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Abstract

In cosmological models where dark energy has a dynamical origin one would expect that a primordial inflationary epoch leaves no imprint on the behavior of dark energy near the present epoch. We show that a notable exception to this behavior is provided by a nonlocal infrared modification of General Relativity, the so-called RT model. It has been previously shown that this model fits the cosmological data with an accuracy comparable to ΛCDM, with the same number of free parameters. Here we show that in this model the dark energy equation of state (EOS) near the present epoch is significantly affected by the existence of an epoch of primordial inflation. A smoking-gun signature of the model is a well-defined prediction for the dark energy EOS, $w_{\rm DE}(z)$, evolving with redshift from a non-phantom to a phantom behavior, with deviations from -1 already very close to the limits excluded by the *Planck* 2015 data. Future missions such as EUCLID should be able to clearly confirm or disprove this prediction.

1 Introduction

Understanding the origin of dark energy (DE) is among the most interesting problems in cosmology. The simplest solution, a cosmological constant, fits the data very well, so ΛCDM is at present the standard cosmological paradigm. However, the accuracy of present and future observations allows us to test ΛCDM against competing theories, so it is clearly interesting to develop alternatives to ΛCDM. At the conceptual level, an especially intriguing possibility is to explain DE by modifying General Relativity (GR) at cosmological distances, see e.g. [1–3] for reviews of different approaches.

Nonlocality opens up new interesting possibilities for building large-distance ("infrared") modifications of GR. While at the fundamental level quantum field theory is local, in the presence of massless or light particles the quantum effective action that includes quantum corrections unavoidably develops nonlocal terms, both at the perturbative and at the non-perturbative level. In this spirit, a phenomenological nonlocal modification of GR was first proposed in [4,5] (see [6] for review), which however has been ruled out by the comparison with structure-formation data [7]. In the last few years our group has introduced a different class of nonlocal models, characterized by the fact that the nonlocal operators are associated to a mass scale. These models, which evolved from previous work related to the degravitation idea [8–10] as well as from attempts at writing massive gravity in nonlocal form [11,12], turn out to work extremely well in the comparison with cosmological observations. A first model of this class, proposed in [13], is based on a nonlocal equation of motion,

$$G_{\mu\nu} - \frac{m^2}{3}\left(g_{\mu\nu}\Box^{-1}R\right)^{\mathrm{T}} = 8\pi G\, T_{\mu\nu}\,, \tag{1.1}$$

where the superscript T denotes the operation of taking the transverse part of a tensor (which is itself a non-local operation), and ensures energy-momentum conservation. We will refer to this model as the "RT" model, where R stands for the Ricci scalar and T for the extraction of the transverse part. A second model was introduced in [14], and is defined by the quantum effective action

$$\Gamma_{\mathrm{RR}} = \frac{M_{\mathrm{Pl}}^2}{16\pi}\int d^4x\sqrt{-g}\left[R - \frac{m^2}{6}R\frac{1}{\Box^2}R\right]\,, \tag{1.2}$$

where again m is the new mass parameter of the model. We will refer to it as the RR model. Conceptual and phenomenological aspects of these models have been discussed in a series of papers [13–31] and recently reviewed in [32]. Once interpreted in terms of the dynamics of the in-in matrix elements induced by a quantum effective actions, the RT and RR models have no problem with ghosts nor with causality (see in particular Sect. 5 of [32], and references therein). The study of their cosmological solutions shows that, at the background level, they generate an effective dark energy and have a realistic background FRW evolution, without the need of introducing a cosmological constant. Their cosmological perturbations are well-behaved both in the scalar and in the tensor sector, and give predictions consistent with CMB, supernovae, BAO and structure formation data. This allowed us to move to the next step, implementing the cosmological perturbations in a Boltzmann code and performing Bayesian parameter estimation and a detailed quantitative comparison with ΛCDM [23,29]. The result is that the RT model fits the data at a level which is statistically indistinguishable from ΛCDM (and in fact even fits slightly better). The

performance of the RR model is also statistically indistinguishable from that of ΛCDM, once neutrino masses are left free to vary, within the existing experimental limits [33]. It should also be appreciated that these two nonlocal models have the same number of free parameters as ΛCDM, with the new parameter m replacing the cosmological constant.[1] At smaller scales, both the RT and RR models have no vDVZ discontinuity and reduce smoothly to GR. Hence, they pass without problems all solar-system tests.[2]

