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From global scaling to the dynamics of individual cities

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Scaling has been proposed as a powerful tool to analyze the properties of complex systems, and in particular for cities where it describes how various properties change with population. The empirical study of scaling on a wide range of urban datasets displays apparent nonlinear behaviors whose statistical validity and meaning were recently the focus of many debates. We discuss here another aspect which is the implication of such scaling forms on individual cities and how they can be used for predicting the behavior of a city when its population changes. We illustrate this discussion on the case of delay due to traffic congestion with a dataset for 101 US cities in the range 1982-2014. We show that the scaling form obtained by agglomerating all the available data for different cities and for different years displays indeed a nonlinear behavior, but which appears to be unrelated to the dynamics of individual cities when their population grow. In other words, the congestion induced delay in a given city does not depend on its population only, but also on its previous history. This strong path-dependency prohibits the existence of a simple scaling form valid for all cities and shows that we cannot always agglomerate the data for many different systems. More generally, these results also challenge the use of transversal data for understanding longitudinal series for cities.

Keywords: Science of cities | Scaling | Path dependency

The recent availability of data for cities opens the fascinating possibility of a science of cities [1, 2] and has led numerous scientists to search for general laws [3, 4] ruling the evolution of various socio-economical and structural indicators such as patent production, personal income or electric cable total length, etc. In [3], it was suggested that assuming the population P to be the most important determinant for cities, we could study the evolution of many different features when P is increasing. In [4], many socio-economic factors were studied versus population indicating the existence of simple scaling laws under the form of power laws. For each indicator Y , Bettencourt et al. [4] found a power law of the form $Y \sim P^\beta$ where the exponent β depends on the quantity considered. Some quantities evolve *superlinearly* with the population ($\beta > 1$), for instance new patents ($\beta = 1.27$), GDP ($1.13 < \beta < 1.26$) or serious crime ($\beta = 1.16$), while some other behave *sublinearly* ($\beta < 1$) as gasoline stations or sales. Quantities that are independent from the size of the city – typically human-related quantities such as water consumption – scale with an exponent $\beta = 1$. The usual explanation for these effects is the impact of interactions (scaling as P^2) for superlinear quantities, and economies of scale for sublinear quantities. This publication [4] was followed by a wealth of other measures such as the abundance of business categories [5], the number of sexually transmitted infection [6], road networks [7], or carbon dioxide emissions [8–12].

Scaling in urban systems has however been criticized in some recent papers [10, 13–16]. A first re-analysis of

the data for the GDP and income [13] showed that the power law could not be distinguished from other functional forms, or that the linear fit is better [14], and in [15] the authors led a rigorous investigation on the statistical quality of scalings for various quantities and found that in many superlinear cases, the linear assumption could in fact not be rejected. They also showed that the fitting results depend crucially on the assumptions about noise. From another point of view, the authors in [16] showed that, for some socioeconomic indicators, those scaling are not universal and could depend on details of urban systems. More precisely, they showed on data of 5,000 french cities that two different definitions of the cities (*Unité urbaine* (Urban Units) and *Aire urbaine* (Metropolitan areas)) lead to different values of the scaling exponent for a given quantity, a result confirmed on transport-emitted CO_2 in [10]. Not only the value of the exponent can change, but in some case, for different definitions of the city, the scaling regime changes: for instance, the number of jobs in the manufacturing sector grows superlinearly with the population of Urban Units, but sublinearly if one considers Metropolitan Areas [16]. We can expect the results to change quantitatively, but here we have changes from the superlinear to the sublinear regime, casting some doubts about this nonlinear scaling and its universality.

In this paper we raise another problem that is the relevance of such a scaling for the individual dynamics of cities. At a more theoretical level, we question here the scaling assumption where a quantity Y (usually exten-

sive) is assumed to be determined by the population only $Y = F(P)$ (where F is in general an unknown function). Even if the population is an important determinant for cities we cannot exclude time effect and path-dependency which would then imply that the quantity Y depends also on time $Y = F(P, t)$ and possibly on all $Y(t')$ for $t' < t$. In other terms, the path-dependency means that it doesn't make sense in general to compare two cities having the same population but at very different dates: both central Paris and Phoenix (AZ) had a population of about 1 million inhabitants, the former in 1840 and the latter in 1990, and it is very likely that the dynamics – for most of the relevant quantities – from 1840 in Paris will be very different from the one starting in 1990 in Phoenix, implying that the usual scaling form does not apply in general. In this paper, we investigate this question and test if a scaling exponent computed by aggregating data for different cities (usually at a same date) is relevant for predicting what will happen at the level of individual cities as their population grow. We illustrate this discussion on the case of congestion-induced delays but our results could have far-reaching consequences on many other scaling results for cities.

