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# On the energy efficiency of base station cooperation under limited backhaul capacity

Mireille Sarkiss · Mohamed Kamoun

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**Abstract** Recently, energy-efficient (EE) communications have received increasing interest specially in cellular networks. Promising techniques, such as multiple input multiple output (MIMO) and base station (BS) cooperation schemes, have been widely studied in the past to improve the spectral efficiency and the reliability. Nowadays, the purpose is to investigate how these techniques can reduce the energy consumption of the systems. In this paper, we address for a single-user scenario, the energy efficiency of two BSs cooperation under limited backhaul capacity. In order to evaluate the EE metric, we provide first an information-theoretical analysis based on the outage probability, for a quantization model over the backhaul. Then, we extend this EE analysis to a more practical approach with data transmission over the backhaul. For both approaches, we identify by numerical/simulation results the cooperation scenarios that can save energy depending on the backhaul capacity.

**Keywords** Energy efficiency · Base station cooperation · Outage probability · Backhaul capacity · Precoding techniques

## 1 Introduction

During the last decades, wireless communications have experienced exponential growth of mobile broadband

applications and multimedia services driving tremendous requirements on high data rates and high quality of service (QoS). However, these prominent demands have come at the expense of increasing substantially the energy consumption and the environmental carbon footprint, specially in the mobile cellular systems: GSM (2G), UMTS(3G), 3G evolution HSPA<sup>+</sup>, and 3GPP long term evolution (LTE).

On one side, for the ecological aspect, according to recent studies, the information and communication technology (ICT) represented about 1.3 % of global CO<sub>2</sub> equivalent (CO<sub>2</sub>e) emissions in 2007, where the contribution of mobile networks was estimated to be 0.2 %. These emissions are expected to more than double between 2007 and 2020 as shown by SMART 2020 Report. One major driver of this overall carbon footprint is the increasing number of worldwide subscriptions, hence the large data traffic volume generated by the broadband services. Another important contribution comes also from the network infrastructure including the radio access network (RAN) sites manufacturing, construction and the electricity consumption of base stations (BS), as well as the manufacturing and operation of the mobile devices, such as regular mobile phones, smartphones, and laptops.

On the other side, for the economical aspect, it was recently reported that energy costs can account for as much as half of a mobile service provider's annual operating expenses (OPEX). Moreover, it is expected that the number of base stations will double between 2010 and 2020, and that the overall RAN energy consumption will increase by about 40 %. With the increasing energy prices, this matter will lead to significant energy costs and thus will affect the revenue models for mobile operators.

Therefore, all these issues have prompted the emergence of energy efficiency (EE) as a new key concept for mobile communications. The main concern has been to reduce the

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energy consumption of the network infrastructure and communications as well as the CO<sub>2</sub> footprint of the network manufacturing and operating devices, for their potential economical benefits and expected environmental impact. In this regard, many academic and industrial researches have been investigating energy-aware architectures and energy-efficient techniques to reduce the energy wastage without compromising the system capacity and the quality of service. Projects have also been funded to address this subject such as the European Project “Energy Aware Radio and neTwork tecHnologies” (EARTH) [7].

Among several advanced technologies, multiple input multiple output (MIMO) schemes have attracted much attention throughout the wireless communications evolution. It is well known now that exploiting the use of multiple antennas at the transmitter and/or receiver can significantly increase the channel capacity, improve the link reliability, and achieve high data rates at no cost of extra bandwidth and power. Thereby, MIMO techniques have been widely investigated for spectral efficiency (SE) and reliability targets. Nowadays, they are adopted in many wireless standards; for example, single-user MIMO (SU-MIMO) and multiuser MIMO (MU-MIMO) modes are supported in the downlink 3GPP LTE. In addition, coordinated multipoint (CoMP) techniques, known also as network-MIMO or cooperative/coordinated MIMO, are proposed in the 3GPP LTE-advanced standard to further improve the cell coverage, the system fairness, and the spectral efficiency.

In CoMP communications, base stations cooperate together to jointly process, transmit, and receive data of one or several users in different cells. This cooperation is enabled by the presence of backhaul links connecting the BSs for information exchange between them or with a central processing unit. Depending on the quality of these backhaul links and the amount of the information which has to be exchanged (control/signalling, channel state, data), different coordination/cooperation schemes can be performed, namely inter-cell interference coordination, diversity techniques, coordinated beamforming, and joint processing. The latter one is the most promising CoMP candidate for future wireless communication standards. It exploits the inter-cell interference rather than only canceling it, thus mimicking the benefits of a large virtual MIMO array. So, many efforts have been concentrated on the multicell cooperative processing in uplink and downlink cellular networks addressing some of the challenging issues [9, 15, 16, 20, 21].

Focusing on backhaul infrastructure, according to [22], LTE-advanced small cells are expected to connect to existing, capacity-limited Internet protocol (IP) backhauls such as digital subscriber line. Therefore, we consider the capacity-limited backhaul solution in this paper. Considering downlink transmission, [20] has provided analytical insights on the impact of finite capacity backhaul links

on the throughput of multicell processing. According to a Wyner-type model and considering that the BSs are connected to a central processor via these limited capacity links, the authors have studied in [20] different transmission strategies that require different amount of information at the BSs regarding the codebook information used to communicate with the mobile stations. They have derived the closed-form achievable rates and evaluated the performance in asymptotic regimes (high backhaul capacity and high/low signal-to-noise ratio (SNR)). Similar analyses have been proposed for BSs cooperation under limited capacity backhaul links (between BSs) [17] and imperfect channel knowledge at base stations and terminals, and per antenna power constraints [18]. They have derived inner capacity bounds and showed that the system should adapt between different BS cooperation schemes depending on the channel conditions in order to optimize the rate-backhaul tradeoff.

So as noticed, the aforementioned studies on MIMO and CoMP schemes aimed at improving the systems' capacity and spectral efficiency. Nevertheless, recently, the potential of these techniques has been reconsidered for energy-efficiency purpose. For instance, [14] has presented a survey on energy-efficient wireless communications including some researches on EE of MIMO techniques that focused mainly on open-loop SU-MIMO schemes (see references therein for more details). In addition, the authors, in [2–4], have provided an information-theoretical analysis of MIMO EE communications. They have determined the optimal precoding matrices, i.e., power allocation strategy, that maximize the energy efficiency measure in different fading channels and different scenarios starting from single-user single input single output (SISO) and extending to MIMO and multiuser scenarios. Moreover, from a system level evaluation, [8] has addressed the tradeoffs between gains in cell throughput obtained from CoMP schemes and increased energy consumption induced in the base stations under different inter-site distances and CoMP cooperation sizes i.e., number of colocated BSs. Highest EE gains were observed for colocated BSs cooperation (cooperation size of three BSs) throughout all site distances [8].

Based on previous approaches of CoMP techniques with limited backhaul capacity for SE enhancement and the EE analysis of MIMO techniques, our objective in this paper is to investigate the energy efficiency of BS cooperation under constrained backhaul capacity from an information-theoretical and practical perspectives. Considering a simple model of two base stations cooperating to transmit to a single user at the cell edge, we aim at identifying the scenarios where this cooperation can be more energy saving.

The paper is organized as follows. Section 2 describes the BS cooperation model with limited backhaul capacity and defines the energy efficiency metric to be used. The information theoretical approach is then derived for

the cooperative scheme and its numerical results are presented in Section 3. The practical approach is described in Section 4 with some simulation results in realistic scenarios. Finally, conclusion is given in Section 5.

## 2 System model

We consider a downlink transmission from two base stations with a single antenna each to a single user with one receiver antenna. The user is located between both cells as presented in Fig. 1. The BSs are assumed to be perfectly synchronized in time and frequency. They are connected via limited capacity backhaul link with capacity  $C_b$  bits per channel use (bits/cu). This latter condition limits the number of bits that can be sent over the backhaul link per transmission block.

