Towards an innovative R_$ij-\epsilon$ model for turbulence in bubbly flows from DNS simulations

A. Du Cluzeau, G. Bois, Adrien Toutant, J-M. Martinez

DE LA RECHERCHE À L'INDUSTRIE

5th International Conference on Turbulence and Interactions

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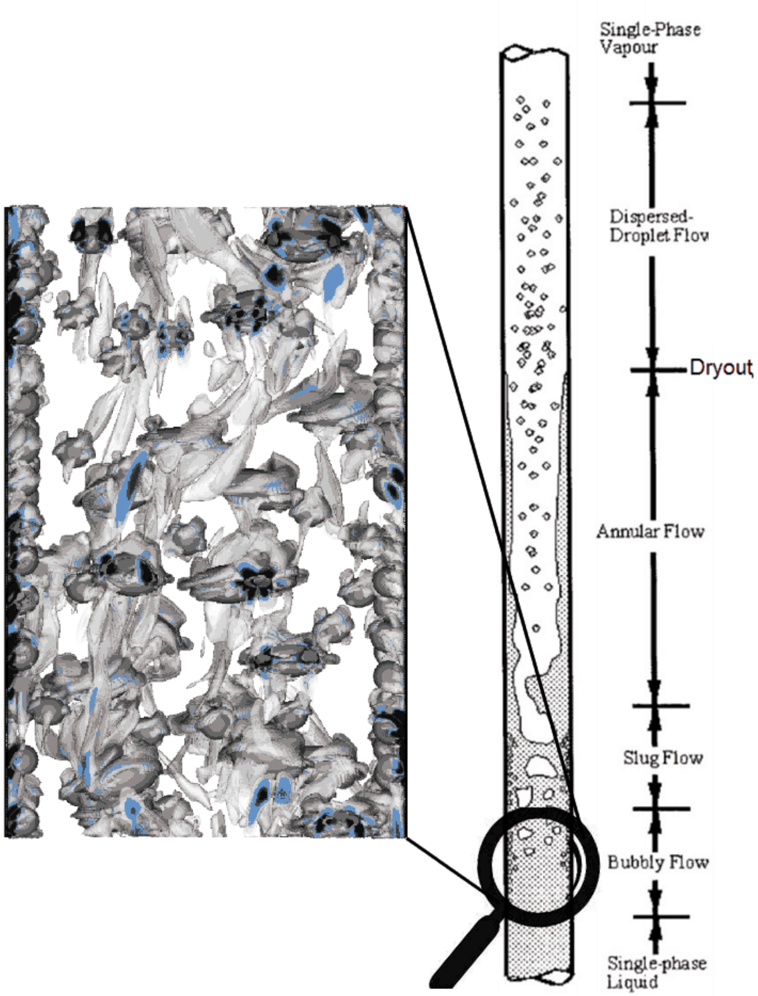
Industrial purposes and boiling flows

Safety issue for PWR : the boiling crisis

- Pressurized water reactor : a very large system
- Critical heat flux : a very local phenomenom
- Solution : the upscaling strategy

Lack of predictive model in two phase flows

- A lot of different regimes
- The importance of the transition between boiling and slug flows
- Bubbles at the wall $\rightarrow$ decrease of heat transfer
- Importance of the void fraction prediction
- Focus on dynamical aspects in bubbly flows



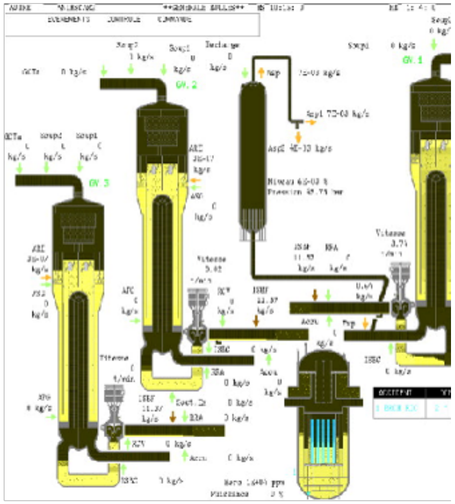
Up-scaling strategy

Towards a numerical reactor

- Need for powerful simulations

TRUST		NEPTUNE		CATHARE
DNS	$\Longrightarrow$	M-CFD	$\Longrightarrow$	System

- How can complex two-phase flows be solved in a system code ?



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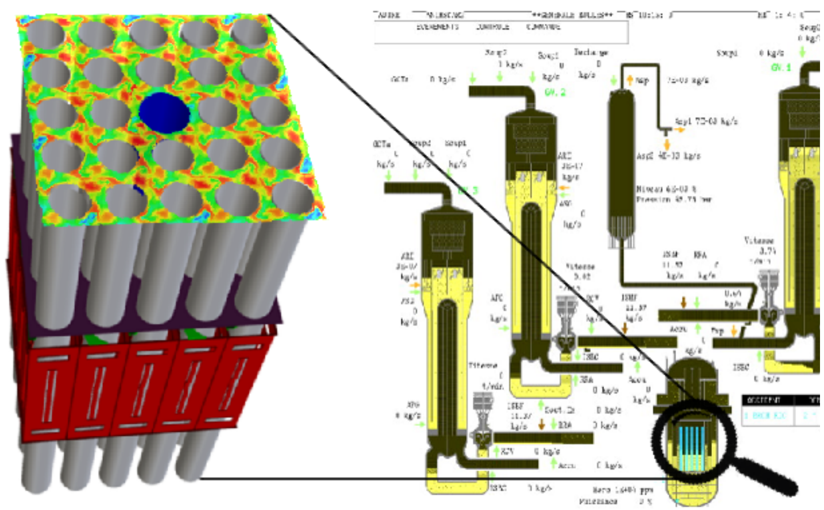