Aggregating all cities: Global scaling

We focus on the particular case of traffic congestion and its impact on time delays. Previous studies have been made in order to empirically test and theoretically explain how traffic congestion scale with the population. In [17, 18] for instance, the authors propose a theory of urban growth which accounts for some of the observed scalings. The theoretical predictions are tested against several data sets, collected by OCDE or by a GPS device company (TomTom) [17]. Here, we study the dataset (freely available at [19]) published by the Texas A&M Transportation Institute (TTI) in the Urban Mobility Report (UMR), obtained for 101 cities in the United States over 33 years (from 1982 to 2014). This database has been investigated in 2017 by [20] and in this study, the authors agglomerate all the data corresponding to different cities and performed the usual power law fit of the form

$$\delta\tau_i = aP_i^\beta \quad (1)$$

where $\delta\tau_i$ is the annual congestion induced delay corresponding to the city i . In this study we take for P_i (also denoted by P in the following) the number of car commuters for the city i rather than the population, because this is the relevant parameter in many models that deal with congestion in cities (see [18]). If we take the population instead of the number of car commuters, our results are qualitatively the same and our conclusions remain unchanged, even if all the exponent values change slightly (a fit for all cities and all years shows that the number

of car commuters is approximately a constant fraction of order 35% of the population). In [20], they used the least square method to estimate β and for the year 2014 (the last available year in the urban mobility report), we find with this method $\beta = 1.23 \pm 0.03$. We plot the data and the corresponding fit on Fig. 1.

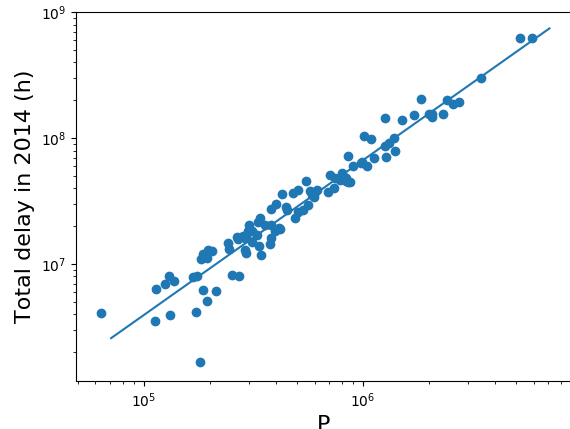


FIG. 1: Plot of the annual delay $\delta\tau$ versus the number of drivers P for all cities in 2014 (data from TTI's Urban Mobility Information website, see [19]). The straight line is a power law fit in this loglog representation and gives an exponent value $\beta \approx 1.23$ (and $R^2 = 0.93$).

The quality of a fit has in general to be carefully checked with the help of statistical methods [15], and computing a good estimation of this exponent values relies on several assumptions: data points are independent, the noise is multiplicative and has a variance independent of P_i (homoscedasticity). It should also be checked that the nonlinear fit that has an additional parameter compared to the linear one, is much better than what would be expected by pure chance. In this case, the trend seems however to fit the data in a reasonably good way with a large $R^2 = 0.93$, even if we have only two decades here. The value of β larger than 1 indicates a superlinear behavior of the traffic congestion, a fact in agreement with recent empirical [20] and theoretical approaches [18, 21].

We can repeat this fit for each year separately, from 1982 to 2014. Formally, we test for each time t the relationship $\log(\delta\tau_i(t)) = \log(a) + \beta(t) \times \log(P_i(t)) + \text{noise}$ where $\beta(t)$ is the scaling exponent to determine. We show the values of $\beta(t)$ versus t in Fig. 2 and we observe that $\beta(t)$ is not constant through time and displays non-negligible fluctuations of order 20%. However all these values are larger than 1 indicating a consistent superlinear behavior. In [20] a least square method has been used on all the points available: they mix all the 33 years available for each of the 101 cities and get $33 \times 101 = 3333$ points leading to a scaling exponent $\beta \approx 1.36 \pm 0.01$,

