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► **To cite this version:**

Vincent Verbavatz, Marc Barthelemy. Critical factors for mitigating car traffic in cities. 2019. cea-02007268

**HAL Id: cea-02007268**

**<https://cea.hal.science/cea-02007268>**

Preprint submitted on 5 Feb 2019

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# Critical factors for mitigating car traffic in cities

Vincent Verbavatz

*Institut de Physique Théorique, Université Paris Saclay, CEA, CNRS, F-91191 Gif-sur-Yvette, France*

Marc Barthelemy\*

*Institut de Physique Théorique, Université Paris Saclay,  
CEA, CNRS, F-91191 Gif-sur-Yvette, France and*

*Centre d'Analyse et de Mathématique Sociales, (CNRS/EHES) 54, Boulevard Raspail, 75006 Paris, France*

Car traffic in urban systems has been studied intensely in past decades but its analysis is often limited to empirical observations and agent-based modelling, and despite the importance and urgency of the problem we have a poor theoretical understanding of the parameters controlling urban car use and congestion. Here, we combine economical and transport ingredients into a statistical physics approach and propose a parsimonious model that predicts the share of car drivers, the  $CO_2$  emitted by cars and the average commuting time. We confirm these analytical predictions on 25 major urban areas in the world, and our results suggest that urban density is not the relevant variable controlling car-related quantities but rather are the city's area size and the density of public transport. Mitigating the traffic (and its effect such as  $CO_2$  emissions) can then be obtained by reducing the urbanized area size or, more realistically, by improving either the public transport density or its access. In general, increasing the population at fixed area would increase the emission of  $CO_2$  in sharp contrast with the commonly accepted paradigm that increasing the density leads to a reduction of car traffic.

## INTRODUCTION

As most humans now live in urban areas and two-third of the world population will live in cities by 2050 [1], understanding urban mobility patterns [2] has become paramount in reducing transport-related greenhouse gas emissions and crucial to efficient environmental policies [3–6]. In a seminal paper, Newman and Kenworthy correlated transport-related quantities (such as gasoline consumption) with a determinant spatial criterion: urban density [7]. Higher population density areas were shown to have reduced gasoline consumption per capita and thus reduced gas emissions. Their result had a significant impact on urban theories over the last decades and has become a paradigm of spatial economics [8]. This study is however purely empirical and has no theoretical foundation, which casts some doubts about the importance of density as the sole determinant of gasoline consumption and other car dependent quantities.

On the other hand, car traffic has been studied at various granularities and with various tools ranging from agent-based modelling (see for example [9, 10]), cellular automata [11], or hydrodynamic approaches (see for example the review [12] on various physics type approaches). Car congestion was also considered from the point of view of economics (see for example [13, 14]). We have however no parsimonious model that is able to point to the critical parameters and dominant mechanisms of traffic in urban areas. In this paper, we address this problem and propose a theoretical model of urban daily commuting relying on two crucial ingredients: the coexistence of cars and mass rapid transit (MRT), and traffic congestion. Combining these ingredients within

a disordered system type approach allows us to derive significant conclusions concerning MRT transport ridership and transport-related greenhouse gas emissions. We compare our predictions to empirical data obtained for 25 major cities in the world (see Sup. Mat. for details about data) showing an excellent agreement given the simplicity of the model and the absence of any adjustable parameter. In this approach we deliberately left out details of these systems and focused on the basic processes that capture the complexity of urban systems while accounting for qualitative and quantitative behaviors [15–19].

## THE MODEL

According to the classical urban economics model of Fujita and Ogawa [20], individuals choose job and dwelling places that maximize their net income after deduction of rent and commuting costs. More precisely, an agent will choose to live in  $x$  and work at location  $y$  such that the quantity

$$Z(x, y) = W(y) - C_R(x) - G(x, y) \quad (1)$$

is maximum. The quantity  $W(y)$  is the typical wage earned at location  $y$ ,  $C_R(x)$  is the rent cost at  $x$ , and  $G(x, y)$  is the generalized transportation cost to go from  $x$  to  $y$ . There is also a similar equation for the profit of companies (that they want to maximize) and that we do not need here: we assume that employment is located at a unique center  $y = 0$  and that wages and rent costs are of the same order for all individuals. Most large cities are polycentric [21, 22] with the existence of many different activity centers but this first approach assumes that

