Polyhedral Dataflow Programming: a Case Study
Romain Fontaine, Laure Gonnord, Lionel Morel

To cite this version:

HAL Id: cea-01855997
https://cea.hal.science/cea-01855997v2
Submitted on 14 Aug 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Polyhedral Dataflow Programming: a Case Study

Abstract—Dataflow languages expose the application’s potential parallelism naturally and have thus been studied and developed for the past thirty years as a solution for harnessing the increasing hardware parallelism. However, when generating code for parallel processors, current dataflow compilers only take into consideration the overall dataflow network of the application. This leaves out the potential parallelism that could be extracted from the internals of agents, typically when those include loop nests, for instance, but also potential application of intra-agent pipelining, or task splitting and rescheduling.

In this work, we study the benefits of jointly using polyhedral compilation with dataflow languages. More precisely, we propose to expend the parallelization of dataflow programs by taking into account the parallelism exposed by loop nests describing the internal behavior of the program’s agents. This approach is validated through the development of a prototype toolchain based on an extended version of the $\Sigma$ language. We demonstrate the benefit of this approach and the potentiality of further improvements on relevant case studies.

Keywords—parallelism; dataflow programming; compilation; runtime system; load-balancing.

I. INTRODUCTION

Multi-core processors are everywhere, from high-end servers to user-oriented embedded systems like cellphones or task-specific accelerators. Applications that need to take advantage of the parallelism offered by these hardware are now numerous and range from latency-sensitive compression/decompression algorithms (eg video processing) to compute-intensive ones like machine-learning algorithms.

Programming these applications and taking advantage of the hardware parallelism is still a considerable challenge from the programmer’s point-of-view. For some of these applications, dataflow programming is a premier choice because this programming style naturally fits the designer abstraction model. As a plus, dataflow is amenable to quite efficient parallelization because it naturally exposes task, data and pipeline parallelism.

In a dataflow programming language, programmers describe their application as a set of side-effect-free actors or agents that communicate solely through First-In-First-Out channels (FIFOs). Agents can be seen as independent processes, ie sequential programs, that interact through reading (resp. writing) data from (resp. to) their input (resp. output) FIFOs. There exists a large panel of dataflow languages, whose characteristics differ notably. One major point of variability is the scheduling of agents and their communications. There is indeed a continuum from the synchronous dataflow languages like Lustre [1] or Streamit [2], where the scheduling is fully static, to general communicating networks like KPNs [3] or RVC-Cal [4] where a dedicated runtime is responsible for scheduling tasks dynamically, when they can be executed.

So far, parallelization techniques for dataflow programs have focused on taking advantage of the decomposition in agents, potentially duplicating some agents to have several instances that work on different data items in parallel [5]. In the presence of big agents, the programmer is left with the splitting (or merging) of these agents by-hand if she wants to further parallelize her program, or at least give this opportunity to the runtime, which in general only sees agents as non-malleable entities. In the presence of arrays and loop-nests or, more generally, some kinds of regularity in the agent’s code, we believe however that the programmer would benefit from automatic parallelization techniques such as those implemented within polyhedral compilation tools. We propose to use such sequential code parallelization techniques to parallelize the sequential code that describes an agents’ behavior.

The work reported in this paper aims at demonstrating the practical advantage of combining the dataflow paradigm with the polyhedral optimization framework. We empirically demonstrate this by building a proof-of-concept tooling approach, using existing tools on existing languages. This paper’s contributions are:

- A tentative approach combining dataflow programming with polyhedral compilation in order to enhance program parallelization by leveraging both inter-agent parallelism and intra-agent parallelism (ie regarding loop nests inside agents).
- An implementation of this approach using a state-of-the-art dataflow language as well as classical polyhedral tools.

Part of this work was carried out while Lionel Morel was with INSA Lyon, Université de Lyon.
• An evaluation of the approach on several example benchmarks.
• A discussion about the costs and benefits of the approach.

The rest of the paper is organized as follows. Section II introduces background on both dataflow programming languages (II-A), including the ΣC dataflow language, and the polyhedral model (II-B). Section III explains our proposition, namely jointly parallelizing loop nests that are found inside dataflow agents. Section IV describes our the experimental setting, the toochain that supports the ΣC language and the Pluto tool we use for automatic loop parallelization. Section V describes our experiments. These comprise three applications: a toy matrix multiplication, an implementation of the Deriche algorithm for image edge detection and a simple instance of artificial neural network parallelization. Section VI concludes and draws perspectives to this work.