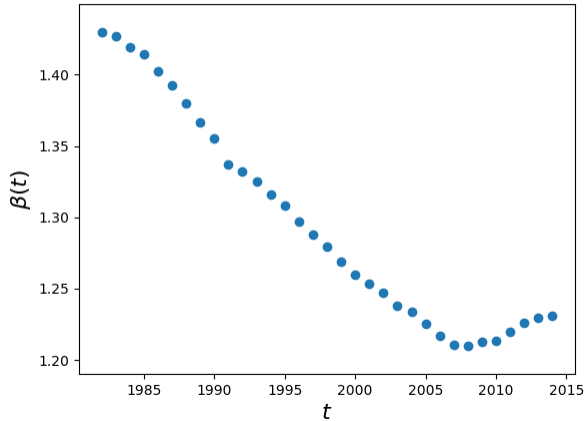


FIG. 2: Scaling exponent $\beta(t)$ for the delay computed for each year separately, from 1982 to 2014. All these values are consistent with a superlinear behavior found in [20].

consistent again with a superlinear relation, as found in [20]. For this dataset, we plot the scatterplot and the corresponding nonlinear fit in Fig. 3(top) (note that we plot here the delay per capita). We observe some variability but the global increasing trend seems to be correct. This way of proceeding with data is common: one mixes data for different cities and for the available years, and then performs a regression over the whole set. The scaling that is obtained – and that we qualify as ‘global’ – is then used for discussing theoretical approaches. For instance, in [21], this approach is used for computing some scaling exponents (for quantities such as land area, wages, etc.) and are compared with the exponent expected from theoretical calculations. In [22], empirical regularities are found by applying this methodology to different indicators, suggesting the existence of a universal socioeconomic dynamics. Beyond statistical problems related to fitting procedures, the exact meaning and the relevance of this global scaling for individual cities is however not clear. In other words, when we know that a certain quantity Y scales for all cities as $Y \sim P^\beta$, what can we say about the evolution of a single city? In the following we address this question on the case of congestion delay and by studying in details the dynamics of every individual city and compare its behavior with the global scaling described above.

The dynamics of individual cities

In Fig. 3(bottom), we show the same plot as in Fig. 3(top) but where we now distinguish cities (one color corresponds to one city). This allows us to compare the evolution of the delay due to congestion in each city when

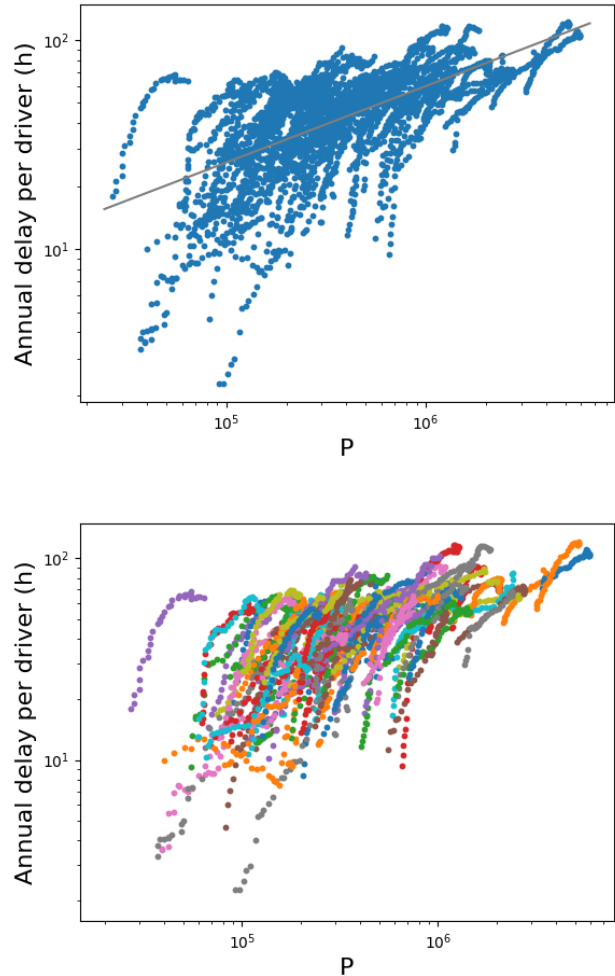


FIG. 3: (Top) Scatterplot of the annual delay per capita $\delta\tau/P$ versus P for all the 101 cities and for all years (1982-2014). The straight line is the power law fit with value $\beta \approx 1.36$ consistent with a superlinear behavior. (Bottom) Same scatterplot but where the points are colored according to the city they describe (one color per city). As we discuss in the text there is no obvious relation between the global power law scaling and the individual behavior of cities.

its population grows. The first striking observation is that for *all* cities in our dataset, the evolution of the congestion delay *does not behave as predicted by the global trend*. They have their own trend which depends on their particular history. In this respect, it is natural to ask what is the individual city dynamics and what does it have in common with the global scaling. In what follows we thus focus on this individual behavior and discuss its relation with the global power law exponent.