The purpose of this paper is to point out another phenomenologically interesting aspect of the RT model (not shared, as we will see, by the RR model). Namely, it predicts a very characteristic form of the equation of state of dark energy, which depends on the existence of an epoch of primordial inflation. This connection between the behavior of DE in the recent cosmological epoch, and the properties of the cosmological model during an epoch of primordial inflation, is by itself quite interesting. Furthermore, we will obtain a prediction for $w_{\mathrm{DE}}(z)$ which is already very close to the region excluded by the *Planck* 2015 data, and should be clearly accessible to future missions such as EUCLID. No tunable parameter is involved in this prediction. Once we fix the model (that we will take to be the RT model supplemented by a Starobinsky inflationary sector) and choose any number of e-folds ΔN during inflation, larger than the minimal value required for solving the standard flatness and horizon problems, the result for $w_{\mathrm{DE}}(z)$ is uniquely fixed, since it goes to an asymptotic value for large ΔN.

2 Cosmological evolution and initial conditions

As discussed in [13] (see also Sect. 7.1.1 of the review [32]), the equations of motion of the RT model can be put into local form by introducing two auxiliary fields U and Y (which, at the quantum level, do not correspond to physical degrees of freedom, in the sense that there are no creation and annihilation operators associated to them). Then, looking for background FRW solutions with scale factor $a(t)$, we get a Friedman equation of the form

$$h^2(x) = \Omega_M e^{-3x} + \Omega_R e^{-4x} + \gamma Y(x), \tag{2.1}$$

where $x \equiv \log a(t)$ is used as the time-evolution variable, $h(x) = H(x)/H_0$ is the Hubble parameter normalized to the present value, Ω_M and Ω_R are the present density fractions of matter and radiation, respectively, and $\gamma \equiv m^2/(9H_0^2)$ is the free parameter of the model in dimensionless form. The two auxiliary fields U and Y satisfy the coupled system of equations

$$Y'' + (3 - \zeta)Y' - 3(1 + \zeta)Y = 3U' - 3(1 + \zeta)U, \tag{2.2}$$

$$U'' + (3 + \zeta)U' = 6(2 + \zeta), \tag{2.3}$$

where $\zeta(x) \equiv h'/h$ and the prime denotes d/dx. We see from eq. (2.1) that there is an effective DE density $\rho_{\mathrm{DE}}(x) = \rho_0 \gamma Y(x)$, where $\rho_0 = 3H_0^2/(8\pi G)$.

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[1]By comparison, in $f(R)$ gravity and in the Deser-Woodard nonlocal model one tunes a whole function, while in bigravity the cosmological constant is replaced by a set of 5 parameters β_n and there is also a new Planck mass associated to the second metric.

[2]See [21] and app. A of [29] for a discussion of a potential issue of the RR model in comparison with Lunar Laser Ranging experiments. This issue in any case does not even arise for the RT model.

Given the initial conditions on the auxiliary fields, the numerical integration of eqs. (2.2) and (2.3) is straightforward. Useful analytic understanding can however be obtained observing that, in any given epoch, the parameter ζ has an approximately constant value ζ_0, with $\zeta_0 = \{0, -2, -3/2\}$ in deSitter (dS), RD and MD, respectively. In the approximation of constant ζ eq. (2.3) can be integrated analytically, and has the solution [13]

$$U(x) = \frac{6(2 + \zeta_0)}{3 + \zeta_0}x + u_0 + u_1 e^{-(3+\zeta_0)x}, \tag{2.4}$$

where the coefficients u_0, u_1 parametrize the general solution of the homogeneous equation $U'' + (3 + \zeta_0)U = 0$. Plugging eq. (2.4) into eq. (2.2) and solving for $Y(x)$ we get

