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# EFFECTIVE FIELD THEORY OF POST-NEWTONIAN GRAVITY INCLUDING SPINS

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We present in detail an Effective Field Theory (EFT) formulation for the essential case of spinning objects as the components of inspiralling compact binaries. We review its implementation, carried out in a series of works in recent years, which leveled the high post-Newtonian (PN) accuracy in the spinning sector to that, recently attained in the non-spinning sector. We note a public package, "EFTofPNG", that we recently created for high precision computation in the EFT of PN Gravity, which covers all sectors, and includes an observables pipeline.

## 1 Introduction

A new era of high precision Gravity has been launched with the recent direct detection of gravitational waves (GWs) from compact binary coalescence [1,2]. The influx of improved GW data is expected to increase, and more accurate GWs templates will be required to optimally analyze the signal. The continuous GW signal comprises various kinds of physics, corresponding to the different phases in the evolution of the radiating binary. The initial inspiral phase, where the components of the binary orbit with a non-relativistic velocity, can only be treated analytically with the post-Newtonian (PN) theory of General Relativity [3,4]. Indeed, in recent years there has been a remarkable progress in high order PN theory, in particular also involving the essential *spins* of the components of the binary. In what follows we detail the formal progress, which has been obtained within an extension of the Effective Field Theory (EFT) for the binary inspiral to spinning objects [5,6]. We also review its specific applications, which we carried out.

## 2 Effective Field Theory of Post-Newtonian Gravity including Spinning Objects

Effective Field Theories (EFTs) and their setup are universal. Once a hierarchy of scales is identified in the physical problem, be it classical or quantum, the robust EFT framework can then be applied. For the compact binary coalescence in the inspiral phase of its evolution, this observation was made by Goldberger *et al.* [7,8]. Indeed, there are three distinct characteristic length scales in the problem: $r_s$, $r$, and $\lambda$, corresponding to the scales of the internal structure of the single compact component of the binary, the orbital separation of the binary, and the wavelength of radiation emitted from the binary, respectively. They are widely separated by powers of $v \ll 1$, the typical non-relativistic orbital velocity at the inspiral phase, as we have that $r_s \sim r\,v^2 \sim \lambda\,v^3$. Goldberger *et al.* [7,8] put forward a program to tackle the binary inspiral problem with a tower of EFTs, corresponding to each of the scales. We note that the only scale in the full theory is the UV scale $m$, the mass of the isolated compact object, where $r_s \sim m$. For the general case that we tackle in our work, where the compact object is *spinning*, and hence also characterized by its spin length [6], $S^2$, we also have that $S \lesssim m^2$.

The construction of an EFT follows one of two general procedures, top-down and bottom-up. These approaches differ on how the effective action, which formally represents the EFT, is obtained. The bottom-up approach constructs the effective action from scratch as an infinite sum of operators, constrained by symmetry considerations. The top-down approach obtains the effective action by explicitly eliminating degrees of freedom (DOFs) from the action of the high energy (small scale) theory. We apply, in fact, both of these approaches in our construction of the EFTs. First, we remove the scale $r_s$, using a bottom-up approach, by eliminating the strong field modes $g^s_{\mu\nu}$, where the gravitational field is decomposed into $g_{\mu\nu} \equiv g^s_{\mu\nu} + \bar{g}_{\mu\nu}$, and $\bar{g}_{\mu\nu}$ represents the field modes in scales above $r_s$. The effective action for an isolated compact object can then be generically written as:

$$S_{\text{eff}}\left[\bar{g}_{\mu\nu}, y^\mu, e^\mu_A\right] = -\frac{1}{16\pi G}\int d^4x\sqrt{\bar{g}}R\left[\bar{g}_{\mu\nu}\right] + \underbrace{\sum_i C_i \int d\sigma O_i(\sigma)}_{S_{pp}\equiv\text{point particle action}} , \tag{1}$$

where we introduce a point particle action, $S_{\text{pp}}$, with an infinite tower of worldline operators, $O_i(\sigma)$. These should contain the DOFs relevant at this scale, and satisfy the symmetries of the theory. Hence, it is crucial to accurately identify the DOFs and symmetries for the general case of spinning gravitating objects as we further elaborate below[6]. Here, $y^\mu$, and $e^\mu_A$, are the particle worldline coordinate, and tetrad DOFs, respectively. All of the UV physics is encapsulated in the Wilson coefficients, $C_i(r_s)$, in the point particle action, $S_{\text{pp}}$.

