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# Simulation of the UT Inspection of Planar Defects Using a Generic GTD-Kirchhoff Approach

Vincent Dorval<sup>1, a)</sup>, Michel Darmon, Sylvain Chatillon and Larissa Fradkin<sup>2</sup>

<sup>1</sup>CEA, LIST, F-91191 Gif-sur-Yvette, France

<sup>2</sup>Sound Mathematics Ltd., Cambridge, CB4 2AS, U.K.

<sup>a)</sup>Corresponding author: vincent.dorval@cea.fr

**Abstract.** The modeling of ultrasonic Non Destructive Evaluation often plays an important part in the assessment of detection capabilities or as a help to interpret experiments. The ultrasonic modeling tool of the CIVA platform uses semi-analytical approximations for fast computations. Kirchhoff and GTD are two classical approximations for the modeling of echoes from plane-like defects such as cracks, and they aim at taking into account two different types of physical phenomena. The Kirchhoff approximation is mainly suitable to predict specular reflections from the flaw surface, whereas GTD is dedicated to the modeling of edge diffraction. As a consequence, these two approximations have distinct and complementary validity domains. Choosing between them requires expertise and is problematic in some inspection configurations. The Physical Theory of Diffraction (PTD) was developed based on both Kirchhoff and GTD in order to combine their advantages and overcome their limitations. The theoretical basis for PTD and its integration in the CIVA modeling approach are discussed in this communication. Several results that validate this newly developed model and illustrate its advantages are presented.

**Keywords:** Ultrasounds, Scattering, Kirchhoff approximation, GTD  
PACS: 43.20.El, 43.35.Bf, 43.35.Cg

## INTRODUCTION

The ultrasonic testing simulation module of the CIVA software offers ways of computing crack echoes, including semi-analytical models [1,2] and a hybrid model using finite element code (ATHENA code)[3]. In the CIVA 10 version, two semi-analytical models were available in the case of cracks: Kirchhoff and GTD [4]. They are based on different approximations and have complementary validity domains. Choosing between the two models required expertise. Moreover, in some inspection configurations, no model was entirely satisfactory. The PTD model, available in CIVA 11, aims at overcoming these limitations.

In this communication, the hypotheses of the Kirchhoff and GTD models are described in terms of Crack Opening Displacement. This formalism is then used to explain the principle of the more complete PTD model, which relies on both Kirchhoff and GTD. PTD simulated results are finally compared to both Kirchhoff and GTD simulations and in addition to finite element simulations.

## CRACK SCATTERING MODELS

The three semi-analytical models used in CIVA for the simulation of crack echoes are presented here, after defining the Crack Opening Displacement which will be used to shed light on the different approximations.

### Crack Opening Displacement

The Crack Opening Displacement (COD) is defined as the difference between the displacements on the two sides of a planar crack. It is a determining factor in the scattering of a planar crack. One way to make it appear is to develop the expression for the scattering signal given by Auld [5] in the frequency domain:

$$\delta\Gamma_{ba} = \frac{1}{4P} \int_{S_F} \left( \underline{\underline{v}}_1 \cdot \underline{\underline{T}}_2 - \underline{\underline{v}}_2 \cdot \underline{\underline{T}}_1 \right) \cdot \underline{\underline{n}} dS, \quad (1)$$

where  $\delta\Gamma_{ba}$  is the difference in the measured signal due to the presence of the crack,  $P$  is the excitation power,  $\underline{\underline{v}}$  a particle velocity,  $\underline{\underline{T}}$  a stress,  $S_F$  the flaw surface and  $\underline{\underline{n}}$  the local outward normal to that surface. The index 1 denotes the field obtained when the emitting probe emits and the flaw is absent. The index 2 denotes the field obtained when the receiving probe emits and the flaw is present.

We assume that the surface of a planar crack behaves as a free surface. Therefore, the presence of the flaw implies that the stress projected on the normal to the surface is zero. This leads to the disappearance of the first term of equation (1). The COD appears in the remaining term when the contributions on the two sides on the crack are combined: the stress  $\underline{\underline{T}}_1$  in the absence of the crack is continuous and the normal  $\underline{\underline{n}}$  has opposed directions on each sides. It is then possible to make the COD appear:

$$\delta\Gamma_{ba} = -\frac{i\omega}{4P} \int_{S_+} \overline{COD} \cdot \underline{\underline{T}}_1 \cdot \underline{\underline{n}} dS, \quad (2)$$

$S_+$  is one of the sides of the crack, arbitrary chosen,  $\omega$  is the angular frequency and the COD is defined here as the difference between the particle displacement on the side  $S_+$  and on the other side at the same location.

