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Modelling of specimen interaction with ferrite cored coils by coupling semi-analytical and numerical techniques

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Abstract. Two numerical techniques, the finite integration technique (FIT) and the surface integral equation (SIE) method are combined with the semi-analytical calculation of the field in planar conducting pieces in order to calculate the interaction of such specimens with ferrite-cored eddy-current probes. Coupling different techniques allows us to choose the best suited for each part of the configuration and to better exploit the existing symmetries.

Keywords. Ferrite-cored probe, finite integration technique, surface integral equation, semi-analytical solutions, coupling of techniques

1. Introduction

Eddy-current testing (ECT) probes feature ferrite cores in a large number of practical situations. The use of ferrite cores enhances the electromagnetic coupling between probe and the specimen, and results consequently in signal amplification and hence improved signal-to-noise ratio. From the modelling point of view, however, the strong coupling between probe and specimen complicates the analysis since the convenient separation of probe and work-piece, which allows the treatment of the latter using semi-analytical tools, is no longer valid (at least rigorously speaking) and the two of them have to be considered simultaneously.

The straight-forward approach to deal with such problems is the application of a numerical technique, like the finite elements method (FEM), to the entire configuration, which may result in a computationally demanding problem.

An alternative approach to the brute-force discretization is to split the problem into a probe and a specimen-related subproblem, similarly with the case of the air-cored probe, coupling the two sub-problems in a way that properly addresses the bi-directional interactions between them. Such approach will enable us to exploit the possible symmetries of the consisting parts, and to apply the best suited technique for each subproblem. Furthermore, relative motion or rotation of the consisting parts may be viewed as a rear-

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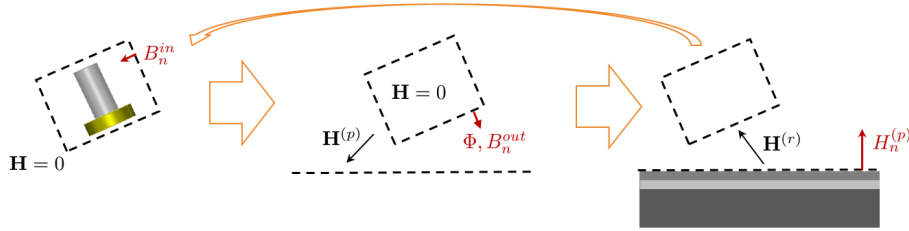


Figure 1. Schematic demonstration of the FIT/semi-analytical coupling approach: decomposition of the initial problem into a probe and a workpiece-related subproblem and solution propagation of one into another.

rearrangement of the same objects, the analysis of which remains the same and hence only their mutual interactions must therefore be modified.

The application of the latter approach for the calculation of ferrite-cored probes in interaction with planar work-pieces, is the subject the present contribution. The probe ensemble is treated using either the finite integration technique (FIT) [1] or the surface integral equation (SIE) [2,3], depending whether the probe is rotationally symmetric or not, whereas the effect of the planar piece is taken into account by means of semi-analytical tools. Basic tool for the realisation of the coupling is the equivalence theorem. The results of the two presented approaches will be compared with plain FEM simulations using the commercial package COMSOL [4].

2. Coupling principle

The working principle for the FIT/semi-analytical coupling is depicted schematically in Fig. 1. We consider an arbitrary fictitious closed surface that entirely encloses the probe ensemble. According to the equivalence principle, the solution of the interior (in respect to the introduced surface) problem is formally equivalent with the one the original problem, provided that the (actual) normal magnetic flux density on the boundary is given. Using the same arguments, we can set an equivalent problem in the external to the surface domain, based on the (assumedly known) normal magnetic flux and scalar magnetic potential distribution (or equivalently of the normal and tangential magnetic flux distribution) on the boundary [5].

Starting with an arbitrary initial magnetic flux density distribution on the boundary, the internal (probe) problem is solved numerically using FIT, and the calculated solution is propagated in the external domain by means of the updated boundary field distribution. Using the semi-analytical solution for the planar piece, the effect of the specimen is propagated back to the probe domain and the procedure is repeated until convergence is reached [5].

A similar procedure is also followed for the SIE/semi-analytical coupling. The main difference here consists in the choice of the domain boundary, which coincides with the surface of the ferrite. Furthermore, the boundary conditions used for the transition between the two domains are intrinsically satisfied by the method formulation.