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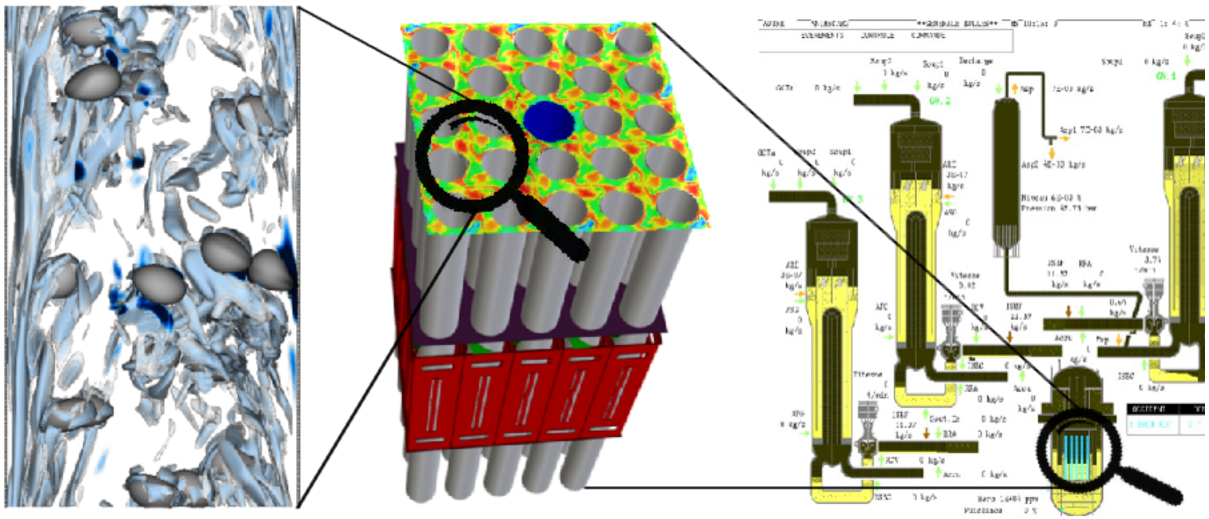
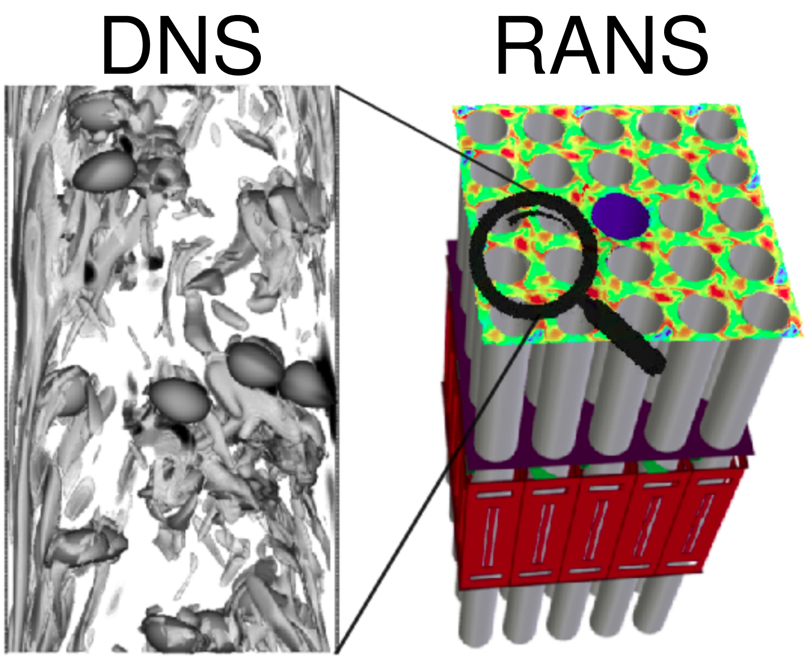


Failures and needs of the averaged scale (RANS)

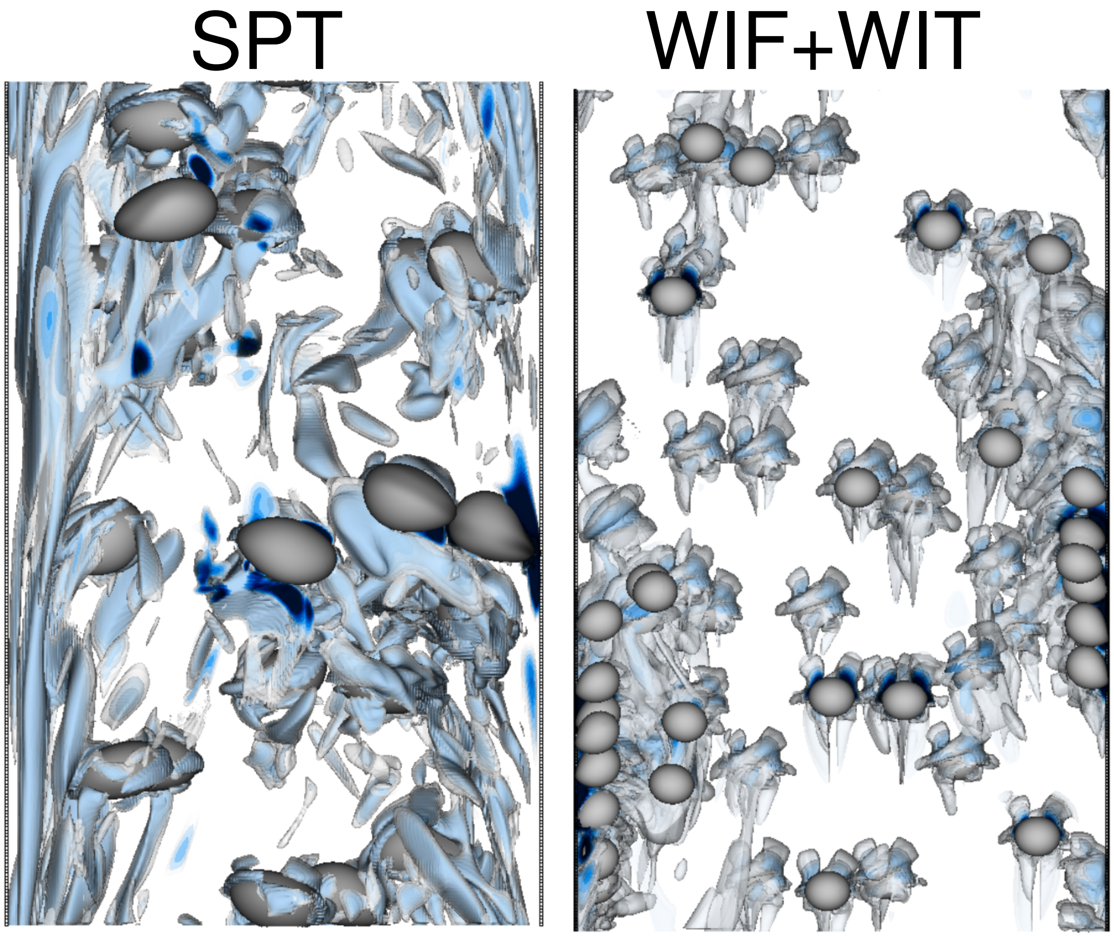


RANS closures limitations

- Empirical correlations built on coupled mechanism simultaneously
  → Interfacial forces :
  - Drag force
  - Lift force
  - Added-mass force
  - Dispersion forces
- Point-size particle approach
  - No surface tension effects
  - One pressure closure
- Theoretical modelling based on questionable parallel with single phase flows