$$Y(x) = -\frac{2(2 + \zeta_0)\zeta_0}{(3 + \zeta_0)(1 + \zeta_0)} + \frac{6(2 + \zeta_0)}{3 + \zeta_0}x + u_0$$
$$-\frac{6(2 + \zeta_0)u_1}{2\zeta_0^2 + 3\zeta_0 - 3}e^{-(3+\zeta_0)x} + a_1 e^{\alpha_+ x} + a_2 e^{\alpha_- x}, \tag{2.5}$$

where

$$2\alpha_\pm = -3 + \zeta_0 \pm \sqrt{21 + 6\zeta_0 + \zeta_0^2}. \tag{2.6}$$

During RD and MD both α_+ and α_- are negative. This is important, since a positive value would have led to a mode growing exponentially in x (i.e., as a power in the scale factor). Any small perturbation would then unavoidably excite this growing mode, and this would have quickly led to an unacceptable background evolution during RD or MD. In contrast, during dS there is a growing mode, with $\alpha_+ = \alpha_+^{\text{dS}} = (-3 + \sqrt{21})/2 \simeq 0.79$. In our previous works on the RT model the cosmological evolution was started deep in the RD phase, with initial values $U(x_{\text{in}}) = U'(x_{\text{in}}) = Y(x_{\text{in}}) = Y'(x_{\text{in}}) = 0$. The initial conditions $\{U(x_{\text{in}}), U'(x_{\text{in}}), Y(x_{\text{in}}), Y'(x_{\text{in}})\}$ are in one-to-one correspondence with the parameters $\{u_0, u_1, a_1, a_2\}$ in eqs. (2.4) and (2.5). From the fact that, in RD, $\alpha_\pm < 0$, it follows that u_1, a_1, a_2 corresponds to stable directions, i.e. the solution obtained starting in RD with generic non-vanishing values of u_1, a_1, a_2 approaches exponentially fast the solution obtained starting with $u_1 = a_1 = a_2 = 0$. In contrast, u_0 is associated to a marginally-stable direction, and setting a non-vanishing initial value for u_0 effectively amounts to reintroducing a cosmological constant [13, 16]. As stressed in particular in [32], the evolution obtained setting $U(x_{\text{in}}) = U'(x_{\text{in}}) = Y(x_{\text{in}}) = Y'(x_{\text{in}}) = 0$ at a value $x = x_{\text{in}}$ in RD must be understood as defining a 'minimal model', which is useful to get a first understanding of the kind of predictions that can be obtained from the RT model. Here we rather explore the non-minimal model that can be obtained by taking into account the existence of an inflationary phase in the primordial Universe, as well as the subsequent reheating phase. In RD and MD any solution approaches exponentially the one obtained setting $u_1 = a_1 = a_2 = 0$. Nevertheless, the fact that during a previous inflationary era Y grows exponentially means that, in practice, given the finite duration of the RD and MD eras, when we reach the recent epoch and DE becomes important, the solution for $Y(x)$ is still sensibly different from that obtained in the minimal model. As we will see, the resulting background evolution is still viable, but its prediction for the recent DE phase will be different from that of the minimal model, and will carry an imprint of the inflationary epoch. Observe that in the RR model all solutions of the homogeneous equations for its auxiliary fields are decaying in dS, RD and MD [14]. Therefore in the RR model there is no such effect.

3 Evolution during inflation

In order to illustrate the above behavior, we consider a specific model of inflation and of the subsequent reheating phase. Similarly to ΛCDM, the nonlocal models can in principle be supplemented by any desired inflationary dynamics in the primordial epoch, by introducing an inflationary sector. In the following we will consider the Starobinsky model of inflation, so that during the inflationary epoch the dynamics is effectively governed by the action

$$S = \frac{M_{\rm Pl}^2}{16\pi} \int d^4x \sqrt{-g} \left[R + \frac{R^2}{6M^2} \right] , \tag{3.1}$$