The focus of this communication is the computation of crack echoes. The ultrasonic fields in the absence of the crack are known. In the applications presented here, they are calculated in CIVA software using a paraxial beam method [6]. In this framework, the modeling of crack scattering can be reduced to the calculation of the COD. In the next sections, the Kirchhoff and GTD models are described with an emphasis on the corresponding CODs.

### The Kirchhoff approximation

The Kirchhoff approximation consists, for each point of the defect surface, in assuming that the surface is an infinite plane tangent to the actual surface. The ultrasonic field at this point is expressed as the sum of an incident plane wave and the plane waves reflected by the infinite plane. This approximation is also sometimes called the ‘‘tangent plane approximation’’. The ultrasonic field is supposed to be zero on the non-insonified side of the crack, therefore the COD coincides with the displacement on the insonified side.

Coefficients for different incident and diffracted waves and angles for the scattering of ultrasonic waves by cracks are given by Chapman [7]. The current CIVA implementation of Kirchhoff model computes the integral of equation (2) numerically based on the amplitudes of incident and reflected waves [8], instead of using expressions such as those of Chapman where the amplitudes of these waves are implicit. This approach has the advantage of being easily adapted to any material.

The Kirchhoff expression of the echo in an integral form over the flaw surface allows a lot of flexibility in practical applications, as the variations of the fields over the surface or complex flaw geometries can easily be taken into account.

The Kirchhoff approximation implies that the COD can be non-zero at the edge of a crack. This is non-physical, as the two sides of a crack join at the edge, therefore there cannot be any difference between their displacements at such locations. This leads to errors in the calculated diffraction when the contribution of the edge is significant. As a consequence, the Kirchhoff approximation will tend to be inaccurate in the calculation of edge diffraction.

Conversely, it will tend to give reliable results for specular echoes, where the contributions of the crack surface dominate compared to the contributions of the edges.

## The Geometrical Theory of Diffraction

The Geometrical Theory of Diffraction (GTD) is based on an exact solution for the COD of a semi-infinite crack. As a consequence, it is complementary to the Kirchhoff approximation: it is appropriate for modeling edge diffraction, but not for specular echoes.

Notable early works on the application of GTD to elastodynamics have been done by Achenbach and Gaudesin [9]. For the purpose of an interpretation in terms of COD, another way of obtaining coefficients [10] is more convenient. This later approach makes use of expressions similar to the integral of equation (2). For incident plane waves, the amplitude of  $\underline{T}_1$  is an oscillating exponential and the integral can be treated as a spatial Fourier transform of the COD. This Fourier transform is calculated asymptotically using the stationary phase method. The asymptotic calculation is accurate for most wave directions, except for those near critical angle where it can cause errors and for those around specular and transmission directions.

The main limitation of the GTD coefficients is their divergence around specular and transmission directions, which is related to the semi-infinite crack assumption.

## The Physical Theory of Diffraction

There are several ways of overcoming the limitations of the Kirchhoff and GTD models. One is to improve the GTD model in order to correct its results in the specular and diffraction regions. This leads to methods such as the Uniform Asymptotic Theory (UAT) of diffraction [11-13]. These methods constitute an improvement over GTD but they are inconvenient for handling complex geometries and taking into account the variations of the fields over the surface of the flaw.