In this system model, the main base station BS1 transmits messages to its user located at the cell edge. In order to improve the transmission rate and coverage, it cooperates with the neighboring BS2 through the backhaul link to convey the same messages to this user. So, at any given time, BS1 and BS2 transmit to the user the symbols  $X_1$  and  $X_2$  with power  $P_1$  and  $P_2$ , respectively. For this downlink transmission, a rate of  $R$  bits/cu is considered and a sum power of  $P = P_1 + P_2$  is fixed, with  $P_1 = \alpha P$  and  $P_2 = (1 - \alpha)P$ ,  $\alpha$  denoting the ratio of the total sum power used by BS1. The inter-site distance between the BSs is given by  $d$  in meter, with  $d_1 = \beta d$  and  $d_2 = (1 - \beta)d$  the distances between the user and each BS.

We assume that the transmission happens over a slow flat Rayleigh fading channel where the channel remains constant over a transmitted block and varies independently from one block to another. The channel fading coefficients  $h_1$ , and  $h_2$  are respectively between BS1, BS2 and the user. They are modeled as i.i.d complex Gaussian random variables with zero mean and unit variance  $\sim \mathcal{N}_c(0, 1)$ . For the

moment, we assume that are only known at the receiver side. The white Gaussian noise  $Z$  is added to the transmitted signal. It has i.i.d complex random components with zero mean and variance  $\sigma^2 \sim \mathcal{N}_c(0, \sigma^2)$ ,  $\sigma^2 = 1$ . Thus, the received signal is given by

$$Y = \sqrt{P_1 A_1} h_1 X_1 + \sqrt{P_2 A_2} h_2 X_2 + Z, \quad (1)$$

where  $A_i, i = 1, 2$  are the path loss coefficient of each base station computed according to  $A_i = K \cdot d_i^{-\theta}$  with  $K = 10^{-3.2}$ , the noise figure of the thermal Gaussian noise is considered equal to NF= 10 dB, and  $\theta = 3.5$  for cellular networks.

In the sequel, we investigate two approaches for the transmission over the backhaul. The first one is an information theoretical approach where Gaussian symbols are transmitted from the base stations and quantization is performed over the backhaul for the cooperation. On the other side, the second approach deals with more practical scenarios where finite modulation schemes are transmitted and precoding strategies are considered with channel state information (CSI) feedback.

For both approaches, we aim at assessing the energy efficiency. Therefore, we present first the different energy efficiency metrics used in mobile communications, and particularly the EE definition to be used for the described system model.

### 2.1 Energy efficiency metrics

Several EE metrics have been defined to assess the energy consumption of the mobile communication systems [5, 11]. They can be divided into three principal categories:

- Component level metrics: They consider the components in a typical wireless equipment including antennas, radio frequency (RF) front end, baseband processor, support system, and power supply.
- Equipment level metrics: They are related to the power/energy consumption of wireless equipments such as radio base stations (RBSs) and wireless terminals.
- Network level metrics: They consider not only energy consumed by the RBS site, but also the features and properties related to capacity and coverage of the network.

For the Network level, two energy efficiency metrics are mainly used [5, 7, 11]:

- Area power consumption metric: It reflects the power consumption  $P$  of cellular cells at different site densities i.e., the coverage area  $A$  of a network in square

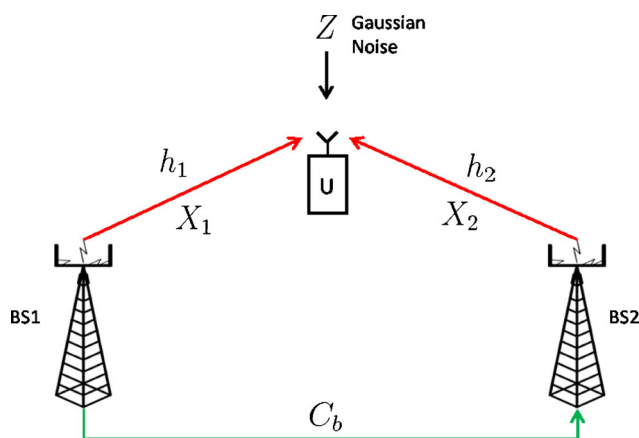


Fig. 1 Downlink base stations cooperation

meters. It is relevant for low traffic loads and is measured in watts per square meter as

$$\varrho = \frac{P}{A} \left( \text{W/m}^2 \right). \quad (2)$$

The European Technical Standards Institute (ETSI) defines also an equivalent metric for the GSM system. It is expressed as an efficiency metric instead of the above consumption metric as

$$\varrho_{A/P} = \frac{A}{P} \left( \text{m}^2/\text{W} \right). \quad (3)$$

- Bit per Joule: This metric is expected as the basic EE metric for beyond 3G cellular systems. It measures the efficiency of the system by the throughput represented by the average data rate  $R$  over the average consumed power  $P$ . It is expressed in bits per Joule or equivalently in bits per second per watt as

$$\xi = \frac{R}{P} \text{ (bit/Joule)}. \quad (4)$$

Alternatively, EARTH project employs the energy per bit metric measured in joules per bit, focusing on the network energy consumption  $E$  for a total number of  $B$  bits that were correctly delivered. Hence, it is an indicator of network bit delivery EE that can be more critical for high traffic load scenarios. It is defined as:

$$\xi_{E/B} = \frac{E}{B} \text{ (Joule/bit)}. \quad (5)$$

In [2–4], an information-theoretic approach is given to describe the energy efficiency (bits per joule) metric of MIMO communications. It is defined by the benefit per cost function i.e., what the system delivers to what it consumes, and it is expressed as

$$\xi(P, R) = \frac{F(P, R)}{P} \text{ (bit/Joule)}, \quad (6)$$

where the benefit  $F(P, R)$  represents the number of bits reliably transmitted at the rate  $R$  while consuming a certain amount of transmit power per Joule.

Following this last definition, we will derive the energy efficiency of our system model in joule per bit from an information-theoretical point of view.

## 2.2 EE metric derivation

We recall that the system model presented previously assumes a slow-fading channel. In this case, a transmitted codeword with finite length spans only one channel realization, and the Shannon capacity is no more a suitable capacity metric. Since the channel is random, there always exists a nonzero probability that a target transmission rate  $R$ , no matter how small, is not supported by a bad channel state  $H$ . The transmitted codeword is decoded successfully if the rate is lower than the instantaneous capacity  $R < C(H)$ .

However, if  $R > C(H)$ , the channel is in outage and the receiver declares a decoding error.

Therefore, in this case, with a given transmit power  $P$ , the capacity metric is defined by the outage probability for a target data rate  $R$  [13] as

$$P_{\text{out}}(P, R) = \Pr\{C(P, H) < R\}. \quad (7)$$

Based on the outage probability, an EE metric was proposed in [2–4] to measure the energy efficiency of slow-fading MIMO channels by optimizing the power allocation policy. It is the ratio between the expected throughput that is the benefit function  $F(P, R)$  and the average consumed power. The expected throughput can be seen as the average system throughput over many transmissions.

In the sequel, an equivalent metric in joules per bit will be used to define the EE of our BS cooperation model for fixed total transmission power  $P$  and under limited backhaul capacity  $C_b$ . It is given by

$$\xi(P, R, C_b) = \frac{P}{R(1 - P_{\text{out}}(P, R, C_b))}. \quad (8)$$

## 3 Information theoretical approach

In this section, a first theoretical approach is described for the cooperation model, where Gaussian symbols are transmitted and quantization is assumed over the backhaul link. The energy efficiency is then analyzed based on outage probability derivation.