3. Solution of the internal problem: probe field calculation

3.1. Modelling of axisymmetrical probes using the finite integration technique

The FIT in its classical formulation [1] is based on direct discretisation of the continuous Maxwell's equations using a pair of mutually orthogonal, staggered (Yee-type) grids (extensions of the basic schema to non-orthogonal grids have also been proposed, yet they will not be considered herein). The so-derived discrete Maxwell's equations (called in FIT jargon as the Maxwell's grid equations) read for the magnetostatics

$$\mathbf{C}\widehat{\mathbf{h}} = \widehat{\mathbf{j}}_s, \quad (1)$$

$$\widetilde{\mathbf{S}}\widehat{\mathbf{b}} = \mathbf{0}, \quad (2)$$

where \mathbf{C} , $\widetilde{\mathbf{S}}$ stand for the discrete curl and div operator, respectively, $\widehat{\mathbf{h}}$ are the magnetic voltages along the primary grid edges and $\widehat{\mathbf{b}}$ the magnetic fluxes across the grid facets [1,6].

We introduce the discrete magnetic scalar potential ϕ by means of the relation $\widehat{\mathbf{h}} = \widehat{\mathbf{h}}_i - \mathbf{G}\phi$, where \mathbf{G} is the discrete grad topological operator and $\widehat{\mathbf{h}}_i$ stands for a partial solution to the inhomogeneous Ampere's equation (1). Substituting to the Gauss' equation for the magnetic field, we obtain the magnetostatic formulation

$$\widetilde{\mathbf{S}}\mathbf{M}_\mu\mathbf{G}\phi = \widetilde{\mathbf{S}}\mathbf{M}_\mu\widehat{\mathbf{h}}_i + \widehat{\mathbf{b}}^{(b)}, \quad (3)$$

with \mathbf{M}_μ being the diagonal permeability matrix, derived by the discretisation of the magnetic constitutive relation. Its detailed expression can be found in [6]. $\widehat{\mathbf{b}}^{(b)}$ denotes the normal component of the magnetic flux on the FIT boundary, i.e. it yields the contribution from the non-homogeneous Neumann conditions, and hence it accounts for the coupling effects.

Let us now restrict ourselves to configurations where the material distribution and the excitation current $\widehat{\mathbf{j}}_s$ are rotationally symmetric. No assumption is made instead concerning the boundary term $\widehat{\mathbf{b}}^{(b)}$. Due to the rotational symmetry of the geometry, the solution can be decomposed into a series of modes along the azimuthal direction φ , namely [7]:

$$\phi = \sum_{m=-N}^N \phi_m e^{jm\varphi}. \quad (4)$$

The sum theoretically extends over an infinite number of modes, however the series can be truncated to a specific number of terms N (which is determined by geometric criteria) with negligible impact to the accuracy of the results. This is a well studied issue in modal techniques and bears standard treatment.

Starting from a conventional 3D FIT discretisation using a cylindrical grid, we introduce (4) into the magnetostatic formulation (3), and, after taking the orthogonality of the exponential functions into account, we arrive at the following set of discrete equations

$$\left[\widetilde{\mathbf{S}}\mathbf{M}_\mu^T\mathbf{G} - \alpha_m^2\mathbf{M}_\mu^N \right] \phi_m = \widetilde{\mathbf{S}}\mathbf{M}_\mu^T\widehat{\mathbf{h}}_i\delta_{m,0} + \widehat{\mathbf{b}}_m^{(b)}, \quad \text{for } m = -N \dots N, \quad (5)$$

with $\alpha_m = m\Delta\varphi$, $\Delta\varphi$ being the angular opening of a FIT cell and $\delta_{m,0}$ the Kronecker delta. Notice that in the above equation the 3D spatial vectors are reduced to their 2D projections for each mode. In other words, the spatial discretisation is now restricted to the normal to azimuthal direction plane (i.e. the $\rho - z$ plane). Finally, the material matrix \mathbf{M}_μ is decomposed into a co-planar \mathbf{M}_μ^T and a normal to the discretisation plane $\rho - z$ one \mathbf{M}_μ^N , and the topological operators \mathbf{G} and $\tilde{\mathbf{S}}$ are the 2D operators corresponding to that plane. Hence for $m = 0$, (5) reduces to the usual 2D FIT formulation.