The Physical Theory of Diffraction (PTD) is another way to overcome these limitations. It has been proposed under several forms by several authors: It was first developed in acoustics [14], and later in elastodynamics [15]. The work presented here is based on another elastodynamic form proposed in [16]. It allows combining all the advantages of the Kirchhoff (suitable for specular and echoes, convenient for complex geometries and field variations) and GTD (suitable for diffraction echoes) models. The PTD echo of a 2D planar crack can be decomposed in the following way:

$$Echo_{PTD} = Diffraction_{GTD}^{Edge1} - Diffraction_{Kirchhoff}^{Edge1} + Echo_{Kirchhoff} + Diffraction_{GTD}^{Edge2} - Diffraction_{Kirchhoff}^{Edge2}, \quad (3)$$

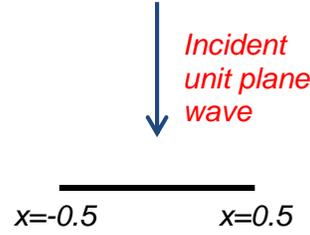
where  $Echo_{Kirchhoff}$  and  $Diffraction_{GTD}$  are the outputs of the Kirchhoff and GTD model.  $Diffraction_{Kirchhoff}$  is a correction term that ensures the consistency of the model. This term corresponds to the diffraction by a semi-infinite crack simulated in the Kirchhoff approximation: it can be obtained by applying the same asymptotic approach as for the GTD model, except the exact COD is replaced by the Kirchhoff approximated COD.

$Diffraction_{Kirchhoff}$  has two notable properties. Around specular directions, it diverges in the same way as the GTD coefficient. For observation directions for which edge diffraction is predominant, it agrees with standard Kirchhoff coefficient. As a result, in expression (3), the two  $Diffraction_{Kirchhoff}$  terms will cancel  $Echo_{Kirchhoff}$  in diffraction directions and they will cancel the singularities of the two  $Diffraction_{GTD}$  terms in specular directions. PTD therefore agrees with GTD for diffraction echoes and with Kirchhoff for specular echoes.

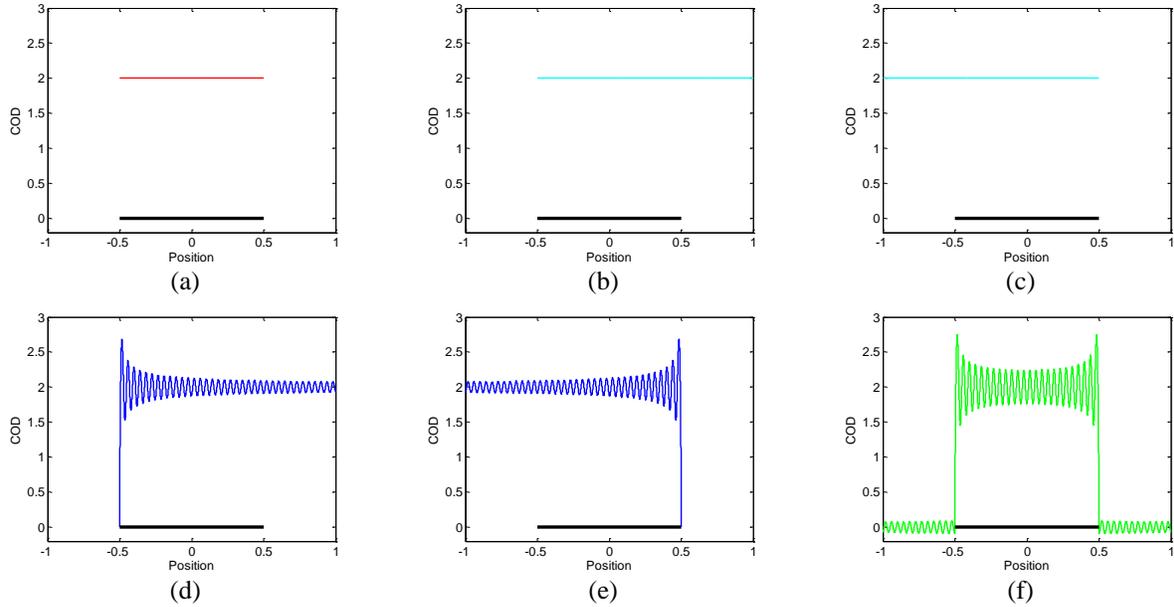
An interpretation in terms of COD clarifies the way PTD combines Kirchhoff and GTD models. As the echoes are proportional to the COD, the echo decomposition of equation (3) translates into a COD decomposition:

$$COD_{PTD} = COD_{GTD}^{Edge1} - COD_{Kirchhoff}^{Edge1} + COD_{Kirchhoff} + COD_{GTD}^{Edge2} - COD_{Kirchhoff}^{Edge2}, \quad (4)$$

where the COD associated to an edge is the COD of a semi-infinite crack starting at the corresponding edge. The case of a 2D crack in normal incidence (figure 1) will be considered as an illustration.