polycentricity does not change the order of magnitude of commuting trip distances (see also [22] on this point). We also assume that the residence location  $x$  is given and random – residence choice is obviously a complex problem and replacing a complex quantity by a random one is a typical assumption made in the statistical physics of complex systems. Within these assumptions, we obtain a simplified Fujita-Ogawa model where the maximization of  $Z(x, 0)$  is equivalent to the minimization of the transport cost  $G(x, 0)$ . Thus, individuals simply tend to minimize their commuting costs and we will discuss their commuting trips between their home and the central business district. All these assumptions are of course approximations to the reality but we claim here that our model captures the essence of the traffic in large urban areas phenomenon. Starting with a model containing all these various parameters would actually be not tractable and would hide the critical ingredients.

We assume that a proportion  $p$  of the population has access to mass rapid transit such as the subway or elevated rail (we neglect buses or tramways here) whereas a share  $1 - p$  of the population has no choice but to commute by car (we assume that all individuals can drive a car if needed). We omit all other forms of commuting (walking, cycling, etc.), and we neglect spatial correlations between the densities of public transport and residence or population, which is an important assumption that certainly needs to be refined in future studies. The fraction  $p$  of individuals that have a choice between car and MRT will choose the transport mode with the lowest generalized cost which takes into account both monetary costs and trip duration (see for example [24, 60]). For cars, we include congestion described by the Bureau of Public Roads function (see [25] and Methods) which captures the main effect: increasing the traffic on a road will decrease the effective speed on it. The corresponding generalized costs for cars and the MRT then read

$$G_{car}(x) = C_c + \frac{d(x)}{v_c} V \left( 1 + \left( \frac{T}{c} \right)^\mu \right) \quad (2)$$

$$G_{MRT}(x) = V \left( f + \frac{d(x)}{v_m} \right) \quad (3)$$

where  $C_c$  is the daily cost of a car,  $v_c$  and  $v_m$  are respectively the car and MRT velocities,  $c$  is the road capacity of the city,  $f$  the walking plus waiting time for transit,  $d(x)$  the distance between home located at  $x$  and the central business district,  $T$  the total car traffic, and  $\mu$  the exponent that characterizes the sensitivity to traffic. The quantity  $V$  is the value of time defined in transport economics as the money amount that a traveler is willing to pay in order to save one hour of time. It is an increasing function of income and is bounded by the hourly wage. We assume here that driving is faster than riding public transport ( $v_c > v_m$ ) but is more expensive ( $C_c > 0$  – we neglect here the cost of a MRT ticket in comparison

with car costs). Once an individual has chosen a mode he sticks to it and will not reconsider his choice even if the traffic evolves. In other words, we assume here that individual habits have a longer time scale than traffic dynamics.

Individual mobility is then governed by comparing these costs  $G_{car}$  and  $G_{MRT}$  and will depend on exogenous parameters (such as car and subway velocities, car costs, etc.) and endogenous parameters such as the commuting distance. In the general mode choice theory (see for example [26]), given the values of the costs  $G_{car}$  and  $G_{MRT}$  there is a probability  $P_C = F(G_{car} - G_{MRT})$  to choose the car. The function  $F$  is in general smooth and satisfies  $F(-\infty) = 1$  and  $F(+\infty) = 0$  and we consider here the simplest case where  $F(x > 0) = 0$  and  $F(x < 0) = 1$ . An individual located at  $x$  with access to the MRT will then choose to use the car if  $G_{car} < G_{MRT}$  which implies a condition on the value of time of the form  $V < V_m(x)$  where  $V_m(x)$  depends on the parameters of the system and on  $d(x)$  and reads

$$V_m(x) = \frac{C_c}{f + d(x) \left( \frac{1}{v_m} - \frac{1}{v_c} \left( 1 + \left( \frac{T}{c} \right)^\mu \right) \right)} \quad (4)$$

Also, writing the equality  $G_{car} = G_{MRT}$  between generalized costs of car and MRT leads to the critical distance given by

$$d(V, T) = \min \left( L, \frac{\frac{C_c}{V} - f}{\frac{1}{v_m} - \frac{1}{v_c} \left( 1 + \left( \frac{T}{c} \right)^\mu \right)} \right) \quad (5)$$

where  $L \sim \sqrt{A}$  ( $A$  is the area size of the city) is the largest extent of the city. This critical distance evolves as traffic increases (Fig. 1), translating the fact that driving is less advantageous when the traffic is large. This expression also shows that individuals with a small value of time are more likely to use public transport, since they are more apt to spend time than money. Distance to the center is pivotal in this decision process: too far from the center – further than a critical distance  $d(V, T)$  – individuals favor driving to avoid lengthy journeys, and the richer they are, the smallest this distance.

