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1 **An investigation of infrasound propagation over mountain ranges**

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## Abstract

8        Linear theory is used to analyze trapping of infrasound within the lower tropospheric  
9 waveguide during propagation above a mountain range. Atmospheric flow produced by the  
10 mountains is predicted by a nonlinear mountain gravity wave model. For the infrasound  
11 component, we solve the wave equation under the effective sound speed approximation using  
12 both a finite difference method and a WKB approach. It is shown that in realistic con-  
13 figurations, the mountain waves can deeply perturb the low level waveguide, which leads  
14 to significant acoustic dispersion. To interpret these results each acoustic mode is tracked  
15 separately as the horizontal distance increases. It is shown that during statically stable sit-  
16 uations, situations that are common during night over land in winter, the mountain waves  
17 induce a strong Foehn effect downstream, which shrinks the waveguide significantly. This  
18 yields a new form of infrasound absorption, that can largely outweigh the direct effect the  
19 mountain induces on the low level waveguide. For the opposite case, when the low level flow  
20 is less statically stable (situations that are more common during day in summer), moun-  
21 tain wave dynamics do not produce dramatic responses downstream. It may even favor the  
22 passage of infrasound and mitigate the direct effect of the obstacle.

## **I. Introduction**

Infrasound, which is defined as sound waves that are lower in frequency than 20 Hz, is characterized by an ability to travel over long horizontal distances in the atmosphere. This is related to the fact that the wind and temperature strongly vary with altitude, providing multiple ducts in which infrasound can propagate efficiently<sup>20</sup>. Although an important duct is potentially in the lower thermosphere, as a result of the steep increase in temperature (e.g. above 90 km), the decrease in mean density produces substantial absorption coefficients there<sup>42</sup>. For this reason, the most efficient ducts are often within the middle atmosphere, e.g. above the tropopause at around 20 km and below the mesopause at 90 km. At lower altitudes, infrasound can also be trapped within tropospheric waveguides over distances that may reach several hundred of kilometers, at least when the weather conditions permit<sup>43;31</sup>. However in this case, the wave interacts with the ground surface much more than wave refracting higher in the atmosphere and topographic features produce quantifiable effects on the recorded data<sup>30</sup>. Furthermore, at these altitudes the absorption coefficient is small and thus, it is neglected in most practical applications.

Although much less studied, the propagation of infrasound over distances of a few tens of kilometers can be controlled by a planetary boundary layer duct<sup>11;45</sup>, which is a region of approximately 1 km depth in which the boundary effects are reflected in the flow<sup>16</sup>. For these relatively short propagation ranges, the upward refraction at higher altitudes (around

42 and above the tropopause) can be ignored<sup>45</sup> and the acoustic field can be described by a  
43 modal expansion involving a few modes. For completeness, it is important to note that  
44 the absorption properties of ground play a significant role<sup>4</sup>, in the sense that vegetation-  
45 covered land absorbs more energy than bare-ground for instance. We know that some modes  
46 are sensitive to such absorptions<sup>44;47</sup> but we will not include these effects here, essentially  
47 because a comprehensive theory of acoustic propagation which accounts for both absorption  
48 by vegetation and turbulence is lacking.

49 The common approach to calculate infrasound propagation in the atmosphere consists  
50 in solving the acoustic equation in a given background atmospheric state that varies with  
51 altitude and horizontal distance. This approach captures the most significant ducts, but  
52 sometimes it fails in predicting important arrivals<sup>17;3;34</sup>. The reason is that the atmospheric  
53 specifications, that are issued from operational numerical weather forecasts (e.g., provided  
54 by the ECMWF Integrated Forecast System or the NOAA Global Forecast System) or atmo-  
55 spheric climate reanalysis (e.g., ERA-Interim or NASA Modern Era Retrospective Reanalysis  
56 for Research and Applications), are associated with spatial resolutions that are much larger  
57 than the typical infrasound wavelength. These products therefore fail in representing im-  
58 portant small-scale atmospheric fluctuations that can substantially modify the larger-scale  
59 ducts<sup>6</sup>, especially for borderline ducts that barely return sound to the ground. The statistics  
60 of these fluctuations, however, are poorly understood, whereas their knowledge is required

61 for infrasound propagation modeling. For instance, Chunchuzov *et al.*<sup>12</sup> have shown the need  
62 to introduce random atmospheric perturbations to adequately represent the acoustic prop-  
63 erties of the boundary layer, but in their work the sources of perturbations are not specified.  
64 As in the troposphere the (unresolved) fluctuations are mainly produced by mountains<sup>18</sup>,  
65 the contribution of these mountains to infrasound propagation remains an important open  
66 question.

67 In a first attempt to incorporate topography effects in acoustic propagation, high-  
68 resolution terrain models have been used to represent the lower boundary by a sequence  
69 of up and down stair steps<sup>2;30</sup>. In this approach, mountains directly modify the altitude  
70 of the lower boundary of the troposphere, which affects the acoustic cut-off frequencies of  
71 the corresponding ducts. This can be viewed as applying a “mask” onto the atmospheric  
72 specifications, and ignoring the direct influence of the mountain ridges on the local wind  
73 and temperature fields. This is an extremely serious limitation, given that mountains can  
74 dynamically produce very intense phenomena, like downslope winds, Foehn, or trapped  
75 lee waves<sup>40;15</sup>. As an illustration, it is worth mentioning that even small “mountains”,  
76 with elevations of a several hundred meters, can develop substantial winds and temperature  
77 disturbances, depending on the incoming flow structure<sup>15</sup>.

78 There are two primary objectives in the present investigation. The first is to compare  
79 the results of the “mask” approximation to that obtained with a wind model that captures

80 the interaction between the topography and the boundary layer. The second objective is  
81 to examine the physical mechanisms that cause a low-level acoustic duct to be affected  
82 and eventually destroyed by mountain-induced disturbances. Here we use the mountain  
83 flow model described by Lott<sup>29</sup>. With respect to our first objective, this model involves a  
84 nonlinear boundary condition, *i.e.* it includes an obstacle that penetrates inside the low level  
85 waveguide and reduces its depth, an effect that potentially recovers the classical “mask”  
86 technique. It is worth while to point out that the model also predicts a mountain wave  
87 field, which compares in amplitude to the background winds and temperature variations  
88 responsible for the waveguide. This inherently affects the trapped acoustic modes, yielding  
89 highly dispersed signals as well as irreversible absorption of the acoustic wave passing over  
90 the ridge.

91 The paper is organized as follows. In section II, the mountain wave model is described  
92 and the dominant features of the mountain wave field are discussed in terms of dimensionless  
93 Richardson and Froude numbers. The effect of mountain wave disturbances on the acoustic  
94 field  $p(x, z)$  is then considered in section III, using a classical range-dependent normal mode  
95 approach<sup>24</sup> to account for flow changes along the source-receiver distance  $x$ . To make the  
96 absorptive properties more transparent, the acoustic modes are also obtained using a WKB  
97 approximation. In section IV, it is found that the interaction between the mountain flow  
98 and the acoustic field gives rise to attenuation or amplification of ground-based signals,

99 depending on the statical stability of the boundary layer flow. The characteristics of the  
100 perturbed acoustic modes such as phase velocities, attenuations, and wave structures in  
101 the  $(x, z)$ -plane are provided and discussed. In section V, the downstream attenuation is  
102 systematically evaluated in terms of dimensionless numbers that control the mountain flow  
103 dynamics. Importantly, it is found that in near-neutral conditions, the mountain wave  
104 dynamics can favor infrasound propagation above the mountain, mitigating the direct effect  
105 of the obstacle.

## 106 **II. Atmospheric mountain flow model**

### 107 **A. Formalism**

108 Mountain waves, that occur when a stably stratified flow is forced by an obstacle, are  
109 often standing or nearly so, at least to the extent that the upstream environmental conditions  
110 are stationary. They can accompany Foehn wind conditions that are characterized by warm  
111 and dry downslope winds on the lee side of mountains<sup>37</sup>. In the present study, we use  
112 the mountain wave model developed by Lott<sup>29</sup>, which is adapted from Long<sup>27</sup>'s model to  
113 incoming shear flows that varies with altitude. Comparisons with nonlinear simulations<sup>13</sup>  
114 demonstrated that this model is well-suited for capturing realistic features of mountain flow  
115 dynamics.

116 In the present study, the mean state consists of an isothermal atmosphere, at temper-



117 ature  $T_0$ , in the presence of a background wind  $U(z)$  which is assumed to be in the shape of  
118 an hyperbolic tangent function. This representation is appropriate to describe the planetary  
119 boundary layers<sup>8;10</sup> and can even be used to initialize mesoscale models<sup>35</sup>. Although such  
120 a profile can occur during strong stratification or above the lowest maximum of the wind  
121 speed<sup>32</sup>, there are many other semi-empirical models that adequately describe the wind  
122 shear. Here, the profile is used to mimic the incoming boundary layer, so that stationary  
123 gravity waves can be generated through interaction with the mountain, as observational  
124 evidence<sup>9;41</sup> suggest. Specifically, the mean flow is given by

$$T(z) = T_0, \quad U(z) = U_0 \tanh(z/\delta), \quad (1)$$

125 where  $\delta$  is the boundary layer thickness,  $U_0$  denotes the maximum wind speed over the  
126 mountain, and  $z$  is the height, which is here typically smaller than 5 km. The thermodynamic  
127 sound speed  $c_0$  is given by  $c_0^2 = \gamma RT_0$ , where  $\gamma$  is the ratio of specific heats and  $R$  is the  
128 specific gas constant for dry air. Hence, in an isothermal atmospheric boundary layer, the  
129 sound speed is constant. Using the ideal gas law and hydrostatic balance we know that in an  
130 isothermal atmosphere the background pressure and density vary as  $\exp(-gz/RT_0)$ , and the  
131 background potential temperature  $\theta$  is related to the Brunt-Väisälä frequency<sup>20</sup>  $N$  through

$$N^2 = \frac{g}{\theta} \frac{d\theta}{dz} = \frac{\gamma - 1}{\gamma} \frac{g^2}{RT_0}, \quad (2)$$

132 where  $g$  is the gravitational constant. Solving (2), we observe that  $\theta$  varies as  $\exp(N^2 z/g)$ ,

133 which provides the stratification needed for internal gravity waves to develop.