II. BACKGROUND

A. Data Flow Programming

First dataflow programming models were proposed in the seminal works of G. Kahn [5] and J. Dennis [6]. The main concept consists in decomposing the application one wants to build as a set of independent processes (we will call them agents in the rest of the paper) that communicate solely through First-In-First-Out (FIFO) channels.

Since the mid 70s, models and languages following the same philosophy have flourished. A main characteristic that distinguishes these models is the ability to determine statically (or not) how many data tokens are exchanged by agents on the FIFO channels. When these quantities can be determined at compile-time, these models are named static (Boolean, static, cyclo-static, etc). This gives nice properties to such programs, namely boundedness of memory usage and static schedulability. When data rates are not fixed statically, agents need to be scheduled at runtime. Such languages are qualified as dynamic dataflow.

ΣC: Our experiments are applied to programs written using the ΣC programming language [7]. ΣC implements the Cyclo-Static Dataflow (CSDF) model [8] where data rates of agents are known statically and can change periodically.

Throughout this paper, we use the example of the Deriche algorithm [9], an optimal edge-detection algorithm for discrete bi-dimensional images. Here we only describe the structure of the program. Written in ΣC, the implementation is composed of 6 agents that are connected as shown in Figure 1. Each of these agents applies a transformation to an image-size matrix. Links between agents represent FIFO channels, which are the unique mean that can be used to share data among agents.

An agent’s behavior is defined using a DSL in which one describes:

- the interface of the agent (its input and output ports)
- a set (at least one) of functions that describe what happens when the agent is activated.

The code given in Figure 2 defines the agent L1 to have one input (resp. output) flow on which the agent, everytime it is triggered, will read (resp. write) HEIGHT float values. The start function is triggered at each of L1’s activations and performs some computations to define the value to be written on its output flow. Here, the computation consists in a for loop iterating over all the elements read from (resp. written to) input and output ports.

The compilation process implemented in the ΣC toolchain is in charge of transforming these dataflow language concepts into runtime concepts. Agent activities are translated into sequential programs encapsulated into runtime threads. Depending on the target architecture, FIFO communication channels can be translated into efficient shared-memory implementations or into distributed communication mechanisms. The ΣC runtime is then in charge of allocating memory regions and scheduling the activities corresponding to agents, either relying on an underlying operating system or through dedicated scheduling policies.

B. Polyhedral model

The second building block of our approach is a compilation and optimisation framework for imperative kernels that perform intensive computations, namely, the polyhedral model. This framework [10] provides exact dependence analysis information where statement instances (i.e., statements executed at different loop iterations) and array elements are distinguished. The exact dependence information

Figure 1. Representation of the dependency graph in Deriche’s algorithm.

```
agent L1()
{
  interface
  {
    in<float> input;
    out<float> output;
    spec{ input[HEIGHT]; output[HEIGHT] };
  }

  void start (void) exchange {input in[HEIGHT], output o(HEIGHT)}
  {
    float ym1 = 0, ym2 = 0, xm1 = 0;
    for(int i = 0; i<HEIGHT; i++){
      o[i] = a1 * in[i] + a2 * xm1 + b1 * ym1 + b2 * ym2;
      xm1 = in[i];
      ym2=ym1;
      ym1=o[i];
    }
  }
}
```

Figure 2. ΣC implementation of the L1 agent.
obtained through this analysis and the use of linear programming techniques to explore the space of legal schedules [11] is what constitutes the base of the polyhedral model for loop transformations.

Figure 3 illustrates the polyhedral representation with an example. The statement \( S \) is executed approximately \( \frac{N^2}{2} \) times during the execution of this loop. The triangular region expressed as a set of constraints, called the domain of \( S \), represents this set of dynamic execution instances. Accesses to array \( A \) from each of these statement instances can be succinctly captured through affine functions of the loop iterators. The dependences are also expressed as a function between two statement instances. From this mathematical description, the framework is capable of extracting a set of valid schedules (i.e. new ways to execute loop statements), compute tiled, parallel, pipelined versions of the same code, depending on the optimization function (locality, maximum parallelization, ...), and then generate their code.