### 3.1 Quantization over the Backhaul link

From information theory perspective, we consider that the transmitted symbols are i.i.d. complex Gaussian random variables such that BS1 transmits symbols  $X_1 = X \sim \mathcal{N}_c(0, 1)$  to the user. For cooperation between base stations, and since the backhaul link has finite capacity  $C_b$ , BS1 performs quantization of the signal  $X$ . It forwards then the quantized version to BS2 via the backhaul link. BS2 tries to reconstruct  $X$  and transmits an estimate  $X_2 = \hat{X}$  to the user. The transmission over the backhaul is subject to quantization noise based on the rate-distortion theory [6].

**Proposition 1** *Given the backhaul rate  $C_b$ , the quantization distortion is defined for independent complex Gaussian random variables  $X \sim \mathcal{N}_c(0, 1)$  by*

$$D(C_b) = 2^{-C_b}, \quad (9)$$

*and the backhaul can be modeled by a forward test channel of the form*

$$\hat{X} = c(X + \eta), \quad (10)$$



where  $X$  and  $\eta$  are independent and  $\eta$  represents the quantization noise with variance  $\sigma_\eta^2$  and  $c$  a constant according to

$$\sigma_\eta^2 = \frac{2^{-C_b}}{1 - 2^{-C_b}} \quad \text{and} \quad c = 1 - 2^{-C_b}. \quad (11)$$

*Proof* See Appendix A. Thus, the received signal can be redefined as

$$\begin{aligned} Y &= \sqrt{P_1 A_1} h_1 X + \sqrt{P_2 A_2} \hat{X} + Z \\ Y &= \sqrt{P_1 A_1} h_1 X + \sqrt{P_2 A_2} (1 - 2^{-C_b}) h_2 (X + \eta) + Z \\ Y &= \left( \sqrt{P_1 A_1} h_1 + \sqrt{P_2 A_2} (1 - 2^{-C_b}) h_2 \right) X + N \end{aligned} \quad (12)$$

where  $N$  is the equivalent noise with variance  $\sigma_N^2$  obtained from (11) and (13)

$$N = \sqrt{P_2 A_2} (1 - 2^{-C_b}) h_2 \eta + Z \quad (13)$$

$$\sigma_N^2 = P_2 A_2 2^{-C_b} (1 - 2^{-C_b}) |h_2|^2 + \sigma^2. \quad (14)$$

□

### 3.2 EE analysis

In order to evaluate the energy efficiency of this channel model as defined by Eq. (8), we need first to compute the outage probability. The model can be seen as a virtual MISO channel, where the user terminal has a single antenna and the BSs are equipped with a single antenna each and are connected through capacity-limited backhaul. The outage probability can be then defined by

$$\begin{aligned} P_{\text{out}}(P, R, C_b) &= \Pr\{\log_2(1 + \text{SNR}) < R\} \\ &= \Pr\left\{\log_2\left(1 + \frac{|\sqrt{P_1 A_1} h_1 + \sqrt{P_2 A_2} (1 - 2^{-C_b}) h_2|^2}{P_2 A_2 2^{-C_b} (1 - 2^{-C_b}) |h_2|^2 + \sigma^2}\right) < R\right\} \end{aligned} \quad (15)$$

$$\begin{aligned} P_{\text{out}}(P, R, C_b) &= \Pr\left\{\left|\sqrt{P_1 A_1} h_1 + \sqrt{P_2 A_2} (1 - 2^{-C_b}) h_2\right|^2 \right. \\ &\quad \left. - \lambda P_2 A_2 2^{-C_b} (1 - 2^{-C_b}) |h_2|^2 < \lambda \sigma^2\right\}. \end{aligned} \quad (16)$$

The expression can be formulated as in Eq. (16) with  $\lambda$  a constant equal to  $\lambda = (2^R - 1)$ .

Let  $S_1$  and  $S_2$  be the variables defined by

$$S_1 = \left|\sqrt{P_1 A_1} h_1 + \sqrt{P_2 A_2} (1 - 2^{-C_b}) h_2\right|^2 \quad (17)$$

$$S_2 = P_2 A_2 2^{-C_b} (1 - 2^{-C_b}) |h_2|^2 \quad (18)$$

Since the channel fading  $h_1$  and  $h_2$  are i.i.d. complex Gaussian random variables  $\sim \mathcal{N}_c(0, 1)$ , the sum of the

weighted coefficients in (17), let denote it  $s = s_1 + s_2$ , is also a complex Gaussian random variable with zero mean and variance  $\sigma_1^2 = P_1 A_1 + P_2 A_2 (1 - 2^{-C_b})^2$ , and  $s \sim \mathcal{N}_c(0, \sigma_1^2)$ . The modulus  $|s|$  of this term is a Rayleigh variable  $\sim \text{Rayleigh}(\sigma_1/\sqrt{2})$ . Then,  $S_1 = |s|^2$  follows a scaled

Chi-squared distribution  $\chi^2$  with  $2^\circ$  of freedom  $S_1 \sim \frac{\sigma_1^2}{2} \cdot \chi^2(2)$ . Similarly for  $S_2$ , the variance of the related Gaussian variable is  $\sigma_2^2 = P_2 A_2 2^{-C_b} (1 - 2^{-C_b})$  and  $S_2 \sim \frac{\sigma_2^2}{2} \cdot \chi^2(2)$ .

It can be noticed that  $S_1$  and  $S_2$  are correlated variables. Hence, the outage probability represents the probability that the difference of these correlated  $\chi^2$ -distributed variables falls below a given threshold

$$P_{\text{out}}(P, R, C_b) = \Pr\{\Delta = S_1 - \lambda S_2 < \lambda \sigma^2\}. \quad (19)$$

In probability theory, this describes the cumulative distribution function (CDF) of the random variable  $\Delta$  evaluated at  $\lambda \sigma^2 \geq 0$ , denoted by  $F_\Delta(\lambda \sigma^2)$ . It is clear that it is necessary to know the probability density function (PDF) of  $\Delta$ ,  $f_\Delta(\delta)$ , to compute its CDF as following

$$F_\Delta(\lambda \sigma^2) = 1 - \int_{\lambda \sigma^2}^{\infty} f_\Delta(\delta) d\delta. \quad (20)$$

It was proved in [12] that the difference of two correlated Gamma variables follows Type II McKay distribution. Since a  $\chi^2$  distribution is a special case of the Gamma distribution, then  $f_\Delta(\delta)$  is given by this Type II McKay distribution.

In fact,

**Definition 1 Gamma Distribution** A random variable  $X$  follows a Gamma distribution with shape parameter  $k > 0$  and scale parameter  $\theta > 0$ , denoted by  $X \sim \Gamma(k, \theta)$ , if the PDF of  $X$  is given by

$$f_X(x) = \frac{x^{k-1} e^{-x/\theta}}{\Gamma(k) \theta^k} H(x), \quad (21)$$

where  $H(\cdot)$  is the Heaviside unit step function [10].

Moreover, we have the properties:

$$X \sim \Gamma(k, \theta), \Rightarrow \epsilon X \sim \Gamma(k, \epsilon \theta) \quad (22)$$

$$E[X] = k\theta, \quad \text{var}[X] = k\theta^2. \quad (23)$$

If  $X$  is a scaled  $\mu \cdot \chi^2$ -distributed variable with  $\nu$  degrees of freedom, it follows the relationship

$$X \sim \mu \cdot \chi^2(\nu) \Leftrightarrow X \sim \Gamma(k = \nu/2, \theta = 2\mu) \quad (24)$$

Consider now two correlated  $\chi^2$ -distributed (Gamma) variables  $X_1$  and  $X_2$ , the bivariate Gamma distribution is then used to take into account their correlation [12].