→ Turbulence closure
- Single-phase turbulence
- Wake structure
- Wake interactions and instabilities

Two different kinds of turbulence

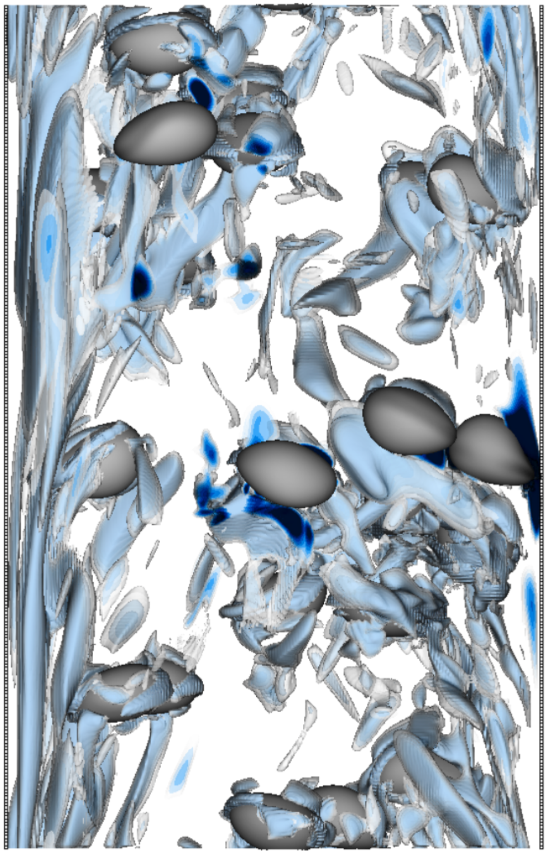


Flow led by the upward pressure gradient

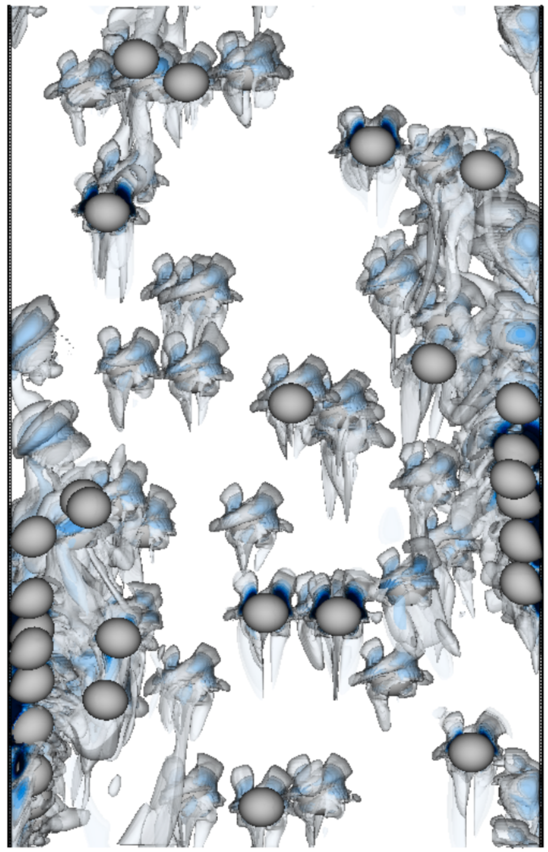
- Classical turbulence with elongated streaks
- Single Phase Turbulence (SPT) due to shear stresses at the wall
- Sufficient existing model

Two different kinds of turbulence

SPT



WIF+WIT



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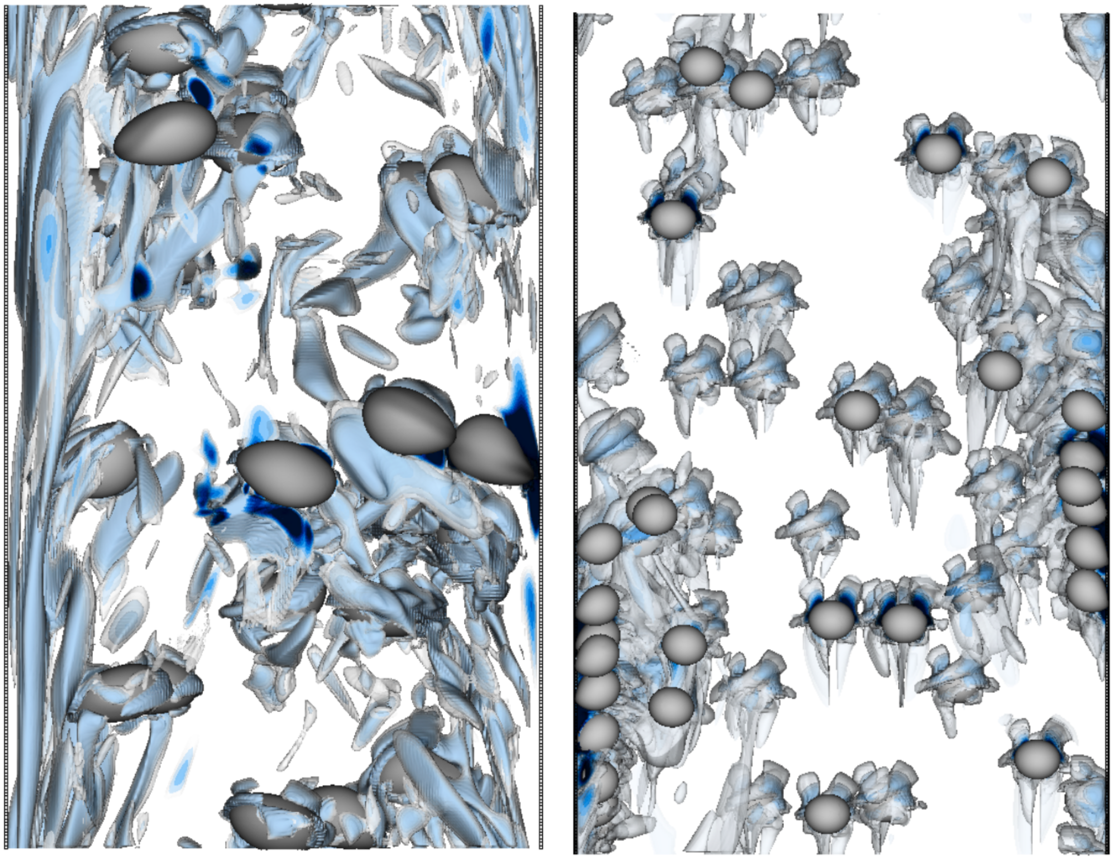
Flow led by bubbles buoyancy (WIF+WIT)