Once the internal problem has been solved and the potential ϕ_m distribution throughout the FIT box is known, the solution has to be propagated into the external region in order to calculate the effect of the workpiece. The external field calculation is based on the second Green's theorem, which in the case of free space outside an arbitrary boundary Γ reads:

$$\mathbf{H}(\mathbf{r}) = -\nabla \int_{\Gamma} \left[g(\mathbf{r}, \mathbf{r}') \frac{\partial \Phi(\mathbf{r}')}{\partial n'} - \frac{\partial g(\mathbf{r}, \mathbf{r}')}{\partial n'} \Phi(\mathbf{r}') \right] dS', \quad (6)$$

where $g(\mathbf{r}, \mathbf{r}') = 1/4\pi|\mathbf{r}-\mathbf{r}'|$ is the scalar Green's function of the free space. The Green's function can be developed onto a basis of cylindrical wave functions, namely

$$g(\mathbf{r}, \mathbf{r}') = \frac{1}{4\pi} \sum_{m=-\infty}^{\infty} e^{im(\varphi-\varphi')} \int_{\kappa} g_o^m(\rho, z, \kappa) g_s^m(\rho', z', \kappa) d\kappa \quad (7)$$

with

$$g_s^m(\rho', z', \kappa) = \begin{cases} J_m(\kappa\rho')e^{\pm\kappa z'}, & z \leq z' \\ \frac{1}{\pi} I_m(\kappa\rho')e^{i\kappa z'}, & \rho \geq \rho' \end{cases}, \text{ and } g_o^m(\rho, z, \kappa) = \begin{cases} J_m(\kappa\rho)e^{\mp\kappa z}, & z \leq z' \\ K_m(\kappa\rho)e^{i\kappa z}, & \rho \geq \rho' \end{cases}.$$

For convenience in the integration, we switch between the upper and the lower expression in the above relations according to the relative position of the source and the integration point. This will permit to remain on the same branch during the integration along Γ . Note that the spectral domain is different for each case, i.e. it is $\kappa \in [0, \infty)$ for the upper expression and $\kappa \in (-\infty, \infty)$ for the lower one. Applying (7) on the FIT solution, we obtain after some algebra:

$$\mathbf{H}(\mathbf{r}) = -\frac{1}{2} \sum_{m=-\infty}^{\infty} \int_{\kappa} Q_m(\kappa) \nabla g_o^m(\rho, z, \kappa) d\kappa, \quad (8)$$

with

$$Q_m(\kappa) = \sum_{i \in \Gamma_T} \left[g_s^m(\rho_i, z_i, \kappa) b_{i,m} - \frac{\partial g_s^m(\rho_i, z_i, \kappa)}{\partial n_i} \phi_{i,m}(\rho_i, z_i) \right], \quad (9)$$

where Γ_T is the intersection of the Γ boundary with that plane, and $\phi_{i,m}$ and $b_{i,m}$ are the FIT vector elements for the m -th mode of the magnetic scalar potential and the magnetic flux density, respectively.

The elements of the boundary vector $\widehat{\mathbf{b}}_m^{(b)}$ in (5) will be thus given by the following relation

$$\widehat{b}_{m,i}^{(b)} = \mu_0 \left[H^{(p)}(\mathbf{r}_i) + H^{(r)}(\mathbf{r}_i) \right] \cdot \mathbf{n} \Delta A_i, \quad \mathbf{r}_i \in \Gamma_T \quad (10)$$

where ΔA_i is the area of the boundary facet centred at \mathbf{r}_i . $H^{(p)}$ stands for the direct contribution to the field calculated by the integration of the FIT solution on Γ_T according to (8), and $H^{(r)}$ gives the reflection from the work-piece. Its calculation will be presented in a next paragraph.

3.2. Solution for probes of arbitrary shape: the surface integral equation method

3.2.1. Formulation

In the box of the sensor, we consider the linear magnetostatic problem

$$\begin{cases} \nabla \times H = J_0 \\ \nabla \cdot \mu H = 0 \end{cases},$$

with $\mu = \mu_f$ in the ferrite and $\mu = \mu_0$ (relative permeability of vacuum) on the outside, where J_0 is the current flowing through a coil, and where H is the magnetic field. The first equation of the system allows decomposition of the magnetic field in the form

$$H = H_0 - \nabla \varphi,$$

where H_0 is the field emitted by the source J_0 with no ferrite (obtained by Biot and Savart). The second equation of the system then imposes the transmission condition

$$\left[\mu \frac{\partial}{\partial n} \varphi \right] = [\mu] H_0 \cdot n, \quad (11)$$

where the normal n is oriented from the inside to the outside of the ferrite and wherein the brackets indicate the jump on either side of its surface.