**FIGURE 1.** Simplified case for the illustrations of the COD. The crack size has been taken as unity.

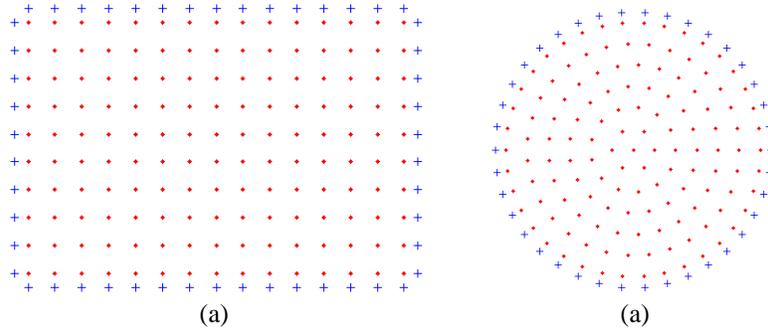


**FIGURE 2.** CODs corresponding to different components of the PTD model. COD is expressed as multiples of the displacement of the incident longitudinal wave. The unit distance is the size of the crack. The location of the crack is represented by a thick black line. (a): Kirchhoff model, (b): Kirchhoff diffraction for the left edge, (c): Kirchhoff diffraction for the right edge, (d): GTD for the left edge, (e): GTD for the right edge, (f): Total PTD.

In this simplified case, the COD corresponding to the Kirchhoff approximation (figure 2(a)) is uniform along the crack. As expected with this approximation, it has an unrealistic discontinuity at the edges of the crack, where the COD should be zero. Figure 2(b) and 2(c) shows the COD corresponding to the Kirchhoff approximation for a semi-infinite crack starting at the left edge. Figure 2(d) and 2(e) show the exact solution for the COD corresponding to a semi-infinite crack starting at the left edge [17]: it illustrates the exact solution for the COD, which is implied but not explicitly given in the GTD model. Figure 2(f) is the COD corresponding to the sum of the 5 previous CODs, as expressed in equation (4). It illustrates how the corrective terms cancel the undesirable effects of each model: the discontinuity due to Kirchhoff disappears, and so do the semi-infinite contributions of GTD. Only small residual oscillations due to GTD remain.

For more complex crack geometries or for non-homogeneous fields, the treatment of the edges is similar as in the simple case: the crack is still assumed to be semi-infinite, whereas the field is assumed to be constant and to have the same value as at the edge everywhere. However, the surface integral of the Kirchhoff approximation is done by taking into account both the specific geometry of the crack and the spatial variations of the field.

In the CIVA implementation of this method, the surface and the edges of the crack are both meshed. The surface mesh is used for the computation the  $COD_{Kirchhoff}$  term in equation (4), and the edge mesh the  $Diffraction_{GTD-Diffraction_{Kirchhoff}}$  terms. Figure 3 illustrates how defects can be meshed.

