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Sensitivity of power spectral density noise techniques to numerical parameters in analyzing neutron noise experiments

E. Gilad^{a,*}, B. Geslot^b, P. Blaise^b, C. Dubi^c

^a*The Unit of Nuclear Engineering, Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel*

^b*DEN/CAD/DER/SPEX/LPE, CEA Cadarache, Saint-Paul-les-Durance 13108, France*

^c*Department of Physics, Nuclear Research Center NEGEV (NRCN), Beer-Sheva 84190, Israel*

Abstract

Power spectral density methods (Cohn- α methods) are well-known and widely used for the analysis of neutron noise experiments and obtaining the reactor core integral kinetic parameters, i.e., the effective delayed neutron fraction β_{eff} and the prompt neutron generation time Λ . The Cohn- α methods are considered as the standard data processing procedure in the case of a current acquisition system that works at high fission rates. Many uncertainties are usually considered in the Cohn- α method, e.g., statistical fluctuations in the neutron count, power drifts, uncertainties in the Diven factor, the integral fission rate, and in the reactivity value. However, the uncertainty associated with the numerical parameters used in the power spectra calculation procedure, e.g., time bin size and buffer size, is hardly discussed in the literature and generally overlooked.

In this paper, The Cohn- α method is implemented to analyze critical and subcritical configurations of the MAESTRO core in the MINERVE zero power reactor in order to measure its β_{eff} and Λ integral kinetic parameters. Both cross-correlation and auto-correlation power spectral densities are calculated and the kinetic parameters are obtained via Lorentzian curve fitting over the calculated spectra. The sensitivity of the obtained kinetic parameters to the choice of numerical parameters used for spectrum calculations is studied and found to be pronounced and comparable with other uncertainties. A novel methodology is proposed for analyzing the kinetic parameters' sensitivity to the power spectra calculations and for quantifying the associated uncertainties.

Keywords: Noise techniques, power spectral density, sensitivity analysis, integral kinetic parameters, spectrum calculations

*Corresponding author

Email address: gilade@bgu.ac.il (E. Gilad)

1. Introduction

A set of neutron noise measurements have been performed on the MINERVE zero power reactor at Cadarache research center in France during September 2014. This experimental campaign was conducted in the framework of a tri-partite collaboration between CEA, PSI and SCK-CEN (Geslot et al., 2015; Perret, 2015; Gilad et al., 2016). Measurements were then also processed and analyzed in the framework of a collaboration between CEA, Ben-Gurion University of the Negev (BGU), and the Israeli Atomic Energy Commission (IAEC). The main purpose of the campaign was to obtain the core kinetic parameters using various existing and novel noise techniques and compare it with recent measurements. The last time a similar campaign was performed in MINERVE was in 1975 and the core configuration was different (Carre and Oliveira, 1975). This campaign is a continuation of a previous one aimed at determining the delayed neutron fraction β_{eff} in the MINERVE reactor using in-pile oscillations technique (Gilad et al., 2015).

Several well-known and widely used neutron noise techniques were implemented for analyzing the experimental measurements, e.g., Cohn- α , Feynman-Y, and Rossi- α methods. These methods were used to obtain the reactor core integral kinetic parameters, i.e., the effective delayed neutron fraction β_{eff} and the prompt neutron generation time Λ (Geslot et al., 2015; Gilad et al., 2016). More specifically, the Cohn- α method is considered as the standard data processing procedure in the case of a current acquisition system that works at high fission rates by digitizing the current signal issued by fission chambers (Diniz and dos Santos, 2002; dos Santos et al., 2006; Geslot et al., 2015). Such a system has recently been developed and qualified by CEA and is able to process signals on line without any data loss (de Izarra et al., 2015).

The statistical uncertainties associated with the Cohn- α method are usually thoroughly analyzed and are propagated to the final results, i.e., the integral kinetic parameters, using well established methodologies and considerations. For example, Geslot et al. (2015) recommend using the values obtained by the Cross-Correlation Power Spectral Density (CPSD) estimator following data processing considerations and final uncertainties associated with the results. This estimator has proved to be very robust and produced minimum uncertainties. The uncertainties usually considered in the Cohn- α method include statistical fluctuations in the neutron count, power drifts, uncertainties in the Diven factor, the integral fission rate, and in the reactivity value. The high-level analysis by Geslot et al. (2015) using Power Spectral Density (PSD) techniques leads to uncertainties of 1.8-2.8 pcm in the value of β_{eff} and 0.7-1.3 μs in Λ (at 1σ).

On the other hand, the uncertainty associated with the numerical parameters used in the power spectra calculation procedure, e.g., time bin size and buffer size, is hardly discussed in the literature and generally overlooked. Despite their conspicuous importance (as demonstrated in this paper), very little considerations are usually given to their values. These values are often determined rather arbitrarily according to the acquisition system technical specifications and the bias degree of the residuals in the curve fitting procedure. Moreover, well-defined criteria or methodologies for setting and tuning these numerical parameters, as well

61 as for evaluating their associated uncertainties, are generally absent.

62 In this paper, the sensitivity of the Cohn- α method to numerical parameters used in the power spectra
63 calculation procedure is studied by analyzing noise measurements performed in the MINERVE reactor core
64 at three different reactivity states. The associated uncertainties are evaluated and a methodology for optimal
65 determination of these parameters is proposed. The experimental setup is described in section 2, the Cohn- α
66 formalism is introduced in section 3, and the CPSD results for the critical state Acq12 are described and
67 discussed in sections 4. The CPSD results for the subcritical states Acq16 and Acq19 are described in
68 sections 4.4. APSD results for the different reactivity states are described section 4.5 and the conclusions
69 are discussed in section 5.

70 2. Experimental setup

71 The MINERVE reactor is a pool-type ($\sim 120 \text{ m}^3$) reactor operating at a maximum power of 100 W
72 with a corresponding thermal flux of $10^9 \text{ n/cm}^2\cdot\text{s}$ (Cacuci, 2010). The core is composed of a driver zone,
73 which includes 40 standard highly enriched MTR-type metallic uranium alloy plate assemblies surrounded
74 by a graphite reflector. An experimental cavity, in which various UO_2 or MOX clad fuel pins can be
75 loaded in different lattices, reproducing various neutron spectra (Cacuci, 2010; pascal Hudelot et al., 2004),
76 is located in the center of the driver zone. During the experimental campaign, the central experimental zone
77 was loaded with 770 3% enriched UO_2 fuel rods arranged in a lattice representative of a PWR spectrum.
78 An oscillator piston, capable of moving periodically and vertically between two positions located inside and
79 outside of the core is located inside the experimental zone. A general view of the MINERVE reactor is shown
80 in Fig. 1, together with schematic drawings of the reactor geometrical configuration and the MAESTRO
81 core configuration (Leconte et al., 2013).