The “traditional” use of polyhedral techniques in optimizing compilers typically focuses on loop transformations of polyhedral kernels. PLuTo [12] is a now widely used push-button tool for automatically parallelizing polyhedral loop nests that tries to optimize locality in addition to parallelization. There is also significant work in data layout optimization for polyhedral programs where analyses are performed to minimize the program’s memory requirement [13]. Polyhedral techniques for loop transformations are now adopted by many production level compilers, such as GCC, IBM XL, and LLVM.

Recently, polyhedral techniques are being applied to many different areas besides loop transformations. One natural application of automatic parallelization techniques is in verification of given parallelizations where the tools take parallelized programs as inputs, and use polyhedral analysis to guarantee the absence of parallel bugs [14], [15]. Moreover, a huge amount of research is done to extend the applicability of polyhedral optimizations [16], such as hybrid techniques that mix static compilation and dynamic tests [17]–[19].

The polyhedral model is becoming a standard to reason about regular programs and to effectively perform kernel optimizations inside SCOPS (static control parts of a program, where it is effectively applicable).

Our proposal, described next, is an integrated language approach to use both task and instruction parallelism within a unique setting.

### III. General Approach

This section describes our arguments in favor of a new integrated approach that would mix in the same language dataflow idioms to express dataflow parallelism and “la polyhedral” model to express instruction parallelism.

The advantages of a dataflow approach are:

- Streaming algorithms [4], scientific workflows [20], and many other applications, are already thought as agents communicating data through FIFOs.
- From Kahn [3] proposition to the Tensorflow [21] language, a wide variety of (more or less semantically well-founded) languages share the common idea of letting the programmer express as much parallelism as he can, dataflow being one (or the main) of the paradigm proposed.
- In some well-defined variants like the Static/Synchronous DataFlow [22], the problem of statically scheduling agents with a maximum parallelism is shown to be decidable. It is the case of the \( \Sigma \subset \) language we use in our experiment.
- In some other variants, the user benefits from clever runtime supports.

But this is theory. All these approaches compile separately (and agnostically) the code inside the agents, thus they may miss some opportunities for static compilation as well as static or dynamic scheduling, like:

- the intrinsic parallelism of a given agent;
- the organization of data inside a given agent.

With this information made explicit, the dataflow compiler and scheduler would take the decision of splitting agents, pipelining or merging them, in order to exploit all the intrinsic parallelism of a given application. We can even imagine being able to express and schedule several applications running on the same parallel machine [23].

However, up to now, the developer was in charge of writing “well-optimized” agents. This approach is clearly error-prone, and lack flexibility. Moreover, the agents’ code then begins to be overspecialized and not portable anymore. We propose to solve this issue by letting the programmer write the application she has in mind, and have the compilation chain harness the application’s potential parallelism.

This integrated approach gives opportunities to the developer to express her knowledge of the application she designs. However the theoretical counterpart would not be trivial as it involves being able to express and compute hierarchical schedules so that the actual aggressive compilations done statically do not interfere with the task scheduling of agents, whether scheduled statically or at runtime. Indeed, there is a huge design space from explicitly expressing all parallel
statements as tasks and schedule them independently (with a possibly unreasonable cost) to individual two-steps scheduling of tasks, then sub-tasks (which may be simpler to design but may lose inter-tasks optimization opportunities).

Our long term objective is to go towards such a formal framework to express, compile and run dataflow applications with intrinsic instruction or pipeline parallelism.

Motivating example: As a motivational example, let’s look again at the Deriche image transformation application, shown in Figure 1.

This program is intended to deal with images in a pipelined fashion. When L1 and L2 have finished manipulating a first image, the combination phase of their result (agent L3) can start while L1 and L2 are fed with data corresponding to a new image. This program is naturally described in a dataflow manner. More importantly, the succession of phases L1, L2, etc. follows the mathematical description (see [9]).

One of the limitations of dataflow programming as it is done today is that the dataflow compiler sees each agent as a separate compilation unit. It is therefore unable to optimize code across agent’s boundaries. As an example, consider the agent L1 of Figure 2. Figure 4 gives the code of agents L2 and L3.

Each of these agents contains a loop iterating on its input data to produce its output data. Intuitively, we would like the compiler to consider these loop nests as a potential source of optimization. It could then decide to 1) fuse actors L1, L2 and L3 as well as 2) apply loop-based parallelization. This would produce an agent L123 such as the one of Figure 5 where for instance a parallel loop has been identified. Of course, this is one possible transformation and our ideal compiler would have many possibilities to choose from.