Writing  $d(V, T = T^*) = L$ , gives the critical maximal traffic  $T^*$  for which driving is less beneficial in the whole agglomeration:

$$T^* = c \left[ v_c \left( \frac{1}{v_m} - \frac{1}{v_c} - \frac{\frac{C_c}{V} - f}{L} \right) \right]^{1/\mu} \quad (6)$$

### Computing the car share

In the following, we will then discuss the traffic dynamics in the two regimes ( $T > T^*$  or  $T < T^*$ ). For  $T < T^*$ , the evolution of the car traffic when the population  $P$

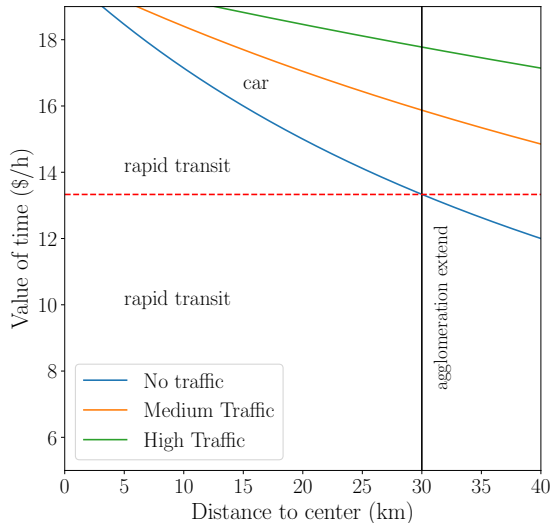


FIG. 1: The most advantageous mode of transport depends on the value of time of individuals and the distance to the urban center. The limit between the two areas evolves with congestion: the larger the traffic (curves from blue to green) and the larger the area in which rapid transit is beneficial compared to car driving. The grey solid vertical line corresponds to the size of the urban area and indicates the critical value of time (dashed red line) below which rapid transit is advantageous in the whole agglomeration whatever the value of congestion. The values of the parameters are chosen here as:  $C_c = 15$  €,  $v_c = 40$  km/h,  $v_m = 30$  km/h,  $f = 30$  minutes (see the Appendix 1 for a precise description of all data).

increases is given by the differential equation:

$$\frac{dT}{dP} = 1 - p + p \left( 1 - \pi \frac{d(V, T)^2}{A} \right) \quad (7)$$

where the first term of the r.h.s. corresponds to the  $1 - p$  share of individuals far from MRT stations, and the second term to individuals living further from the center than  $d(V, T)$  (we consider here statistically isotropic cities). Plugging in the expression Eq. 5 of  $d(V, T)$  leads to an equation that can be solved exactly at least for  $\mu = 1$  and  $\mu = 2$ . However, for all  $\mu$ , we obtain at dominant order in  $T/c$

$$T \simeq \left( 1 - \frac{p}{Ab^2} \right) P + \mathcal{O} \left( \left( \frac{T}{c} \right)^\mu \right) \quad (8)$$

where  $b$  is a function of the exogenous parameters

$$b = \left( \frac{1}{v_m} - \frac{1}{v_c} \right) \frac{1}{\sqrt{\pi}(C_c/V - f)} \quad (9)$$

For  $P > P^* = T^*/(1 - \frac{p}{Ab^2})$ , since  $d(V, T) > L$  the only source of car traffic comes from individuals who do

not have access to the mass rapid transit, which leads to  $dT/dP = 1 - p$  implying

$$T = (1 - p)(P - P^*) + T^* \quad (10)$$

We note that even if this result seems somewhat simple, it derives from non-trivial considerations such as the comparison of the critical distance and the area size. The important fact to retain here is that cost considerations in the case where a mode choice is available usually lead for large urban areas to leave the car and take the MRT.