**Definition 2 Bivariate Gamma Distribution** Two random Gamma variables  $X_1$  and  $X_2$  are governed by a

Bivariate Gamma distribution and denoted by  $\{X_1, X_2\} \sim \Gamma(k, \theta_1, \theta_2, \rho)$ , with  $\rho$  the correlation coefficient

$$\rho = \frac{\text{cov}[X_1, X_2]}{\sqrt{\text{var}[X_1]\text{var}[X_2]}}, \quad 0 \leq \rho \leq 1 \quad (25)$$

if their joint PDF is defined by

$$\begin{aligned} f_{X_1 X_2}(x_1, x_2) &= \frac{(x_1 x_2)^{\frac{k-1}{2}}}{\Gamma(k)(\theta_1 \theta_2)^{\frac{k+1}{2}} (1-\rho) \rho^{\frac{k-1}{2}}} \\ &\times \exp\left(-\frac{\frac{x_1}{\theta_1} + \frac{x_2}{\theta_2}}{1-\rho}\right) \\ &\times I_{k-1}\left(\frac{2\sqrt{\rho}}{1-\rho} \sqrt{\frac{x_1 x_2}{\theta_1 \theta_2}}\right) \\ &\times H(x_1)H(x_2), \end{aligned} \quad (26)$$

where  $I(\cdot)$  is the modified Bessel function of the first kind and of order  $(k-1)$  [12].

Therefore, we can conclude that  $S_1$  and  $S_2$  are bivariate Gamma variables,

$$\{S_1, S_2\} \sim \Gamma\left(k=1, \theta_1=\sigma_1^2, \theta_2=\sigma_2^2, \rho\right), \quad (27)$$

then  $S_1$  and  $\lambda S_2$  are also bivariate Gamma variables

$$\{S_1, \lambda S_2\} \sim \Gamma\left(1, \sigma_1^2, \lambda \sigma_2^2, \rho\right), \quad (28)$$

as

$$\frac{\text{cov}[S_1, \lambda S_2]}{\sqrt{\text{var}[S_1]\text{var}[\lambda S_2]}} = \frac{\text{cov}[S_1, S_2]}{\sqrt{\text{var}[S_1]\text{var}[S_2]}} = \rho.$$

Let us compute this correlation factor. From Definition 1, we have

$$\begin{aligned} E[S_1] &= \sigma_1^2 \quad \text{and} \quad \text{var}[S_1] = \sigma_1^4, \\ E[S_2] &= \sigma_2^2 \quad \text{and} \quad \text{var}[S_2] = \sigma_2^4. \end{aligned} \quad (29)$$

Since  $S_1$  and  $S_2$  are real random variables, their covariance is given by

$$\begin{aligned} \text{cov}[S_1, S_2] &= E[(S_1 - E[S_1])(S_2 - E[S_2])] \\ &= E[S_1 S_2] - E[S_1]E[S_2]. \end{aligned} \quad (30)$$

$$\begin{aligned} E[S_1 S_2] &= E\left[\left|\sqrt{P_1 A_1 P_2 A_2 2^{-C_b}} (1 - 2^{-C_b}) h_1 h_2\right.\right. \\ &\quad \left.\left.+ P_2 A_2 \sqrt{2^{-C_b}} (1 - 2^{-C_b})^{3/2} h_2^2\right|^2\right] \\ &= P_2 A_2 2^{-C_b} (1 - 2^{-C_b}) \\ &\quad \left(P_1 A_1 + 2 P_2 A_2 (1 - 2^{-C_b})^2\right). \end{aligned} \quad (31)$$

Developing this expression, we find  $E[S_1 S_2]$  as given in Eq. (31).

Then,

$$\text{cov}[S_1, S_2] = (P_2 A_2)^2 2^{-C_b} (1 - 2^{-C_b})^3, \quad (32)$$

and the correlation factor is equal to

$$\rho = \frac{\text{cov}[S_1, S_2]}{\sigma_1^2 \sigma_2^2} = \frac{P_2 A_2 (1 - 2^{-C_b})^2}{P_1 A_1 + P_2 A_2 (1 - 2^{-C_b})^2}. \quad (33)$$

As mentioned earlier, the difference between these bivariate Gamma variables, namely  $\Delta = S_1 - \lambda S_2$  follows Type II McKay distribution [12].

**Proposition 2** *In our Rayleigh fading case, the CDF of Type II McKay distribution reduces to*

$$P_{\text{out}}(P, R, C_b) = F_{\Delta}(\lambda \sigma^2) = 1 - \frac{1-c}{2} \exp\left(-\frac{1+c}{b} \lambda \sigma^2\right) \quad (34)$$

with the parameters  $B$  and  $c$  defined as

$$\begin{aligned} b &= \frac{2\lambda \sigma_1^2 \sigma_2^2 (1-\rho)}{\sqrt{(\sigma_1^2 + \lambda \sigma_2^2)^2 - 4\lambda \sigma_1^2 \sigma_2^2 \rho}} \\ c &= -\frac{\sigma_1^2 - \lambda \sigma_2^2}{\sqrt{(\sigma_1^2 + \lambda \sigma_2^2)^2 - 4\lambda \sigma_1^2 \sigma_2^2 \rho}} \end{aligned} \quad (35)$$

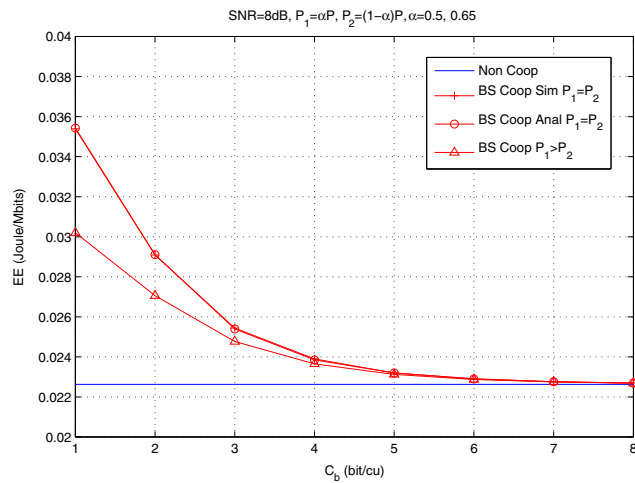
*Proof* : See Appendix B.  $\square$

### 3.3 Numerical results

In this section, we evaluate the energy efficiency of the BS cooperation model of Section 3 with limited backhaul capacity  $C_b$  and fixed total transmission power  $P$  via numerical results. The EE performances are represented in terms of joule per megabits versus the backhaul capacity  $C_b$  bits/cu or the transmit power  $P$  expressed in terms of the SNR in decibels. For all the results, we consider a transmission rate of  $R = 2$  bits/cu, a bandwidth of  $W = 10$  MHz and a distance between the BS and the user equal to  $d_1 = d_2 = 200$  m for the path loss computation.

Figure 2 demonstrates the accuracy of the analytical EE metric, hence the analytical outage probability by comparing to simulation results. In addition, it compares the BS cooperation model to a noncooperative model where only BS1 is transmitting to the user with the total power  $P$ . It is evident that cooperation with any power allocation policy between BS1 and BS2 ( $P_1 = P_2$  or  $P_1 > P_2$ ) cannot be as energy-efficient as noncooperation unless the backhaul capacity is unlimited (perfect backhaul).