Absence of a single scaling

With this dataset, we can monitor the evolution of each city when its population grows. The first thing that we observe on the examples in Fig. 4(top) is that the annual delay is not a simple function of P only. The value of the number of drivers (or the population) is not enough to determine the delay. We also note in this figure that the slopes are different (a power law fit gives $\beta \approx 3.20$ for Bakersfield and $\beta \approx 1.45$ for Sarasota) showing that even when a power law exists it is not with the same exponent (see below for a further analysis of this point). In order to test further the existence of a scaling of the form $\delta\tau \sim P^\beta$ we plot in Fig. 4(Bottom) for all cities $\delta\tau(t)/\delta\tau(t_1)$ versus $P(t)/P(t_1)$ where t_1 is the first available time. Even if

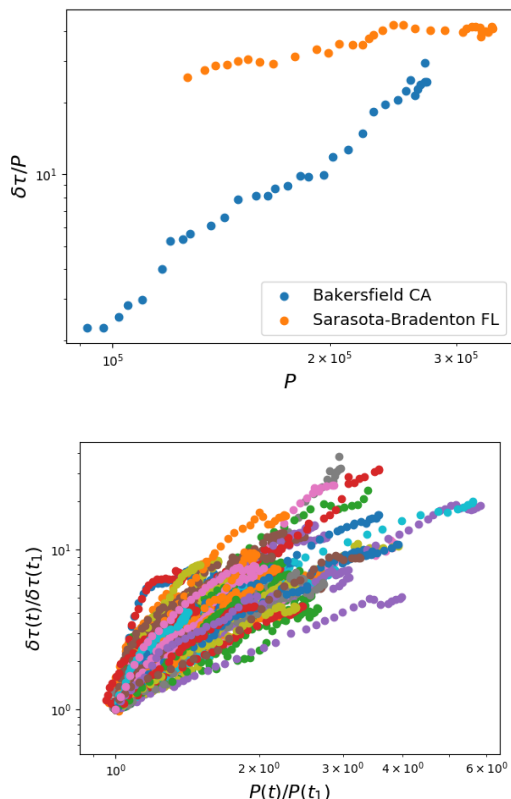


FIG. 4: (Top) Loglog plot of the annual delay per capita $\delta\tau/P$ versus P for two different cities: Bakersfield (CA) and Sarasota (FL). For the same range of P values, the delay is different, and the slopes are different as well. (Bottom). Plot of the rescaled delay $\delta\tau(t)/\delta\tau(t_1)$ versus $P(t)/P(t_1)$. The curves correspond to different cities and the fact that they do not collapse indicates the absence of a unique scaling determined by a single exponent.

the prefactor changes from a city to another this rescaling allows to test the existence of a unique power law scaling.

As we can see in this figure 4(bottom), the curves for different cities do not collapse signalling the absence of a scaling form governed by a single exponent. In the following we will focus on the different behaviors observed for this set of cities.

Different categories of cities

We analyze the behavior of each of the 101 cities in the dataset and we observe a variety of behaviors. More precisely, there are two main categories characterized by different time evolutions:

- The delay increases with P and in most cases can be fitted by a power (see Fig. 5(top)) and we refer to this set as ‘type-1’ cities and which represent over 30% of our cases. We note here that for the dataset studied here, the time range (from 1982 to 2014) does not allow to have a very large variation of the number of drivers (the ratio $P(2014)/P(1982)$ varies from 1.2 to 6 approximately) and a much larger dataset would be needed in order to have a better accuracy for these exponent values.
- The other cities (about 40% of all cities) display two regimes separated by a change of slope that is in general abrupt. The second regime for these ‘type-2’ cities can be in some cases a ‘saturation’ where the delay stays constant. We show in Fig. 5(bottom) an example of such city that displays saturation with a zero slope in the second regime.
- The rest of cities ($\approx 30\%$) do not display a common behavior (for instance some present 3 changes of slope, etc.)