82 During the measurement campaign, neutron noise experiments have been conducted in three reactor
83 states; one very close to critical state (marked as “Acq12”) and two different subcritical states (marked as
84 “Acq16” and “Acq19”). The different criticality states were obtained by inserting one of the four control
85 rods into the core. The reactor configuration was that of the MAESTRO program (Leconte et al., 2013),
86 representing a PWR spectrum in the central experimental cavity, as shown in Fig. 1. Two large fission
87 chambers with approximately 1g of ^{235}U have been installed next to the driver zone (denoted n°670 and
88 n°671 in Fig. 1). In order to minimize flux disturbances in the detectors during measurement, reactor
89 criticality was controlled by control rod B1, which is far from the two detectors. During the measurements,
90 the power was regulated by an automatic piloting system that makes use of a low efficiency rotating control
91 rod with cadmium sectors. The only slightly subcritical measurement Acq12 has been conducted at a power
92 of 0.2 W with detectors’ count rate around $5.5 \times 10^5 \text{ cps}$. The subcritical measurements Acq16 and Acq19
93 have been conducted with detectors’ count rate around $4 \times 10^4 \text{ cps}$. The signals were acquired using fast

94 amplifiers and CEA-developed multipurpose acquisition system X-MODE. The signals were acquired in time
 95 stamping mode with a resolution of 25 ns. A 1-second sample segment of the detectors' signal is shown in
 96 Fig. 2. More details on the experimental setup and acquisition systems can be found in (Geslot et al., 2015).
 97 The measurements analyzed in this paper are described in Table 1.

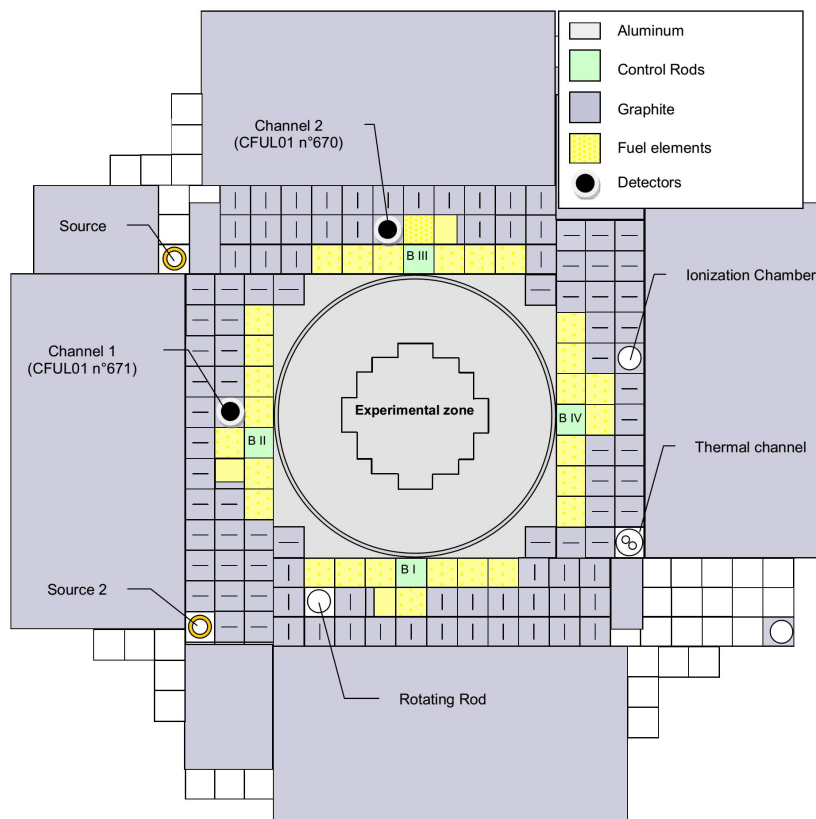


Figure 1: Schematic layout of the MINERVE zero power reactor core during the noise measurements campaign in Sep. 2014.

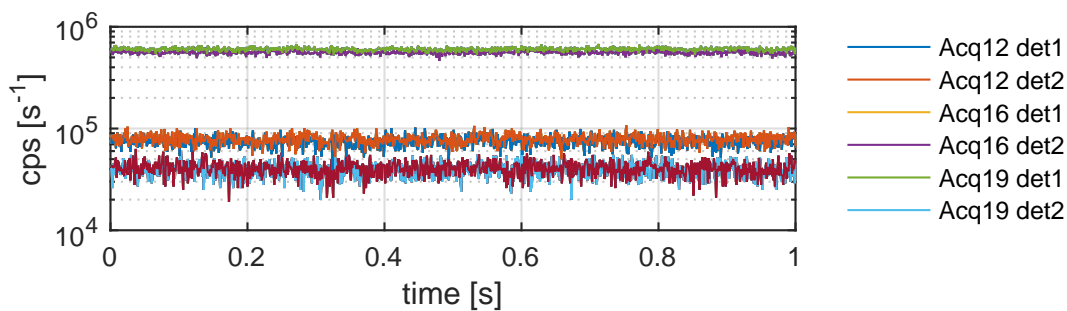


Figure 2: A sample segment of the detectors' signals for the different reactivity states of the core.

Table 1: Pile noise measurements during the Sep. 2014 experimental campaign that are analyzed.

Data set	Acq12	Acq16	Acq19
Control rod height [mm]	B1@499	B1@399	B1@449
Core power [W]	0.2	0	0
Duration [s]	5400	5500	5500
Integral fission rate F [s^{-1}]	6.45×10^9	4.00×10^8	7.91×10^8
Reactivity [pcm]	~ 0	-230	-117

3. The Cohn- α formalism

The transfer function of the reactor links the reactor neutron population to the neutron source fluctuations. The zero power transfer function can be derived from point kinetic equations, where the source noise is considered to be entirely due to fluctuations in the core reactivity, in the neutron population and in the precursors concentration (Keepin, 1965; Williams, 1974; Santamarina et al., 2012). For large enough frequencies, i.e. $\omega \gg \lambda_j$, the transfer function amplitude takes the following form

$$|H(\omega)|^2 = \frac{1}{1 + (\omega/\omega_c)^2}, \quad (1)$$

where ρ is the reactivity of the core and $\omega_c = \frac{\beta_{\text{eff}} - \rho}{\Lambda}$ is called the cutoff frequency.

The Cross-correlation Power Spectral Density (CPSD) is defined as the Fourier transform of the cross-correlation between two detectors, i.e.,

$$\text{CPSD} \equiv \mathcal{F}\langle c_1(t), c_2(t) \rangle = \mathcal{F}\{c_1(t)\} \mathcal{F}^*\{c_2(t)\} = \mathcal{F}\left\{ \frac{1}{2T} \int_{-T}^T c_1(t) c_2(t + \tau) dt \right\}, \quad (2)$$

where \mathcal{F} is the Fourier transform operator, $c_i(t)$ stands for the readings of detector i as a function of time, $\langle x, y \rangle$ is the temporal correlation function between x and y , T represents the buffer size, dt represents the time bin size, and \mathcal{F}^* represents the complex conjugate of \mathcal{F} . Similarly, the Auto-correlation Power Spectral Density (APSD) is defined as the Fourier transform of the auto-correlation of a single detector i , i.e.

$$\text{APSD}_i \equiv \mathcal{F}\langle c_i(t), c_i(t) \rangle = \mathcal{F}\{c_i(t)\} \mathcal{F}^*\{c_i(t)\} = \mathcal{F}\left\{ \frac{1}{2T} \int_{-T}^T c_i(t) c_i(t + \tau) dt \right\}. \quad (3)$$

The discrete form of the correlation function between two detectors $R_{12}(n)$ and its Fourier transform are as follows

$$R_{12}(n) = \langle c_1, c_2 \rangle(n) = \frac{1}{N-n} \sum_{k=0}^{N-n-1} c_{1,k} c_{2,k+n} \quad (4)$$