In Fig. 3, the focus is on the case of uniform power allocation between BS1 and BS2 ( $\alpha = 0.5$ ), for different  $C_b$  values. We can see that cooperation with low capacity  $C_b$

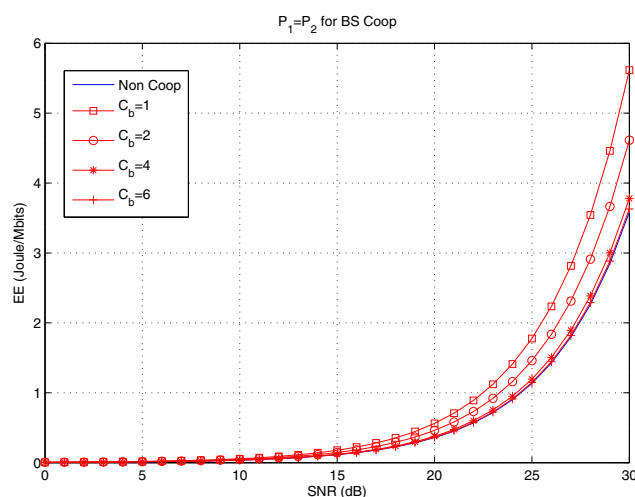


**Fig. 2** EE vs  $C_b$  for a fixed SNR

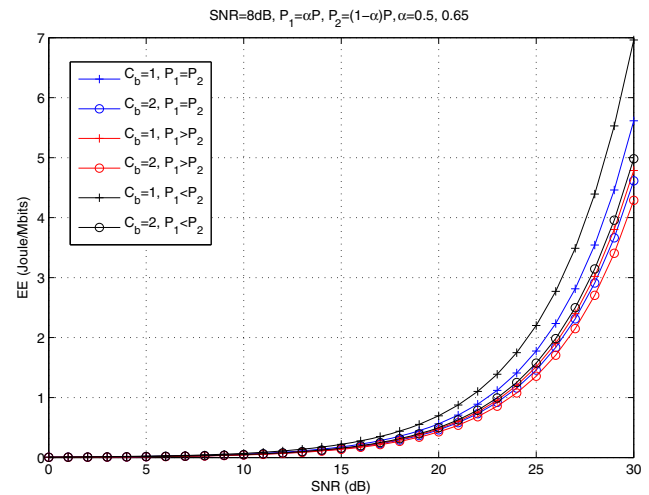
increases the EE in joule per megabits. In fact, the  $P_{out}$  is increased, so we need more energy (power) to transmit reliably the same number of bits.

Now, comparing the power allocation between BS1 and BS2, we can observe in Fig. 4 that with limited  $C_b$ , BS1 transmitting at a higher power  $P_1 > P_2$  is more energy saving than both BSs transmitting at the same power or BS2 transmitting at  $P_2 > P_1$ . It is obvious that this last case is subject to more outage events since  $C_b$  is low. Increasing the power at BS2 results then in wastage of energy. Whereas, with unlimited backhaul capacity, for instance  $C_b = 6$  bits/cu, all the power configurations are almost equivalent as it is illustrated in Fig. 5.

Finally, we represent in Fig. 6 the energy efficiency as a function of the distance  $d$  between both BSs with  $d_1 = \beta d$  and  $d_2 = (1 - \beta)d$  the user's distances to BS1 and BS2,

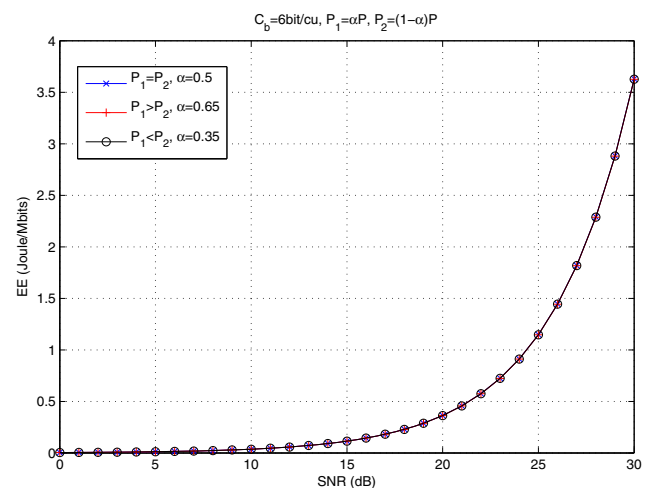


**Fig. 3** EE vs SNR for different  $C_b$  and with  $P_1 = P_2$



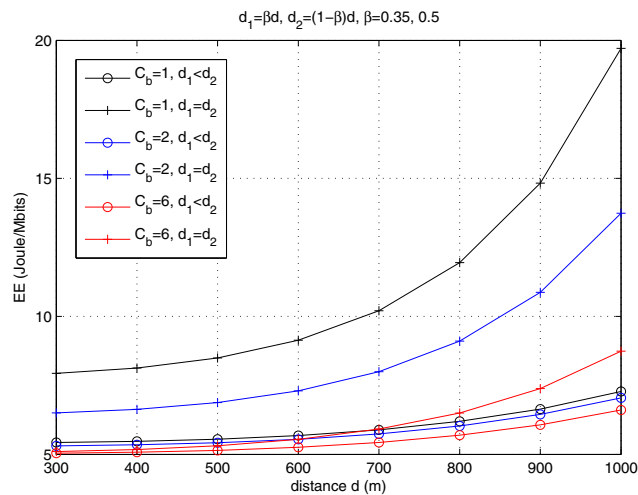
**Fig. 4** EE vs SNR for limited  $C_b$

respectively, and where  $\beta$  can take values 0.35 and 0.5. The total transmit power is fixed to  $P = 10$  W with uniform power allocation between both base stations ( $\alpha = 0.5$ ). Figure 6 shows that EE in joule per megabits increases, for any  $d$ , with limited backhaul capacity due to more outage events. It also increases, for any  $C_b$ , with larger distances due to higher path loss, thus higher outage. Moreover, the figure confirms that asymmetric configuration where the user is at a distance  $d_1 < d_2$  is more energy saving than the case where it is at equal distance from base stations. This EE gain is more significant for low  $C_b$ , specially at high distances  $d > 700$  m. In addition, for  $d_1 < d_2$ , the energy consumption for limited  $C_b$  is relatively close to the unlimited backhaul one since better signal is received from BS1 in both cases.



**Fig. 5** EE vs SNR for unlimited  $C_b$





**Fig. 6** EE vs distance for different  $C_b$

## 4 Practical approach

In this section, the EE analysis based on outage probability derivation is extended to practical scenarios by considering concrete modulations schemes, realistic precoding strategies, and LTE-based channel state information feedback.

### 4.1 Data transmission over the backhaul

The system model is the same as described in Section 2, but we assume here that base stations BS1 and BS2 transmit quadrature amplitude modulation (QAM)-modulated symbols  $X_1$  and  $X_2$  to the user located at the cell edge. Considering the transmission rate of  $R$  bits/cu and the backhaul link with capacity  $C_b$  bits/cu, we can distinguish two cases depending on the transmitted data on the backhaul.

- If  $C_b \geq R$ , i.e., with unlimited backhaul capacity, both base stations can cooperate to transmit such that the received signal is given by

$$Y_2 = \sqrt{P_1 A_1} h_1 X_1 + \sqrt{P_2 A_2} h_2 X_2 + Z. \quad (36)$$

- If  $C_b < R$  i.e., with limited backhaul capacity, the number of bits that can be sent over the backhaul link per transmission block is limited. In this case, there is  $C_b/R$  bits that are transmitted by both BSs and  $(1 - C_b/R)$  bits transmitted only by BS1 with the total power  $P$ . The received signals are respectively

$$\begin{aligned} Y_2 &= \sqrt{P_1 A_1} h_1 X_1 + \sqrt{P_2 A_2} h_2 X_2 + Z, \\ Y_1 &= \sqrt{P A_1} h_1 X + Z. \end{aligned} \quad (37)$$

### 4.2 Outage probability

Let  $p = \min\left(\frac{C_b}{R}, 1\right)$  the probability of bits transmitted by the two BSs, then the outage probability can be defined by

$$P_{\text{out}}(P, R, C_b) = (1 - p)P_{\text{out},1} + pP_{\text{out},2} \quad (38)$$

where  $P_{\text{out},1}$  and  $P_{\text{out},2}$  are relative to  $Y_1$  and  $Y_2$  in Eqs. (36) and (37), and  $p = 1$  if  $C_b \geq R$  while it is  $p = \frac{C_b}{R}$  if  $C_b < R$ .