- Potential flow and averaged Wake Induced Fluctuations (WIF)
- Wake Induced Turbulence due to interactions and instabilities of wakes (WIT)
- Unsatisfactory models

Two different kinds of turbulence

SPT | WIF+WIT



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Modelling conclusion : Bubble turbulence cannot be modelled as a kinetic energy source term

Overview

PROMES

Introduction

Direct Numerical Simulations

- Governing equations
- Limitation of DNS
- Numerical setup

Results and modelling on swarm calculations

Local scale (DNS) / Averaged scale (RANS)

Local instantaneous description

- Navier-Stokes equations :

$$\frac{D\rho_k \mathbf{u}_k}{Dt} = -\nabla P_k + \rho_k \mathbf{g} + \nabla . \underbrace{\left[\mu_k \left(\nabla \mathbf{u_k} + \nabla^T \mathbf{u_k}\right)\right]}_{\tau_k}$$

- Interfacial jump conditions :

$$\mathbf{u_l^i} = \mathbf{u_v^i}, \qquad \sum_k \left(p_k \mathbf{n_k} - \tau_k . \mathbf{n_k}\right) = \sigma \kappa \mathbf{n_k}$$

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Extension to full space

- Multiply by phase indicator function :
  $\chi_k = 1$ in phase k, 0 otherwise.

$$\frac{D\chi_k \rho_k \mathbf{u_k}}{Dt} = -\nabla \left(\chi_k P_k\right) + \chi_k \rho_k \mathbf{g} + \nabla . \left(\chi_k \tau_k\right) - \left(p_k \mathbf{n_k} - \tau_\mathbf{k} . \mathbf{n_k}\right) . \nabla \chi_k$$

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One-fluid formulation

- 'one-fluid' variables : $\phi = \sum_k \chi_k \phi_k$

$$\frac{D\rho\mathbf{u}}{Dt} = -\nabla P + \rho\mathbf{g} + \nabla\left[.\mu\left(\nabla\mathbf{u} + \nabla^T\mathbf{u}\right)\right] + \sigma\kappa\mathbf{n}\delta^i$$

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Definition of averaging operators

- Statistical average = temporal average :

$$\overline{\phi\left(x, y, z\right)}^{TX} = \frac{1}{\Delta t} \int_{t-\Delta t/2}^{t+\Delta t/2} \phi\left(x, y, z, \tau\right) d\tau$$

- Phase average :

$$\overline{\phi}^k = \frac{\overline{\chi_k \phi_k}^{TX}}{\overline{\chi_k}^{TX}}$$

Local scale (DNS) / Averaged scale (RANS)

Local instantaneous description

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- Phase average :

$$\overline{\phi}^k = \frac{\overline{\chi_k\phi_k}^{TX}}{\overline{\chi_k}^{TX}}$$

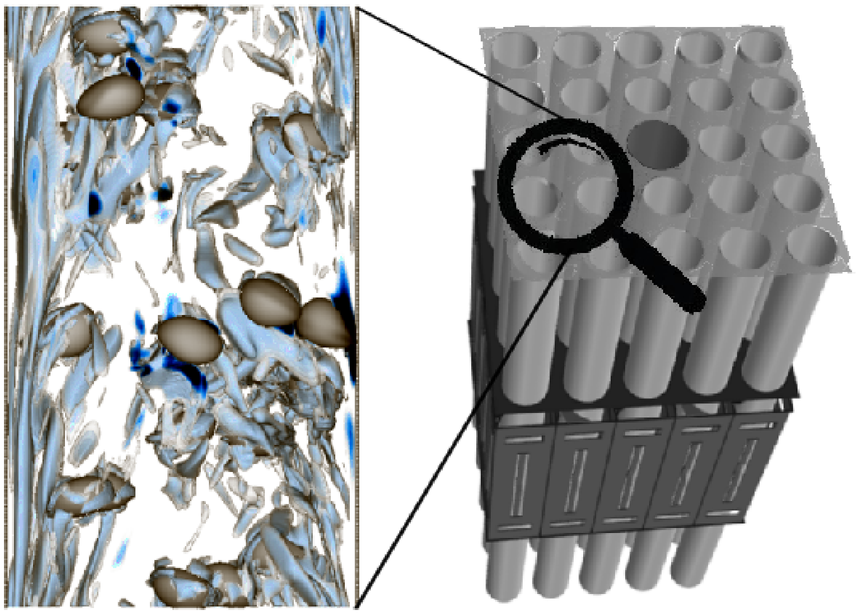
Two-fluid averaged formulation

- Note $\alpha_k = \overline{\chi_k}^{TX}$

$$\frac{D\alpha_k\rho_k\overline{\mathbf{u_k}}^k}{Dt} = -\nabla\left(\alpha_k\overline{P_k}^k\right) + \alpha_k\rho_k\mathbf{g} + \nabla.\left(\alpha_k\overline{\tau_k}^k\right) + \nabla.\left(\alpha_k\overline{\mathfrak{R}_k}^k\right) \underbrace{-\overline{\left(p_k\mathbf{n_k} - \tau_\mathbf{k}.\mathbf{n_k}\right).\nabla\chi_k}^{TX}}_{\mathbf{M_k}}$$