134 Now, given this stratification (through  $N$ ), it is conventional to neglect the vertical  
 135 changes of background density for relatively small  $\delta$  (typically less than 1 km). This is the  
 136 classical Boussinesq approximation, that we can adopt here because our focus lies on the  
 137 low level waveguide. Within the framework of the above hypothesis, and following previous  
 138 works<sup>29</sup>, the vertical perturbation in the velocity is given by the (inverse) Fourier transform

$$w'(x, z) = \int_{\mathbb{R}} f(k) \hat{w}_c(k, z) e^{ikx} dk, \quad (3)$$

139 where  $f(k)$  is an amplitude function that depends on the wavenumber  $k$ , and  $\hat{w}_c(k, z)$  is a  
 140 canonical solution satisfying the Taylor-Goldstein equation

$$\frac{d^2 \hat{w}_c}{dz^2} + \left[ \frac{N^2}{U^2} - \frac{U_{zz}}{U} - k^2 \right] \hat{w}_c = 0, \quad (4)$$

141 with the condition  $\hat{w}_c(k, z) \sim e^{-\lambda(k)z}$ , as  $z \rightarrow \infty$ , and where the square-root function  $\lambda$  is  
 142 defined by

$$\lambda(k) = \left[ k^2 - \frac{N^2}{U_0^2} \right]^{\frac{1}{2}}. \quad (5)$$

143 In order that the boundedness or outgoing-wave condition be satisfied as  $z \rightarrow \infty$ , the branch  
 144 cuts of  $\lambda$  are inserted such that we have  $\hat{w}_c(k, z) \sim e^{i\epsilon\lambda(k)z}$  where  $\epsilon = \text{sign}(k)$  is to ensure  
 145 upward propagation for  $|k| < N/U_0$ .

146 The boundary condition at  $z \rightarrow \infty$  and the choice of branch cuts allow the solution of (4)  
 147 to be expressed in terms of hypergeometric functions. A dynamically consistent horizontal

148 velocity field  $u'$  can be obtained in spectral space using a polarization relation<sup>18</sup>. The  
 149 amplitude  $f(k)$  is then determined through inversion of the “free-slip” nonlinear boundary  
 150 condition

$$w'(x, h(x)) = [U(h) + u'(x, h(x))] \frac{dh}{dx}, \quad (6)$$

151 with the witch of Agnesi profile

$$h(x) = \frac{H}{1 + \frac{x^2}{2L^2}}, \quad (7)$$

152 where  $H$  is the ridge top height and  $L$  is a characteristic length scale. Application of the  
 153 Fourier transform to (7) leads to  $\hat{h}(k) = HLe^{-k\sqrt{2}L}/\sqrt{2}$ , which implies that the dominant  
 154 horizontal wavelength is given by  $k = 1/\sqrt{2}L$ . In the following, this profile will be centred  
 155 at  $x_0 = 25$  km and we will use  $h(x)$  instead of  $h(x - x_0)$  for notational conciseness.

156 To describe the flow response, it is also worthwhile to use the three dimensionless  
 157 parameters

$$J = \frac{N^2\delta^2}{U_0^2}, \quad H_N = \frac{NH}{U_0}, \quad \text{and} \quad F = \frac{NL}{U_0}. \quad (8)$$

158 While the Richardson number  $J$  measures the background flow stability<sup>33;23</sup>, the other pa-  
 159 rameters are related to the shape of the mountain. The parameter  $H_N$  is a dimensionless  
 160 mountain height that measures the degree of nonlinearity in the flow response<sup>39</sup>. The classi-  
 161 cal Froude number  $F$  compares the advective time-scale to cross the ridge and the buoyancy  
 162 oscillation time-scale. This last parameter measures the significance of non-hydrostatic ef-  
 163 fects<sup>36</sup>. In the following we will fix  $N$ ,  $U_0$  and  $L$  such that  $F = 10 \gg 1$ , a value that

164 guarantees that no substantial trapped lee waves are forced. We will vary the boundary  
 165 layer depth  $\delta$  and/or the mountain height  $H$ .

## 166 **B. Effective sound speed disturbances**

167 Following Waxler<sup>44</sup> we next use the effective sound speed approximation<sup>19</sup>, in which  
 168 the component of the horizontal wind speed in the direction of propagation is added to the  
 169 thermodynamic sound speed. For an isothermal atmosphere, with a varying background  
 170 wind  $U$ , this approximation yields  $c(z) = c_0 + U(z)$  and thus, ducting is only due to the  
 171 change in altitude of  $U$ . Hence, in presence of temperature and wind fluctuations, the  
 172 perturbed effective sound speed is given by

$$c_0 \sqrt{1 + \frac{T'}{T_0}} + U + u'. \quad (9)$$

173 where the temperature and the horizontal wind perturbations, which are denoted by  $T'$  and  
 174  $u'$ , respectively, are obtained from the vertical velocity  $w'$  using polarization relations<sup>18</sup>.

175 In order to illustrate how mountain waves can perturb the background state, the moun-  
 176 tain wave model described in section II.A is used with parameters that are representative  
 177 of the lower troposphere. Here, and in the following, we consider a boundary layer flow at  
 178  $U_0 = 10 \text{ m}\cdot\text{s}^{-1}$  in a stratified medium characterized by  $N = \sqrt{2}\cdot 10^{-2} \text{ s}^{-1}$ , and take  $L = 10 \text{ km}$   
 179 to enforce  $F = 10$ . For illustrative purposes, the height of the mountain and the boundary  
 180 layer thickness are fixed to  $H = 350 \text{ m}$  and  $\delta = 860 \text{ m}$ , respectively. For these parameters,

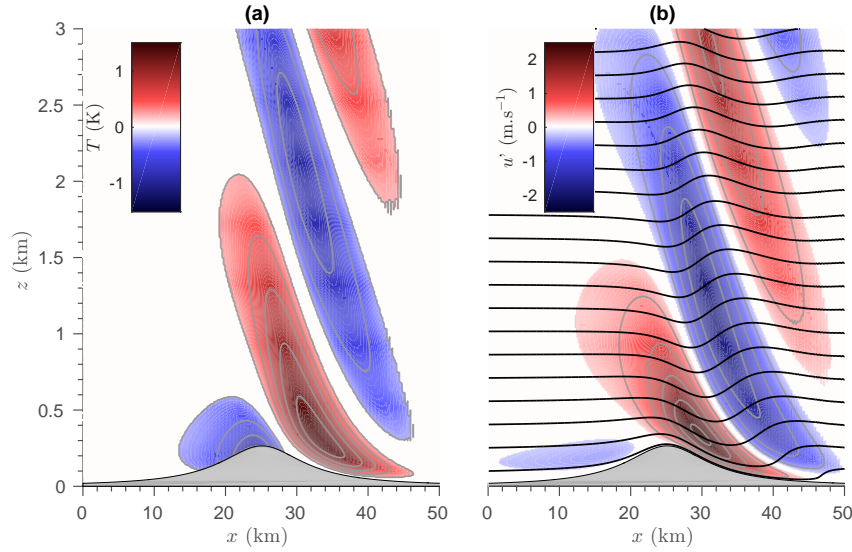


Figure 1: (a) Temperature fluctuations  $T'$  and (b) horizontal wind fluctuations  $u'$  resulting from interaction between a mountain and an incoming boundary layer. The streamlines are given by black lines superimposed to the wind fluctuations. The dimensionless parameters used are  $H_N = 0.5$  and  $J = 1.5$ . The mountain is represented in gray.

181 we obtain  $J = 1.5$ , which corresponds to a moderately stable situation. Finally, the dimen-  
 182 sionless value  $H_N = 0.5$  is sufficiently small to guarantee that the near-linear mountain flow  
 183 theory applies and produces realistic downslope winds and Foehn.