117 and

$$118 \quad \text{CPSD}(N, dt) \equiv \mathcal{F} \{R_{12}(n)\} = \sum_{k=0}^N \tilde{c}_{1,k} \tilde{c}_{2,k}^* , \quad (5)$$

119 where $Ndt = T$ and $c_{x,k}$ and $\tilde{c}_{x,k}$ are the number of detections in detector x in time bin dt_k and its discrete
120 Fourier transform, respectively, i.e., $\tilde{c}_{x,k} = \mathcal{F} \{c_{x,k}\}$. Similarly, the discrete form of the APSD of detector i
121 is

$$122 \quad \text{APSD}_i(N, dt) \equiv \mathcal{F} \{R_{ii}(n)\} = \sum_{k=0}^N \tilde{c}_{i,k} \tilde{c}_{i,k}^* . \quad (6)$$

123 The zero power transfer function amplitude $|H(\omega)|^2$ can also be written in terms of two detectors'
124 readings $c_1(t)$ and $c_2(t)$ (Cohn, 1960; Santamarina et al., 2012) in the following form

$$125 \quad |H(\omega)|^2 = \frac{\mathcal{F} \langle c_1(t), c_2(t) \rangle}{\bar{c}_1 \bar{c}_2} \frac{1}{|\mathcal{F}(\delta\rho)|^2} = \frac{\text{CPSD}}{\bar{c}_1 \bar{c}_2} \frac{1}{2D/F} , \quad (7)$$

126 where \bar{c}_i is the average count rate of detector i , $\delta\rho$ is the reactivity perturbation, $D = \frac{\nu(\nu-1)}{\bar{p}^2}$ is the Diven
127 factor (Diven et al., 1956), and F is the integral fission rate in the core. Hence, by combining Eqs. (1)
128 and (7), the expression linking the CPSD with the effective delayed neutron fraction β_{eff} and the prompt
129 neutron generation time Λ is (Cohn, 1960; Carre and Oliveira, 1975; Diniz and dos Santos, 2002; dos Santos
130 et al., 2006) straightforward:

$$131 \quad 2 \frac{D}{F} \frac{1}{(\beta_{\text{eff}} - \rho)^2} \frac{1}{1 + (\omega/\omega_c)^2} = \frac{\text{CPSD}}{\bar{c}_1 \bar{c}_2} . \quad (8)$$

132 Similarly, for the APSD, one gets:

$$133 \quad 2 \frac{D}{F} \frac{1}{(\beta_{\text{eff}} - \rho)^2} \frac{1}{1 + (\omega/\omega_c)^2} = \frac{\text{APSD}_i}{\bar{c}_i \bar{c}_i} + B_i , \quad (9)$$

134 where B_i is some constant due to the fact that unlike CPSD, the APSD does not asymptotically tends
135 to zero due to detections produced by the randomly (uncorrelated) arriving neutrons. In any case, for all
136 practical purposes the RHS of Eqs. (8) and (9) is fitted with a function of the form

$$137 \quad f(\omega) = \frac{x_1}{1 + \left(\frac{\omega}{x_2}\right)^2} + x_3 \quad (10)$$

138 and

$$139 \quad \beta_{\text{eff}} - \rho = \sqrt{\frac{2D}{F} \frac{1}{x_1}}, \quad \Lambda = \frac{\beta_{\text{eff}} - \rho}{2\pi x_2} . \quad (11)$$

140 4. Standard PSD analysis

141 A total of three measurements were analyzed using the Cohn- α method (see Table 1). The Diven factor
142 for thermal fission of ^{235}U is set to $D = 0.8$. The integral fission rate F is obtained by calculation of the flux
143 distribution in the core and its calibration using the readings of a dedicated fission chamber located at the

144 core center during the experiment. All measurements were recorded using the X-MODE acquisition system
 145 in time stamping mode with resolution of 25 ns. The reactivity worth of the control rod B1 was calculated
 146 using rod-drop experiment and inverse kinetics analysis.

147 The standard Cohn- α procedure usually continues by evaluating the fission rate F by calculation and
 148 measurement, calculating the CPSD in Eq. (5) from the measurement, and calculating the Diven factor D .
 149 Then, Eq. (8) is used to obtain $\beta_{\text{eff}} - \rho$ and the cutoff frequency $\omega_c = \frac{\beta_{\text{eff}} - \rho}{\Lambda}$ by Lorentzian curve fitting to
 150 the right hand side of the equation. An example for CPSD and APSD spectra for the different reactivity
 151 states and a fitted Lorentzian curve are shown in Fig. 3.

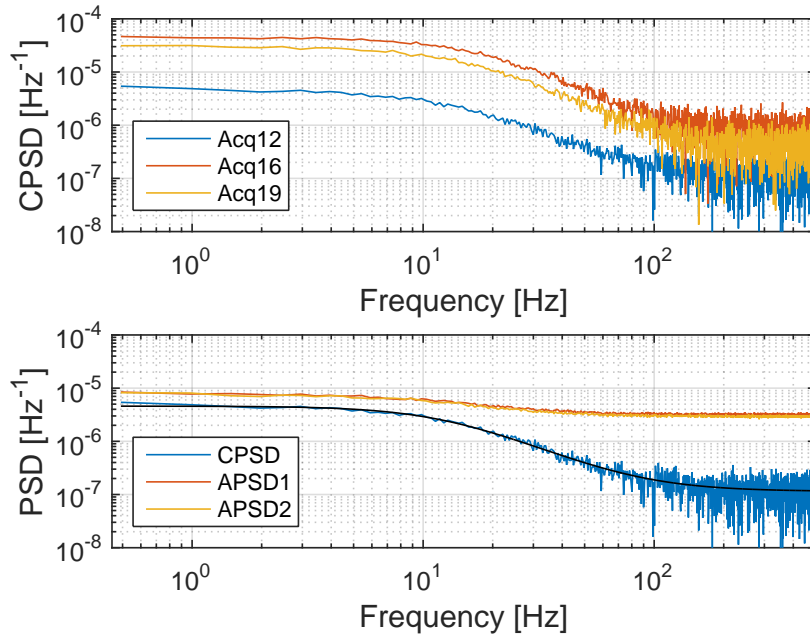


Figure 3: Upper panel: An example of CPSD spectra for the different reactivity states. Lower panel: An example of APSD and CPSD spectra for Acq12 and the fitted Lorentzian curve marked by solid black line. The spectra were calculated using $N = 2000$ and $dt = 1.02$ ms.

152 4.1. The numerical parameters

153 Power spectral density methods inevitably introduce additional purely numerical parameters. Unlike
 154 physical parameters of the experimental system, these parameters should have little or no effect on the
 155 results of the analysis. The acquisition method of time stamps records the time of each detection within a
 156 resolution of 25 ns. This time resolution is too fine and not adequate for power spectral density calculations
 157 due to the relevant signal bandwidth of 1-80 Hz. Therefore, the detector signal is binned on a coarser time
 158 resolution. Furthermore, the power spectral density is not calculated for the entire signal at once due to
 159 computer memory limitations. The long signal is divided into shorter segments (or buffers), each of duration

160 $T = Ndt$, where dt is the size of the time bin and N is the number of bins (buffer size) considered for a
 161 single spectrum calculation. For each segment, the power spectral density (CPSD or APSD) is calculated
 162 and averaged with the rest of the spectra calculated for the other segments.