In order to take into account finite modulation schemes, mutual information approximation is considered to evaluate the capacity. In fact,

**Proposition 3** *The capacity of QAM constellations of  $R$  bits/cu can be approximated by the Shannon capacity of Gaussian symbols by choosing a lower rate  $R_{\text{QAM}} < R$ .*

*In other words, depending on the SNR region, the effective selected constellation is the one for which the mutual information is the closest to the  $\log_2(1 + \text{SNR})$  value and thus has lower rate.*

*Proof* See Appendix C for more details.  $\square$

The outage probability derivation is adapted to take into account this approximation. It is thus defined by

$$\begin{aligned} P_{\text{out}}(P, R, C_b) &= \Pr\{C(H) < R_{\text{QAM}}\} \\ &= \Pr\{\log_2(1 + \text{SNR}) < R_{\text{QAM}}\} \end{aligned} \quad (39)$$

When no CSI is available at BSs, the transmitted symbols by the cooperating BSs are  $X_i = s$ ,  $i = 1, 2$  where  $s$  are the modulated symbols, and the SNR is given by

$$\text{SNR} = \frac{|\sqrt{P_1 A_1} h_1 + \sqrt{P_2 A_2} h_2|^2}{N_0} \quad (40)$$

In order to improve the performance in terms of outage probability, hence to increase the energy efficiency, precoding strategies will be investigated assuming channel feedback to the base stations.

### 4.3 Precoding schemes

The precoding schemes are based on cooperative beamforming with channel state information at the transmitter side (CSIT). The channel state information is obtained through a feedback channel from the mobile user to the serving base stations. The user selects the best precoder from a predefined limited size codebook. Codebooks specified by LTE standard as well as larger codebooks are presented in the following.

With precoding, the transmitted symbols are  $X_i = w_i s$  with  $w_i$  the components of the precoder vector  $\mathbf{W}$ . The received signal is then given as

$$Y = \sum_i \sqrt{P_i} A_i h_i w_i s + Z \quad (41)$$

#### 4.3.1 First precoder

The first precoder is the beamforming precoder when we assume perfect feedback. It is defined by

$$\mathbf{W} = \begin{bmatrix} \frac{h_1^*}{\sqrt{|h_1|^2 + |h_2|^2}} \\ \frac{h_2^*}{\sqrt{|h_1|^2 + |h_2|^2}} \end{bmatrix} \quad (42)$$

For imperfect feedback, we need to quantize the channel and thus generate a codebook of different precoders. Then, the user chooses the best precoder among them that minimizes the chordal distance defined by

$$\hat{\mathbf{W}} = \arg \min_{\mathbf{U}} (d_{\text{ch}}(\mathbf{W}, \mathbf{U})) = \arg \max_{\mathbf{U}} |\mathbf{W}^H \mathbf{U}| \quad (43)$$

For this study, we consider two codebooks, and accordingly two precoders satisfying this equation.

#### 4.3.2 Second precoder

The second precoder is obtained from the LTE codebook specified by 3GPP specifications [1], Section 6.3.4.2 “Precoding for spatial multiplexing,” with transmission on  $N_t = 2$  or 4 antennas and one single layer. For instance, for two antennas, the codebook is of size 4, and the four possible vectors are

$$\mathbf{U} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & i & -i \end{bmatrix} \quad (44)$$

#### 4.3.3 Third precoder

In order to optimize the precoder, we aim at generating a larger codebook. Hence, we quantize the channel modulus and phase over more bits, namely  $\log_2 N_p$  and  $\log_2 N_m$  bits, respectively. All the possible precoder vectors can be obtained according to

$$\mathbf{U} = \frac{1}{\sqrt{2}} \begin{bmatrix} \tau \\ \sqrt{1 - \tau^2} \exp\left(\frac{2i\pi k}{N_p}\right) \end{bmatrix} \quad (45)$$

with  $\rho = \frac{l}{N_m}$ ,  $0 \leq l \leq N_m$ ,  $0 \leq k \leq N_p$ .

#### 4.4 Simulation results

For simulation results, we evaluate the performance of the different precoders used in the system model of Section 4.1 and we compare them to the cases of no cooperation and

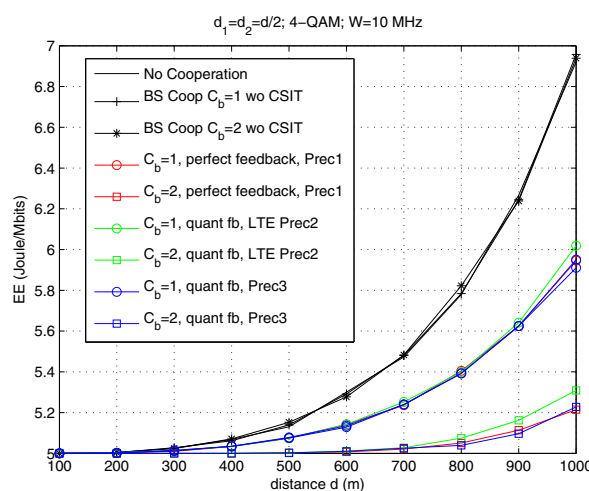


Fig. 7 EE vs distance for 4-QAM,  $N_t = 2$ , different  $C_b$  and precoders

no CSIT. We consider 4-QAM and 16-QAM modulations and thus we choose  $R_{\text{QAM}} = 1.5, 3$  bits/cu to approximate Shannon capacity while the transmission rate is  $R = 2, 4$  bits/cu, respectively. The performances are shown in terms of EE in joule per megabits as function of the inter-site distance with  $d_1 = d_2 = d/2$ , and the powers  $P_1 = P_2 = P/2$  with the total power  $P = 10$  W and the bandwidth  $W = 10$  MHz.

In Figs. 7 and 8, we consider  $N_p = N_m = 8$  (3 bits) for the third precoder to quantize the channel modulus and phase. It can be seen that precoding can increase EE even for limited  $C_b$  compared to no cooperation and no CSIT. Limited  $C_b$  decreases the EE compared to perfect backhaul due to more outage events since some of the transmitted bits cannot benefit from this diversity gain. For high SNR i.e., for low inter-site distance  $d$ , LTE precoder (second precoder)

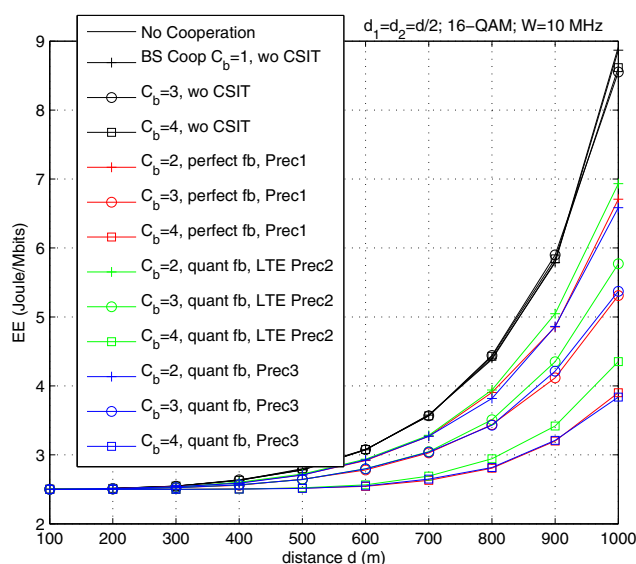
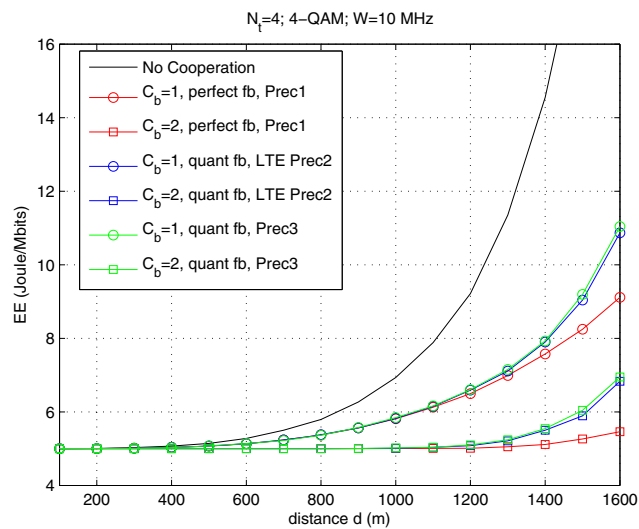