Compiling data from 25 megacities in the world (see Appendix 1) for which we found an estimate of the population having access to the MRT (and therefore the quantity  $p$ ), we compute the critical levels  $T^*$  and  $P^*$  (we note that for all these cities the definition of  $p$  is the same: it is the share of individuals living within 1km from a MRT station). For reasonable values of time [59, 60], most of cities have  $T^* = 0$  and all have  $T^* \ll P$  ( $T^*$  ranges from 0 to 50% of  $P$ ). Cases where  $T^* = 0$  mean that even at zero traffic the critical distance  $d(V, T = 0)$  is larger than the typical size of the city  $L$ . This depends a bit on the value of time but in most cases we do observe small values of  $T^*$  indicating that cities are mostly in the saturated regime where the MRT is always more advantageous than the car. Public transports are so economical (compared to cars) that people living near rapid transit stations are highly likely to ride them. Thus, traffic does not appear as a determinant parameter in individual mobility choices as it concerns mostly individuals who have no choice but to drive and who suffer from onerous commuting costs and unavoidable time-consuming trips as traffic increases. We note here that correlations between the neighborhoods with low MRT density (small  $p$ ) where individuals have expensive commuting costs and their revenue is an interesting field of study and appears as an important source of urban segregation [24]. Since we have in general  $T^*, P^* \ll P$ , we obtain the simple prediction that  $\frac{T}{P} \simeq 1 - p$ , a non-trivial consequence of rapid transit cheapness and individual choices of mobility. We compare the empirical car modal shares  $\frac{T}{P}$  of these cities (see data description in Appendix 1) to our prediction on Figure 2 and observe – considering the simplicity of the model – a very good agreement and a relevant linear trend highlighting the efficiency of public transportation in reducing traffic. In particular, most of the European cities are well described by our prediction and we observe a few deviations. These discrepancies can probably find their origin in the existence of other modes of commuting, lower car ownership rates (e.g. in Buenos Aires [28]), lower road capacities, higher cost of MRT, or a high degree of polycentrism.

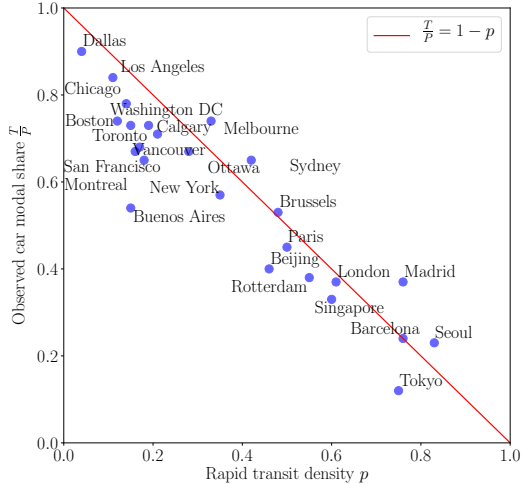


FIG. 2: Comparison between the observed car modal share  $T/P$  and the share of population  $p$  living near rapid transit stations (less than 1 km) for 25 metropolitan areas in the world. The red line is the prediction of our model ( $R^2 = 0.69$ ). Discrepancies to the predicted value are probably mostly due to the existence of other modes of transport (walking or cycling), lower car ownership rates, or a higher cost of the MRT, etc.

### Emitted $CO_2$

Our model also provides a prediction for the transport-related gas emissions and we will focus on the  $CO_2$  case for which we obtained data. We make the simplest assumption where these emissions are proportional to the total time spent on roads. The quantity of  $CO_2$  emitted for a driver residing at  $x$  is then given by  $Q_{CO_2}(x) \propto d(x)[1 + (T/c)^\mu]$  which leads to a total

$$Q_{CO_2} \propto \sum_{\text{drivers } i} d(x_i) \left[ 1 + \left( \frac{T}{c} \right)^\mu \right] \quad (11)$$

$$\propto g\sqrt{A}(1-p)P \left[ 1 + \left( \frac{T}{c} \right)^\mu \right] \quad (12)$$

where we used our result for the total traffic  $T = (1-p)P$ . We assumed that the sum  $\sum_i d(x_i)$  is of the form  $g\sqrt{A}T$  where  $\sqrt{A}$  sets the scale of displacement and where the prefactor  $g$  encodes the geometrical aspect of car mobility in the city, including the spatial distribution of residences and activities, and the transport infrastructure. Its estimation probably requires a more detailed, specific calculation but the important aspect here is the scaling with  $\sqrt{A}$  (see also the appendices). We thus obtain that the annual  $CO_2$  emitted by car and per capita is given by