163 This standard calculation method introduces two numerical, *not physical*, parameters into the proce-
 164 dure, i.e., the buffer size N and the time bin size dt . Hence, the discrete form of the PSD should be written
 165 as

$$\text{PSD} \equiv \text{PSD}(\omega; N, dt) . \quad (12)$$

167 The explicit dependence of the CPSD and APSD on these parameters is demonstrated in Eqs. (2)–(6). In
 168 order to evaluate the sensitivity of the calculated kinetics parameters β_{eff} and Λ to these numerical param-
 169 eters, the calculation procedure is repeated using the same data but with different numerical parameters.
 170 The results for the critical state Acq12 are shown in Fig. 4.

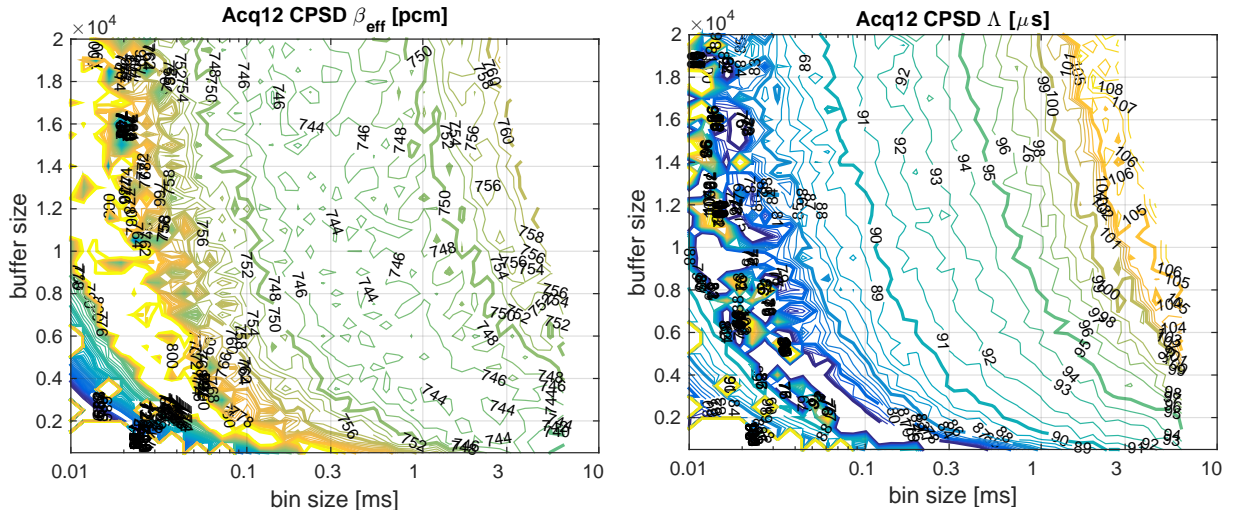


Figure 4: Sensitivity of the obtained effective delayed neutron fraction β_{eff} and the prompt neutron generation time Λ to the buffer size N and the time bin size dt for the critical state Acq12 using CPSD spectra.

171 It is clear from Fig. 4 that the numerical parameters N and dt have a pronounced effect on the obtained
 172 kinetic parameters. Moreover, methodologies for tuning these parameters are not usually addressed. Initially,
 173 no compelling physical arguments favoring a specific set of values for the buffer size N and the time bin size
 174 dt were found. These parameters are usually set such that the sensitivity of the obtained results is minimized
 175 and the residuals are normally distributed without any trend at low or high frequencies. Examination of
 176 Fig. 4 indeed reveals areas in the numerical parameters space where the value of β_{eff} is only weakly sensitive
 177 to the parameters values, but no such areas are found for Λ . The upper right corner is empty since the
 178 signals recorded by the acquisition system for the critical state Acq12 were segmented into files containing
 179 55 seconds each, hence no spectra were generated for $Ndt > 55$ seconds.

180 4.2. The relevant frequency range

181 The irregular and erratic behavior of the fit results in the left and lower left parts of the parameter space
 182 (mainly small dt) is due to the fact that the relevant frequency range, where the PSD possesses physical
 183 meaning (i.e., the transfer function and not white noise), is roughly between 1-80 Hz (Geslot et al., 2015).
 184 This range depends of course on the physical properties of the specific core and can assume different values.
 185 Moreover, the kinetic parameters presented in Fig. 4 are fitted over the entire spectrum and not confined to
 186 some predefined frequency range.

187 The buffer size N and the time bin size dt determine the frequency range and resolution of the derived
 188 spectra. The maximal frequency is determined by the Nyquist frequency, $f_{max} = \frac{1}{2dt}$, and the minimal
 189 frequency, which is equivalent to the spectrum resolution, is determined according to $df = \frac{1}{T} = \frac{1}{Ndt}$. Hence,
 190 the requirement $f_{max} \geq 80$ Hz means $dt \leq \frac{1}{160}$ s and $f_{min} \leq 1$ Hz means $Ndt \geq 1$ s. The values of f_{min} as
 191 a function of dt and N are illustrated in Fig. 5, where the line $dt = \frac{1}{160}$ s is also marked. Any pair of dt and
 192 N which define a frequency range that includes the range 1-80 Hz should be considered as relevant for PSD
 193 fit procedures.

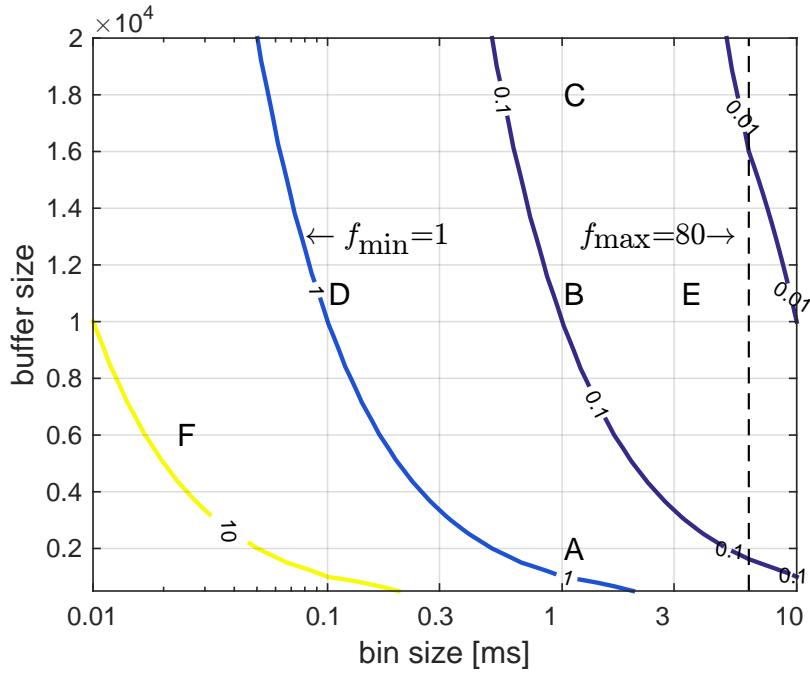


Figure 5: The values of f_{min} as a function of dt and N . The line $dt = \frac{1}{160}$ s is marked with dashed black line. Points A-E represent appropriate sets of values (dt, N) , i.e., the PSD range spans the 1-80 Hz range. Point F represent inappropriate set of values for PSD fit.