184 Figure 1a and 1b show the temperature and wind fluctuations produced by the mountain  
 185 flow model, respectively. In figure 1a we observe that the strongest temperature anomaly is  
 186 reached on the lee side, which is the “Foehn” effect. Figure 1b shows that the wind intensity  
 187 on the lee side is larger than that on the windward side, which is characteristic of downslope

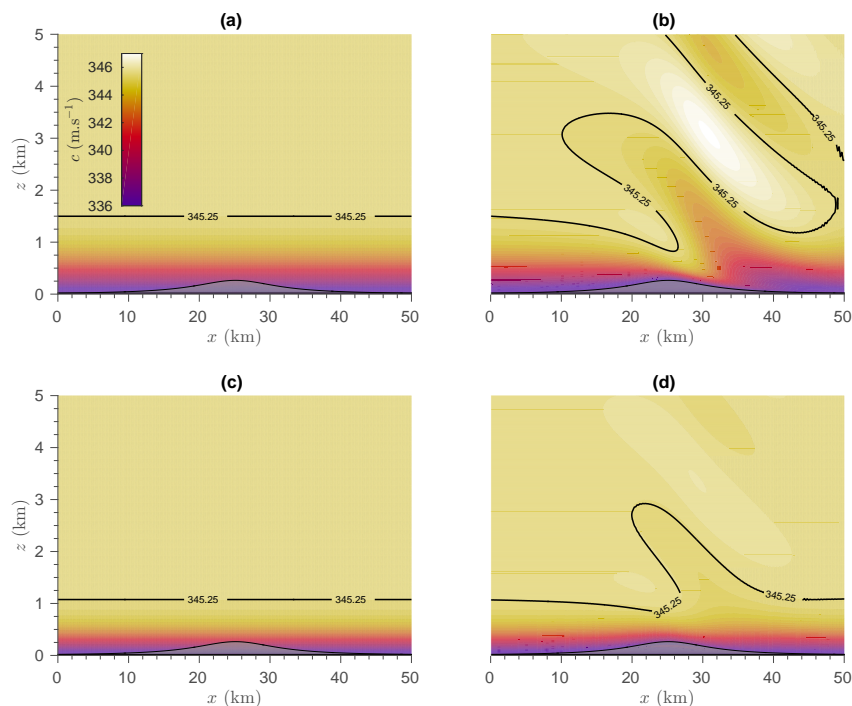


Figure 2: Effective sound speed field without (a,c) and with (b,d) interaction between a mountain and an incoming boundary layer. The dimensionless parameters used are  $H_N = 0.5$  (a,b,c,d) and  $J = 1.5$  (a,b) or  $J = 0.5$  (c,d).

188 windstorms. The streamlines are represented in figure 1b to illustrate the so-called isentropic  
 189 drawdown mechanism often used to explain Foehn. In this dry mechanism the Foehn results  
 190 from warm air masses that slightly ascend on the windward side before descending abruptly  
 191 on the leeward side. From Lott<sup>29</sup> we know that this effect and the intensity of the downslope  
 192 winds are not that strong for significantly smaller values of  $J$ .

193 Figure 2 shows various effective sound speed fields that will be used in sections IV

194 and V. In figure 2a,c we just keep the incident waveguide unaltered and chopped it by the  
195 mountain height for  $J = 1.5$  (figure 2a) and  $J = 0.5$  (figure 2c). This is representative of the  
196 “mask” technique used in the literature<sup>2</sup>, and to which we will systematically compare our  
197 results to in the following. From figure 2a we can expect its effect to be substantial since  
198 this mask potentially excludes from trapping all the waves with phase speed between around  
199  $336 \text{ m.s}^{-1}$  and  $339 \text{ m.s}^{-1}$ . This exclusion is not as strong when the mountain wave field is  
200 included as figure 2b shows, and indeed the effective sound speed “follows” the ground as  
201 the air passes over the mountain (see, for instance, isoline  $c = 336 \text{ m.s}^{-1}$ ). Nevertheless,  
202 it is clear that even in this case, the depth of the lower atmospheric duct substantially  
203 decreases as we move from the upstream side of the mountain to its top. This shrinking also  
204 manifests on the lee side, before that the flow reaches an abrupt expansion at around mid-  
205 slope to return to its upstream depth. Hence, for lower altitudes, these two effects produce  
206 a waveguide contraction as the flow passes over the mountain. Far above the mountain, the  
207 disturbances take the form of gravity waves that propagate upward. In the effective sound  
208 speed approximation framework, these gravity waves may be regarded as several acoustic  
209 waveguides in which relatively low-frequency acoustic waves can potentially propagate.

210 It is worthwhile noting that both the distorsion of the low level waveguide and the  
211 mountain wave field are not as intense for less stable situations (e.g.  $J = 0.5$ , figure 2d).  
212 This is consistent with the fact that large values of  $J$  favor downslope winds and Foehn.

213 Comparison with the “mask” technique (figures 2c,d) demonstrates that for  $J = 0.5$  the  
 214 lowest effective sound speed isoline follows the global curvature of the terrain, instead of  
 215 being chopped by the mountain. As discussed in section IV, this effect helps infrasound  
 216 signals to travel across the hill.

### 217 III. Acoustic propagation in range-dependent media

#### 218 A. Normal mode approach

219 The approach follows the formulation of the initial-value problem adopted by Bertin *et*  
 220 *al.*<sup>6</sup>, among others, for range-dependent environments. Assuming that the modes couple  
 221 *adiabatically*<sup>24</sup>, the solution for the Fourier transform  $\tilde{p}(z; x, \omega)$  of the infrasound pressure  
 222 fluctuations can be written as

$$\tilde{p}(z; x, \omega) \sim \sum_j \frac{a_j(\omega)\phi_j(z; x, \omega)}{\sqrt{k_j(x, \omega)}} e^{i\theta_j(x, \omega)}, \quad (10)$$

223 where  $\phi_j$ ,  $k_j$ ,  $a_j$  and  $\theta_j$  are respectively the  $j$ th mode function, the corresponding modal  
 224 wave number, amplitude and phase function. For a localized point-source at  $x = z = 0$  that  
 225 emits a signal  $s(t)$  we simply have  $a_j(\omega) = \phi_j(0; 0, \omega)$ , and the pressure fluctuation reads as

$$p(z; x, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{s}(\omega) \tilde{p}(z; x, \omega) e^{-i\omega t} d\omega, \quad (11)$$

226 where  $\tilde{s}(\omega)$  is the Fourier transform of  $s(t)$ . For convenience, we denote the derivative of  
 227  $\theta_j(x, \omega)$  by  $k_j(x, \omega)$  and the frequency dependence is dropped for conciseness. Physically  $k_j$   
 228 is the local (acoustic) wavenumber and the local phase speed is given by  $c_j = \omega/k_j$ .



229 It is worth noting that the pressure fluctuation can generally be decomposed into prop-  
 230 agating modes (along the  $x$ -axis direction) and evanescent modes, for which the imaginary  
 231 part of  $\theta_j$  is positive. Far downstream of the acoustic point-source, at a distance large  
 232 compared to the wavelength, the evanescent modes are negligible and (10) is the correct  
 233 expression to consider.

234 Substitution of (10) into the classical Helmholtz equation gives, to order unity,

$$\frac{\partial^2 \phi_j}{\partial z^2} + \left[ \frac{\omega^2}{c^2} - k_j^2 \right] \phi_j = 0, \quad (12)$$

235 with the Neumann boundary condition expressing that the derivative of  $\phi_j$  at  $z = h(x)$  van-  
 236 ishes. For unbounded boundary layers, (12) must be supplemented by requiring a bounded-  
 237 ness or outgoing-wave condition as  $z \rightarrow \infty$ . The solution of (12) then becomes  $p \sim e^{-\mu_\infty z}$  as  
 238  $z \rightarrow \infty$ , with the square-root function

$$\mu_\infty(k) = \left[ k^2 - \frac{\omega^2}{c_\infty^2} \right]^{\frac{1}{2}}, \quad (13)$$

239 and  $c_\infty$  denotes the effective sound speed in the limit  $z \rightarrow \infty$ . The function (13) depends  
 240 on the variable  $k$ . Thus in the complex  $k$ -plane the branch cuts are to be inserted such that  
 241  $-\pi/2 < \arg(\mu_\infty) \leq \pi/2$ . This choice of the branch cuts assures that as  $z \rightarrow \infty$  the solution  
 242 of (12) either goes to zero or represents an outgoing wave for all values of  $k$  in the complex  
 243 plane.

244 The branch cuts extend from the branch points  $k = \omega/c_\infty$  and  $k = -\omega/c_\infty$  to infinity in

245 the complex  $k$ -plane. Over the initial region, far upstream of the mountain, the gravity wave  
 246 field vanishes and we have  $c_\infty = c_0 + U_0$ , whereas at ground-level the effective sound speed  
 247 reaches its minimum  $c(0, 0) = c_0$ . For a right-propagating wave the condition of trapping  
 248 therefore imposes that the initial eigenvalues  $k_j$  lie initially along the interval  $c_0 < \omega/k <$   
 249  $c_0 + U_0$ . As each mode propagates downstream both the vertical sound speed profile and the  
 250 branch points  $\omega/c_\infty(x)$  vary, and the local eigenvalues  $k_j(x)$  slowly adapts to these changes.  
 251 In this process, the phase velocity of some acoustic modes eventually becomes larger than  
 252  $c_\infty$  and the associated trajectories terminate at a branch cut. Since it is not allowed to cross  
 253 the branch cut it is therefore not possible to continue the eigensolution downstream of this  
 254 point (and still satisfy the boundedness condition as  $z \rightarrow \infty$ ). Therefore the corresponding  
 255 modes are simply suppressed from the expansion (10).

256 In this work, the eigenfunctions and eigenvalues of (12) are calculated at a discrete set  
 257 of ranges  $x = x_n$  using the finite difference scheme used by Waxler *et al.*<sup>48</sup>, among others.  
 258 The eigenvalues are obtained using a QR decomposition for  $x = 0$  and, for other ranges, the  
 259 eigenvalues are tracked by using an iterative approach.