In most cases however, the individual behavior of a city does not correspond to the global scaling $\delta\tau \sim P^{1.36}$. In the following we focus on each of these classes and try to characterize them more precisely.

Type-1 cities: power law growth

This particular class comprises cities that display an individual scaling law that can be fitted by a power law of the form $\delta\tau(t) \simeq P(t)^{\beta_i}$, where $P(t)$ is the number of commuters at time t and $\delta\tau(t)$ the corresponding annual congestion-induced delay. The quantity β_i depends in general on the city i and we show in Fig. 6 the histogram for this exponent computed for all type-1 cities. We clearly see that very few cities behave as the ‘global trend’ predicted: only 2 cities over 31 have an exponent < 1.5 , while 13 cities have an exponent > 2.5 . This result shows that when we observe a power law behavior at the individual city level, it is generally with an exponent that is much larger than 1 and much larger than

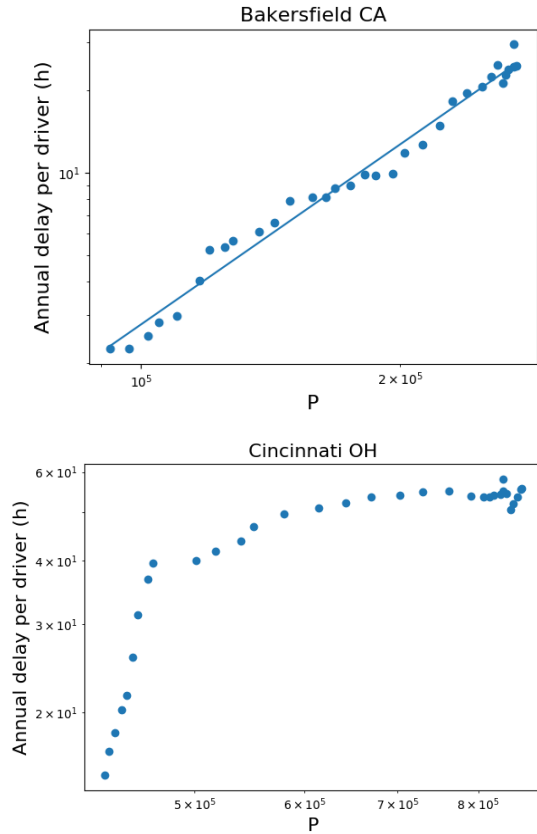


FIG. 5: Loglog plot of the annual delay per capita $\delta\tau/P$ versus P from 1982 to 2014. (Top) An example of a type-1 city where the delay grows with P and that can be reasonably fitted by a power law (Bakersfield, CA). (Bottom). Example of a type-2 city with two power law regimes characterized by two different exponents (Cincinnati, OH).

the result found for the global scaling. In other words there seems to be no correlation between the global observation made on all cities and the individual behavior of cities when its population evolves.

Type-2 cities: existence of two regimes

For about 40% of the cities in the dataset, the delay versus the number of car commuters displays a change of slope and $\log(\delta\tau)$ is a piecewise linear function of $\log(P)$. Formally one could write:

$$\log(\delta\tau) = \begin{cases} a_1 + \beta_1 \times \log(P) & \text{when } P < P^* \\ a_2 + \beta_2 \times \log(P) & \text{when } P > P^* \end{cases} \quad (2)$$

This behavior indicates that the dynamics of the traffic congestion in those cities followed successively two different scaling laws with two different exponents β_1 and β_2 and we plot the histograms for both these exponents in

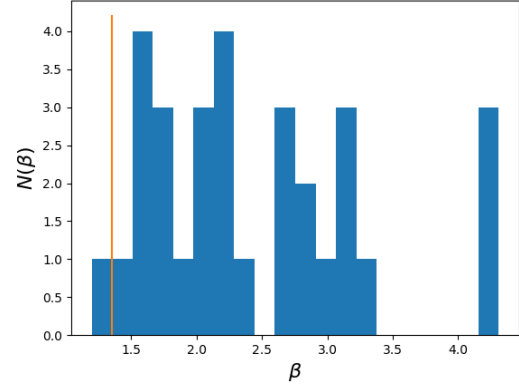


FIG. 6: Empirical histogram of β for type-1 cities. The vertical line indicates the value of the global scaling $\beta \approx 1.36$.