For the next EFT in the tower, where the orbital scale $r$ is removed, we use the top-down approach. First, we decompose the field into $\bar{g}_{\mu\nu} \equiv \eta_{\mu\nu} + H_{\mu\nu} + \widetilde{h}_{\mu\nu}$, where $H_{\mu\nu}$ represents the field modes at the orbital scale, and $\widetilde{h}_{\mu\nu}$, the radiation modes. These different field modes scale with a definite power of the small PN parameter $v$: $\partial_t H_{\mu\nu} \sim \frac{v}{r}H_{\mu\nu}$, $\partial_i H_{\mu\nu} \sim \frac{1}{r}H_{\mu\nu}$, $\partial_\rho \widetilde{h}_{\mu\nu} \sim \frac{v}{r}\widetilde{h}_{\mu\nu}$. Then, we use the effective action of two compact objects, with a copy of $S_{\text{pp}}$ for each object, to integrate out the field modes $H_{\mu\nu}$ by computing the following functional integral:

$$e^{iS_{\text{eff(composite)}}\left[\widetilde{h}_{\mu\nu}, y^\mu, e^\mu_A\right]} \equiv \int \mathcal{D}H_{\mu\nu}\; e^{iS_{\text{eff}}\left[\bar{g}_{\mu\nu}, y^\mu_1, y^\mu_2, e_{(1)}{}^\mu_A, e_{(2)}{}^\mu_A\right]}, \tag{2}$$

taking only the tree level for the classical limit. This defines the effective action of the composite object with $y^\mu$, and $e^\mu_A$, now being its worldline coordinate, and tetrad, respectively. In general, we should proceed to integrate out the radiation modes, $\widetilde{h}_{\mu\nu}$, to get the final EFT. However, this is not required in the conservative sector, where no radiation modes are present. Therefore, we stress that the final effective action at this stage should consist of no remaining field modes at the orbital scale[9,10,6].

As we noted, in order to construct the point particle action in Eq. 1, it is crucial to accurately identify the DOFs and symmetries of the theory[6]. As for the DOFs for a spinning object, of which we have three kinds, it is important to note the following: 1. As for the gravitational field we should consider the tetrad field, $\eta^{ab}\tilde{e}_a{}^\mu(x)\tilde{e}_b{}^\nu(x) = g^{\mu\nu}(x)$, rather than just $g_{\mu\nu}(x)$; 2. As for the particle worldline coordinate, $y^\mu(\sigma)$, the particle worldline position does not in general coincide with the object's "center"; 3. As for the particle worldline rotating DOFs, from the worldline tetrad, $\eta^{AB}e_A{}^\mu(\sigma)e_B{}^\nu(\sigma) = g^{\mu\nu}$, we define the angular velocity, $\Omega^{\mu\nu}(\sigma) \equiv e^\mu_A \frac{De^{A\nu}}{D\sigma}$, and add its conjugate, the worldline spin, $S_{\mu\nu}(\sigma) \equiv -2\frac{\partial L}{\partial \Omega^{\mu\nu}}$, as further DOFs. Later, we switch to the worldline Lorentz matrices, $\eta^{AB}\Lambda_A{}^a(\sigma)\Lambda_B{}^b(\sigma) = \eta^{ab}$, and the conjugate local spin, $S_{ab}(\sigma)$.

Regarding the symmetries of the theory, it is crucial to note the additional symmetries that play a role for the spinning case, i.e. beyond general covariance, and worldline reparametrization invariance. These additional symmetries are: 1. Parity invariance; 2. Internal Lorentz invariance of the local frame field; 3. $SO(3)$ invariance of the worldline spatial triad; 4. Spin gauge invariance, that is invariance under the choice of completion of the worldline spatial triad through a timelike vector. This is a gauge of the rotational variables, i.e. of both the worldline tetrad,

and spin. In addition to these symmetries, we assume that the isolated object has no intrinsic permanent multipole moments beyond the mass monopole, and the spin dipole.