260 Since eigenfunctions are determined only up to a multiplicative constant, for definiteness  
 261 we impose the normalization condition<sup>24</sup>

$$\int_{h_n}^{\infty} \phi_j^2(x_n, z) dz = 1, \quad (14)$$

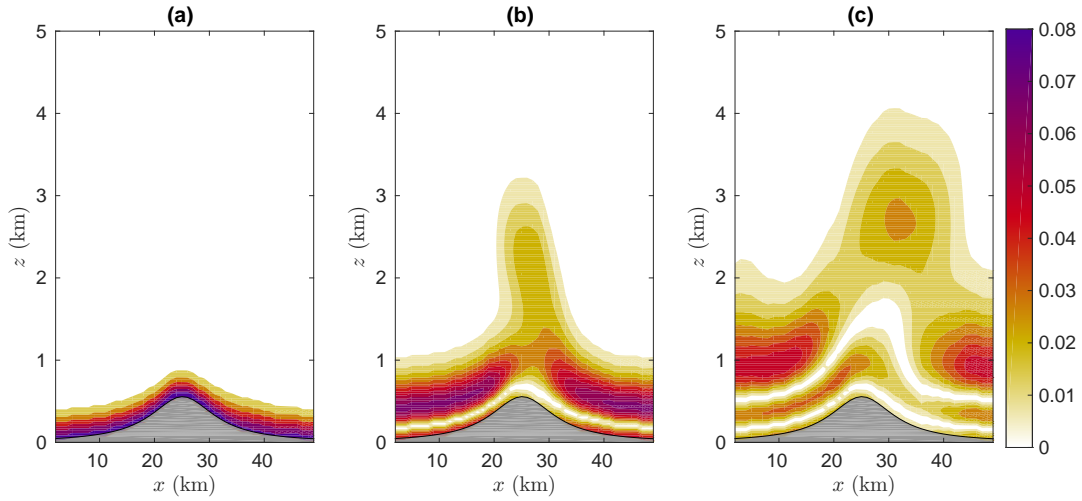


Figure 3: Eigenfunctions  $|\phi_j|$  as functions of  $x$  and  $z$  for the first three modes (from left to right), and for a fixed frequency of 2.8125 Hz. The background state is computed for  $H_N = 0.5$  and  $J = 1.5$  (cf. figure 2b). (a)  $j = 1$ , (b)  $j = 2$  and (c)  $j = 3$ .

262 where  $h_n = h(x_n)$ , together with the orthogonality condition

$$\int_{h_n}^{\infty} \phi_j(x_n, z) \phi_l(x_{n+1}, z) dz = \delta_{jl}. \quad (15)$$

263 Anticipating the presence of upper level waveguides, the upper bound of integrals (14)  
 264 and (15) was set to a sufficiently large value  $z_{\text{top}}$  and the effective sound speed profile  
 265  $c(x_n, z)$  was smoothly continued to higher altitudes when necessary. To assess the validity of  
 266 the numerical results, it has been checked that the eigenvalues were not sensitive to changes  
 267 in  $z_{\text{top}}$ , or to the choice of the continuation of  $c(x_n, z)$  above  $z_{\text{top}}$ .

268 For illustrative purposes, figure 3 shows the eigenfunctions of the first three modes as  $x$   
 269 increases along the source-receiver path, for a fixed frequency  $\omega_0 = 2\pi \times 2.8125 \text{ rad.s}^{-1}$ . For

270 this frequency, these modes carry the dominant part of sound intensity over long distances  
 271 and the expansion (10) can be truncated to  $j \leq 3$ , as discussed by Bertin *et al.*<sup>6</sup>. The first  
 272 mode (figure 3a) is weakly sensitive to changes of the atmospheric flow as  $x$  varies. The  
 273 other two modes in figures 3b,c are clearly affected by the presence of gravity waves and  
 274 indeed, ground-based attenuation is clearly visible in the vicinity of the ridge top elevation.  
 275 Furthermore, figure 3c shows that the presence of mountain waves aloft allows the modes to  
 276 be trapped in an upper duct. This result is discussed further in section IV.

## 277 B. WKB treatment of the low-level waveguide

278 To distinguish the effect due to the boundary layer shrinking from that due to the  
 279 mountain wave at upper levels, and to gain insight onto the behavior of the trapped modes,  
 280 we have obtained solutions to (12) using the following profile of effective sound speed

$$\bar{c}(z) = \bar{c}_0 + \bar{c}_1 \tanh [z/\bar{\delta}], \quad (16)$$

281 and the WKB approximation of (12). In (16) the parameters  $\bar{c}_0$ ,  $\bar{c}_1$  and  $\bar{\delta}$  are chosen to  
 282 minimize the integrated squared error between  $c$  and  $\bar{c}$  over the domain  $h < z < z_{\max}$ ,  
 283 where  $z_{\max}$  is the depth of the low level waveguide, e.g. the lowest altitude such that

$$\frac{dc}{dz}(z_{\max}) = 0. \quad (17)$$

284 This definition ensures that the mountain wave is filtered out from the sound speed field and  
 285 that the resulting waveguide width  $z_{\max}$  varies slowly in the flow direction, as required by

286 the classical asymptotic methods for modeling infrasound propagation<sup>24</sup>.

287 For fixed  $x$ , the filtered effective sound speed  $\bar{c}$  in (16) is a strictly decreasing function  
 288 of height and thus,  $\omega^2/\bar{c}^2 - k^2$  is a continuous function which involves a single turning point<sup>5</sup>  
 289 at  $z = z_0(k)$ . This choice allows us to use the Langer's formula<sup>26</sup> to build the uniformly  
 290 valid WKB approximation

$$\bar{\phi}(z) = 2\sqrt{\pi}C \left[ \frac{3}{2} \frac{S_0(z)}{\mu^3(z)} \right]^{\frac{1}{6}} \text{Ai} \left\{ \left[ \frac{3}{2} S_0(z) \right]^{\frac{2}{3}} \right\}, \quad (18)$$

291 where Ai is the Airy function, and where the phase is given by

$$S_0(z) = \int_{z_0}^z \mu(s) ds, \quad (19)$$

292 and the turning point  $z_0$  is the unique root of  $\mu(z_0) = 0$ , where

$$\mu(z) = \left[ k^2 - \frac{\omega^2}{\bar{c}^2(z)} \right]^{\frac{1}{2}}. \quad (20)$$

293 To ensure that the boundary condition at  $z \rightarrow \infty$  is satisfied, the branch cuts are defined  
 294 as for the function (13). Hence, using the leading asymptotic behavior of the Airy function  
 295 for large  $z$ , (18) may be approximated by  $\bar{\phi} \sim C\sqrt{\mu}e^{-S_0}$ , which is the leading order of  
 296 the classical WKB approximation and the constant  $C$  is determined by the normalization  
 297 condition (14). It is important to point out at this time that this normalization condition  
 298 plays a central role, especially when estimating the ground-based pressure  $\bar{\phi}_j/\sqrt{k_j}$  as  $x$   
 299 varies. In many cases, it can easily be verified that the closer to the ground the turning  
 300 point is located, the greater the amplitude of the pressure field at ground level.

301 Below the turning point, the path of integration must be deformed such that the square-  
 302 root function is continued into  $\mu^2 = -\mu^2 e^{i\pi}$  for  $z < z_0$ . On substituting this into (19), we  
 303 note that  $S_0^{2/3}$  is large and negative, and (18) can be simplified for  $z \ll z_0$  by using the  
 304 asymptotic behavior of the Airy function for negative argument<sup>1</sup>. To leading order, (18)  
 305 may then be written as

$$\bar{\phi}(z) \sim 2C [-\mu^2(z)]^{-\frac{1}{4}} \cos \left\{ \int_z^{z_0} [-\mu^2(s)]^{\frac{1}{2}} ds - \frac{\pi}{4} \right\}. \quad (21)$$

306 Now for  $z = h$ , on account of the (Neumann) boundary condition  $\bar{\phi}_z(h) = 0$ , we obtain the  
 307 constraint

$$\int_h^{z_0(k)} \sqrt{-\mu^2(s)} ds = \frac{\pi}{4} + j\pi, \quad (22)$$

308 where  $j$  is a nonnegative integer. Since  $\mu^2$  depends on  $k$ , it appears that (22) determines the  
 309 approximate value of  $k_j$ . In other words (22) defines the local dispersion relation where the  
 310 streamwise station  $x$  only appears as a parameter (which is not specified here for conciseness).

311 As an additional bonus, equation (22) may be used to evaluate the effect of either  
 312 downslope winds or mountain height on the local wavenumber. Upon totally differentiating  
 313 the implicit function (22) for fixed  $\omega$ , and equating to zero, we obtain

$$\frac{dk}{dx} = - \frac{\int_h^{z_0} \frac{\omega^2 \bar{c}_x}{\bar{c}^3 \sqrt{-\mu^2}} ds + \frac{dh}{dx} \sqrt{-\mu^2(h)}}{\int_h^{z_0} \frac{k}{\sqrt{-\mu^2}} ds}, \quad (23)$$

314 Where the terms of this ratio are the derivatives of (22) with respect to  $k$  and  $x$  and  $\bar{c}_x$  is the

315 derivative of  $\bar{c}$  with respect to  $x$ . Similarly, we use the notations  $k_x$  and  $h_x$  for the derivatives  
 316 of  $k$  and  $h$  with respect to  $x$ , respectively. Application of (23) for  $\bar{c}_x > 0$  and  $h_x > 0$  leads  
 317 to  $k_x < 0$ . This means that the phase speed  $\omega/k$  increases as the flow speeds up or when  $h$   
 318 increases.