Proposed approach: This paper is a first proposition of a combination of the polyhedral model framework with a production dataflow language, namely $\Sigma C$. Through the implementation of three non trivial case studies, we explore the relationships between dataflow, pipeline and instruction parallelisms, how they interfere at compile time and at runtime.

This relationship is non trivial to predict, since the polyhedral model is able to capture a potentially infinite parallelism that neither the $\Sigma C$ compiler, or runtime, is capable to reason on. The scheduling of a “parallel” agent has to make a compromise between the number of resulting “sub agents” and the intrinsic cost of having too many agents to orchestrate at runtime. Moreover, the benefit of the potential parallelism may be lost if there is too many FIFOs that increase memory pressure.

In essence, what we state in this paper is that the polyhedral model and the dataflow paradigm are going toward two different directions that will be reconciliated only if we express all their capabilities in a unique formal framework. This paper is a first experimental step to validate...
the potentiality of the approach.

IV. IMPLEMENTATION

To experiment on the idea of combining polyhedral loop parallelization with agent-level parallelism, we have combined the ΣC language and the ΣC toolchain together with the Pluto optimizing tool.

The complete workflow is depicted in Figure 6. This workflow has been applied identically on all our running examples. First we have written the original ΣC program, defining agents, some with loop nests as required by the application’s implementation choices. As the ΣC language only allows one-dimensional arrays, we then have delinearized this version (this could have been done automatically with a technique adapted from [24], for instance) and added Pluto annotations on loop nests for which we have identified a potential benefit.

The loop nests are then fed to Pluto individually. For each nest, Pluto returns a semantically-equivalent loop nest along with OpenMP annotations. The original loop nest is then replaced by the new one in the ΣC agent. The ΣC program is then compiled to C and then through GCC to an optimized C program. The original loop nest is operated on loop nests is straightforward, consisting, as shown in Figure 8 of simply adding scop/endscop pragmas around the loop nests that need to be parallelized.

V. EXPERIMENTS

A. Use cases

We demonstrate our proposal on three non-trivial example programs.

The first one is the Deriche edge detection implementation already described in Section II-A. This application comprises several agents among which six include loop nests on which polyhedral compilation can be applied.

Our second program is an instance of an artificial neural network (ANN) program that can be used to classify images. ANNs are composed of layers that are pipelined. Each layer comprises a static number of perceptrons that produces a single dataflow output while reading a number of input flows coming from the preceding layer. The architecture of such an ANN is depicted in Figure 7.

Figure 7 shows the ΣC implementation of a layer agent that gathers the activity of column perceptrons. Each perception of a given layer computes a linear combination of its input [i] with its weight configuration [i][j]. All the computations are gathered into a unique loop nest of depth 2. At the end, a non linear function (fns_activ) is applied to the output vector.

This network corresponds to the decision phase of the ANN only. The training of our ANN was performed with another program that exports the weights and the settings of the trained network. This is out of the scope of this paper, but suffice it to say that this training program has been implemented in Python, using the Keras library [25]. The specification of the network (weights and settings) is written by the python training program to a plain-text file. This file is then parsed, at compile time, by the root graph of the ΣC program. It specifies the number of layers, and for each layer, its size, its weight matrix, and the activation function to use. Hence, the whole structure (level of task parallelism, depth) of the ΣC program is configurable by the learning phase.

Finally, we implemented a block-based matrix multipli-
Concerning applications, we have made the following implementation and experimentation choices. To further study the capabilities offered by our approach, these choices should be extended to consider larger data and more agents as well, where applicable. In the case of the Matrix Multiplication, we have only manipulated matrices of size 1000*1000 elements. The multiply-accumulate grid consists of only 4 agents. This seemed a reasonable trade-off to limit the amount of data exchanged by agents on FIFO channels. The downside is that it limits the potential parallelism of the application. Concerning the Deriche case study, images used were of size 7786*3000 (ie 23 MPx). The number of agents is fixed by the global design choice and does not depend on data sizes, as it is the case for MatMult. Also, we have implemented a second version of Deriche, where agents L1, L2 and L3 have been manually replaced by agent L123 of fig. 3. Finally, ANN was used to classify 1000 images of 784 pixels each. The network considered contains 3 dense layers of 784 neurons each and an output layer of 10 neurons. This topology has been chosen in order to have several layer agents pipelined as it allows to take advantage of pipeline parallelism.