where $M_{\rm Pl}^2 = 1/G$ and M is the Starobinsky mass scale. This choice is first of all motivated by the fact that, when combined with ΛCDM, the predictions of the Starobinsky model for the tensor-to-scalar ratio r and the tilt n_s of scalar perturbations are the ones that fit better the data, among a large class of inflationary models [34]. As we will see below, the inflationary epoch is not affected by addition of the nonlocal term, so the same predictions for r and n_s hold in our case.[3] Furthermore, as first discussed in [26], Starobinsky inflation is very natural in the context of the nonlocal models. Indeed, the action (1.2) already contains a nonlocal term quadratic in the Ricci scalar. Once we admit nonlocal terms involving R^2, it is also natural to admit a local R^2 term, as in eq. (3.1).[4] We will defer to a future work a detailed study of the dependence of our results from the choice of the inflationary sector.

We start from generic initial values for Y at the beginning of inflation, i.e. we do not fine-tune Y to zero at the initial time. We denote by $a_{\rm in}$ the value of the scale factor when inflations begins, and by $a_{\rm end}$ the value when inflation ends and reheating starts. We will further denote by $a_{\rm reh}$ the value of a when reheating ends and RD begins, by $a_{\rm eq}$ the value at RD-MD equilibrium, and by a_0 the present value (that we will eventually fix to $a_0 = 1$). We write $x_{\rm end} - x_{\rm in} = \log a_{\rm end}/a_{\rm in} \equiv \Delta N$, i.e. ΔN is the number of e-folds during a quasi-deSitter phase of inflation. Thus, if $Y(x_{\rm in})$ has a generic value of order one (i.e., is not fine-tuned to zero), by the end of inflation $Y(x_{\rm end}) \simeq \exp\{\alpha_+^{\rm dS}\Delta N\} \simeq \exp\{0.79\Delta N\}$.

Note that, despite this exponential growth, even for very large values of ΔN the corresponding DE density $\rho_{\rm DE}(x) = \rho_0 \gamma Y(x)$ has no effect on the inflationary dynamics. This is due to the fact that $\rho_0 = 3H_0^2/(8\pi G) \sim (10^{-3}{\rm eV})^4$ is extremely small compared to the energy density during inflation. For instance, if $Y(x_{\rm in}) = \mathcal{O}(1)$ and we take $\Delta N = 60$, at the end of inflation we get $Y(x_{\rm end}) = \mathcal{O}(10^{20})$. Even with such a large value of Y, $[\rho_0 Y(x_{\rm end})]^{1/4} \sim 10^{-3}{\rm eV} \times Y^{1/4}(x_{\rm end}) = \mathcal{O}(10^2)$ eV (furthermore, we will see below that $\gamma \ll 1$, and decreases with ΔN so to keep $\gamma Y(x = 0)$ fixed). This is totally negligible compared to the inflationary scale M, which has typical values, say, of order 10^{13} GeV. Thus, during the inflationary phase the evolution of the scale factor is the same as in standard GR without the nonlocal term; the auxiliary fields U and Y are at this stage

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[3]Of course, in order to assess how well these predictions fit the data within the nonlocal model, we need to perform the corresponding Bayesian parameter estimation. For the minimal RT model with initial condition in the RD era this has been done in [29]. We will present elsewhere the result of Bayesian parameter estimation for the RT model, with initial conditions in the inflationary era.

[4]The action of the RT model is not explicitly known but, when one linearizes the model over flat space, it turns out to be equivalent to the RR model, so its full action is expected to be a nonlinear extension of eq. (1.2).

'spectator fields', that do not influence the evolution of the scale factor, and whose evolution can be computed inserting in eqs. (2.2) and (2.3) the solution for $H(t)$ computed from the inflationary solution, neglecting the nonlocal term. Observe that eqs. (2.2) and (2.3) are not affected by the inclusion of the inflationary sector, since they just express the definition of the auxiliary fields (e.g $U = -\Box^{-1}R$), needed to put the original nonlocal equations in local form.

