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Elastodynamic models for extending GTD to penumbra and finite size scatterers

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Abstract

The Geometrical Theory of Diffraction (GTD) is one classical method used for modelling edge diffraction. GTD is theoretically valid for canonical infinite edges and diverges around the direction of specular reflection. To deal with finite flaws, 3D incremental models using both GTD and secondary sources have been developed. Experimental validation of these models has been performed. A GTD uniform correction, the UTD (Uniform Theory of Diffraction), has also been developed in elastodynamics in the view of designing a generic model able to correctly simulate both specular reflection and diffraction. Some UTD numerical results are presented.

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Keywords: Elastodynamics, GTD, Incremental methods, UTD, UAT

1. Introduction

The scattering of elastic waves by an obstacle involves phenomena such as specular reflection and diffraction. Specular reflection can be modelled by Geometrical Elastodynamics (GE), a ray model which just considers incident and reflected waves. The Kirchoff Approximation (KA) [1, Ch. 3] which is an integral method can model both specular reflection and diffraction. However, the diffraction amplitude produced by KA is not accurate as the one of GTD (Geometrical Theory of Diffraction), a ray method, initially developed in electromagnetism [2] which models only diffraction.

GTD is an extension of GE. Indeed, it adds diffracted rays to usual incident and reflected rays. These diffracted rays propagate in shadow regions. The GTD diffracted field is the product of the incident field with a diffraction coefficient and a divergence factor. GTD can be obtained only for canonical configurations as infinite half-planes or wedges. Therefore, GTD does not take into account the finite length of the diffracting edges. Furthermore, GTD fails in the zones where edge diffracted waves interfere with incident or reflected waves. For this reason, GTD solution is said to be non-uniform.

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To overcome these limitations of GTD, in a first step, incremental methods have been developed in electromagnetism to take into account the finite edge size: Incremental Theory of Diffraction (ITD) [3], Incremental Length Diffraction Coefficient (ILDC) [4] and Equivalent Edge Currents (EEC) [5]. In such approachs, the points of the scatterers are considered as secondary sources generating a spherical wave, called "incremental field". In this paper, it is shown that ITD can be extended from electromagnetism to elastodynamics and another incremental model based on Huygens' principle has been developed.To avoid the second GTD drawback (failure at specular direction), a uniform GTD correction, the Uniform Theory of Diffraction (UTD) developed in electromagnetism by [6] based on the Pauli-Clemmow process, has been extended to elastodynamics. UTD is preferred to UAT (Uniform Asymptotics Theory of diffraction) [7], another uniform GTD correction, because UAT requires artificial extension of the scattering surface and fictitious reflected rays contrary to UTD [8, Ch. 3, pp. 265].

This paper focuses on the development of incremental models and of UTD in elastodynamics. In section 2, the two incremental models, ITD and Huygens-based, are developed in elastodynamics. They are validated by experimental results. In section 3, UTD is developed in elastodynamics and some numerical results are presented. Conclusions are provided in section 4.

2. Elastodynamic incremental models

In the following, the symbols α and β are used to denote the wave type, i.e. α,β = L, TV or TH (Longitudinal, Transverse Vertical or Transverse Horizontal, respectively). α is used for the incident wave and β for reflected and diffracted waves.

The geometry of the problem is presented in Fig. 1. A stress-free obstacle is irradiated by a plane wave

$$\mathbf{u}^{\alpha}(\mathbf{x}) = A\,\mathbf{d}^{\alpha} e^{i(-\omega t+\mathbf{k}^{\alpha}\cdot\mathbf{x})}, \tag{1}$$

where A is the wave displacement amplitude, $\mathbf{d}^{\alpha}$ its polarization, $\mathbf{k}^{\alpha}$ its wave vector, whose magnitude is $k_{\alpha} = \omega/c_{\alpha}$, with ω - the circular frequency and c_{α} - the speed of the corresponding mode; t is time and $\mathbf{x}$ is the position vector. The exponential factor $\exp(-i\omega t)$ is implied but omitted everywhere.