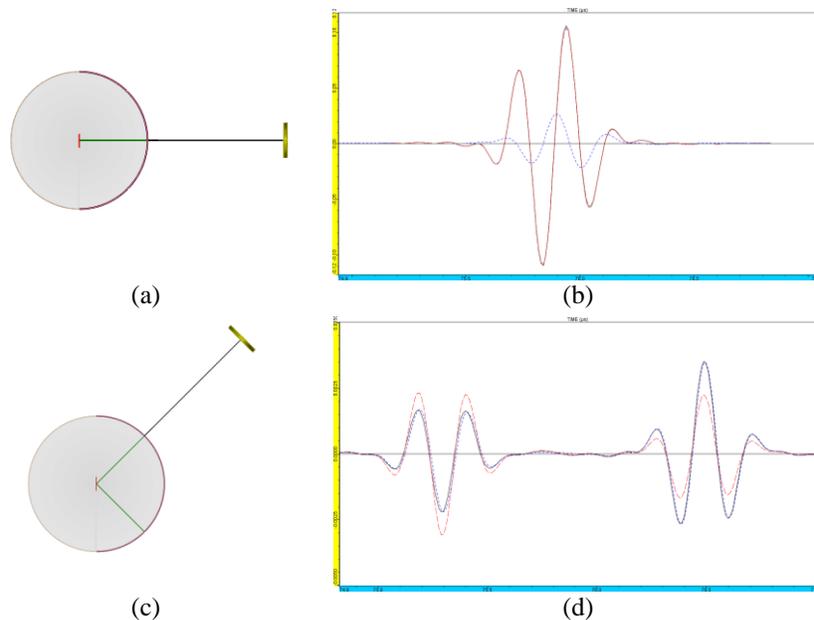


**FIGURE 3.** Examples of crack meshes for the calculation of echoes in the PTD approximation. Red dots are the mesh points for the Kirchhoff model, blue crosses are the mesh points for the edge diffraction. (a): rectangular crack, (b): circular crack.

## COMPARISON OF RESULTS OBTAINED WITH THE DIFFERENTS MODELS

In this section, simulated echoes obtained for a crack seen from different incident angles in a pulse-echo configuration are shown. They will illustrate how PTD behaves compared to Kirchhoff and to GTD for different directions, and compared to a finite elements result.

### Comparison echo calculated with the three models

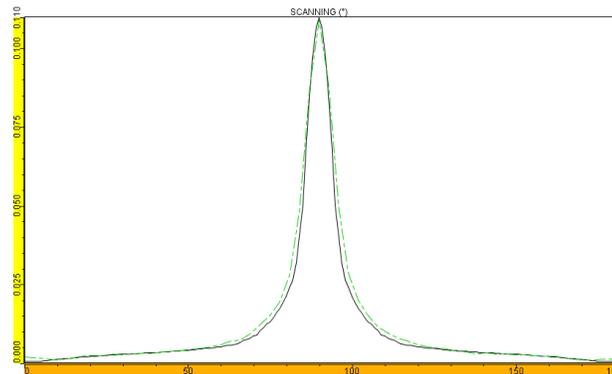


**FIGURE 4.** Two measurement pulse-echo configurations and echo signals for a 5mm crack in a cylinder inspected using compressional waves, in a 2D configuration. (a): normal incidence configuration, (b): normal incidence echoes, (c): 45° incidence configuration, (d): 45° incidence echoes. For (b) and (d), the solid black lines are the PTD results, the red dashed lines (larges dashes) are the Kirchhoff results and the blue dashed lines (small dashes) are the GTD results.

The echoes of figure 4 (b) correspond to specular reflection. As expected, PTD agrees with Kirchhoff and not with GTD. It should be noted that for practical use in CIVA, the divergence of the GTD coefficients is attenuated by preventing the coefficient to rise too much around specular directions. It explains why the GTD result is low. An unaltered GTD model would yield an echo with extremely high amplitude, and would still not agree with PTD.

The echoes of figure 4 (d) correspond to edge diffraction. As expected, PTD agrees with GTD and not with Kirchhoff.

## Comparison of echoes obtained using the PTD model and a finite elements approach



**FIGURE 5.** Variations of the echo amplitude in the pulse-echo configuration of Figure 4 for a wider range of incident angles. The horizontal axis is the angle (in degrees) corresponding to the incidence on the crack,  $90^\circ$  being the normal incidence.

The results of figure 5 have been obtained by computing the echo in the configuration of figure 4 for a wider range of probe positions around the cylinder. Computations have been performed using the CIVA implementation of the PTD model and the CIVA-Athena finite element module [3]. A satisfactory agreement is obtained for all angles, regardless of whether the predominant phenomenon is edge diffraction or specular reflection. Significant differences, such as those of figure 4, would have been obtained for diffraction angles between PTD and Kirchhoff and for specular angles between PTD and GTD.

## CONCLUSION

The PTD model combines the Kirchhoff and GTD models to cover the validity domain of both of them. The interpretation in terms of Crack Opening Displacement presented in this paper provides insight in the hypothesis of the model and could be used as a framework for new model developments.

PTD has been implemented in the CIVA 11 software as a way to improve the simulation of crack echoes. It makes the results more accurate in cases where Kirchhoff or GTD were not sufficient, and removes the need for the user to select the appropriate echo model. Limitations remain in cases that could not be treated by Kirchhoff or by GTD (namely, scattering at critical angles or echoes of very small cracks) and are of interest for additional developments.

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