Fig. 8 EE vs distance 16-QAM,  $N_t = 2$ , different  $C_b$  and precoders

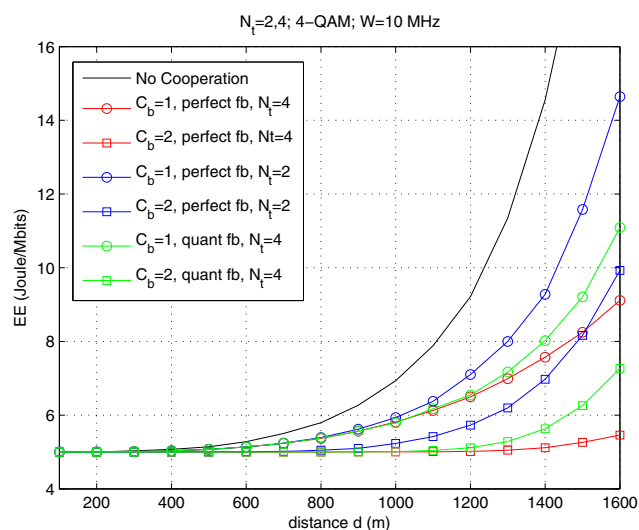


**Fig. 9** EE vs distance for 4-QAM,  $N_t = 4$ , different  $C_b$  and precoders

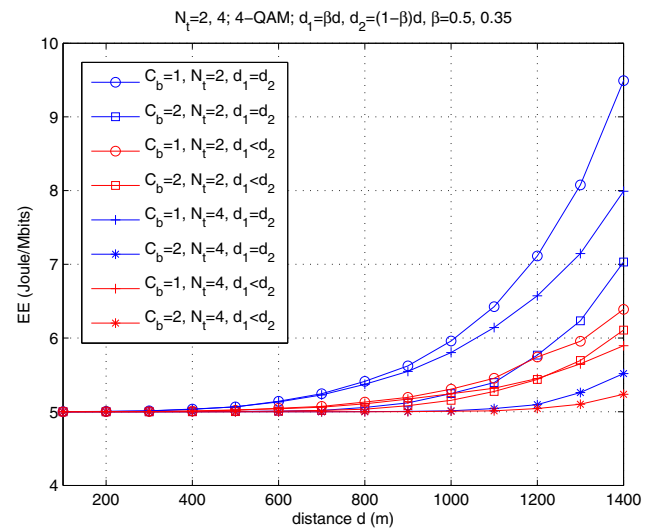
is good. For high  $d$  (low SNR), we need to quantize better the channel, so the third precoder is better than the second one and achieves the beamforming performance of perfect feedback.

In Figs. 9 and 10, we consider precoders for  $N_t = 4$  transmitting antennas, two from each BS. For LTE precoders, we have 16 possible vectors and for the third precoder, we use  $N_p = N_m = 1$  (2 bits) for channel quantization which gives 64 possible vectors. Thus, this last precoder is more energy saving than LTE one. We can also observe that increasing  $N_t$ , thus the diversity increases the energy efficiency.

Finally, we compare in Fig. 11 the energy efficiency for different user to BSs distances, with  $d_1 = \beta d$  and  $d_2 = (1 - \beta)d$  where  $\beta$  can take values 0.5 and 0.35. For



**Fig. 10** EE vs distance for 4-QAM,  $N_t = 2, 4$ , different  $C_b$  and precoders



**Fig. 11** EE vs distance for different  $C_b$  and  $N_t$

this simulation, 4-QAM modulation is considered and the modulated symbols are precoded using the second precoder. Performance for  $N_t = 2$  and 4 transmitting antennas are illustrated, respectively. We observe as for the first studied approach (Section 3.3), that the case when the user is at a distance  $d_1 < d_2$  is more energy saving than the case when it is at equal distance from base stations. This EE gain is more important for limited  $C_b$ , and specially at high distances  $d > 800$  m. In addition, it is less important for higher number antennas  $N_t = 4$ , since there is more diversity gain decreasing hence the outage events.

## 5 Conclusion

In this paper, we have addressed the energy efficiency of cellular communications. In a slow-fading channel, we have considered a system where two base stations are cooperating, under limited backhaul capacity  $C_b$  and fixed total power  $P$ , to transmit to a single user. Then, we have derived the EE metric of this model based on the outage probability analysis. First, we have considered a theoretical approach where Gaussian symbols are transmitted and quantization is performed on the backhaul. We have shown by numerical results that, in this case, cooperation cannot be as energy-efficient as noncooperation unless the backhaul has unlimited capacity. Then, we have identified that this BS cooperation can be more energy saving for limited  $C_b$  in asymmetric configuration where  $P_1 > P_2$ ,  $d_1 < d_2$ , while all power and inter-site distances configurations behave similarly under perfect backhaul.

In order to investigate more realistic scenarios, we have extended the EE analysis to a practical approach where

finite modulations are used and data transmission is considered over the backhaul. Then, to improve the EE of the system, we have studied different precoding strategies based on cooperative beamforming with CSIT, obtained through perfect or quantized channel feedback. We have shown through simulations that cooperation improves EE even for limited  $C_b$ . In fact, it exploits the system diversity thanks to precoding. This EE gain is further improved when using multiple antennas at the BSs. It also becomes more significant for asymmetric user BSs distances.

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## Appendix A: Proof of Proposition 1: rate-distortion theory

In rate-distortion theory, the quantization problem consists of representing a source  $X$  by an estimate  $\hat{X}$  subject to some distortion measure or quantization error such that

$$D = E[|X - \hat{X}|^2]. \quad (\text{A.1})$$

For complex Gaussian source, the rate-distortion  $R(D)$  metric is defined by

$$R(D) = \begin{cases} \log_2 \frac{\sigma_X^2}{D}, & 0 \leq D \leq \sigma_X^2 \\ 0, & D > \sigma_X^2. \end{cases} \quad (\text{A.2})$$

It is modeled by the backward test channel

$$X = \hat{X} + Z_q, \quad (\text{A.3})$$

where  $\hat{X}$  and  $Z_q$  are independent,  $X \sim \mathcal{N}_c(0, \sigma_X^2)$ ,  $Z_q \sim \mathcal{N}_c(0, D = \sigma_X^2 - \sigma_{\hat{X}}^2)$  and  $\sigma_{\hat{X}}^2 = \sigma_X^2 - D$ .