At the diffraction point x_{β}^{α}, the crack is locally approximated by a canonical shape, a half-plane in the case of Fig. 1, tangent to the crack. The diffraction point x_{β}^{α} is the origin of the orthonormal basis vector $\{\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3}\}$ associated to the half-plane. (s, ϕ, θ) are the spherical coordinates of the position vector $\mathbf{x}$ and $(k_{\alpha}, \Omega_{\alpha}, \theta_{\alpha})$ are similarly the spherical coordinates of the incident wave vector $\mathbf{k}^{\alpha}$. The diffraction angle Ω_{β} is linked to the incidence angle Ω_{α} by the Snell's law of diffraction

$$k_{\beta}\cos\Omega_{\beta} = k_{\alpha}\cos\Omega_{\alpha}. \tag{2}$$

Incremental methods supposed that points of the diffracting edge are fictive sources of a field called incremental field $\mathbf{F}_{\beta}(x_{\beta}^{\alpha}, \mathbf{x})$. Thus, the field diffracted by the contour $\mathcal{L}$ at an observation point is

$$\mathbf{u}_{\beta}^{\alpha}(\mathbf{x}) = \int_{\mathcal{L}} \mathbf{F}_{\beta}(x_{\beta}^{\alpha}, \mathbf{x})\, dl. \tag{3}$$

This incremental field has been found hereafter using ITD or the Huygens' principle.

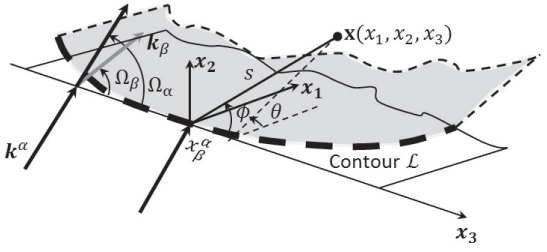

Fig. 1. A plane wave of propagation vector $\mathbf{k}^{\alpha}$ incident on a stress free obstacle (in gray) of contour $\mathcal{L}$. Thick black arrow - direction of the incident wave; thick gray arrow - direction of the wave ($\mathbf{k}_{\beta}$) scattered by the tangential half-plane at the flash point x_{β}^{α}.

2.1. ITD in elastodynamics

ITD has been developed in electromagnetism by [3]. It has just been extended to elastodynamics [10]. The ITD incremental field is

$$\mathbf{F}_\beta(x_\beta^\alpha, \mathbf{x}) = \frac{u^\alpha(\mathbf{x}_\beta^\alpha)}{\sqrt{2i\pi}} \sin\phi\, D_\beta^{\alpha(\mathrm{GTD})}(\theta, \Omega_\alpha(\phi), \theta_\alpha) \frac{e^{ik_\beta s}}{s} \mathbf{d}_\beta\left(-q_\beta \cos\theta\right) \tag{4}$$

where $u^\alpha(\mathbf{x}_\beta^\alpha) = \mathbf{u}^\alpha(\mathbf{x}_\beta^\alpha) \cdot \mathbf{d}^\alpha$ with $\mathbf{x}_\beta^\alpha$ being the position vector of the diffraction point. In (4), $D_\beta^{\alpha(\mathrm{GTD})}$ is the diffraction coefficient [9, Ch. 5], $\Omega_\alpha(\phi)$ is the incidence angle which would lead to the diffraction angle ϕ (see Fig. 1) when using the Snell's law $k_\beta \cos\phi = k_\alpha \cos\Omega_\alpha(\phi)$, $\mathbf{d}_\beta$ is the polarisation vector of the scattered wave and $q_\beta = \kappa_\beta \sin\Omega_\beta$, $\kappa_\beta = c_L/c_\beta$ - the dimensionless slowness of the scattered wave. This incremental field, valid in far field $k_\beta s \gg 1$, is a spherical wave weighted by a coefficient. It has been checked that ITD gives back the GTD solution in the case of the plane wave scattering from a half-plane. Another incremental model based on the Huygens' principle is developed in next section.