194 Points A-E in Fig. 5 represent sets of values (dt, N) appropriate for PSD fit, i.e., the PSD range includes
 195 the 1-80 Hz range. Point F represent inappropriate set of values for PSD fit. The parameters of points A-F

196 are given in Table 2 and the corresponding spectra and Lorentzian fits are shown in Fig. 6.

Table 2: Parameters of points A-F in Fig. 5.

Point	dt [ms]	N	$f_{min} - f_{max}$ [Hz]	β_{eff} [pcm]	Λ [μs]
A	1.02e-03	2000	4.92e-01 – 4.92e+02	744	91
B	1.02e-03	11000	8.94e-02 – 4.92e+02	747	97
C	1.02e-03	18000	5.46e-02 – 4.92e+02	750	99
D	1.01e-04	11000	9.01e-01 – 4.96e+03	751	90
E	3.23e-03	11000	2.82e-02 – 1.55e+02	755	105
F	2.28e-05	6000	7.30e+00 – 2.19e+04	869	106

197 The striking observation from Fig. 6 and Table 2 is that all points A-E cover very well the transition
 198 section of the transfer function, i.e., the relevant bandwidth 1-80 Hz, and they all exhibit excellent fits with
 199 uniform distribution of the normalized residuals. Hence, all these point are appropriate for PSD analysis
 200 and derivation of the kinetic parameters. However, as shown in Table 2, the obtained values of the kinetic
 201 parameters vary significantly between the different points, where no point is favored over the next one.

202 Once we have established some guide rules for selecting proper sets of (dt, N) values, Fig. 4 is redrawn
 203 in Fig. 7 only for appropriate parameters which enable proper fit procedure.

204 This representation of the sensitivity of the obtained kinetic parameters is much more insightful since
 205 although any point in Fig. 7 is legitimate for the Lorentzian fit procedure, the variance in the obtained
 206 results is significant. For example, Geslot et al. (2015) calculated the PSD using time resolution of 1 ms and
 207 frequency resolution of 0.5 Hz, which correspond to $dt = 1$ ms and $N = 2000$. Looking at Fig. 7, this point
 208 is part of a large set of equivalent points where none are physically favored, but produce different results.

209 4.3. Quantifying the uncertainty

210 One possible course of action in determining the value of β_{eff} and Λ is to average their values in the
 211 relevant frequency area, since they are all physically equivalent with appropriate spectrum for fit. The
 212 standard deviation over this set will give a measure of the uncertainty originating from the choice of numerical
 213 parameters such as dt and N . In the case of CPSD analysis of the critical state Acq12 shown in Fig. 7, the
 214 mean and standard deviation are $\beta_{eff} = 756.7 \pm 3.8$ pcm and $\Lambda = 91.7 \pm 3.6$ μs . It should be noted that
 215 in this paper, the precise and exact values of the kinetic parameters are of less importance or experimental
 216 validity (this point is further discussed in later sections). Instead, the important result is the estimation of
 217 the associated uncertainty and the fact that it is of significant magnitude compared to other uncertainties
 218 and should not be ignored in the future.

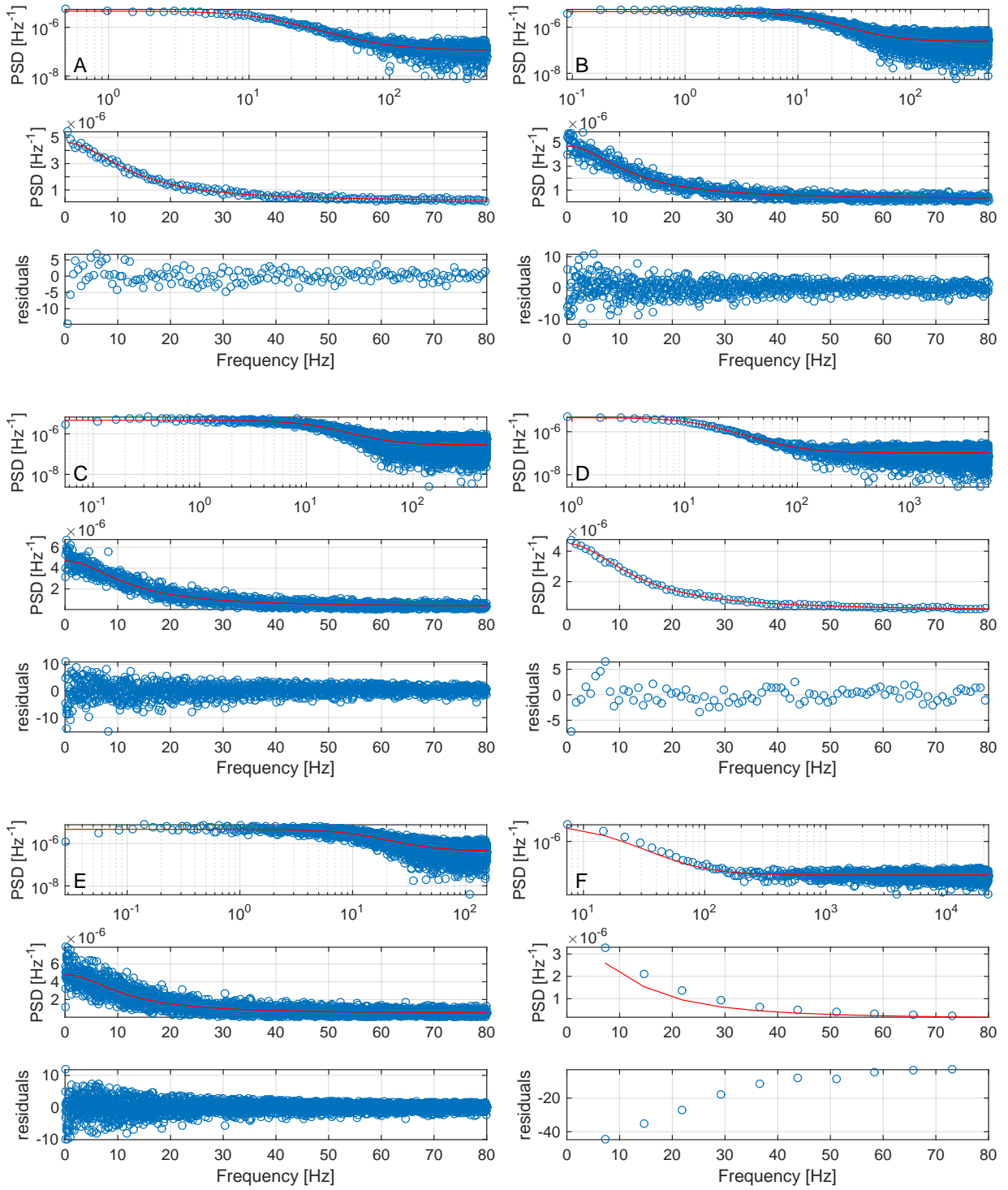


Figure 6: Full spectra (upper panel), Lorentzian fit on the range 1-80 Hz (middle panel), and the normalized residuals (lower panel) for the points A-F detailed in Table 2 and shown in Fig. 5.