Following this model, a forward test channel can be equivalently constructed as

$$\hat{X} = c(X + \eta), \quad (\text{A.4})$$

where  $X$  and  $\eta$  are independent,  $\eta$  being the equivalent quantization noise  $\eta \sim \mathcal{N}_c(0, \sigma_\eta^2)$  and  $c$  a scaling constant such that

$$\sigma_\eta^2 = \frac{\sigma_X^2 D}{\sigma_X^2 - D} \quad \text{and} \quad c = \frac{\sigma_X^2 - D}{\sigma_X^2}. \quad (\text{A.5})$$

It can be readily verified that according to this model, we have,

$$\begin{aligned} E[|X - \hat{X}|^2] &= E\left[\left|\frac{D}{\sigma_X^2}X - \frac{\sigma_X^2 - D}{\sigma_X^2}\eta\right|^2\right] = D \\ E[|\hat{X}|^2] &= \left(\frac{\sigma_X^2 D}{\sigma_X^2 - D}\right)^2 E[|X + \eta|^2] \\ &= \sigma_X^2 - D. \end{aligned} \quad (\text{A.6})$$

For our system model, BS1 quantizes  $X$  and forwards it to BS2 over the backhaul link with capacity  $C_b$ . BS2 decodes then an estimate  $\hat{X}$  of  $X$  with some distortion entailed by the backhaul. This distortion can be expressed in terms of the backhaul rate according to (A.2)

$$D(C_b) = \sigma_X^2 2^{-C_b}. \quad (\text{A.7})$$

then we can define the forward test channel from Eqs. (A.4), (A.5) with

$$\sigma_\eta^2 = \frac{\sigma_X^2 2^{-C_b}}{\sigma_X^2 - 2^{-C_b}} \quad \text{and} \quad c = 1 - \frac{2^{-C_b}}{\sigma_X^2}. \quad (\text{A.8})$$

This concludes the Proof of Proposition 1.

## B Proof of Proposition 2: difference of correlated gamma variables

As presented in [12], Type II McKay Distribution is defined as follows

**Definition 3 Type II McKay Distribution** A random variable  $\Delta$  follows Type II McKay distribution with parameters  $a > -(1/2)$ ,  $b > 0$  and  $|c| < 1$  when the PDF of  $\Delta$  is

$$f_\Delta(\delta) = \frac{(1 - c^2)^{a+\frac{1}{2}} |\delta|^a}{\sqrt{\pi} 2^a b^{a+1} \Gamma(a + \frac{1}{2})} e^{-\delta \frac{c}{b}} K_a\left(\frac{|\delta|}{b}\right), \quad (\text{A.9})$$

where  $K_a(\cdot)$  is the modified Bessel function of the second kind and of order  $a$ .

If  $X_1, X_2$  are two correlated Gamma variables i.e., Bivariate Gamma variables,  $\{X_1, X_2\} \sim \Gamma(k, \theta_1, \theta_2, \rho)$ , the authors of [12] proved that their difference  $\Delta = X_1 - X_2$  follows Type II McKay distribution with the parameters

$$\begin{aligned} a &= k - \frac{1}{2} \\ b &= \frac{2\theta_1\theta_2(1 - \rho)}{\sqrt{(\theta_1 - \theta_2)^2 + 4\theta_1\theta_2(1 - \rho)}} \\ c &= -\frac{\theta_1 - \theta_2}{\sqrt{(\theta_1 - \theta_2)^2 + 4\theta_1\theta_2(1 - \rho)}} \end{aligned} \quad (\text{A.10})$$

where the conditions  $b > 0$  and  $|c| < 1$  are met as long as  $\rho < 1$ .

In the case of  $k = 1$ ,  $a = \frac{1}{2}$  and this McKay distribution reduces to

$$f_{\Delta}(\delta) = \frac{1 - c^2}{2b} e^{-\frac{\delta c + |\delta|}{b}}. \quad (\text{A.11})$$

Thus, it is possible to find a closed-form expression for the CDF of the McKay Type II distribution. It can be written as following

$$F_{\Delta}(\delta) = \begin{cases} \frac{1+c}{2} \exp\left(\frac{1-c}{b}\delta\right), & \delta < 0 \\ 1 - \frac{1-c}{2} \exp\left(-\frac{1+c}{b}\delta\right), & \delta \geq 0. \end{cases} \quad (\text{A.12})$$

In our system model, we have Rayleigh fading channel, thus  $k = 1$ . We have also  $\delta = \lambda\sigma^2 \geq 0$ , so we can apply the second line of the CDF formula to calculate the outage probability.

$$F_{\Delta}(\lambda\sigma^2) = 1 - \frac{1-c}{2} \exp\left(-\frac{1+c}{b}\lambda\sigma^2\right). \quad (\text{A.13})$$

This concludes the Proof of Proposition 2.

### C Proof of Proposition 3: mutual information of finite constellations

The mutual information of finite constellations over Gaussian channel  $Y = \sqrt{\text{SNR}}X + Z$  is defined by

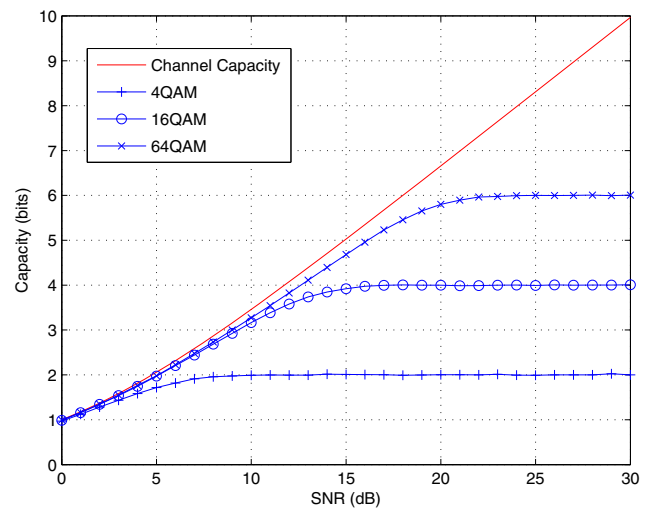
$$I(X, Y) = H(Y) - H(Y|X) = H(Y) - H(Z) \quad (\text{A.14})$$

with  $X \in M$ -QAM, and  $Z$  the Gaussian noise  $Z \sim \mathcal{N}_c(0, N_0)$  with entropy given by

$$H(Z) = \log_2(\pi e N_0) \quad (\text{A.15})$$

On the other hand, the entropy of  $Y$  is computed as follows

$$\begin{aligned} H(Y) &= \mathbb{E}_{X,Y}\{-\log_2(p(y))\} \\ &= \sum_{x_i} p(x_i) \mathbb{E}_{Y|X=x_i}\{-\log_2(p(y))\} \\ &= \sum_{x_i} \frac{1}{M} \mathbb{E}_{Y|X=x_i}\{-\log_2(p(y))\} \\ &= \mathbb{E}_{Y|X}\{-\log_2(p(y))\} \\ &= \mathbb{E}_{Y|X}\left\{-\log_2\left[\sum_x p(y|x)p(x)\right]\right\} \\ &= \mathbb{E}_{Y|X}\left\{-\log_2\left[\frac{1}{M} \sum_x p(y|x)\right]\right\} \\ &= \mathbb{E}_{Y|X}\left\{-\log_2\left[\frac{1}{M} \sum_x \frac{1}{\pi N_0} \exp\left(-\frac{|Y - \sqrt{\text{SNR}}X|^2}{N_0}\right)\right]\right\} \end{aligned} \quad (\text{A.16})$$



**Fig. 12** Mutual information of QAM constellations for the Gaussian channel

Note that the third and the sixth lines follow from the fact that we have uniform probability  $p(x_i) = \frac{1}{M}$ , for all symbols  $x_i \in M$ -QAM.

Therefore, this mutual information can be represented in Fig. 12 for several  $M$ -QAM modulations, with  $M = 4, 16$  and  $64$ .

In order to approximate the Shannon capacity  $C = \log_2(1 + \text{SNR})$ , it can be noticed that for a  $M = 2^m$ -QAM constellation with a rate of  $R = m$  bits/cu, a lower rate corresponding to a lower order modulation  $\sim (m - 1)$  should be used.

For instance, we can choose for

- 4-QAM,  $R_{\text{QAM}} \leq 1.5$
- 16-QAM,  $R_{\text{QAM}} \leq 3$
- 64-QAM,  $R_{\text{QAM}} \leq 5$

This concludes the Proof of Proposition 3.

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