2.2. Huygens method

The second developed method based on the Huygens' principle, also supposes that points on the diffracting edge are fictive sources of spherical waves. The Huygens incremental field [10] is

$$\mathbf{F}_\beta(x_\beta^\alpha, \mathbf{x}) = \frac{u^\alpha(x_\beta^\alpha)}{\sqrt{2i\pi}} \sin\Omega_\beta\, D_\beta^{\alpha(\mathrm{GTD})}(\theta, \Omega_\alpha, \theta_\alpha) \frac{e^{ik_\beta s}}{s} \mathbf{d}_\beta\left(-q_\beta \cos\theta\right). \tag{5}$$

It depends on the diffraction angle Ω_β linked to the incidence angle Ω_α by the Snell's law (2) of diffraction whereas ITD incremental field (4) depends on the angle ϕ between the observation point and the discretization point (see Fig. 1). These incremental models can be applied to GTD and also to uniform GTD corrections. They have been validated against experiments in the following.

2.3. Experimental validation

Diffracted echoes generated by the top tip (edge) of a 40 mm large and 10 mm high backwall breaking planar notch simulated by incremental methods are compared to experimental results for various flaw orientations with respect to the probes incidence plane.

Diffraction echoes have been measured in the TOFD (Time Of Flight Diffraction) inspection of a ferritic steel component (see Fig. 2) using two 6.35 mm diameter mono-element probes emitting P45 waves at 2.25 MHz with a 60 mm PCS (Probe Centre Spacing). The defect is initially perpendicular to $x-$axis so that it is inspected in a 2D configuration. Then, the skew angle (angle between the flaw top edge and the z axis) is increased from O°(2D configuration) to 50°(see Fig. 2) in order to be in a 3D configuration by rotating the specimen around the z-axis. The echo generated by a 2 mm diameter and 40 mm length side-drilled hole (see Fig. 2) is employed for calibration.

The results of measurements and of simulation using ITD and Huygens models are presented in Table 1. In this table, ITD and Huygens give the same results. The errors between ITD/Huygens simulations and experimental results are at most or around 1dB and are less than the measurements incertitudes (around 2dB).

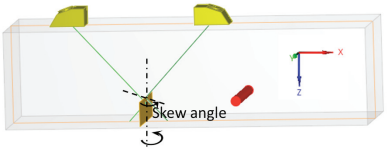

Fig. 2. TOFD simulation configuration. Dash points - position of the top tip when the defect is skewed.

Table 1. Amplitude of the top tip diffracted echoes in dB.

Skew angle(°)	experiments	ITD	Huygens
0	-13.1	-12.9	-12.9
10	-12.6	-13.0	-13.0
20	-14.2	-13.1	-13.1
30	-13.3	-13.3	-13.3
40	-13.7	-13.7	-13.7
50	-13.6	-14.3	-14.3

3. Uniform Theory of Diffraction (UTD) in elastodynamics

The classical edge-diffracted GTD ray field is not valid at the vicinity of shadow boundaries (directions of specular reflection and direct transmission). Indeed GTD evaluates asymptotically the exact solution of the scattering from a half-plane which is a Sommerfeld integral [11, Ch. 3] and just takes into account the contribution of the integral stationary phase point. The contribution of this point corresponds to the diffracted field whereas the integrand's poles contribution corresponds to the geometrical field. To handle the coalescence of stationary phase points and integrand's poles which corresponds to the interference of diffracted waves with incident or/and reflected waves, uniform methods are used such as the Van Der Waerden one which gives rise to UAT and the Pauli-Clemmow one which gives rise to UTD.