The mass scale M in eq. (3.1) is fixed by requiring that the inflationary model reproduces the observed value of the amplitude of the scalar perturbations at a pivot scale k_* used by CMB experiments (with $k_* = 0.05\,\mathrm{Mpc}^{-1}$ being a typical choice by *Planck*). This gives

$$M \simeq 2.9 \times 10^{13}\,\mathrm{GeV}\,(60/N_*)\,, \tag{3.2}$$

where N_* is defined as the number of e-folds to the end of inflation when the scale k_* leaves the horizon. Its value is observationally determined by comparing the prediction for (n_s, r) to the data. For the Starobinsky model this gives $53 < N_* < 64$ at 95% c.l. [34]. Finally, we will also need a model for reheating. For Starobinsky inflation a simple model of reheating has been discussed in [35] (see also the review [1]). For $t > t_{\mathrm{end}}$, it leads to the solution

$$H(t) = \left[\frac{3}{M} + \frac{3}{4}(t - t_{\mathrm{end}}) + \frac{3}{4M}\sin[M(t - t_{\mathrm{end}})]\right]^{-1}\cos^2\left[\frac{M}{2}(t - t_{\mathrm{end}})\right]. \tag{3.3}$$

In the regime $M(t - t_{\mathrm{end}}) \gg 1$, averaging over the oscillations one finds

$$\langle H(t) \rangle = \frac{2}{3}\frac{1}{(t - t_{\mathrm{end}})}\,, \tag{3.4}$$

just as in MD. The total duration of the reheating phase is [1, 35]

$$t_{\mathrm{reh}} - t_{\mathrm{end}} \simeq \frac{400 M_{\mathrm{Pl}}^2}{g_* M^3}\,, \tag{3.5}$$

where g_* is the number of relativistic degrees of freedom at time of reheating (with $g_* \simeq 106.75$ in the Standard Model). Thus, at the time $t = t_{\mathrm{reh}}$ corresponding to end of reheating, we have $H(t_{\mathrm{reh}}) \simeq (M/3)(g_*/200)(M/M_{\mathrm{Pl}})^2$ while, from eq. (3.3), $H(t_{\mathrm{end}}) = M/3$. Since during reheating $\langle H \rangle \propto a^{-3/2}$, as in MD, we get

$$\frac{a_{\mathrm{end}}}{a_{\mathrm{reh}}} \simeq \left(\frac{g_*}{200}\right)^{2/3}\left(\frac{M}{M_{\mathrm{Pl}}}\right)^{4/3} \equiv e^{-2C}\,. \tag{3.6}$$

Using eq. (3.2) and taking $g_* = 106.75$ we get

$$C \simeq 8.84 - (2/3)\log(60/N_*)\,. \tag{3.7}$$

The reheating temperature can be estimated as $T_{\mathrm{reh}} \simeq 3 \times 10^{17} g_*^{1/4}\,(M/M_{\mathrm{Pl}})^{3/2}\,\mathrm{GeV}$ [1]. Using eq. (3.2) gives

$$T_{\mathrm{reh}} \simeq 3.5 \times 10^9\,(60/N_*)^{3/2}\,\mathrm{GeV}\,. \tag{3.8}$$

The value of $Y(x_{\mathrm{end}})$ at the end of inflation depends on the total number of e-folds ΔN from the beginning of inflation. In principle, ΔN is not a directly observable quantity.

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However, a first obvious bound that we have on it is $\Delta N \geq N_*$. Another bound comes from the condition that the inflationary expansion should be long enough to solve the flatness problem. Since the precise numbers will be relevant for us, let us repeat this standard computation (see e.g. [36]), including the effect of the above reheating phase. Defining as usual the energy fraction associated to the curvature as $\Omega_K(t) = -K/[a^2(t)H^2(t)]$, one sees that $\Omega_K \propto a^2$ during RD, $\Omega_K \propto a$ during MD, and $\Omega_K \propto a^{-2}$ during a quasi-deSitter epoch of inflation. Observing that during the reheating phase discussed above the scale factor basically behaves as in MD, the value of the curvature today, $\Omega_K(t_0)$, is related to the value at the beginning of inflation, $\Omega_K(t_{\rm in})$, by

$$\Omega_K(t_0) = \Omega_K(t_{\rm in}) \left(\frac{a_{\rm in}}{a_{\rm end}} \right)^2 \left(\frac{a_{\rm reh}}{a_{\rm end}} \right) \left(\frac{a_{\rm eq}}{a_{\rm reh}} \right)^2 \left(\frac{a_0}{a_{\rm eq}} \right) . \tag{3.9}$$