- $\mathbf{M_k}$ is the interfacial force exerted on phase k

$$\mathbf{M_l} + \mathbf{M_v} = \overline{\sigma\kappa\mathbf{n}\delta^i}^{TX}$$



Framework hypotheses

- incompressible
- isothermal

Limitation of DNS

- Low void fraction → to avoid coalescence (No contact line models)
- Limited Reynolds number → moderate resolution of the mesh
  → Unreachable industrial conditions for bubble columns
  → Reachable industrial conditions for bubble swarms
- Models extracted from the DNS need a posteriori validation on experiments in reactor conditions

Numerical setup

Physical setup

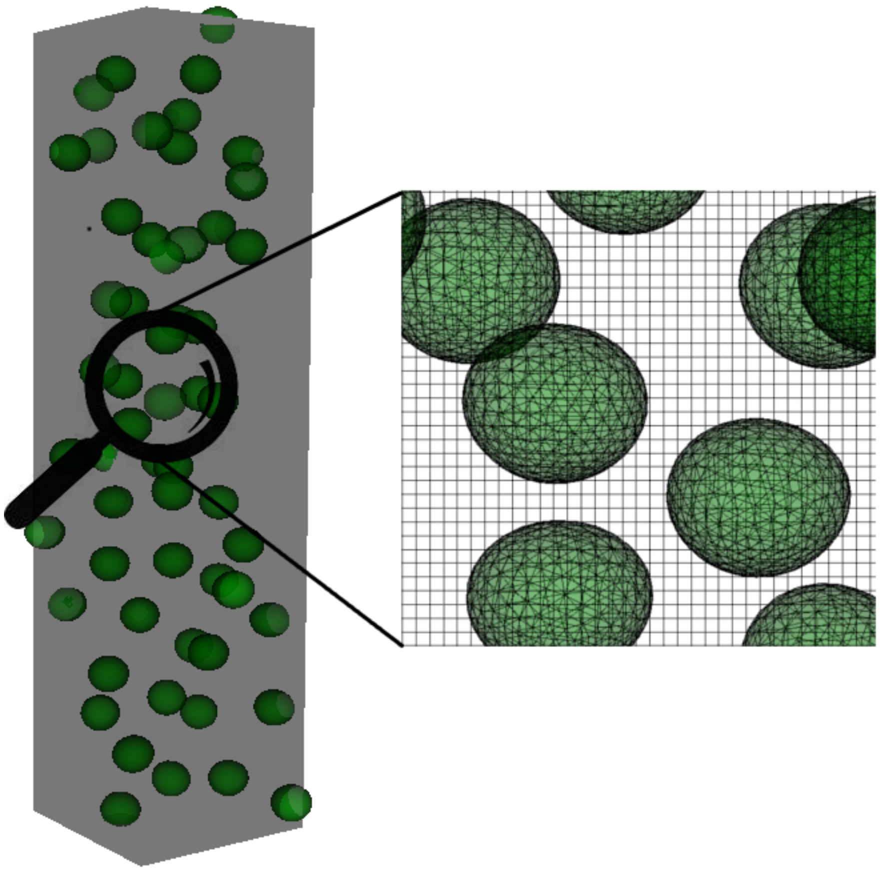
- Bubble swarm in a flow initially at rest
- Fixed bubble simulation $\rightarrow$ Spring force
- periodicity in x,y,z
- Different predefined population of bubbles
- Computation in typical reactor condition based on DEBORA experiments (freon 24 bar)
- $Re_b = 1176$ and $Eo = 0.59$

A Front-Tracking algorithm

- One-fluid equations are solved on an Eulerian / Cartesian mesh
- Interfaces are tracked on a Lagrangian mesh
- Validated by comparison with works of J.Lu and G.Tryggvason

HPC calculations

- $\rightarrow \approx$ 80 millions of cells
- $\rightarrow \approx$ 1.000.000 h CPU a case



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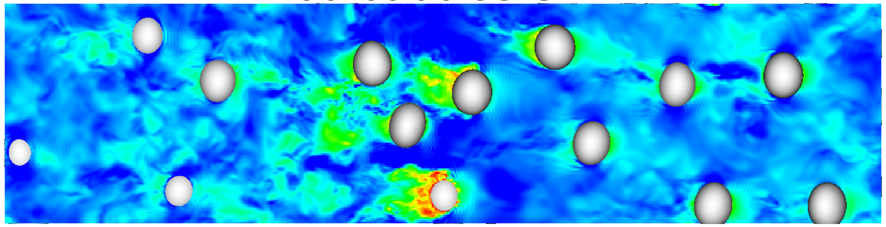
Results and modelling on swarm calculations

- Turbulent and non turbulent fluctuations
- Spectral analysis
- New turbulence modelling

Instantaneous velocity field from DNS with fixed bubbles *SPT*

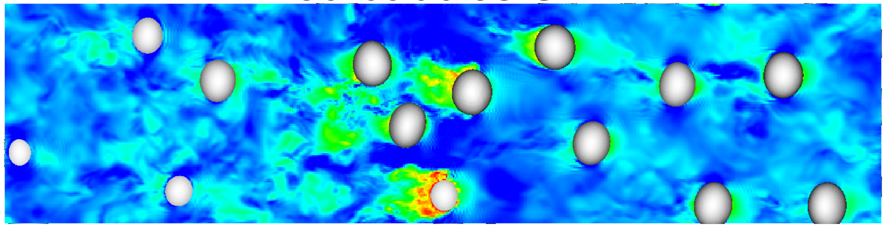


Following F. Risso et al.