We use the single layer potential formulation to solve this problem [2,3]. The unknown is the left-hand side of the transmission condition (11),

$$\omega := \left[\mu \frac{\partial}{\partial n} \varphi \right],$$

and is the solution of the integral formulation

$$\left(\frac{\mu_f + \mu_0}{2} I - (\mu_f - \mu_0) K' \right) \omega = (\mu_f - \mu_0) H_0 \cdot n \quad (12)$$

where K' is the normal derivative of the single layer potential (Γ denotes the surface of the ferrite core):

$$K' \omega(x) := \frac{1}{4\pi} \frac{\partial}{\partial n_x} \int_{\Gamma} \frac{1}{\|x - y\|} \omega(y) dy.$$

The magnetic field can then be calculated at any observation point x outside the ferrite core like

$$H(x) = H_0(x) - \frac{1}{4\pi} \nabla_x \int_{\Gamma} \frac{1}{\|x - y\|} \omega(y) dy. \quad (13)$$

In its variational form and after discretization by a suitable Galerkin method [3], the linear system and the computation of the magnetic field are written like

$$\mathbf{S}\mathbf{X} = \mathbf{Y}_0, \mathbf{H} = \mathbf{H}_0 + \mathbf{R}\mathbf{X},$$

with

$$\begin{aligned} [\mathbf{S}]_{ij} &:= \langle t_i, \mathcal{S}b_j \rangle, & [\mathbf{Y}_0]_i &:= \langle t_i, \mathcal{Y}_0 \rangle, \\ \omega(x) &\sim \sum_j [\mathbf{X}]_j b_j(x), & [\mathbf{R}]_{kj} &:= \mathcal{R}b_j(x_k), \end{aligned}$$

where t_i and b_j are the i -th test and j -th basis functions, x_k is the k -th observation point, \mathcal{S} and \mathcal{Y}_0 correspond to the left and right terms of formulation (12), \mathcal{R} to the integral term of formulation (13), respectively.

3.2.2. Coupling process

Within the coupling, it is necessary to compute

$$\mathbf{H}_{n+1/2} = \mathbf{H}_n + \mathbf{R}\mathbf{S}^{-1}\mathbf{Y}_n \quad (14)$$

for each iteration (n) of the iterative process the term, where $\mathbf{H}_{n+1/2}$ stands for the new incident field emitted by the ferrite core on the workpiece, \mathbf{H}_n (and \mathbf{Y}_n) stands for the previous incident field emitted by the workpiece on the ferrite core (whose incident field was $\mathbf{H}_{n-1/2}$...)

The advantage of using a boundary element method lies in the significant decrease of the number of unknowns in comparison to methods such as the finite element method. When this reduction allows the use of a direct solver, the computation of $\mathbf{R}\mathbf{S}^{-1}$ (or the computation of \mathbf{R} and of the LU decomposition of the matrix \mathbf{S}) is performed only once and reused for each iteration.

Therefore, minimizing the number of unknowns of the boundary element method (while maintaining an acceptable accuracy) stands for a major issue. To this end, higher order (quadrilateral) Lagrangian elements have been used to approximate the unknown and geometry. That is to say, the shape, basis and test functions are high degree interpolation polynomials (rather than constant or linear functions). The increasing order allows to converge faster to the solution with the increase of the number of unknowns, and thus to reduce the number of unknowns while achieving a given accuracy.

4. Solution of the external problem: probe field interaction with the work-piece

The field perturbation due to the presence of the specimen can be obtained by the following Fourier integral [8]:

$$\mathbf{H}^{(r)}(\mathbf{r}) = -\frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\tilde{H}_z^{(p)}(u, v)}{a} R(u, v) e^{-az} (iu\mathbf{e}_x + iv\mathbf{e}_y - a\mathbf{e}_z) e^{iux} e^{ivy} dudv \quad (15)$$

where $\tilde{H}_z^{(p)}$ is the 2D Fourier transform of the normal to the piece interface primary magnetic field calculated by (8) or (13), \mathbf{e}_x , \mathbf{e}_y and \mathbf{e}_z denote the unit vectors in x , y and z axis and $a = \sqrt{u^2 + v^2}$. R stands for the generalised reflection coefficient of the planar piece and its expression for the case of a multilayered medium is given in [9]. Note that all variables are given in the reference frame of the plane.