We now require that the initial value of the curvature is not fine-tuned to zero, $\Omega_K(t_{\rm in}) = \mathcal{O}(1)$, and we impose the observational constraint $\Omega_K(t_0) < 0.005$ [37]. We use $a_0/a_{\rm eq} = (1+z_{\rm eq}) \simeq 3500$ and we determine $a_0/a_{\rm reh}$ by requiring entropy conservation, $g_* a_{\rm reh}^3 T_{\rm reh}^3 = g_{*,0} a_0^3 T_0^3$, where $g_{*,0} \simeq 3.909$ is the present value of the effective number of relativistic species, $T_0 \simeq 2.7$ K, and $T_{\rm reh}$ is given by eq. (3.8). Plugging the numerical values we get $\Delta N \gtrsim 60$. A similar, but slightly weaker bound, is obtained requiring the solution of the horizon problem. In conclusion

$$\Delta N \gtrsim \max(60, N_*) , \tag{3.10}$$

with N_* bounded observationally in the range $53 < N_* < 64$. We now have all the elements for following the evolution of the auxiliary fields Y and U through inflation and reheating. At the end of inflation, $Y(x_{\rm end}) \simeq \exp\{0.79\Delta N\}$. During reheating the scale factor evolves basically as in MD, so $Y(x)$ will decrease as $\exp\{\alpha_+^{\rm MD}(x - x_{\rm end})\}$, where $\alpha_+^{\rm MD} = (-9 + \sqrt{57})/4 \simeq -0.36$. Thus, at the end of reheating and beginning of RD,

$$Y(x_{\rm reh}) \simeq e^{0.79\Delta N - 0.72C} , \tag{3.11}$$

with C given in eq. (3.7). Observe that the dependence on N_* that enters through C only affects the prefactor in $Y(x_{\rm reh})$, with a variation at most of order one, given that N_* is constrained in the range $53 < N_* < 64$. This dependence is therefore irrelevant and can be reabsorbed in the initial value $Y(x_{\rm in}) = \mathcal{O}(1)$. Therefore, in the factor C, we can simply set $N_* = 60$.

The evolution of U can be computed similarly, using eq. (2.4). During a quasi-deSitter phase of inflation, starting from a value of order one, we get $U(x_{\rm end}) \simeq 4\Delta N$. It continues to grow during reheating, where $\zeta_0 = -3/2$, as $U(x) \propto 2x$. Thus, during reheating

$$U(x) \simeq 4\Delta N + 2(x - x_{\rm end}) , \tag{3.12}$$

and in particular $U(x_{\rm reh}) \simeq 4\Delta N + 4C$. We now freeze the inflaton, and use eqs. (2.1)–(2.3) to further evolve numerically the system through RD, MD and the present DE-dominated epoch, using $Y(x_{\rm reh})$ and $U(x_{\rm reh})$ as initial values for the subsequent evolution, together with $Y'(x_{\rm reh}) = \alpha_+^{\rm MD} Y(x_{\rm reh}) \simeq -0.36 Y(x_{\rm reh})$, and $U'(x_{\rm reh}) = 2$, from eq. (3.12).