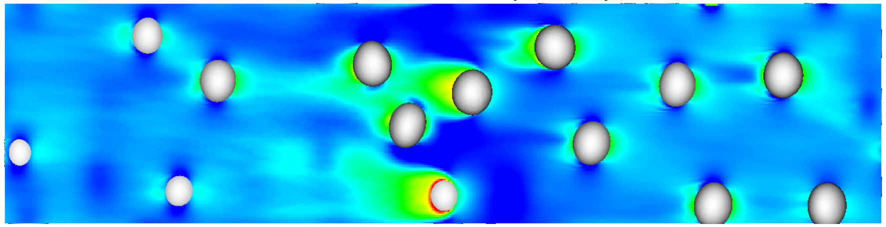
- Frame of reference of bubbles
  - → Fixed bubble simulations
  - → Spring force
- Total fluctuations

Turbulent and non turbulent fluctuations

Instantaneous velocity field from DNS with fixed bubbles ~~*SPT*~~



Temporal average → Wake-induced fluctuations (WIF)



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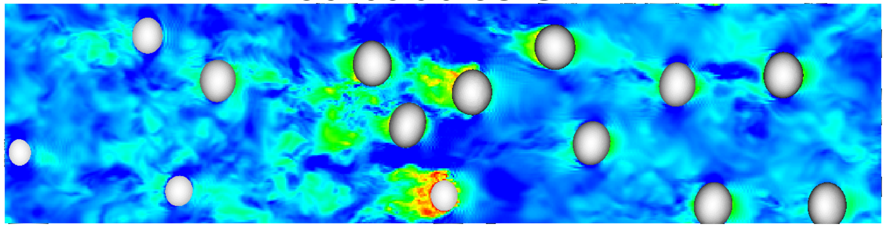
Wake Induced Fluctuations (WIF)

- Temporal average → spatial fluctuations
- Due to the averaged wake
- Due to potential flow around bubbles
- It is not turbulence !

Turbulent and non turbulent fluctuations

Instantaneous velocity field from DNS with fixed bubbles ~~SPT~~



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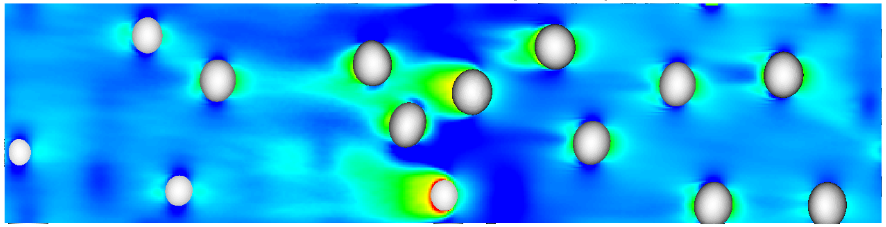
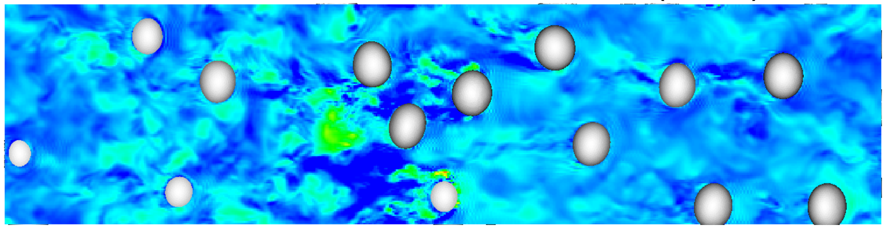


Wake-induced Turbulence (WIT)



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Wake Induced Fluctuations (WIF)

- Temporal average → spatial fluctuations
- Due to the averaged wake
- Due to potential flow around bubbles
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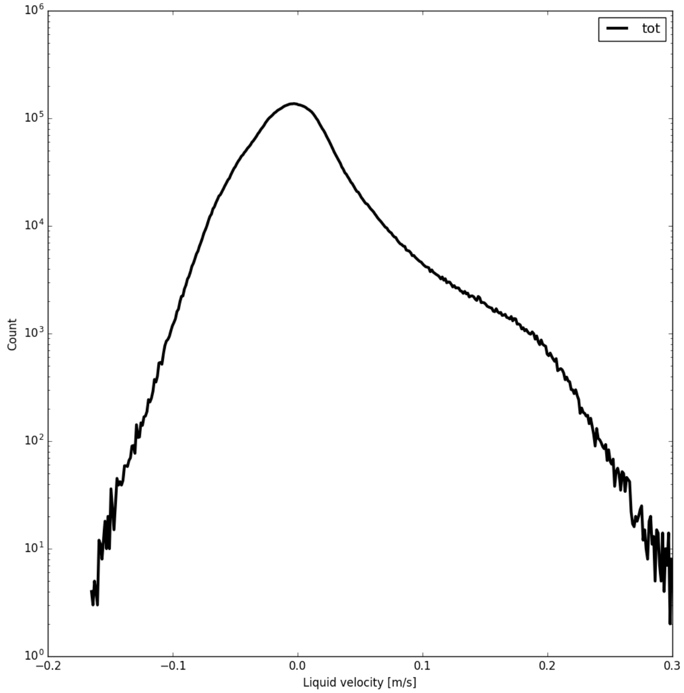
Wake Induced Turbulence (WIT)

- Temporal fluctuations
- Interactions and instabilities of wakes
- Is WIT similar to classical turbulence ?

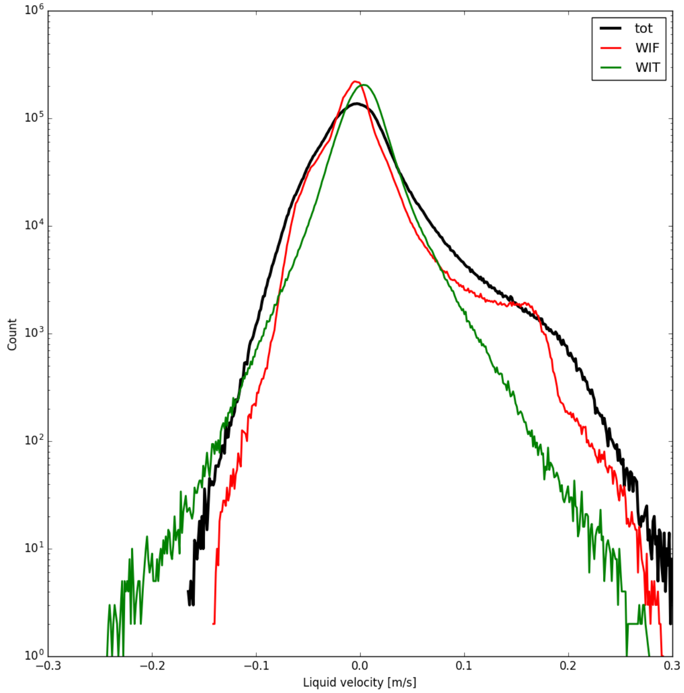
Modelling conclusion : WIF and WIT have to be modelled separately