Fig. 7. We note that the average of β_1 is around 5.3, while

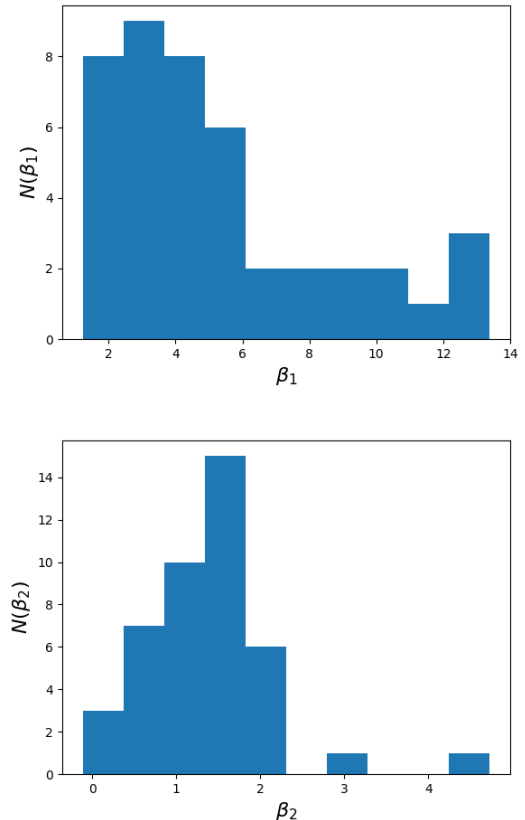


FIG. 7: Empirical histograms for the two exponents β_1 and β_2 that describe the two regimes of type-2 cities. (Top) Histogram for β_1 and (Bottom) the histogram for β_2 . For most cities we have $\beta_1 > \beta_2$.

the average of β_2 drops to 1.32, closer to the 'global ex-

ponent’ (but with a large dispersion around this value). Beyond averages, we have that for almost every case, $\beta_1 > \beta_2$. Almost all the exponents of the first regime β_1 are above 2 (indicating a strong superlinearity) while the second exponents β_2 are mostly < 2 . For this second regime, some cities do not exhibit superlinear behaviour. Indeed for some cities ($\sim 30\%$), the exponent β_2 is very close to 1, indicating a linear behavior and equivalently a delay per capita constant – that we coined ‘saturation’. The cities of Akron (see Fig. 8), or Pittsburg for instance fall into that subcategory. We also observe that in some

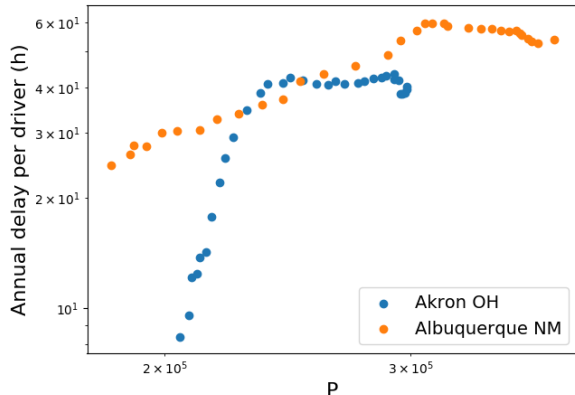


FIG. 8: Example of two different type-2 cities with two regimes characterized by two exponents β_1 and β_2 . In the case of Akron (OH) we observe a ‘saturation’ with a constant delay per capita ($\beta_2 \approx 0$), while for Albuquerque (NM) the delay per capita decreases with the population ($\beta_2 < 0$).

cases a crossing between the curves corresponding to different cities can occur (such as Akron and Albuquerque in Fig. 8). This crossing is another sign that the posterior evolution of a city is not uniquely determined by the population and the delay at a certain time (if it did the evolution after the crossing should be identical for the two cities).