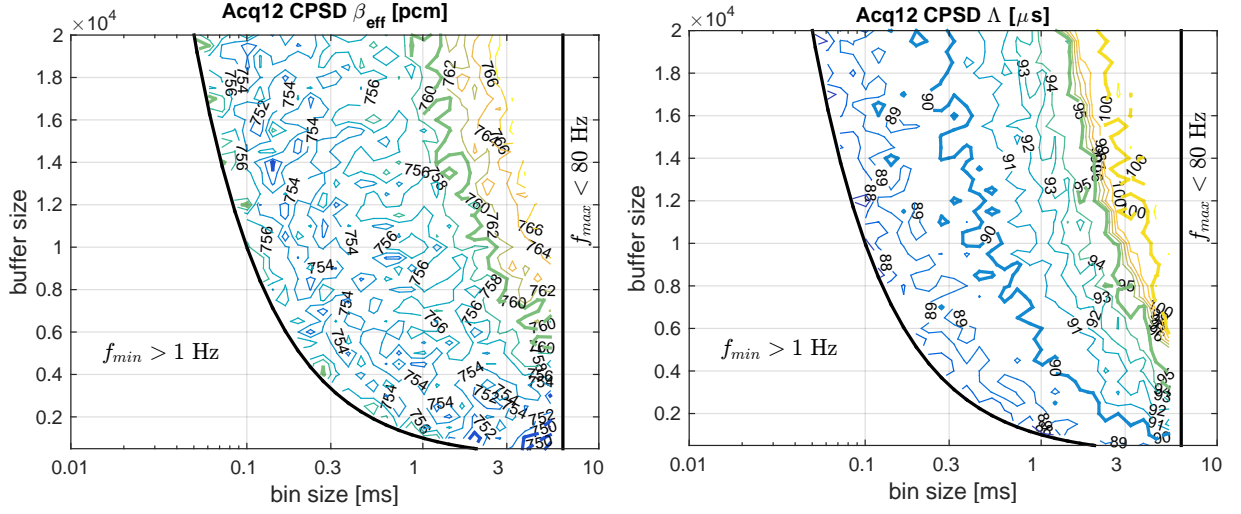


Figure 7: Sensitivity of the obtained kinetic parameters to the buffer size N and the time bin size dt over the relevant frequency range for the critical state Acq12 using CPSD spectra.

219 4.4. CPSD results for subcritical states

220 The CPSD analysis described in section 4 was also applied to the two subcritical states Acq16 and Acq19.
 221 The results are shown in Fig. 8 and show similar qualitative behavior of the kinetic parameters sensitivity
 222 to the choice of dt and N . Quantitatively, however, the uncertainties are smaller compared to the critical
 223 state analysis (Acq12), and the mean and standard deviation are $\beta_{\text{eff}} = 734.4 \pm 4.4$ pcm and $\Lambda = 91.6 \pm 3.0$
 224 μs for Acq16 and $\beta_{\text{eff}} = 715.4 \pm 3.1$ pcm and $\Lambda = 89.6 \pm 3.2$ μs for Acq19.

225 **EREZ:** Moreover, comparing to Fig. 7, it seems that the sensitivity of the CPSD to the numerical param-
 226 eters is not only weaker in subcritical states than in the critical state, but also that the kinetic parameters
 227 exhibit smoother and more homogeneous behavior over the parameter space. This could be related to the
 228 fact that the statistical errors associated with higher moments of the count rate (used in estimators like
 229 CPSD, APSD, Fyenman- α , etc.) converge faster for subcritical measurements than for critical ones. As a
 230 general rule, the convergence rate of the variance of higher moments is proportional to the inverse of the
 231 reactivity. More specifically, the statistical variance of moment M_n of order n converges at a rate inversely
 232 proportional to the reactivity to the power of $2n$, i.e., $\text{Var}(M_n) \sim \frac{1}{\rho^{2n}}$ (Dubi and Kolin, 2016).

233 4.5. APSD results

234 An APSD analysis was carried out on all three reactivity states along the guidelines that were phrased in
 235 section 4 regarding the relevant range of numerical parameters to be used for fit procedure. The sensitivity
 236 of the obtained kinetics parameters to the choice of dt and N show similar qualitative behavior as exhibited

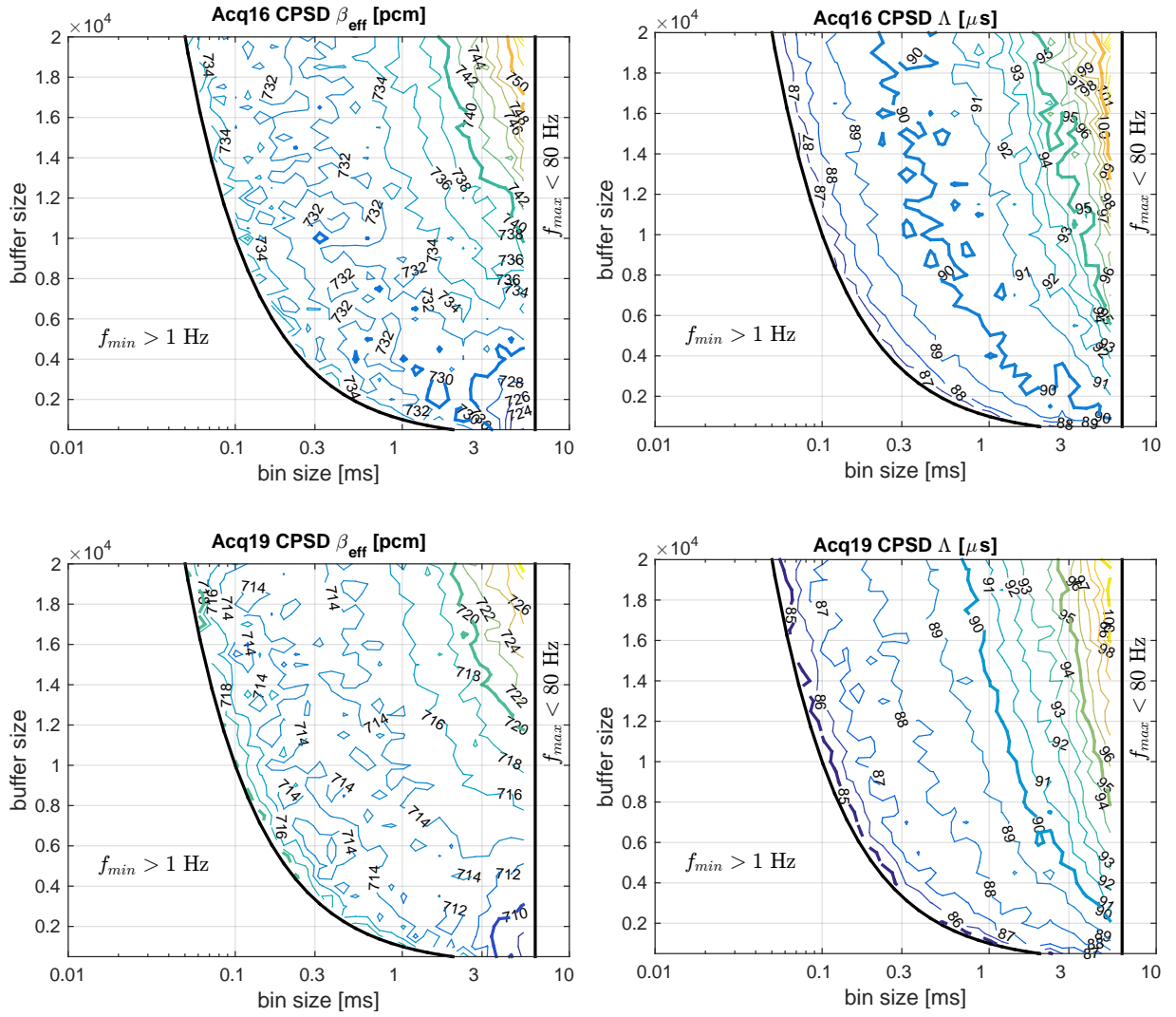


Figure 8: Sensitivity of the obtained kinetic parameters to the buffer size N and the time bin size dt over the relevant frequency range for the subcritical states Acq16 and Acq19 using CPSD spectra.