Spectral analysis of WIT and WIF



PDF of velocity fluctuations
(On $\approx$ 8 million points)

Spectral analysis of WIT and WIF



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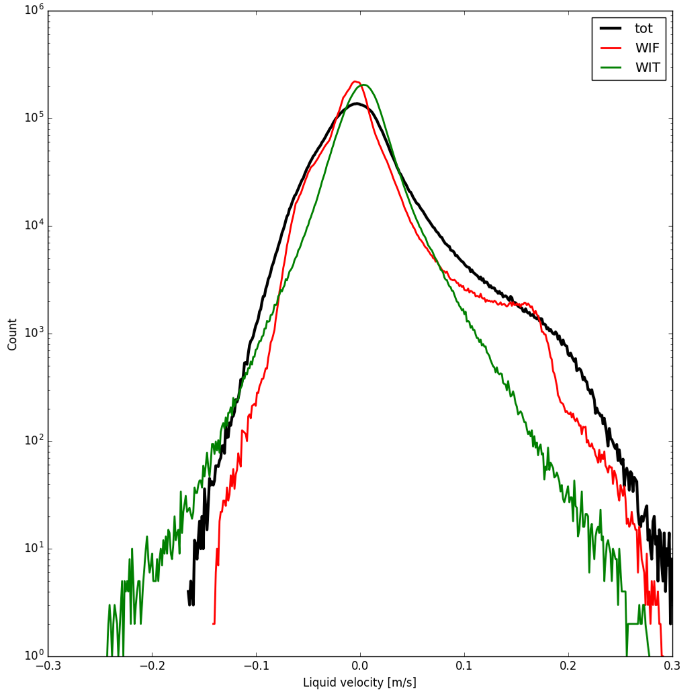
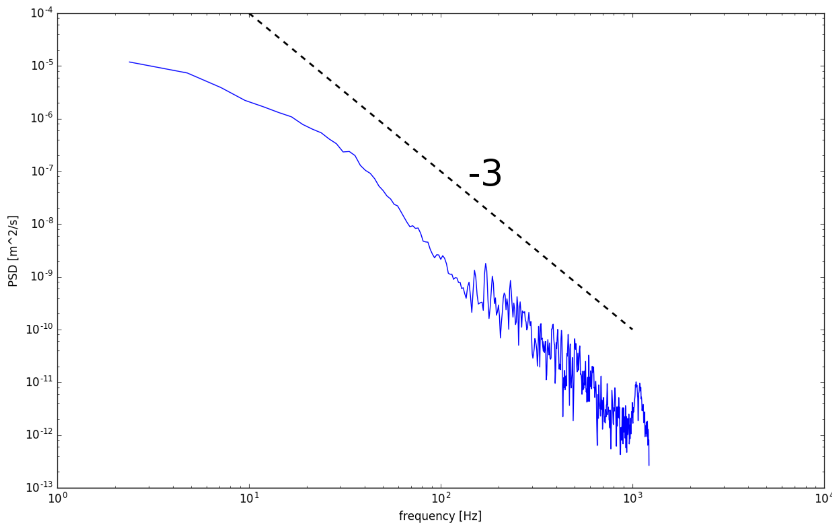


PDF of velocity fluctuations
(On ≈ 8 million points)

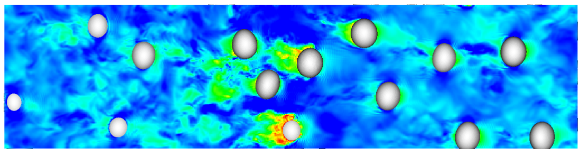


Mean temporal spectrum
(on ≈ 120 probes)

Modelling conclusion : WIT needs its own transport equation because its statistical signature is different than SPT

Splitting of the transport equation

Splitting of the fluctuations



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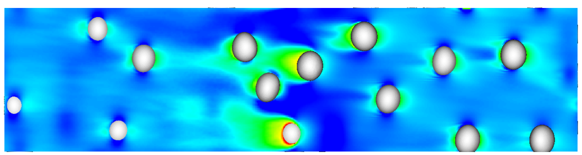
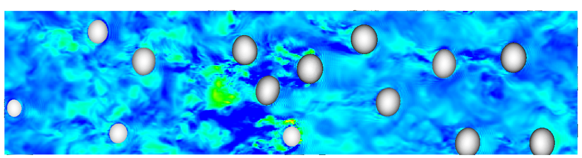


$$+$$



An innovative splitting of the R_{ij} transport equation

$$\frac{D\mathbf{R_{ij}}}{Dt} = -\mathbf{P_{ij}} + \mathbf{D_{ij}} + \phi_{\mathbf{ij}} + \epsilon_{\mathbf{ij}}$$

$$=$$

$$\frac{D\mathbf{R_{ij}^{WIF}}}{Dt} = -\mathbf{P_{ij}^{X}} + \mathbf{D_{ij}^{X}} + \phi_{\mathbf{ij}}^{\mathbf{X}} + \epsilon_{\mathbf{ij}}^{\mathbf{X}}$$

$$+$$

$$\frac{D\mathbf{R_{ij}^{WIT}}}{Dt} = -\mathbf{P_{ij}^{T}} + \mathbf{D_{ij}^{T}} + \phi_{\mathbf{ij}}^{\mathbf{T}} + \epsilon_{\mathbf{ij}}^{\mathbf{T}}$$

Exemple with the dissipation tensor

$$\epsilon_{\mathbf{11}} = -2\frac{\mu_l}{\rho_l}\overline{\chi_l\frac{\partial u_l}{\partial x_b}\chi_l\frac{\partial u_l}{\partial x_b}}^{TX} \quad (100\%)$$

$$=$$

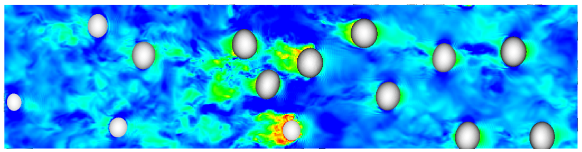
$$\epsilon_{\mathbf{11}}^{\mathbf{X}} = -2\frac{\mu_l}{\rho_l}\overline{\overline{\chi_l\frac{\partial u_l}{\partial x_b}}^{T}\overline{\chi_l\frac{\partial u_l}{\partial x_b}}^{T}}^{X} \quad (23\%)$$

$$+$$

$$\epsilon_{\mathbf{11}}^{\mathbf{T}} = -2\frac{\mu_l}{\rho_l}\overline{\chi_l\frac{\partial u_l'}{\partial x_b}\chi_l\frac{\partial u_l'}{\partial x_b}}^{TX} \quad (77\%)$$