$$\frac{Q_{CO_2}}{P} \propto \sqrt{A}(1-p)(1+\tau) \quad (13)$$

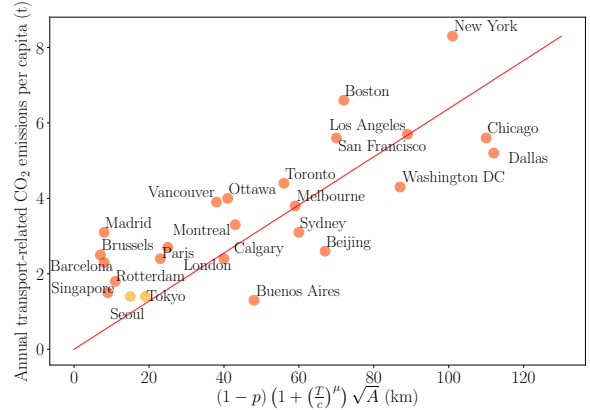


FIG. 3: Comparison between the annual transport-related  $CO_2$  emissions per capita and the effects of congestion, area size and rapid transit density predicted by our model. The red line is the linear fit of the predicted form  $y = ax$  where  $a \approx 0.064$   $CO_2$ tons/km/hab/year (the Pearson coefficient is 0.79). We had no congestion estimate for Seoul and Tokyo and we used an average congestion rate  $\tau = 50\%$ .

where  $\tau = \left(\frac{T}{c}\right)^\mu$  is the average delay due to congestion and is empirically accessible from the TomTom database [61] (while the capacity  $c$  is usually not empirically accessible). It is interesting to note that  $Q_{CO_2}$  is then the product of three main terms: size of the city  $\times$  fraction of car drivers  $\times$  congestion effects, which correspond indeed to the intuitive expectation about the main ingredients governing car traffic. Also, we note here that the dominant term of  $Q_{CO_2}$  is proportional to the population indicating a simple linear scaling, and that nonlinear term could possibly appear as corrections, in contrast with previous results [22, 30], and which could explain the difficulties to reach a consensus about the behavior of this type of quantities (see for example [31]). We compare our prediction Eq. 13 to disaggregated values of urban  $CO_2$  emissions on Figure 3 and observe a good agreement. We observe some outliers like Buenos Aires which has a very small car ownership rate and thus lower than expected  $CO_2$  emissions and areas such as New York which appears to be one of the largest transport  $CO_2$  emitter in the world [45]. This result illustrates the role played by public transport and traffic in modulating transport-related  $CO_2$  emissions. Most importantly, we identify urban sprawl ( $\sqrt{A}$ ) as a major criterion for transport emissions. We note that if we introduce the average population density  $\rho = \frac{P}{A}$ , we can rewrite our result as

$$\frac{Q_{CO_2}}{P} \propto \frac{\sqrt{P}}{\sqrt{\rho}}(1-p)(1+\tau) \quad (14)$$

i.e.  $\frac{Q_{CO_2}}{P} \propto \rho^{-\frac{1}{2}}$  since  $\sqrt{P}$  is a slowly-varying function within the scope of large urban areas. We understand here how Newman and Kenworthy [7] could have ob-

tained their result by assuming the density to be the control parameter. However, even if fitting data with a function of  $\rho$  is possible, our analysis shows that it is qualitatively wrong: the area size  $A$  and the public transport density  $p$  seem to be the true parameters controlling car-related quantities such as  $CO_2$  emissions. Mitigating the traffic is therefore not obtained by increasing the density but by reducing the area size and improving the public transport density. Increasing the population at fixed area would increase the emission of  $CO_2$  (due to an increase of traffic congestion leading to an increase of  $\tau$ ) in contrast with the naive Newman-Kenworthy assumption where increasing the density leads to a decrease of  $CO_2$  emissions.

We also note that  $\frac{\partial \log Q_{CO_2}}{\partial \log p} = -\frac{p}{1-p}$  can be relatively large (in absolute value) while  $\frac{\partial \log Q_{CO_2}}{\partial \log A} = \frac{1}{2}$  is small which suggests that the increase of transport density is in general more efficient than an area decrease (which is also less feasible). We also understand that if mass rapid transit is efficient in reducing transport-related greenhouse emissions, on the other hand reducing the road capacity  $c$  (to deter individuals from driving) may not be a good remedy, as we identified drivers with individuals with no other solution. Worse, by making their trips longer, diminishing  $c$  indirectly contributes to higher emissions and pollution as well as potentially socially segregating situations.