237 in the CPSD analysis. An example is shown in Fig. 9 for both APSD₁ and APSD₂ analysis of the critical
 238 state Acq12.

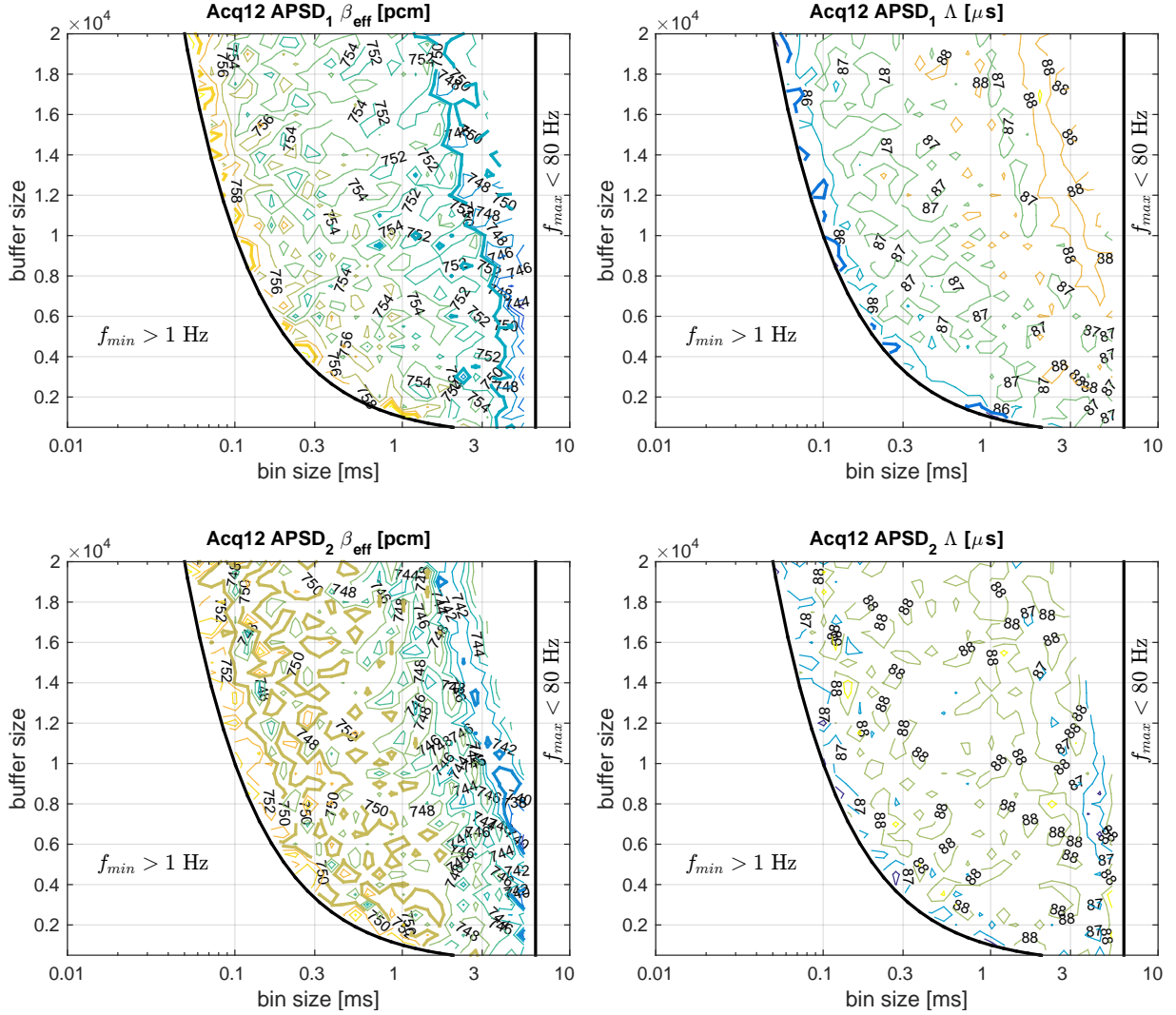


Figure 9: Sensitivity of the obtained kinetic parameters to the buffer size N and the time bin size dt over the relevant frequency range for the critical state Acq12 using APSD₁ (upper panels) and APSD₂ (lower panels) spectra.

239 Qualitatively, the sensitivity of the kinetic parameters to the numerical parameters obtained via APSD
 240 analyses for both subcritical states, Ac16 and Acq19, exhibit very similar behavior to the one showed in
 241 Fig. 9, although quantitatively the APSD analyses produce different results for β_{eff} and Λ . The results
 242 of both CPSD and APSD analysis of all three reactivity states, including the mean values and standard
 243 deviation are summarized in Table 3. It should be noted that the uncertainties presented in Table 3 under
 244 the “Current work” column are associated *only* with the numerical parameters dt and N disregarding any

245 other sources of uncertainty.

Table 3: Mean and standard deviation (1σ) of the kinetic parameters values over the relevant frequency range for both CPSD and APSD analysis of three reactivity states. The emphasized values are the mean and RMS of the different CPSD/APSD results.

Reactivity state	PSD method	Current work		Geslot et al. (2015)	
		β_{eff} [pcm]	Λ [μs]	β_{eff} [pcm]	Λ [μs]
Acq12	CPSD	756.7 ± 3.8	91.7 ± 3.6	746.8 ± 1.8	94.5 ± 0.7
	APSD ₁	753.2 ± 3.3	87.1 ± 0.8	750.4 ± 2.8	94.8 ± 1.3
	APSD ₂	748.0 ± 3.7	87.7 ± 0.6	749.1 ± 2.4	93.7 ± 1.1
		752.6 ± 3.6	88.8 ± 2.2	748.8 ± 2.4	94.3 ± 1.1
Acq16	CPSD	734.4 ± 4.4	91.6 ± 3.0	–	–
	APSD ₁	769.0 ± 4.9	93.4 ± 1.3	–	–
	APSD ₂	683.6 ± 4.4	86.8 ± 0.9	–	–
		729.0 ± 4.6	90.4 ± 2.0	–	–
Acq19	CPSD	715.4 ± 3.1	89.6 ± 3.2	–	–
	APSD ₁	724.5 ± 3.2	89.1 ± 1.0	–	–
	APSD ₂	694.3 ± 2.8	84.0 ± 0.9	–	–
		711.4 ± 3.0	87.6 ± 2.0	–	–

246 ^{EREZ}: The discrepancies of the results is two fold. First, the CPSD and the APSD results are in well
247 agreement for the critical state Acq12, and also agree well with the results obtained by Geslot et al. (2015) for
248 β_{eff} (but less for Λ). However, the discrepancies between the results associated separately with each detector,
249 i.e., APSD₁ and APSD₂, increase as the core becomes more subcritical. Generally, results obtained using
250 counts from detector 1 clearly exceed the results obtained using counts from detector 2 for both β_{eff} and Λ .
251 This disagreement was also observed by Gilad et al. (2016), where the subcritical states were analyzed using
252 a completely different method, i.e., the Feynman-Y method. Second, it seems that the subcriticality level
253 of the core during the experiment significantly influences the results and neither the CPSD nor the APSD
254 methods produce consistent results for the kinetic parameters by analyzing the different reactivity states.