Splitting of the transport equation

Splitting of the fluctuations



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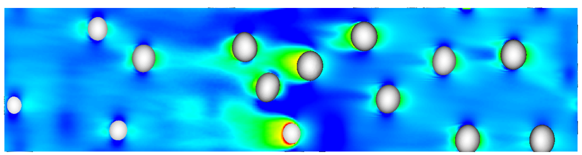

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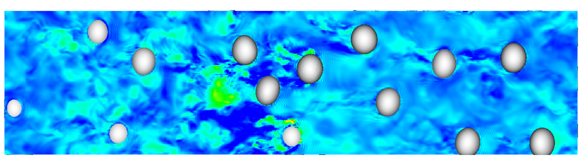

An innovative splitting of the R_{ij} transport equation

$$\frac{DR_{ij}}{Dt} = -P_{ij} + D_{ij} + \phi_{ij} + \epsilon_{ij}$$

$$=$$

$$\frac{DR^{WIF}_{ij}}{Dt} = -P^{X}_{ij} + D^{X}_{ij} + \phi^{X}_{ij} + \epsilon^{X}_{ij}$$

$$+$$

$$\frac{DR^{WIT}_{ij}}{Dt} = -P^{T}_{ij} + D^{T}_{ij} + \phi^{T}_{ij} + \epsilon^{T}_{ij}$$

Exemple with the dissipation tensor

$$\epsilon_{11} = -2\frac{\mu_l}{\rho_l}\overline{\chi_l \frac{\partial u_l}{\partial x_b} \chi_l \frac{\partial u_l}{\partial x_b}}^{TX} \quad (100\%)$$

$$=$$

$$\epsilon^{X}_{11} = -2\frac{\mu_l}{\rho_l}\overline{\overline{\chi_l \frac{\partial u_l}{\partial x_b}}^{T} \overline{\chi_l \frac{\partial u_l}{\partial x_b}}^{T}}^{X} \quad (23\%)$$

$$+$$

$$\epsilon^{T}_{11} = -2\frac{\mu_l}{\rho_l}\overline{\chi_l \frac{\partial u'_l}{\partial x_b} \chi_l \frac{\partial u'_l}{\partial x_b}}^{TX} \quad (77\%)$$

With DNS → post-treatment of all the statistical correlations in order to find scaling laws for the interfacial production, the dissipation etc...

Modelling conclusion :

1 - BIF cannot be modelled as a kinetic energy source term
2 - WIF and WIT have to be modelled separately
3 - WIT needs its own transport equation because its statistical signature is different than SPT

New Turbulence model

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Formal summary

$$\mathbf{R_{ij}} = \mathbf{R_{ij}^{SPT}} + \mathbf{R_{ij}^{WIT}} + \mathbf{R_{ij}^{WIF}}$$

$$\frac{D\mathbf{R_{ij}^{SPT}}}{Dt} = \textbf{single-phase transport equation}$$

$$\frac{D\mathbf{R_{ij}^{WIT}}}{Dt} = -\mathbf{P_{ij}^{WIT}} + \mathbf{D_{ij}^{WIT}} + \phi_{\mathbf{ij}}^{\mathbf{WIT}} + \epsilon_{\mathbf{ij}}^{\mathbf{WIT}}$$

$$\mathbf{R_{ij}^{WIF}} = \textbf{algebraic closure}$$

- Sufficient existing models for $\mathbf{R_{ij}^{SPT}}$

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3 - WIT needs its own transport equation because its statistical signature is different than SPT

Formal summary

$$\mathbf{R_{ij}} = \mathbf{R_{ij}^{SPT}} + \mathbf{R_{ij}^{WIT}} + \mathbf{R_{ij}^{WIF}}$$

$$\frac{D\mathbf{R_{ij}^{SPT}}}{Dt} = \textbf{single-phase transport equation}$$

$$\frac{D\mathbf{R_{ij}^{WIT}}}{Dt} = -\mathbf{P_{ij}^{WIT}} + \mathbf{D_{ij}^{WIT}} + \phi_{\mathbf{ij}}^{\mathbf{WIT}} + \epsilon_{\mathbf{ij}}^{\mathbf{WIT}}$$

$$\mathbf{R_{ij}^{WIF}} = \textbf{algebraic closure}$$

- Sufficient existing models for $\mathbf{R_{ij}^{SPT}}$

On going work

- Computation of bubble swarms at different void fraction and bubble Reynolds number
- Model for $\mathbf{R_{ij}^{WIF}}$ based on previous works (F.Risso and al.) on averaged wake and potential flow solution around bubbles
- Post-treatment of all the statistical correlations of the $\mathbf{R_{ij}^{WIT}}$ transport equation in order to find scaling laws for $\mathbf{P_{ij}^{WIT}}$, $\phi_{\mathbf{ij}}^{\mathbf{WIT}}$ and $\epsilon_{\mathbf{ij}}^{\mathbf{WIT}}$

Conclusion and prospect

Up-scaling process from DNS database

- Answer to complex industrial applications (PWR core evolution)
  → RANS Euler calculations need predictive model
  → DNS used as numerical experiments
- DNS of channel bubbly flows (not presented here)
  → Lots of data available for processing
  → Already used to model interfacial forces
- DNS of swarms with fixed bubbles for the study of bubble-induced turbulence
  → (prospect) Characterise the impact of fixed bubbles
  → (prospect) Comparison with calculation of free bubbles
  → (prospect) Parametric analysis with different Reynolds number and different void fraction

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Turbulence modelling

- Bubble-induced turbulence is different than single-phase turbulence
  → Bubble-induced turbulence has a part of turbulent fluctuation (WIT) and a part of non turbulent fluctuations (WIF)
  → WIF is related to the averaged wake and to the potential flow around bubble
  → WIT is related to wakes interactions and instabilities
- Those three fluctuations have to be modelled separately
  → SPT : classical transport equation
  →(prospect) WIT : new transport equation modelling
  →(prospect) WIF : algebraic closure

Thank you for your attention !

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