Applying the Pauli-Clemmow process to the exact scattering solution [9, Ch. 5], the approximate UTD-based total field [12] in elastodynamics is expressed as

$$\mathbf{u}^{\mathrm{tot(UTD)}}(\mathbf{x}) = \mathbf{u}_{\beta}^{\alpha(\mathrm{GE})}(\mathbf{x}) + \sum_{\beta} F(k_{\beta}L_{\beta}\,a)\,\mathbf{u}_{\beta}^{\alpha(\mathrm{GTD})}(\mathbf{x}) \tag{6}$$

where $\mathbf{u}_{\beta}^{\alpha(\mathrm{GE})}(\mathbf{x})$ is the geometrical field at the position vector $\mathbf{x}$, F is a transition function, $L_{\beta} = s\sin^2\Omega_{\beta}$ is a distance parameter and the parameter a describes the proximity of the observation point to a shadow boundary. When the observation point is far away from the shadow boundaries, the transition function tends to 1 and then, UTD is equal to GTD. When the observation point is close to the shadow boundaries, the transition function tends to 0 and removes the singularity of the GTD diffraction coefficient; it also introduces a discontinuity which is cancelled by the GE one so that the total UTD field is continuous at SB contrary to GTD.

The UTD just modified the amplitude of the diffracted rays. It does not require fictitious rays as UAT and is consequently simpler to implement. UTD asymptotics do not include all terms of the order $(k_{\beta}L_{\beta})^{-1/2}$ as UAT [8, Ch. 3]. Therefore, it is theoretically less accurate than UAT.

Simulations of the scattering from a half-plane using GTD, UAT and UTD are presented in Fig. 3 in the $(\mathbf{e}_1, \mathbf{e}_2)$ plane, which is perpendicular to the edge, since the problem is invariant in the x_3 direction. The observation point is specified using the polar coordinates (r, θ) The solid material used for simulations is ferritic steel with Poisson's ratio $\nu = 0.29$, longitudinal speed $c_L = 5900$ m.s^{-1} and transversal speed $c_T = 3230$ m.s^{-1}.

In Fig. 3, the used configuration is 3D since the incidence is oblique to the edge crack ($\Omega_{\alpha} \neq 90^{\circ}$) . The incident wave is a longitudinal wave. There are three shadow boundaries in Fig. 3, the incident L shadow boundary (ISB) $\theta = 45^{\circ}$, reflected L shadow boundary (RSB) $\theta = 330^{\circ}$ and reflected TV shadow boundary $\theta \approx 290^{\circ}$. As expected, UTD is continuous at all shadow boundaries and gives back the GTD solution far away from the shadow boundaries. Moreover, UTD leads to very satisfying results similar to UAT ones.

4. Conclusion

This paper focuses on recent advances in modelling the scattering of elastic waves from an obstacle. The GTD ray method, classically used to model edge diffraction, is valid for an infinite edge and not for a finite size scatterer. Moreover, GTD is discontinuous for observation directions of specular reflection and direct transmission (called shadow boundaries). To overcome these limitations of GTD, two incremental models and a uniform correction of the

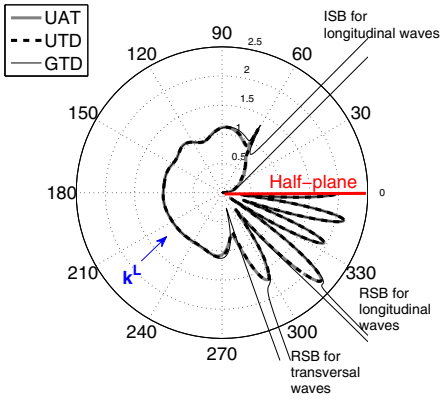

Fig. 3. Scattering of a plane wave from a half-plane: directivity pattern of the total field predicted by different models (GTD and UTD) a $\Omega_L = 60^\circ$, $\theta_L = 45^\circ$ and $k_{TV}\, r = 4\pi$. Each circle represents amplitude of the total field normalized by the incident amplitude.

GTD have been derived in elastodynamics. The two developed ITD and Huygens models allow to take into account the finite size of the scatterer edges. These two models give back the GTD solution in the case of a straight infinite edge. They have also been both successfully validated against experimental results. The UTD uniform correction of GTD has been developed and allows to simulate a continuous total field even at shadow boundaries. UTD is simpler to implement than UAT, another GTD uniform correction, which requires the determination of fictitious rays and leads to results close to UAT ones. Incremental methods can be coupled with UTD to build a generic model both uniform and for finite size flaws.

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