### Average commuting time

Finally, we can also estimate the average commuting time (details are given in Appendix 2) and obtain for the one-trip commuting time  $\tau_c$  averaged over the population the following expression

$$\bar{\tau}_c = p \left( f + \frac{g\sqrt{A}}{v_m} \right) + (1-p) \frac{g\sqrt{A}}{v_c} (1 + \tau) \quad (15)$$

where  $g$  is a geographical factor that encodes the spatial complexity of trips. The comparison with the data displays relatively large fluctuations, but our analysis seems to capture the main trend and the one-parameter fit over  $g$  using our expression Eq. 16 leads to the average value  $g \approx 0.203$ . We note that for a uniform distribution of residences, a simple calculation leads to  $g = 2/3\sqrt{\pi} \approx 0.376$  and in the simple isotropic case where the density decreases with the distance  $r$  to the center as  $\rho(r) = \rho_0(1 - r/L)$ , we obtain  $g = 1/3\sqrt{\pi} \approx 0.188$ . The average value  $g$  obtained by the fit is in between these two theoretical estimates. However, polycentrism and more generally the spatial organization of the city and its infrastructure certainly play an important role here and our model can only provide a first approximation to the average commuting time.

## DISCUSSION

We presented a simple model for the car traffic and its consequences in large cities. This model illustrates how a combination of statistical physics, economical ingredients and empirical validation can lead to a robust understanding of systems as complex as cities. In particular, this approach is in contrast with previous works where it was a priori assumed that urban density was pivotal, and our aim here was to capture the essence of the urban mobility phenomenon. Our analysis shows that traffic related quantities are governed by three factors: access to mass rapid transit, congestion effects and the urban area size. In order to reduce  $CO_2$  emissions for example, our model suggests to increase public transport access either by increasing the density around MRT stations or to increase the density of public transport (in contrast with the conclusions of an econometric study in the US [14]), or to reduce the urban area size (impossible in most contexts). Increasing the cost of car use seems actually unable to lower car traffic in the absence of alternative transportation means. Also, it is important to note that increasing in general the density at fixed both area size and MRT access would actually increase  $CO_2$  emissions, in sharp contrast with the commonly accepted paradigm about the effect of urban density. Finally, we insist on the fact that in this model we voluntarily left out a number of parameters such as other transportation modes (buses, tramway), polycentrism, the transport network structure, fuel price and tax, dynamic road pricing, etc. but our main point was to fill a gap for understanding traffic in urban areas by proposing a parsimonious model with the smallest number of parameters and the largest number of predictions in agreement with data. Given the simplicity of this model we cannot expect a perfect agreement with data for various and different cities, but it seems that this approach captures correctly all the trends and identifies correctly the critical factors for car traffic. This seems to be a basic requirement before adding other factors and increasing the complexity of the model. Also, it seems at this stage necessary to first encourage the measure and sharing of data such as the density of public transport in order to propose further tests of our theoretical framework.

**Acknowledgments.** VV thanks the IPhT for support during his internship.

### Appendix 1: Data sources

We studied 25 metropolitan areas from Europe, America, Asia and Australia. The number of cities was limited by the availability of data on MRT accessibility and reliable modal share estimates. All the data is freely available and we list here their sources.

- The definition of a metropolitan area varies from one country to another but we aimed at assembling a statistically coherent set of agglomerations. Populations and areas were collated from Wikipedia data on metropolitan statistical areas in accordance with national definitions.
- Various indicators were compiled from diverse sources: the quantity  $p$  is taken from transit accessibility reports [33–37, 41]. A common metric is the share of individuals living within 1 km from a MRT station (this is the assessed maximal walking distance). We note however that this is an unusual indicator of mobility that is not always easy to find and was here the main bottleneck for getting a larger set of cities. Modal shares were obtained from [38–44], CO<sub>2</sub> emissions from [45–49], and commuting time from [50–58].
- Values of time were assessed by taking half of the hourly wage after tax for each city [59, 60].
- Congestion delays were taken from the TomTom index [61], except for Seoul and Tokyo where we had no data (we assumed an average delay of 50%).
- The road capacity was computed from the congestion delay  $\tau$  by  $c \sim P/\tau^{1/\mu}$  with the value  $\mu = 2$  that we used for all cities.
- For the velocities  $v_m$  and  $v_c$  and the costs  $C_c$  and  $f$  we choose the same values for all cities. For  $v_m$  wikipedia data [62] displays values in the range 25 – 35 km/h depending on the city and we took  $v_m \approx 30$ km/h. The free flow car velocity  $v_c$  also depends a bit on the city and varies from 30 km/h (without congestion effect) in European cities such as Paris to 56 km/h in some american cities [63]. We decided to take an average value  $v_c \approx 40$  km/h.
- For  $C_c$ , we used a cost simulator [64] which gives on average a value of  $C_c \approx 15$  USD per trip.
- For  $f$  we counted on average 10 minutes to reach the station, 10 minutes to reach the office and 10 minutes for transit for a total of about  $f \approx 30$  minutes. This value can certainly be improved but it is difficult to get and we expect it not to vary too much (at least not over more than one order of magnitude).