In other cases ($\sim 10\%$), the exponent β_2 is clearly < 1 , which indicates sublinearity and that the delay per capita decreases with the population. We show the example of the city of Albuquerque (New Mexico) in Fig. 8. This phenomenon is very counter intuitive, even if we can point out some elements of explanation. Indeed, in addition to the congestion induced delay, we also have the data for the total driven length L_{tot} (in *miles* \times *commuters*) for each city and each year. We can check if this quantity can explain, even partially, the behavior of the total delay. For some type-2 cities with two regimes, we plot the driven length per commuter against the number of drivers and we observe that this curve displays a change of regime at the exact same point for the delay. In Fig. 9(top), we see that for the case of Birm-

ingham, from 1998, the delay remains almost constant, whereas it increased constantly at a high rate before that (more precisely we have here $\beta_1 \simeq 4$ and $\beta_2 \simeq 0$). In Fig. 9 (bottom), we observe that in the same year, the curve for L_{tot}/P experienced a change of slope: the length per capita increased before 1998, and slowly decreases after that date. This could explain partially why the delay does not evolve after this date: there are certainly more people on the road after 1998, and therefore more likely some congestion, but each commuter drives less on average which decreases the occurrence of traffic jams: these two effects can compensate each other. This is one possible partial explanation, which however does not hold for all the cities. The change of slope in L_{tot}/P vs P is common in this dataset and in most cases happens simultaneously with the change of regimes of the delay, pointing to the existence of correlations between these quantities, even if not in a causal manner. The simultaneous change of regime for these two quantities might also be the sign that the city experienced a large scale structural change.

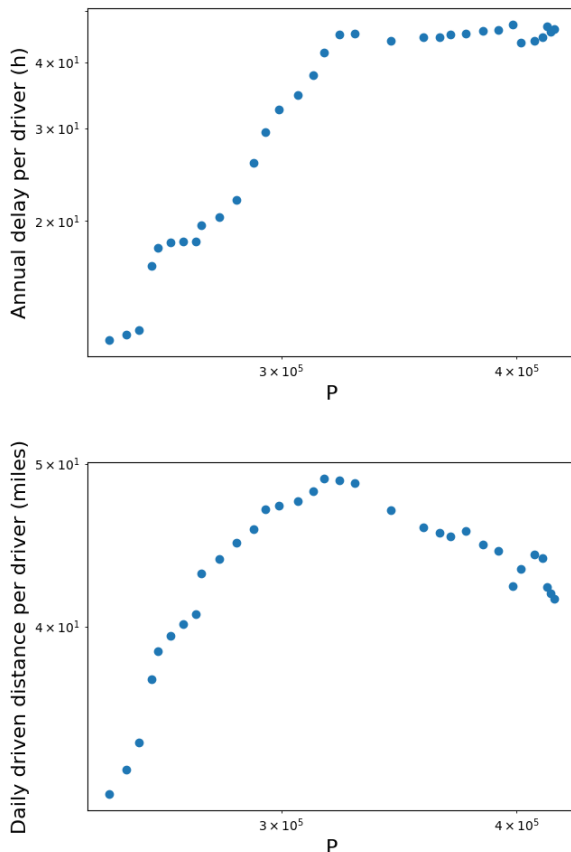


FIG. 9: Birmingham case. (Top) Loglog plot of $\delta\tau/P$ versus P . (Bottom) Loglog plot of the total driven length per capita L_{tot}/P vs P .

For this category of cities, beyond the two exponents

β_1 and β_2 , we can also study (i) at what *time* T^* the change of slope happened, (ii) what was the *population* of the city when it happened (P^*), and (iii) what was the delay per capita when it happened $((\delta\tau/P)^*)$. We represent the histograms for these three quantities in Fig. 10. The distribution of T^* is difficult to interpret and do not display a typical date at which the slope changes. The change of slopes do not occur at the same time for these cities, which would have been the case for instance if there had been a national plan in the US to rebuild the whole road system, or any other federal decision. The histogram for P^* seems clearer to interpret with the existence of a clear maximum around 200,000 commuters and a quick decay for larger values. The average of the distribution is 394,000, while the standard deviation is 367,000. Finally, the delay per capita $(\delta\tau/P)^*$ displays a histogram that has a relatively small compact support, with an average of about 39 hours per year, and a standard deviation about 18 hours per year. This relatively small variation of $(\delta\tau/P)^*$ suggests that it is the congestion that triggers the change of regime signalled by different exponents. Further studies are however certainly needed in order to clarify this important point.

Discussion

We focused in this paper on the dataset for congestion-induced delay in some US cities. This is a particularly interesting dataset as it is both transversal (it contains many cities), and longitudinal (for each city we have the temporal evolution of the delay). This is a rather rare case at the moment, but this type of data will certainly become more abundant in the future and will allow to test our results on other quantities. Our observations about scaling might therefore have far reaching consequences for the quantitative study of urban systems, well beyond the case of congestion induced delays.