5. Results

The proposed coupled approach is applied for the calculation of a cylindrical, ferrite-cored coil interaction with a planar conducting and non-magnetic half-space. The considered ferrite is rod-shaped and had negligible losses. The coil and core characteristics are given in Table 1. The half-space conductivity is taken 18 MS/m.

Table 1. Coil and core parameters

Coil parameters		Core parameters	
Inner radius r_{in}	3 mm	Radius r	2 mm
Outer radius r_{out}	6 mm	Length l	5 mm
Length l	2 mm	Relative permeability	10
Number of turns N	2000		

Fig. 2 illustrates the normal component of the induced magnetic flux density on the half-space interface calculated using the two coupled approaches (i.e. SIE/semi-analytic and FIT/semi-analytic) versus the fully numerical solution obtained by a commercial FEM package (COMSOL) [4] at 1 KHz. The probe axis is tilted in respect to the normal of the plane by an angle of 20 degrees. The convergence of the iterative solution has been reached after 5 iterations for the FIT/semi-analytical coupling and 3 iterations for the SIE/semi-analytical one. The calculation time was ca. 5 minutes for both approaches on a standard PC whereas FEM simulation time was 10 minutes on a 24 GB, Xeon PC with 8 cores.

A second comparison between the FIT/semi-analytical results and the full numerical FEM simulation has been carried out for the calculation of the probe impedance as a function of the tilt angle. The results are shown in Fig. 3

6. Conclusions

A coupled approach combining numerical with semi-analytical methods has been proposed for the calculation of ferrite-cored probes interaction with planar work-pieces. The restriction of the discretisation to a small part of the structure and the rapid convergence of the iterative procedure provide a computational efficient treatment of the interaction problem. Furthermore, the fact that the coupling method does not depend on the specific geometry of the work-piece, make the approach also applicable to other geometries for which analytical solutions are available, such as tubes, boreholes, right-angle corners etc.

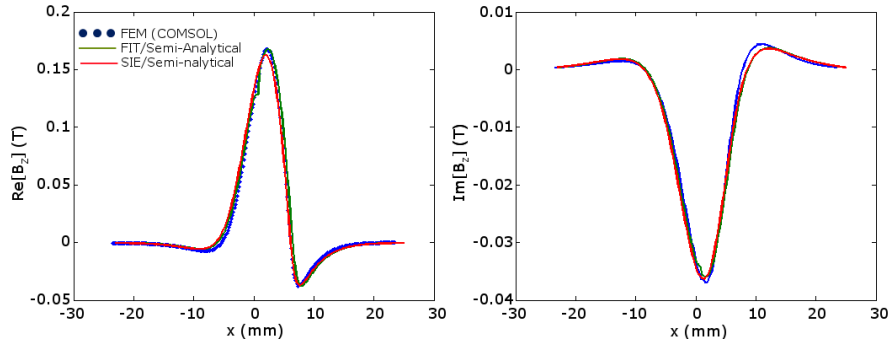


Figure 2. B_z field component on the half-space interface calculated using the two coupled approaches versus FEM results obtained with COMSOL.

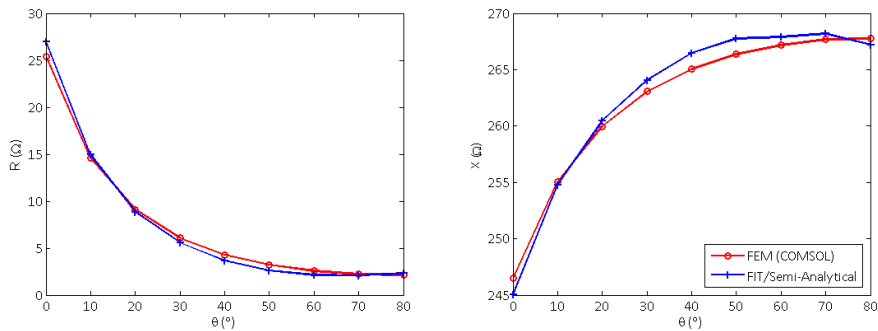


Figure 3. Real and imaginary part of the probe impedance variation as a function of the tilt angle.

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