm which corresponds exactly to the result obtained by the uncertainty calculation using SPT (Tab.3). The uncertainty without correlation (replacing  $\underline{\underline{\Omega}}$  by the identity matrix) would give  $\varepsilon(\rho) = 390$  pcm. So, even if taking into account the temperature coefficient does not change the uncertainty, we showed that for reactivity coefficients producing important uncertainties, it is necessary to take into account the correlations.

#### 4.5 Tabulation of Pearsons coefficients

The Pearson correlations have certain stability according to the configurations. We precise that:

- The second order Pearsons coefficients like  $r(\Delta\rho_i, \Delta\rho_j)$  present important variations. However, their impact on the total uncertainty remains negligible because the uncertainties linked to reactivity coefficients are less important than uncertainties on a reactivity value. Then, taking the previous example and neglecting these coefficients, we get:

$$\left\{ \begin{array}{l} \underline{Z} = [350 \quad 169 \quad 34] \\ \underline{\underline{\Omega}} = \begin{bmatrix} 1 & 0.41094 & 0.07628 \\ 0.41094 & 1 & 0 \\ 0.07628 & 0 & 1 \end{bmatrix} \end{array} \right. \text{ Then, } \varepsilon(\rho) = 450 \text{ pcm}$$

The uncertainty is then conserved.

- The first order correlations like  $r(\rho_0, \Delta\rho_i)$ , impact more the total uncertainty but they can be represented by a model. For example, those coming from the boron reactivity coefficient are a power function of the boron concentration. Moreover, variations of 25% of these coefficients do not modify a lot the final uncertainty. If we take for example the boron correlation of an amount of 100 ppm instead of this coming from an amount of 2500 ppm, we have the following system:

$$\left\{ \begin{array}{l} \underline{Z} = [350 \quad 169 \quad 34] \\ \underline{\underline{\Omega}} = \begin{bmatrix} 1 & 0.31860 & 0.07628 \\ 0.31860 & 1 & 0 \\ 0.07628 & 0 & 1 \end{bmatrix} \end{array} \right\} \text{Then, } \varepsilon(\rho) = 438 \text{ pcm}$$

This gives an error of 2.5% on the final uncertainty.

- In this particular case, the temperature correlation can be neglected:

$$\left\{ \begin{array}{l} \underline{Z} = [350 \quad 169 \quad 34] \\ \underline{\underline{\Omega}} = \begin{bmatrix} 1 & 0.31860 & 0 \\ 0.31860 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \right\} \text{Then, } \varepsilon(\rho) = 448 \text{ pcm}$$

This way of calculating uncertainty from reactivity coefficients and associated correlations can be extended to other modifications in the configuration, such as, for example, the introduction of absorbing element. In this case, when new reactivity coefficients are introduced, the dimensions of both  $\underline{\underline{\Omega}}$  matrix and  $\underline{Z}$  vector are increased.

## 5 CONCLUSIONS

In this paper, we have detailed a particular application of nuclear data uncertainty propagation on reactivity coefficients, and used calculated Pearson's correlations coefficients to extrapolate reactivity effects and uncertainties to different core configurations. These correlations are necessary for rigorous uncertainty propagation. We have shown on a very simple case that they cannot be neglected, with the exception of some values of low reactivity coefficient uncertainties or for second order correlations. The reactivity uncertainty, calculated without taking into account these correlations is underestimated by about 13 %.

Of course, values obtained here should be different for different cores. However, these correlation coefficients can be tabulated and models for interpolating reactivity effects and associated uncertainties using these correlations can easily be built, as we showed that perturbations of these correlations do not induce important errors on the final propagated uncertainty.

The calculation of these correlations can be extended for other core parameters such as local power factors or isotopic concentrations in the case of burnup calculations. The knowledge of all these uncertainties and correlations could, in the future, feed an "uncertainty data base" associated to a cumulating model, dedicated to actual MTR or NPP. This would allow an easy and direct access to ND propagated uncertainties of all local and global core parameters for any configuration.



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