In Fig. 1 we show the result for the DE density $\rho_{\rm DE}(x)$ as a function of $x = \log a$, for $\Delta N = 50$ and $\Delta N = 60$, compared to the 'minimal scenario' where the evolution
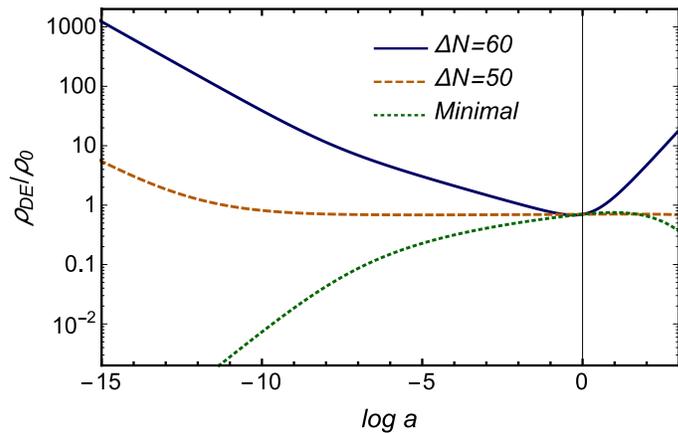
Figure 1: Evolution of the dark energy density for $\Delta N = 50$ and $\Delta N = 60$, compared to the "minimal" scenario where the evolution is started in RD with vanishing initial conditions on the auxiliary fields.
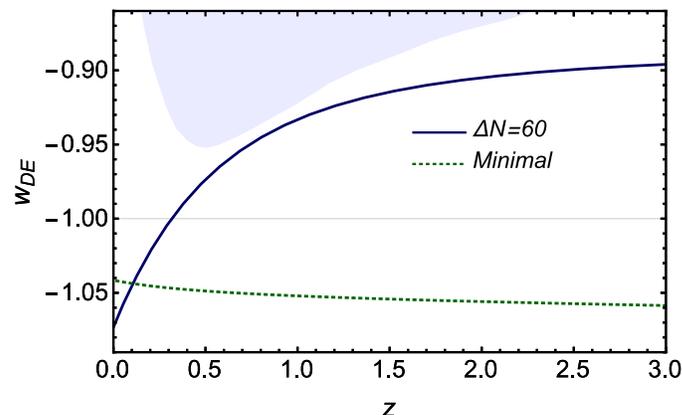

Figure 2: The prediction for $w_{\mathrm{DE}}(z)$ for $\Delta N \gtrsim 60$. The shaded region is excluded by the Planck data [38].

is started in RD with vanishing initial conditions. Observe that RD-MD equilibrium is at $x_{\mathrm{eq}} \simeq -8.1$, while $x = 0$ corresponds to the present epoch, and we extended the plot into the cosmological future. We see that, in the curve for $\Delta N = 50$, during the evolution from the last stages of RD up to the present epoch, ρ_{DE} is basically constant, and therefore indistinguishable from ΛCDM. However, we have seen that such a value of ΔN is too small to solve the flatness and horizon problem. Once we consider values of ΔN that satisfies the bound (3.10), we rather get a behavior for $\rho_{\mathrm{DE}}(x)$ markedly different from the constant DE density of ΛCDM, as well as from the minimal model, such as the line marked $\Delta N = 60$ in the figure. A remarkable result is that, if we increase further ΔN, the result for $\rho_{\mathrm{DE}}(x)$ basically does not change, so the curve for $\Delta N = 60$ is already essentially equal to the asymptotic limit for large ΔN. For instance, the curves for $\Delta N = 70$ or $\Delta N = 100$ are indistinguishable from that for $\Delta N = 60$, on the scale of the figure. This happens because, each time we change ΔN, we must also readjust γ so

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to obtain the desired value of $\rho_{\rm DE}(x = 0)/\rho_0$, that here we have fixed to 0.7 (of course, in a full analysis including the cosmological perturbations, this value will be determined self-consistently by Bayesian parameter estimation). For sufficiently large ΔN, an increase in the initial values of Y at the beginning of RD is exactly compensated by a decrease in γ, and we end up on the same solution. This is due to the fact that, for large ΔN, after inflation $U \ll Y$ and we can set to zero the right-hand side in eq. (2.2). Then we get a homogeneous equation for Y and, for the subsequent evolution, an increase in the values $Y(x_{\rm reh})$ and $Y'(x_{\rm reh}) \simeq -0.36 Y(x_{\rm reh})$ can be exactly compensated by a decrease in γ, such that $\rho_{\rm DE} \equiv \rho_0 \gamma Y$ is unchanged. For instance, we find $\gamma \simeq \{0.05, 0.005, 3 \times 10^{-4}, 10^{-7}\}$ for the minimal model and $\Delta N = 50, 60, 70$, respectively.