The general scaling form $Y \sim P^\beta$ indicates that if the population is multiplied by a factor λ the quantity Y is then multiplied by a factor λ^β . This scaling form relies however on a strong implicit assumption which is the ‘logarithmic population translation’ invariance. In other words, this scaling form implies that for any times t and t' we have $Y(t')/Y(t) = (P(t')/P(t))^\beta$ and then depends on the ratio of populations only (or the difference of logarithms). As we observed in this study, there is no such scaling at the individual city level but a variety of behaviors. In the language of statistical physics, the quantity Y (here equal to $\delta\tau$) is not a state function determined by the population only, and displays some sort of aging effect where the delay in a city depends not only on the population but also on the time, and probably on the whole history of the city. In any case we cannot make for a given city a prediction for time $t_2 > t_1$ knowing only its state for t_1 . This idea of path-

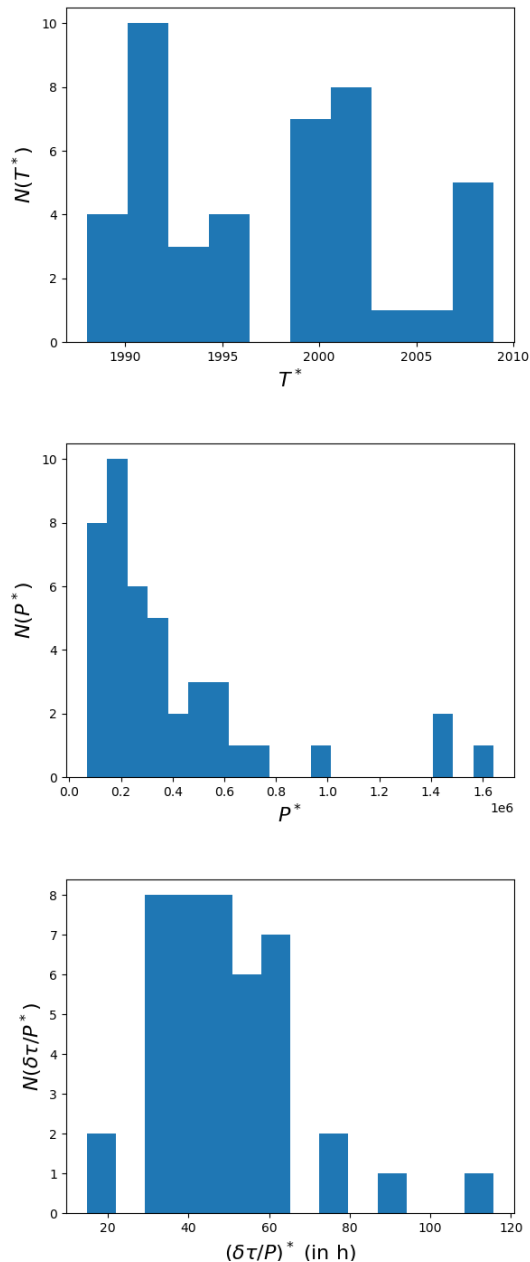


FIG. 10: Empirical histograms for T^* , P^* (in unit of million inhabitants) and $(\delta\tau/P)^*$. In particular the histogram for $(\delta\tau/P)^*$ shows that the changes of slope in type-2 cities appears approximately at the same value of about 40 hours per year and per capita of congestion delay.

dependency is natural for many complex systems, and in statistical physics, we know that spin-glasses [23] for example display aging which means that some features of the system (for instance the relaxation time) evolves with the age of the system and does not depend on the state of the system only. This in particular implies that

we do not have time translation invariance but that most functions of two times t and t' do not depend on $t - t'$ only. This aging theory has been applied to many other complex systems, from ‘soft material’ [24] to superparamagnet [25], and it would be interesting to understand it in the framework of the evolution of urban systems.

The results presented in this paper illustrated on the case of congestion-induced delays could in principle be applied to any other quantity. They highlight the risk of agglomerating data for different cities and to consider that cities are scaled-up versions of each other (as questioned in [26] for example): there are strong constraints for being allowed to do that such as path-independence, which is apparently not satisfied in the case of congestion, and which should be checked in each case.

Beyond scaling, these results also pose the challenging problem of using transversal data (ie. for different cities) in order to get some information about the longitudinal series for individual cities. This is a fundamental problem that needs to be clarified when looking for generic properties of cities.

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