#### 319 **IV. Impact of mountain waves on the normal modes**

320 In order to obtain the pressure signal from (10), the modal wave numbers  $k_j$ , or equiva-  
 321 lently the phase velocities  $c_j = \omega/k_j$  are required. For range-dependent environments, these  
 322 quantities are obtained as functions of  $\omega$  and  $x$  either by solving (12) numerically or by using  
 323 the WKB approximation, as described in section III.B. Figure 4 show contours of the phase  
 324 velocity in the  $(\omega, x)$ -plane for the first three modes [ $j \leq 3$  in (10)] and the two effective  
 325 sound speed fields considered in figures 2a and 2b. In figures 4a,b,c the numerically obtained  
 326 results are represented in colors, when the mountain “mask” is applied, and the black curves  
 327 give the corresponding WKB values. Figures 4d,e,f show the results obtained when the  
 328 mountain flow dynamics is considered. Two important curves are also plotted as red and  
 329 blue curves. These curves are obtained for each eigenvalue by decreasing  $\omega$ , the location  $x$   
 330 being fixed. Starting from an initial value, the phase velocity  $c_j$  increases up to the maxi-  
 331 mum sound speed  $c_{\max}$  as  $\omega$  decreases. This behavior allows to identify the so-called cut-off  
 332 frequency of the low-level waveguide, for which we have  $c_j(x, \omega) = c(x, z_{\max})$ , and which  
 333 is referred to as  $\omega_j^+(x)$  in the following (blue curve). For  $\omega = \omega^+$  the eigenvalue obtained

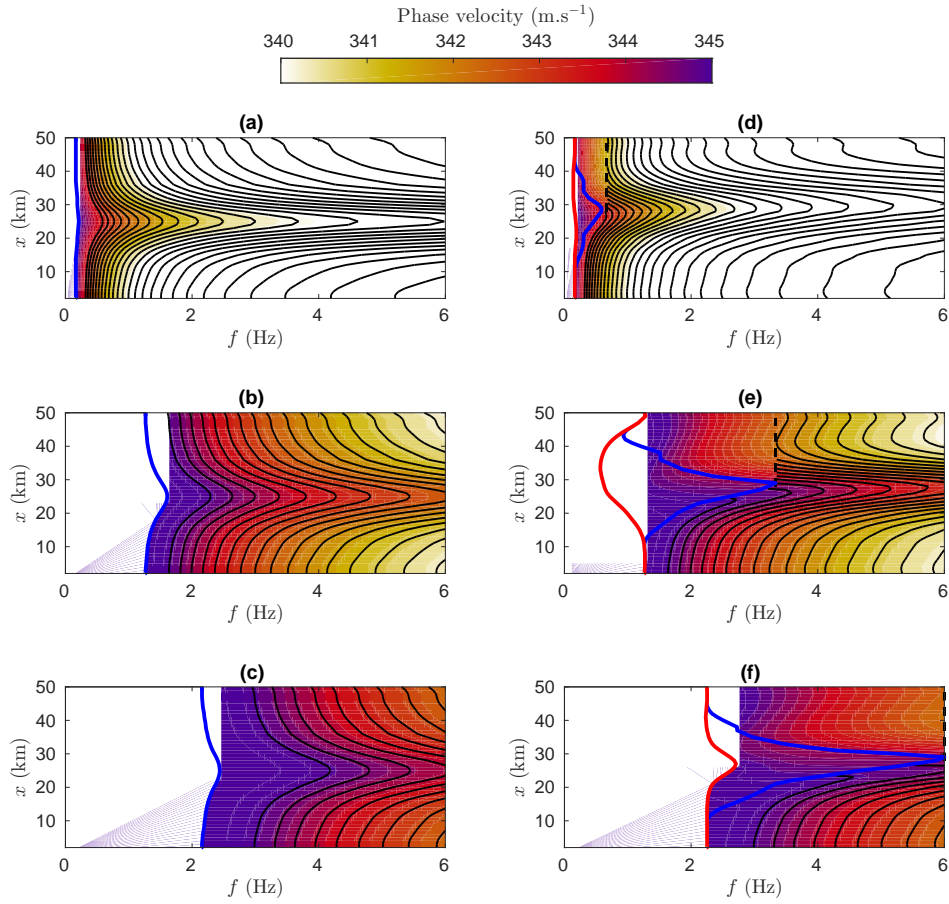


Figure 4: Phase velocity  $c_j$  for the first three modes (from top to bottom) as a function of streamwise location and frequency, for  $H_N = 0.5$  and  $J = 1.5$ . (a,d)  $j = 1$ ; (b,e)  $j = 2$ ; (c,f)  $j = 3$ . The results obtained by applying a “mask” onto the effective sound speed field are given on the left (a-c). The figures on the right (d-f) show the impact of mountain wave dynamics. Black, red and blue curves give the WKB prediction and the cut-off frequencies  $\omega^-$  and  $\omega^+$  as defined in section IV.



334 from (22) crosses a branch cut of (13) and thus, the WKB approximation fails to give a  
 335 result for  $\omega < \omega^+(x)$ . Physically this condition may be interpreted as the requirement that  
 336 the mode is not to be trapped in the low level duct. On the other hand the eigenvalue can be  
 337 computed directly from (12) for lower frequencies, so that the eigenvalue reaches a terminal  
 338 value for which we have  $c_j(x, \omega) = c_{\max}(x)$ . This value is referred to as  $\omega_j^-(x)$  (red curve).  
 339 It turns out that  $\omega^-$  is not defined when considering the “mask” effect alone (figures 4a,b,c),  
 340 essentially because in this case we have a single waveguide. Therefore, when mountain waves  
 341 are present (figures 4d,e,f), the region  $\omega_j^- < \omega < \omega_j^+$  corresponds to frequencies for which  
 342 the eigenfunction  $\phi_j$  penetrates up to the mountain wave field and can be confined within  
 343 an upper level waveguide, as depicted in figure 3c for  $x$  lying in the range 25-35 km. This  
 344 is an indication that at sufficiently low frequencies strong interaction between modes and  
 345 mountain waves may occur.

346 As detailed in section III.A, a mode is not allowed to cross the branch cut, a situation  
 347 that occurs for  $\omega < \omega^-(x)$ . The basic problem here is that as soon as the phase velocity of  
 348 the locally wave solution becomes larger than the maximum effective sound speed, it is not  
 349 possible to find a solution that remains bounded in the limit  $z \rightarrow \infty$ . Within the framework  
 350 of slowly varying media, this condition translates into  $a_j(\omega) = 0$  for  $\omega < \omega^-(0)$ . Physically  
 351 this condition may be interpreted as the requirement that the mode does not propagate  
 352 along the source-receiver path, for  $x > 0$ . The corresponding regions in the  $(\omega, x)$  plane are

353 represented by blank areas in figures 4d,e.

354 For fixed but quite high frequencies (greater than 1 Hz typically), the phase velocity  
355 of the first mode, which is also the slowest mode (figure 4a,d), increases as we approach  
356 the ridge before decreasing in the lee side. Although this effect is less strong in presence of  
357 mountain flow, this behavior can be captured qualitatively using the “mask” technique and the  
358 WKB approximation. In fact this mode, which is confined in the vicinity of the ground, find  
359 its way through the ridge, even when the waveguide is substantially shrunk by mountain  
360 wave dynamics (figure 4d) or chopped by the ridge (figure 4a). The excellent agreement  
361 with the WKB approximation suggests that the mode essentially adjusts to the vertical  
362 shrinking of the waveguide, the increasing in its phase velocity being correctly predicted by  
363 equation (23) with  $\bar{c}_x = 0$ . For lower frequencies (less than 1 Hz), the discrepancies between  
364 the results obtained with the “mask” technique and the mountain flow are more pronounced,  
365 and essentially occur in the region where the low level duct fails to trap the modes, e.g. when  
366 lines of constant phase velocity intersect the blue curve. This is detailed in the following  
367 for the next two modes. For  $j = 1$ , we observe an overall agreement between the results  
368 obtained by solving (12) numerically or by using the WKB approximation. From a practical  
369 standpoint, this demonstrates that the interaction between infrasound and mountain flows  
370 can adequately be predicted at a low numerical cost, through finding the first maximum  
371 in the local effective sound celerity and using the WKB approximation. This approach,

372 however, is justified only if we can neglect the contribution of other modes ( $j > 1$ ).