The function $w_{\rm DE}(x)$ is defined as usual from the conservation equation

$$\dot{\rho}_{\rm DE} + 3(1 + w_{\rm DE}) H \rho_{\rm DE} = 0, \tag{3.13}$$

i.e.

$$w_{\rm DE} = -1 - \frac{\rho'_{\rm DE}}{3\rho_{\rm DE}}. \tag{3.14}$$

In Fig. 2 we plot the prediction of the model, valid in the asymptotic regime $\Delta N \geq 60$, as a function of redshift. The results qualitatively agree with that obtained in Sect. 7.4.1 of [32], where however reheating was treated as instantaneous. We see that our prediction for $w_{\rm DE}(z)$ has quite distinctive features: for large z, $w_{\rm DE}(z)$ is above -1. It then crosses the phantom divide at $z \simeq 0.315$, and in the more recent epoch it becomes phantom, $w_{\rm DE}(z) < -1$. This behavior is very specific, and its observation would be a smoking-gun signature of the RT nonlocal model. At the same time, the observation of an equation of state of this form would be a signature of inflation.

The fact that the result saturates and reaches an asymptotic regime for $\Delta N \gtrsim 60$ is quite satisfying because it means that the prediction for $w_{\rm DE}(z)$ is robust, and does not really depend on the value of ΔN (once the observational bound (3.10) is respected). It also means that even the value of $Y(x_{\rm in})$, which here has been chosen equal to one, is irrelevant, and we can change it by several orders of magnitudes without essentially affecting the prediction.

Another natural question is how the result depends on the choice of inflationary sector and on the details of reheating. Actually, the growth of $Y(x) \propto e^{0.79 \Delta N}$ will be the same for any quasi-deSitter inflationary phase. We see from eq. (3.11) that the effect of reheating is to reduce the value of $Y(x_{\rm reh})$, thought the constant C. With ΔN sufficiently large, we will always be in the asymptotic regime studied in this paper. The issue is whether, for some inflationary model (in particular, models with a low-energy inflationary scale), one could solve the flatness and horizon problems with a minimum number of e-folds $(\Delta N)_{\rm min}$ sufficiently small (or one could have reheating mechanisms producing a constant C sufficiently large) such that, choosing ΔN close to its minimum possible value $(\Delta N)_{\rm min}$, we will not be in the asymptotic regime discussed above. In that case, the prediction for $\rho_{\rm DE}(x)$ will rather be intermediate between the curve shown in Fig. 1 for $\Delta N = 60$, and the curve for the 'minimal model', and possibly quite close to ΛCDM, just as the curve $\Delta N = 50$. Even for such models, however, for ΔN sufficiently larger than $(\Delta N)_{\rm min}$, we will be in the asymptotic regime described above, which is therefore quite generic. A more systematic study on the dependence on the inflationary model and reheating mechanism will be presented elsewhere.

A noteworthy aspect of the prediction shown in Fig. 2 is that it is already very close to the region excluded by the *Planck* 2015 data. Indeed, by performing a principal component analysis, one can get upper and lower limits on the deviation of $w(z)$ from -1, see Fig. 5 of the *Planck* dark energy paper [38]. The upper limit is shown as the shaded area in Fig. 2 (while the lower limit is below the scale of our figure). We see that the *Planck* sensitivity is already very close to that needed for testing our prediction. It is worth stressing that, within the model considered, no parameter was tuned to obtain a result so close to the Planck exclusion limit. The prediction shown in the figure follows unavoidably, within the RT model supplemented by Starobinsky inflation, once we start the evolution in the inflationary phase with generic, non fine-tuned initial conditions, and we let the system evolve for a number of e-folds larger or equal than the minimum required for solving the flatness and horizon problems. With EUCLID a further significant improvement in sensitivity to $w_{\rm DE}(z)$ is expected, by about one order of magnitude compared to *Planck* [39]. Thus, EUCLID should be able to clearly confirm or disprove the above prediction.

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