VI. RESULTS

Figure 11 depicts a comparison of performance results for our three programs running on the full Cluster environment (16 physical cores, 32 threads). Figure 12 shows a similar comparison for the Desktop environment (2 physical cores, 4 threads).

For each program, we use as baseline the timing performance of the initial \( \Sigma C \) program where agents have been implemented following the “dataflow informal semantics of the algorithm” (each agent implements a functionality). We then selected one or two agents as candidates for automatic optimization with tiling, and also compare with automatic parallelization of the same agent(s) with Pluto. Results are reported for execution on 4 cores on the desktop platform and 16 cores on the cluster platform.

The most notable result is that with little effort, essentially consisting in adding straightforward Pluto annotations to loop nests, the user can obtain non-negligible performance improvement on the execution time of her dataflow programs.
Performance are improved by tiling alone and can be further slightly more improved by using both tiling and OpenMP parallelization. Again, using these options is straightforward. We can then surely hope to obtain more interesting results by pursuing our goal of tightly combining polyhedral compilation with dataflow programming languages.

Two performance retreats can be observed, for MatMult and ANN, when using OpenMP on the desktop platform. This is explained by the fact that when using tiling all 4 cores of machines are already fully busy. So when trying to parallelize more by creating more threads at runtime, the OpenMP runtime tends to overload the system’s scheduler: global performance suffers from the negative impact of too many thread context switches.

Finally, we show in figure 13 the speed-up (in %) that is gained by applying the transformations suggested in section III over the Deriche usecase. The fusion of agents L1, L2 and L3 presented in figure 4 leads to a gain in performance of at least 11% when using tiling on the code of the resulting agent. The gain is limited to a few percents when using both tiling and automatic parallelization. At the time of writing, this still needs to be further investigated in order to understand why both optimizations conflict in this manner. We also need to pursue experiments on cluster-like machines.

In the polyhedral model community, the dataflow paradigm is more of an intermediate representation than an actual programming feature. The polyhedral process networks (PPN) [26], communicating regular processes (CRP) [27], and data-aware process networks (DPN) [28] are generated from a unique sequential program that is fully polyhedral, or at least, where SCOPs have been correctly identified. Further optimizations are made, like hierarchical polyhedral scheduling for CRPs, or efficient implementation of communicating buffers for DPNs [29]. The Compaan compiler [30] transforms applications in the field of signal and image processing (thus inherently dataflow applications [31], written in Matlab) to PPNs, from which they can automatically derive an hardware description for FPGA platforms. Although there is a significant amount of research for deriving efficient control for PPNs with particular constraints coming from the hardware [32], all this process remains polyhedral. Like the synchronous proposition of [33], we believe that these polyhedral works should be carefully integrated in our setting in order to gain benefit of both task and instruction parallelism; however we do not want to restrict the input language to a particular subclass of statically optimizable kernels, like in [34].

The approach described in [35] tries to conciliate what the authors call “macro dataflow” and the polyhedral model in order to benefit from the optimization facilities of the state-of-the-art polyhedral frameworks. This paper is the most related to ours, and is as far as we know the only other attempt at combining explicit agents and polyhedral kernels. Contrarily to our proposition to use an existing language (ΣC) and its ecosystem, they propose to use a more restrictive language (DFGL) to define the “macro” dataflow part as well as kernels, for which they propose a polyhedral compilation (“intra-step” optimization). The coordination of the “inter-step” parallelism is left to the underlying runtime system. The expressivity of the language actually compiled remains however in the classical polyhedral domain, since the scheduling problem is resolved by encoding the whole graph (macro and micro parts) as polyhedral dependencies that are solved (and scheduled) classically. We believe that a more general language approach should at least be able to have the same polyhedral expressivity, but be general enough to express non-regular behaviors inside and outside agents.