373 In computing the phase velocity for the other modes ( $j > 1$ ), we observe that the  
 374 mask technique fails in predicting important changes. Primarily, figures 4b,c show that the  
 375 cut-off frequency  $\omega^+$  substantially increases as we approach the ridge top, and reaches its  
 376 maximum at  $x_0 = 25\text{km}$ . As discussed in section III.A, when the condition  $\omega < \omega^+(x_0)$   
 377 is satisfied downstream  $x_0$ , the mode is simply suppressed. In presence of mountain waves  
 378 (figures 4e,f) the low level waveguide is extremely shrunked, and the penetration of  $\omega^+(x)$   
 379 into the  $(\omega, x)$ -plane is very pronounced. This effect is essentially due to Foehn, which  
 380 shifts the maximum cutt-off frequency  $\omega^+(x)$  on the leeward side of the ridge, at a distance  
 381 of approximately 30 km (figure 4e,f). Hence, immediately downstream this location, the  
 382 mode obtained with the one-turning-point WKB approximation (*i.e.* when mountain waves  
 383 are filtered out) must be suppressed, as shown in figures 4e,f. This is not the case when  
 384 mountain waves are considered and indeed, the fact that the mode remains propagating in  
 385 the horizontal direction for  $x > x_0$  is essentially due to the emergence of multiple possible  
 386 upper ducts above the mountain. For  $j = 2$ , we even see that the cut-off frequency of the  
 387 upper duct  $\omega^-$  decreases as we pass over the ridge and thus, the contribution of the mode  
 388 has to be maintained in (10). This finding is in strong contrast with that obtained using the  
 389 “mask” technique. On the other hand, and for  $j = 3$ , figure 4f shows that  $\omega^-$  increases as we  
 390 move closer to the ridge. This means that the mountain wave pattern failed in ducting the

391 mode that escapes from the low-level duct. Finally it is important to notice that for smaller  
 392 values of  $J$  ( $H_N$  being constant), the boundary layer tends to follow the global curvature  
 393 of the terrain, thereby yielding a significant number of modes to travel over the mountain,  
 394 whereas the upper bound  $\omega^+$  obtained with the “mask” technique penetrates much more  
 395 into the  $(\omega, x)$ -plane.

396 Figure 5 shows the sound intensity  $|\phi_j/\sqrt{k_j}|$  at ground level  $z = h(x)$ , as a function of  $x$   
 397 and  $\omega$ , for the first three modes. The magnitude of the contours are labelled in decibel, with  
 398 a reference sound intensity computed at  $x = 0$ . Results are given for the two effective sound  
 399 speed profiles defined above and depicted in figures 2a,b. The contours in color are for the  
 400 results obtained by solving numerically (12), and the black curves give the one-turning-point  
 401 WKB approximation. Red curves and blue curves represent the cut-off frequencies  $\omega_j^-$  and  
 402  $\omega_j^+$ , as in figure 4. Figures 5d,e,f, essentially show strong attenuation in the region  $\omega_j^- < \omega <$   
 403  $\omega_j^+$ . These attenuations are due to strong interactions between the acoustic waves and the  
 404 mountain waves, the latter creating new acoustic waveguides at higher altitudes, as discussed  
 405 previously. The energy leaks that follow the tunneling effect for sound waves (cf. figure 3b,c)  
 406 and the standard requirement that the integral of  $\phi_j^2$  is fixed to one [condition (14)], lead  
 407 to strong attenuations at ground level. These attenuations are more pronounced for higher  
 408 indices, simply because the corresponding turning points, at  $x = 0$ , are closer to  $z_{\max}$ .  
 409 Since the phase velocity adapts to the local environment encountered by the sound wave,

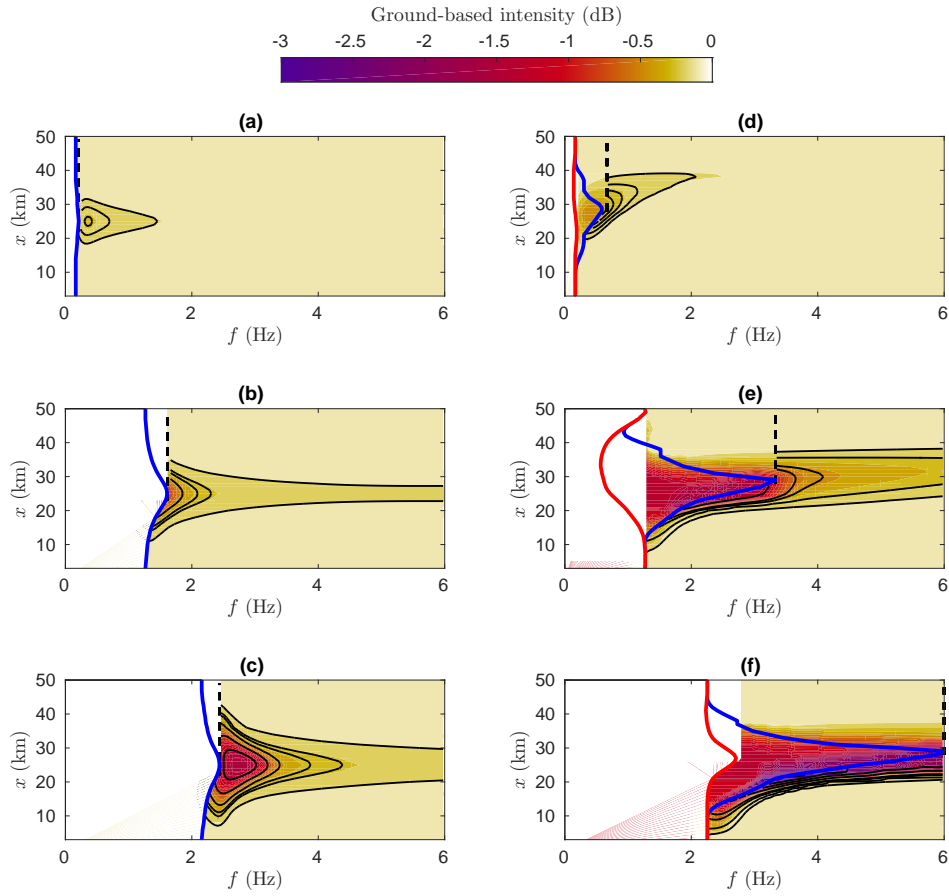


Figure 5: Ground-based sound intensity  $|\phi_j/\sqrt{k_j}|$  for the first three modes as a function of streamwise location and frequency, for  $H_N = 0.5$  and  $J = 1.5$ . (a,d)  $j = 1$ ; (b,e)  $j = 2$ ; (c,f)  $j = 3$ . The WKB prediction is superimposed in black contours. Red and blue curves correspond of those of figure 4.

410 the highest modes are more likely to leave the low-level waveguide.

411 While the ground-based attenuation of sound intensity can be qualitatively understood

412 when the mode shifts to upper-level waveguides (for  $\omega_j^- < \omega < \omega_j^+$ ), it is less clear why

413 it occurs when the waveguide shrinks, as figures 5a,b,c show for  $\omega > \omega^+$ . This behavior  
 414 appears to contradict the normalization requirement, which *a priori* results in surface am-  
 415 plifications rather than surface attenuations. However, using the WKB approximation, the  
 416 apparent contradiction is resolved by the recognition that the proper measure of the size of  
 417 the dispersive region,  $z_0 - h$  is always smaller than its value at  $x = 0$ . Based on the above  
 418 discussions, clearly the surface attenuation is a combination of the emergence of upper-level  
 419 waveguides as well as depth reduction of the low-level waveguide.

## 420 V. Impact of mountain waves on signals

421 In the previous section we have seen how the normal modes, in which the structure  
 422 over the whole  $(\omega, x)$ -plane can be delineated, are attenuated by mountain waves, and we  
 423 have given a general condition by which this interaction can be characterized, in terms of the  
 424 cut-off frequencies  $\omega^-$  and  $\omega^+$ . To measure the extent to which these effects are significant  
 425 when the sources of infrasound are localized in both space and time, we next calculate  
 426 ground-based waveforms, using the FFT algorithm<sup>7</sup>. A source function is introduced in the  
 427 form

$$s(t) = K e^{-\frac{t-T_0}{\sigma^2}} \cos(2\pi f_c t), \quad (24)$$

428 where  $T_0 = 10$  s,  $f_c = 3$  Hz and  $\sigma = 1/5$ . The parameters are adjusted such that the  
 429 maximum frequency is 6 Hz, with a leading frequency of 3 Hz.  $K$  is a suitable coefficient  
 430 that yields a normalized pulse. This source transfers most of its energy onto the first three

431 modes [ $j \leq 3$  in (10)] which are the modes of greatest contribution when the frequency  
 432 is relatively low. For this reason, the modal expansion is truncated to these modes in the  
 433 following.

434 The normalization of signals obtained for different locations downstream the mountain  
 435 is fixed so that the amplitudes can be compared to each other. The global effect of the  
 436 mountain can be summarized by means of the attenuation

$$1 - \frac{I(x)}{I_0(x)}, \quad (25)$$

437 where the sound intensity at ground level ( $z = h$ ) is defined as

$$I(x) = \int_0^\infty [p(h; x, t)]^2 dt, \quad (26)$$

438 where the waveform  $p(h; x; t)$  is obtained from (11) and (10), with  $j \leq 3$ , and by solving  
 439 the Helmholtz equation (12) numerically. Here  $I_0$  is a reference sound intensity obtained  
 440 by taking  $x = 0$  in (26). This choice allows the results to be compared with the classical  
 441 mountain-free range-independent case (*i.e.* when the effective sound speed is given by  $c_0 +$   
 442  $U(z)$ ), which is used in the infrasound research community.

443 In this section, we proceed systematically and vary the Richardson number between  
 444 0.25 and 2 and the non-dimensional mountain height  $H_N$  between 0.2 and 0.8. As discussed  
 445 in section IV, the sound speed modifications are intimately linked to the mountain flow  
 446 situations. To measure the downslope wind amplitude and Foehn, and following Lott<sup>29</sup>, we

447 use

$$A = \max_{\substack{2z < H_N \\ 0 < x < 2F}} \left[ \frac{u'(x, z)}{U(H)} \right], \quad (27)$$

448 which is the maximum of the ratio between the horizontal wind disturbance along the foothill  
 449 and the background wind at the top of the hill. Typically, when  $A$  approaches and exceeds 1,  
 450 the dynamics induces wind amplitudes that are either equal to or exceed the amplitude of  
 451 the winds at the summit of the ridge. In other words, the flow speeds up along the ridge,  
 452 and this occurs easily for  $J > 1$ .