## Appendix 2: Average commuting time

The commuting time for MRT users is given by  $f + g\sqrt{A}/v_c$  while for car users it is  $g\sqrt{A}(1 + \tau)/v_c$  where  $g$  is a geographical factor. We therefore obtain for the one-trip commuting time  $\tau_c$  averaged over the population the

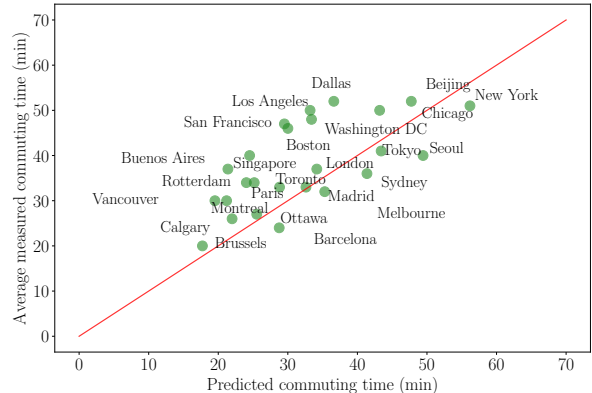


FIG. 4: Average commuting time  $\bar{\tau}_c$  measured for different cities versus the predicted value of our model (Eq. 16). We perform the fit on the single parameter  $g$  using Eq. 16, leading to an average value  $g \approx 0.203$  (the Pearson correlation is here 0.65).

following expression

$$\bar{\tau}_c = p \left( f + \frac{g\sqrt{A}}{v_m} \right) + (1 - p) \frac{g\sqrt{A}}{v_c} (1 + \tau) \quad (16)$$

The comparison of this result with empirical data is shown on Figure 4. Even if we observe relatively large fluctuations, our analysis seems to capture the main trend and the one-parameter fit using our expression Eq. 16 leads to the average value  $g \approx 0.203$ . We note that for a uniform distribution of residences, a simple calculation leads to  $g = 2/3\sqrt{\pi} \approx 0.376$  and in the simple isotropic case where the density decreases with the distance  $r$  to the center as  $\rho(r) = \rho_0(1 - r/L)$ , we obtain  $g = 1/3\sqrt{\pi} \approx 0.188$ . The average value  $g$  obtained by the fit is in between these two theoretical estimates. From Fig. 4 we can compute the ‘effective  $g_{\text{eff}}$ ’ for each city and compare it to the average value  $g$ . This quantity  $g_{\text{eff}}$  (and the ratio  $\eta = g_{\text{eff}}/g$ ) encodes both the complexity of the population and activity densities, and of the transportation infrastructure. For our set of cities, the average ratio is of order 1.37 and we observe outliers with small values such as Barcelona ( $\eta = 0.04$ ), Seoul ( $\eta = 0.62$ ), and large ones (Buenos Aires 1.92, Rotterdam 2.32, Singapore 3.38). Most of the cities (76%) have however here a ratio  $\eta$  larger than 1 which implies that our model underestimates in general the commuting distance. Many local effects can explain the variations observed in the commuting distance: the existence of polycentricity could reduce the commuting distance (for example, it seems that we overestimate the commuting time for New York which might be a consequence of polycentrism), but other factors such as poor transportation infrastructures (or traffic bottlenecks due to geographical constraints) could have the opposite effect of increasing this quantity. In general



we expect that the transport network structure will have an important impact, especially if it is very anisotropic in space. The determination of the commuting distance for each city probably implies to take into account a large number of specific details, but our analysis shows that it is consistent to assume that the order of magnitude of this quantity is  $\sqrt{A}$ .

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\* Electronic address: Corresponding author. Email: marc.barthelemy@ipht.fr

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