Finally, the authors of [36] propose an execution model for single program multiple data (SPMD) on GPUs, based on a polyhedral model based formulation. They propose a way to extract “thread parallelism” from actual (non fully polyhedral) applications. This approach is more a runtime approach than a language approach, and is specific to GPUs. In [37], the authors propose a polyhedral-based prec Compilation phase for their runtime system DAGuE, in order to expose data exchange information that is further used by the runtime. Our approach is more “friendly” to the compiler since the SigmaC language makes some of these
communications explicit. However both approaches share
with us the idea of the integration of polyhedral techniques
inside more pragmatic compilers and runtime that target full
applications.

VIII. CONCLUSION

We have proposed an approach for parallelizing \( \Sigma C \)
programs that takes advantage of the language’s constructs
to deal with task, pipeline and data parallelism and uses
polyhedral compilation techniques to further parallelize loop
nests inside the application’s agents. The approach is val-
ified with a set of non-trivial case studies. These case-
studies show, that the use of polyhedral compilation to
parallelize the internals of some agents increases programs’
performance by a factor between 1.3 and 3.5, depending
on the application and the parallelization technique used
on loop nests. We also show in one example case that
the combination of agent fusion and loop optimization can
improve performance by 11 %.

These results have been obtained with a simple experi-
mental approach only using off-the-shelf tools. These result
encourage us to pursue research on combining expertise
from dataflow programming languages and polyhedral com-
ilation. Our long term objective is to go towards a formal
framework to express, compile and run dataflow applications
with intrinsic instruction or pipeline parallelism.

We plan to investigate the following directions:

- A language approach: propose new stream programing
  models where all kinds of parallelisms are expressed
  explicitly, and where all activities from code design to
  compilation and scheduling can be cleanly expressed.

- An experimental approach: explore various areas of
  applications from classical dataflow examples like radio
  signal and video processing to more recent applications
  in deep learning algorithmic. This will enable us to
  identify some potential (intra and extra) agents’ op-
  timization patterns that could be leveraged into new
  languages idioms.

ACKNOWLEDGMENT

We are grateful to people at Kalray for allowing us to
experiment with their \( \Sigma C \) compiler. Some of the experiments
presented in this paper were carried out using the Grid’5000
testbed, supported by a scientific interest group hosted by In-
ria and including CNRS, RENATER and several Universities
as well as other organizations (see https://www.grd5000.fr).

This work was also partially funded by the French Na-
tional Agency of Research in the CODAS Project (ANR-17-
CE23-0004-01).

REFERENCES

synchronous data flow programming language lustre,” Pro-
cedings of the IEEE, vol. 79, no. 9, pp. 1305–1320, Sep

[2] W. Thies, “Language and compiler support for stream pro-
grams,” Ph.D. dissertation, Massachusetts Institute of Tech-
nology, 2009.

programming,” in Information processing. North-Holland,
1974.

J.-F. Nezan, and O. Deforges, “Reconfigurable video
coding on multicore,” Signal Processing Magazine, IEEE,
http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=5230810

of stream programs on multicore architectures,” Ph.D. dis-
sertation, Massachusetts Institute of Technology. Dept. of


L. Cudennec, V. David, P. Dore, P. Dubrulle, B. D. d.
Dinechin et al., “Extended cyclostatic dataflow program
compilation and execution for an integrated manycore pro-
cessor,” Procedia Computer Science, vol. 18, pp. 1624–
com/science/article/pii/S1877050913004730

[8] G. Bilsen, M. Engels, R. Lauwereins, and J. Peper-
straete, “Cyclo-static dataflow,” Signal Processing, IEEE
jsp?arnumber=485935

implemented optimal edge detector,” International Journal
[Online]. Available: https://doi.org/10.1007/BF00125164

[10] P. Feautrier, “Dataflow analysis of array and scalar re-
ferences,” International Journal of Parallel Programming,

problem, II, multi-dimensional time,” International Journal

[12] U. Bondhugula, A. Hartono, J. Ramanujam, and P. Sa-
dayappan, “A practical automatic polyhedral parallelizer and
locality optimizer,” in Proceedings of the 29th ACM SIGPLAN
Conference on Programming Language Design and Imple-

allocation,” IEEE Transactions on Computers, vol. 54, no. 10,

[14] V. Basupalli, T. Yuki, S. Rajopadhye, A. Morvan, S. Derrien,
P. Quinton, and D. Wonnacott, “ompVerify: Polyhedral anal-
ysis for the OpenMP programmer,” in Proceedings of the 7th
International Workshop on OpenMP, ser. IWOMP ’11, Jun.
2011, pp. 37–53.