453 Before proceeding systematically we present here three cases that illustrate the general  
 454 results that will conclude the paper. Case I is defined by  $\delta = 600$  m and  $H = 250$  m,  
 455 which corresponds to a relatively small mountain and a pronounced shear. In terms of  
 456 dimensionless parameters, we have  $J = 0.75$  and  $H_N = 0.3$  so that the downslope wind  
 457 amplitude is  $A = 0.75$ . Case II is associated with a larger depth  $\delta = 1$  km and a higher  
 458 mountain  $H = 600$  m. The corresponding dimensionless parameters are given by  $J = 2$  and  
 459  $H_N = 0.8$  so that  $A = 3$ , which reflects intense downslope winds. In order to estimate the  
 460 role of stability, we keep  $H_N = 0.8$  and consider a much less stable flow with a Richardson  
 461 number  $J = 0.3$ , as a third Case III. This last case corresponds to a situation for which we  
 462 have  $H/\delta = H_N/\sqrt{J} \simeq 1.5$  and thus, we can expect that most of the modes are obstructed  
 463 by the mountain when the “mask” technique is used.

464 The resulting acoustic signals associated with cases I, II and III are shown in figure 6



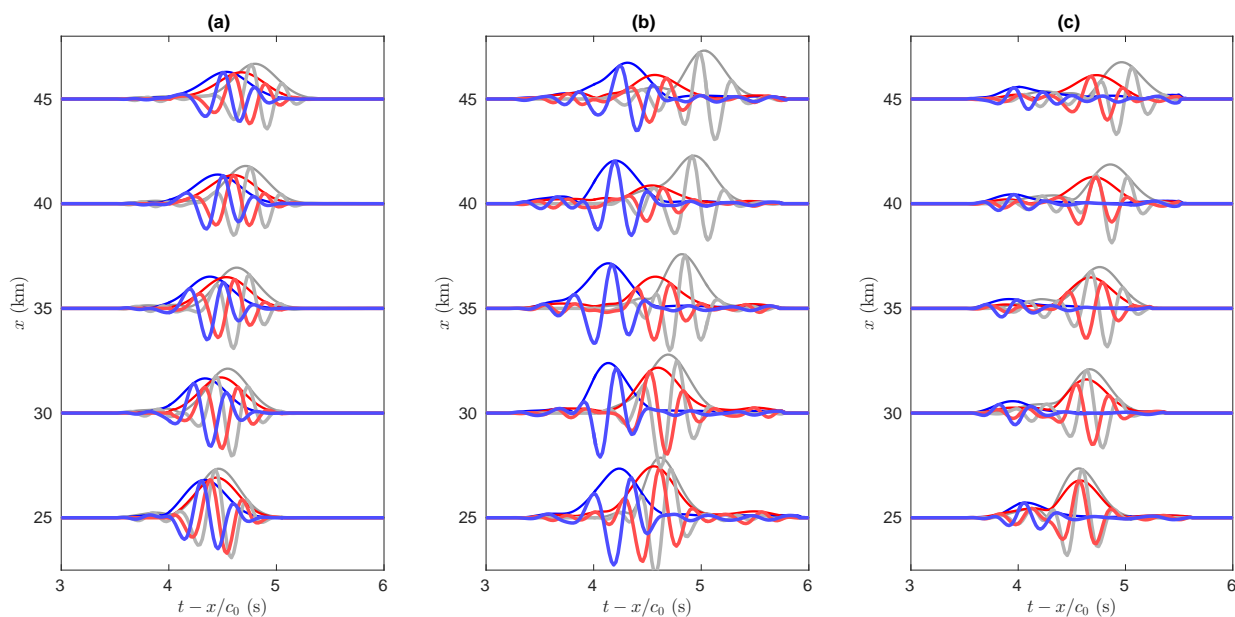


Figure 6: Waveforms obtained for cases I (a), II (b) and III (c) as functions of the retarded time  $t - x/c_0$  without (blue) and with (red) interaction between the mountain and the boundary layer. The signals obtained for an unperturbed range-independent case (without mountain and mask) are plotted in gray, for reference. Case I:  $J = 0.75$  and  $H_N = 0.3$ ; case II:  $J = 2$  and  $H_N = 0.8$ ; case III:  $J = 0.3$  and  $H_N = 0.8$ . The source is defined by (24).

465 for different locations downstream the mountain. The blue and red colors correspond to  
 466 waveforms computed by applying the “mask” technique and by solving mountain flow dy-  
 467 namics, respectively. Waveforms obtained for the unperturbed range-independent profile  
 468  $c_0 + U(z)$  are plotted in gray. The envelope of signals is plotted in thinner line, using the  
 469 Hilbert transform. Figure 6 shows evidence that the interaction between the mountain flow

470 and the acoustic wave may give rise to attenuation or amplification of ground-based signals,  
471 depending on the Richardson number. While the impact of the mountain on the ground-  
472 based signals is moderate for Case I (figure 6a), for which the attenuation does not exceed  
473 13%, Case II (figure 6b) gives rise to attenuations as large as 48% at  $x = 40$  km. This  
474 attenuation is mainly due to the fact that the first mode is no longer trapped in the low level  
475 waveguide and thus, a large part of the energy is lost at higher altitudes through interac-  
476 tions with mountain waves. On the other hand, case III (figure 6c) shows that the mountain  
477 wave dynamics may favor the passage of acoustic waves, mitigating the “mask” effect. The  
478 essential contrast with Case II is that, despite a strong reduction of its height, the incoming  
479 waveguide slips over the mountain rather than being destroyed over the windward side. The  
480 acoustic path then follows the global curvature of the terrain and the sound intensity is 80%  
481 larger than that obtained with the “mask” technique.

482 As discussed above, the signals obtained for the three cases considered in figure 6 do  
483 not cover all situations. In order to estimate how the mountain wave dynamics impacts the  
484 infrasound measurement, the ground-based attenuations (25) are first computed as functions  
485 of  $x$ , and then averaged over two intervals  $x_0 < x < x_1$  and  $x_1 < x < x_2$ , with  $x_0 = 25$  km,  
486  $x_1 = 40$  km and  $x_2 = 50$  km. The process is repeated for different values of  $J$  and  $H_N$   
487 so as to obtain a complete portrait of averaged attenuations in time domain. Figures 7  
488 and 8 show typical results for the first and second intervals, respectively. The first interval

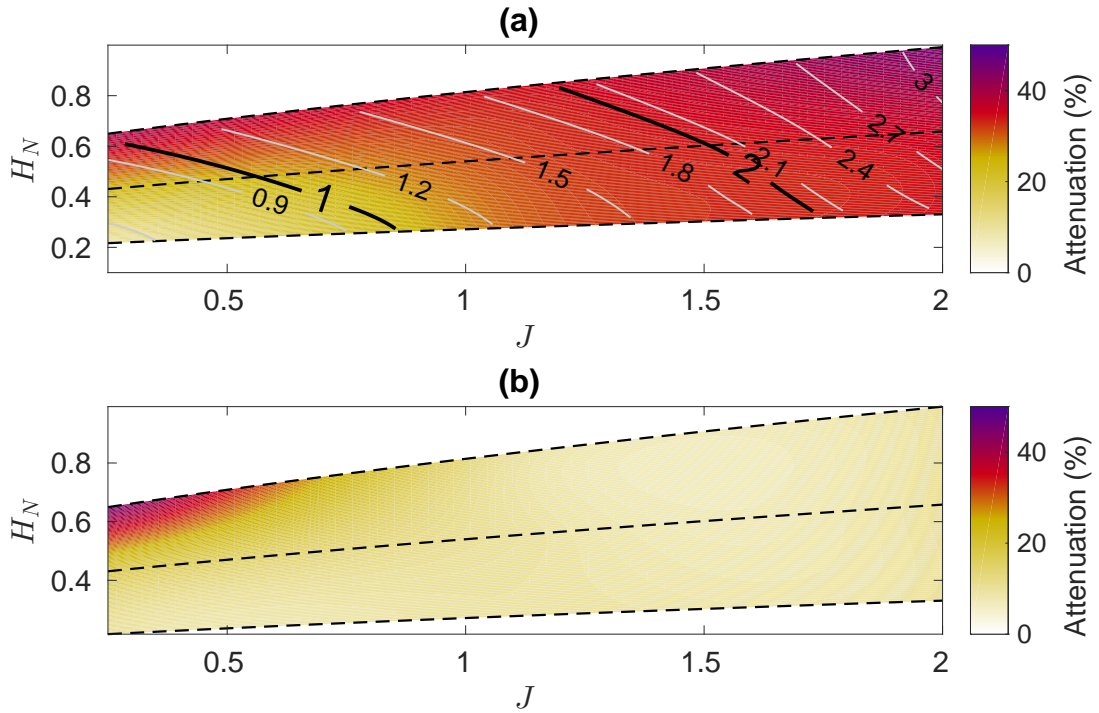


Figure 7: Far-field averaged attenuation downstream the mountain, in the range 25 – 40 km as a function of  $J$  and  $H_N$  with mountain wave disturbances (a) and the “mask” effect alone (b). The downslope wind amplitude  $A$  is given by black and white contours.