255 ^{EREZ}: Several possible sources for the dispersion of the results from the two detectors comes to mind.
256 The detection efficiency is different between the two detectors, which lead to small discrepancies between
257 the statistical characteristics of their associated neutron counts. Although these discrepancies a small,

258 the fact that no dead-time correction was applied to any of the detectors' counts may increase the observed
259 inconsistency (the CPSD is somewhat less sensitive to dead-time corrections than APSD). Moreover, different
260 geometrical positions of the detectors may give rise to small spatial effects. Finally, inconsistencies in the
261 evaluation of the subcriticality levels of the different states (as suggested by Gilad et al. (2015)) or in the
262 evaluation of the integral fission rates can bear significant deviations in the obtained kinetic parameters.

263 ^{EREZ:} The dispersion of the results is important, real, and no obvious trend can be identified, which makes
264 the use of average results a bit unreliable. Having said that, a thorough analysis of these discrepancies is
265 beyond the scope of this paper, which focuses more on the numerical uncertainties associated with the PSD
266 techniques and less with the absolute values of the obtained kinetic parameters.

267 5. Conclusions

268 Power spectral density methods (Cohn- α methods) are well-known and widely used for the analysis
269 of neutron noise experiments and obtaining the reactor core integral kinetic parameters, i.e. the effective
270 delayed neutron fraction β_{eff} and the prompt neutron generation time Λ . The Cohn- α methods are considered
271 as the standard data processing procedure in the case of a current acquisition system that works at high
272 fission rates. These methods have proved to be very robust and produced minimum uncertainties.

273 Uncertainty analysis of Cohn- α techniques usually considers important source for uncertainty, e.g., sta-
274 tistical fluctuations in the neutron count, power drifts, uncertainties in the Diven factor, the integral fission
275 rate, and in the reactivity value. These uncertainties are then properly propagated through the Cohn- α
276 procedure in order to evaluate the total uncertainty in the obtained kinetic parameters. For example, the
277 analysis by Geslot et al. (2015) on the same data for the critical state using Power Spectral Density (PSD)
278 techniques leads to uncertainties of 1.8-2.8 pcm in the value of β_{eff} and 0.7-1.3 μs in Λ (at 1σ).

279 However, the uncertainty associated with the numerical parameters used in the power spectra calcu-
280 lation procedure, e.g., time bin size and buffer size, is hardly discussed in the literature and generally
281 overlooked, whereas these parameters are often determined rather arbitrarily according to the acquisition
282 system technical specifications. Moreover, well-defined criteria or methodologies for evaluating their associ-
283 ated uncertainties are not addressed.

284 In this paper, The Cohn- α method is implemented to analyze critical and subcritical configurations of
285 the MAESTRO core in the MINERVE zero power reactor in order to measure its integral kinetic parameters,
286 i.e. effective delayed neutron fraction β_{eff} and the prompt neutron generation time Λ .

287 The sensitivity of the obtained kinetic parameters to the choice of numerical parameters used for spectra
288 calculations is studied and found to be pronounced. Examination of this sensitivity (Fig. 4) reveals extremely
289 sensitive and erratic behavior of the fit results for small dt and a wide range of N values due to improper
290 frequency range for the PSD, i.e., the PSD does not contain the physically viable frequency range of the

291 zero power transfer function, which is estimated roughly to be between 1-80 Hz for the MAESTRO core
292 configuration.

293 This extremely sensitive and erratic behavior is eliminated once the numerical parameter space (dt, N) is
294 restricted to values which include the proper frequency range for the PSD. However, although the sensitivity
295 of the obtained kinetic parameters to the numerical parameters is reduced dramatically, it does not become
296 negligible and show pronounce changes over the (dt, N) space (Figs. 7, 8, 9).

297 **EREZ:** It should be noted that the choice to fit the Lorentzian curve using spectra in the range 1-80
298 Hz, although based on physical considerations, is rather arbitrary and this arbitrariness is inflicted on the
299 uncertainty. This frequency range should be set according to the form of the reactor's transfer function
300 and the signal-to-noise ratio. The sensitivity of the obtained kinetic parameters to the fitting range was
301 superficially examined by using the range 1-120 Hz for comparison, which yielded no significant differences.

302
303 **EREZ:** Essentially, the different spectra, which are derived from the same measured data (e.g., Acq12,
304 Acq16, or Acq19) using different sets of numerical parameters (dt, N) , encapsulate the same amount of
305 information. The different choices of numerical parameters simply distribute this information differently in
306 the spectra. Large buffers (large N) lead to finer resolution in the frequency domain but less statistics on
307 each point, whereas small buffers lead to coarser spectral resolution but better statistics on each point. This
308 trade-off affect the fit procedure, as nicely demonstrated by considering points A (small buffer) and C (large
309 buffer) in Figs. 5 and 6.

310 A novel methodology is proposed for analyzing the kinetic parameters' sensitivity to the power spectra
311 calculations and for quantifying the associated uncertainty. Since any point in the numerical parameter space
312 that satisfies the requirements for physically viable frequency range (Fig. 5) is adequate for Lorentzian fit,
313 the values of the kinetic parameters and the associated uncertainty are determined by the mean and standard
314 deviation of these parameters over the proper numerical parameter space. It should be noted that the fit
315 results exhibit rather smooth and robust behavior over the numerical parameter space.

316 The uncertainties originate from the sensitivity of the kinetic parameters to the numerical parameters
317 used for PSD calculation are summarized in Table 3. The uncertainty value for the critical state (Acq12)
318 in β_{eff} is 3.8 pcm for CPSD and ~ 3.5 pcm for APSD analyses, and in Λ is $3.6 \mu\text{s}$ for CPSD and $\sim 0.7 \mu\text{s}$ for
319 APSD analyses. These values are significant and non-negligible comparing to the corresponding 1.8-2.8 pcm
320 and 0.7 - $1.3 \mu\text{s}$ uncertainty values calculated by Geslot et al. (2015), where the PSD spectra were calculated
321 at a single point in the (dt, N) space, i.e., $dt = 1$ ms and $N = 2000$.

322 **EREZ:** The discrepancies between the results associated separately with each detector increase as the
323 core becomes more subcritical and results obtained using counts from detector 1 clearly exceed the results
324 obtained using counts from detector 2 for both β_{eff} and Λ . This disagreement was also observed by Gilad
325 et al. (2016), where the subcritical states were analyzed using the Feynman-Y method, which is different

326 from PSD methods in that it is not based on the reactor transfer function. Several possible sources for the
327 dispersion of the results from the two detectors are discussed in section 4.5, e.g. the absence of dead-time
328 correction, spatial effects, inconsistencies in the evaluation of the subcriticality levels and in the evaluation
329 of the integral fission rates. Although the dispersion of the results is important and real, a thorough analysis
330 of these discrepancies is beyond the scope of this paper.

331 We conclude by stating that the uncertainties in the kinetic parameters (β_{eff} and Λ) calculated using
332 Cohn- α methods, which are associated with the numerical parameters time bin size and buffer size used for
333 spectra calculations, are significant and should not be neglected.

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