489  $x_0 < x < x_1$  is adopted here to quantify the infrasound attenuation on the leeward flank  
 490 of the ridge. Firstly, figure 7a shows that the mountain flow produces larger attenuations  
 491 than that obtained with the “mask” technique (in figure 7b). Furthermore, even though  
 492 the shrinking of the waveguide by the Foehn produces strong attenuations ( $A$  is almost  
 493 everywhere larger than 1), a significant fraction of the attenuation is indeed associated

494 with sound propagation within upper level waveguides, through local adjustments of few  
495 normal modes, as discussed in length in section IV. This is typically the case for relatively  
496 large  $J$  ( $J > 1.5$ ) and small  $H_N$ , in the range  $0.2 < H_N < 0.4$ . In this region the sound  
497 intensity on the lee-side flank of the ridge is attenuated by a factor of 30% (figure 7a)  
498 and decreases down to about 20% far downstream (figure 8a). Secondly, comparisons of  
499 figures 7a and 7b show that at low Richardson numbers ( $J < 0.5$ ) and for high mountains  
500 ( $0.6 < H_N < 0.7$ ) attenuation is mainly due to the “mask” effect, which produces a strong  
501 reduction of the waveguide height. The second interval is used to capture the far-field  
502 sound attenuation downstream the mountain without including the constructive/destructive  
503 interference effects associated with local changes of phases. In fact, at about two or three  
504 mountain half-widths downstream of the maximum height location, the modes recover their  
505 initial characteristics for  $x \rightarrow \infty$ , unless they reach a branch cut as discussed in section II.A.  
506 Owing to these changes in the resulting modal expansion (10), a residual attenuation is  
507 expected far downstream the mountain. This attenuation is irreversible in the sense that the  
508 full set of eigenvalues at  $x = 0$  is not recovered downstream the mountain. Comparison of  
509 figures 7a and 8a shows that this effect is apparent at relatively high Richardson numbers, in  
510 the top right corner of figure 7a. Finally, it is important to point out that another striking  
511 result here is that for large values of  $H_N$  and narrow waveguides (small  $J$ ), the mountain  
512 flow dynamics favor infrasound propagation, as discussed in section IV.

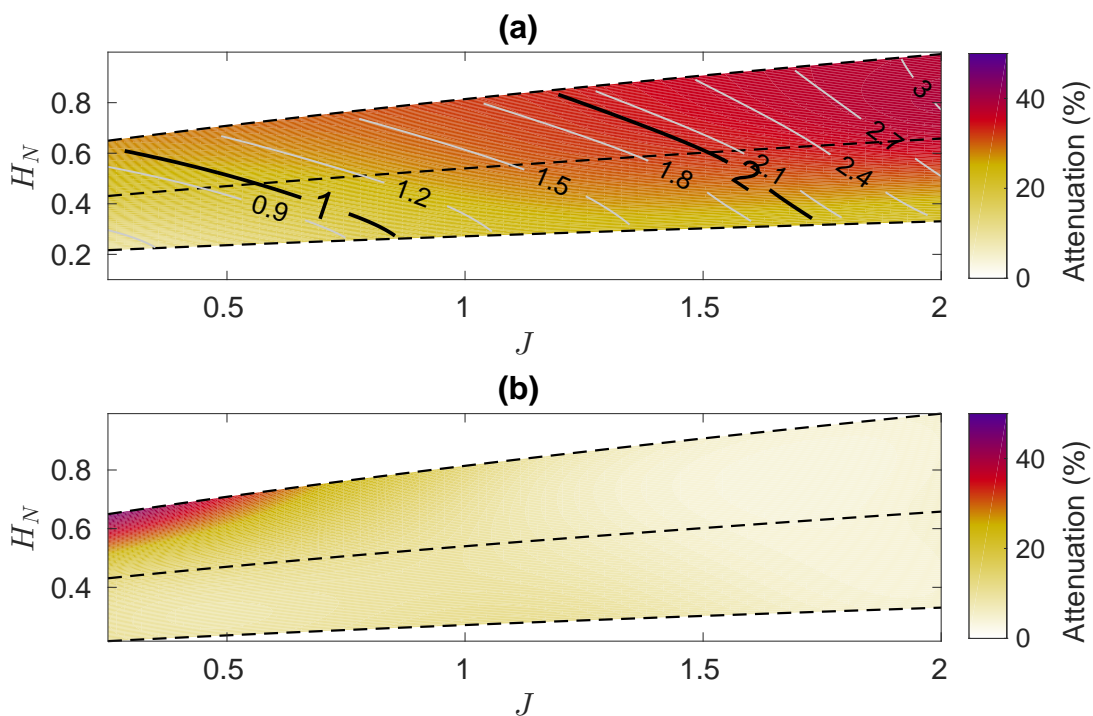


Figure 8: Far-field averaged attenuation, in the range 40 – 50 km as a function of  $J$  and  $H_N$  with mountain wave disturbances (a) and the “mask” effect alone (b). The downslope wind amplitude  $A$  is given by black and white contours.

513 **VI. Conclusions**

514 In this paper, we have examined the propagation of sound within mountain flows.  
 515 The mountain flow model is based on the integration of the linear inviscid Taylor-Goldstein  
 516 equation, forced by a nonlinear surface boundary condition. To calculate infrasound signals,

517 we also used a range-dependent normal mode approach, which allows the decomposition of  
518 the acoustic pressure field into distinct normal modes. The basic assumption introduced in  
519 the present work is that the acoustic modes couple adiabatically, *i. e.* without any transfer  
520 of energy to higher or lower modes. Ground-based signals were computed using Fourier  
521 synthesis of frequency-domain solutions, for a given ground-based broadband acoustic source.

522 The central result of this paper is that mountain wave dynamics may lead to strong  
523 attenuation or amplification of upcoming acoustic waves, regarding to the direct “mask”  
524 effect the mountain has on acoustic propagation. For a stable flow ( $J \geq 1$ ) the mountain  
525 wave dynamics produces large horizontal winds and buoyancy disturbances at low level that  
526 result in intense downslope winds and Foehn. When the downslope wind is less intense  
527 ( $J < 1$ ), the flows can reinforce the acoustic waveguide over the mountain and lead to a  
528 signal of greater amplitude compared to that obtained by the “mask” effect. The acoustic  
529 waveguide is then strongly impacted which leads to a new kind of acoustic (reversible)  
530 absorption that can be related to local adjustments of few normal modes. It is worthwhile to  
531 point out here that acoustic absorption is mainly governed by the Richardson number, and  
532 more precisely by the critical value  $J \simeq 1$ , which is also a transition regime for mountain  
533 wave dynamics.

534 In striking contrast to this local behavior of acoustic modes is the sound attenuation  
535 far downstream from the mountain. This second type of absorption is due to irreversible

536 processes that are intimately connected to leaking modes along the source-receiver path.  
537 While leaking modes are known to play a role in the transient waveform, the classical practice  
538 is to neglect the contributions from these modes at large horizontal distances from explosions  
539 in the atmosphere. This approach, however, ignores range-dependence of the environment.  
540 These modes may be “activated” by a point-source in the form of classical waveguide modes  
541 and then decay exponentially with increasing distances far downstream from the mountain,  
542 as a result of the atmospheric state evolution. In this way, the corresponding component  
543 involving these modes vanishes far downstream from the mountain. This results in absorption  
544 farther downstream from the mountain, even though the background state recovers its initial  
545 state (e.g. upstream from the mountain).

546       The present work presents our current understanding of acoustic absorption due to  
547 mountain wave dynamics with emphasis upon a modal description of the acoustic field, in-  
548 cluding static stability effects. Though the present analysis does not answer all the questions  
549 regarding the complex phenomenon of absorption, it has shown how a range-dependent anal-  
550 ysis can provide some insight into the interaction of acoustic waves and mountain wave fields.  
551 Other aspects that may give rise to additional dissipation have not been fully explored, such  
552 as, interaction of infrasound waves with ground and turbulence. However, the mountain  
553 wave model used in this study cannot predict the turbulence associated with GW breaking,  
554 a process that occurs for small  $J$ . While a rough estimate of the complex impedance effect

555 gives an absorption of 1 % for the cases considered in this study, the role of turbulence is  
556 more complex to quantify. The main difficulty is that the adiabatic approximation ceases to  
557 apply when the turbulence correlation length and acoustic wavelength are of the same order  
558 of magnitude. Some preliminary calculations have been made by the authors for estimating  
559 far-field absorption, using the techniques described in this paper, but with mode couplings  
560 and complex imaginary part of the grounding impedance. The results show good agree-  
561 ment with that obtained in the present work, except for cases where fine-grained turbulence  
562 dominates.

563 The present work is also related to the more general issue of incorporating unresolved  
564 GW variability in infrasound propagation calculations. Recent works<sup>25;21;14</sup> suggest that the  
565 mismatch between simulated and observed signals is related to the fact that the atmospheric  
566 specifications used in most studies do not adequately represent internal gravity waves. In  
567 the Atmospheric General Circulation Models which are used to produce the atmospheric  
568 specifications, these GWs are represented by parameterizations and in return, these param-  
569 eterizations can be used to predict the GWs field used in infrasound studies. This is the  
570 approach followed by Drob *et al.*<sup>14</sup>, in which the global spectral scheme of Hines<sup>22</sup> is used to  
571 estimate the effect of GWs on infrasound time arrivals. The interesting aspect of using the  
572 model proposed by Hines<sup>22</sup> is that it allows obtaining GW fields that give rise to the right  
573 climate<sup>28</sup>.



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