For a spinning object we can write the point particle action in Eq. 1 in the form [11,12,6]:

$$S_{\rm pp} = \int d\sigma \left[ -m\sqrt{u^2} - \frac{1}{2} S_{\mu\nu}\Omega^{\mu\nu} + L_{\rm SI}\left[u^\mu, S_{\mu\nu}, \bar{g}_{\mu\nu}\left(y^\mu\right)\right]\right], \tag{3}$$

where $u^\mu \equiv dy^\mu/d\sigma$, and $L_{\rm SI}$ denotes the Lagrangian part, which is nonminimal coupling, and as we assume, contains only spin-induced higher multipoles. While the minimal coupling here is fixed only from covariance and reparametrization invariance, it can still be worked out to further incorporate the other symmetries related with the worldline tetrad. As for nonminimal coupling, parity invariance also plays a role in constraining it. Indeed, in [6] we work out the point particle action of a spinning object, required for the first EFT of a single spinning particle.

In the spirit of Stueckelberg, we first want to introduce the gauge freedom of the rotational variables into the effective action. We do this by applying a 4-dimensional covariant boost-like transformation on the worldline tetrad [6]. This introduces new gauge DOFs, $w_\mu$, as the timelike vector of the tetrad, $\hat{e}_{[0]\mu} = w_\mu$, and also leads to a generic gauge condition for the spin:

$$\hat{S}^{\mu\nu}\left(p_\nu + \sqrt{p^2}\hat{e}_{[0]\nu}\right) = 0. \tag{4}$$

All in all, these constitute the $3+3$ necessary gauge conditions for the redundant unphysical DOFs, that are contained in the 4-dimensional antisymmetric angular velocity, and spin tensors. The minimal coupling term then yields:

$$\frac{1}{2}S_{\mu\nu}\Omega^{\mu\nu} = \frac{1}{2}\hat{S}_{\mu\nu}\hat{\Omega}^{\mu\nu} + \frac{\hat{S}^{\mu\nu}p_\nu}{p^2}\frac{Dp_\mu}{D\sigma}, \tag{5}$$

where an extra term appears in the action. This extra term, originating from minimal coupling, is of course not preceded by any Wilson coefficient, although it contributes to finite size effects. As for the spin that appears in spin-induced nonminimal couplings in the action, it is transformed to the generic spin variable as:

$$S_{\mu\nu} = \hat{S}_{\mu\nu} - \frac{\hat{S}_{\mu\rho}p^\rho p_\nu}{p^2} + \frac{\hat{S}_{\nu\rho}p^\rho p_\mu}{p^2}. \tag{6}$$

As we noted, the nonminimal coupling action with spin should also be constrained, using the full set of symmetries of the theory, that we detailed above. Based on these symmetries, and further considerations and properties of the problem, the leading order (LO) nonminimal couplings are indeed fixed to all orders in spin [6]:

$$\begin{aligned} L_{\rm SI} =& \sum_{n=1}^{\infty}\frac{(-1)^n}{(2n)!}\frac{C_{ES^{2n}}}{m^{2n-1}}D_{\mu_{2n}}\cdots D_{\mu_3}\frac{E_{\mu_1\mu_2}}{\sqrt{u^2}}S^{\mu_1}S^{\mu_2}\cdots S^{\mu_{2n-1}}S^{\mu_{2n}} \\ &+ \sum_{n=1}^{\infty}\frac{(-1)^n}{(2n+1)!}\frac{C_{BS^{2n+1}}}{m^{2n}}D_{\mu_{2n+1}}\cdots D_{\mu_3}\frac{B_{\mu_1\mu_2}}{\sqrt{u^2}}S^{\mu_1}S^{\mu_2}\cdots S^{\mu_{2n-1}}S^{\mu_{2n}}S^{\mu_{2n+1}}, \end{aligned} \tag{7}$$

where new spin-induced Wilson coefficients precede each of the nonminimal coupling terms. These operators are composed from either the electric, or magnetic curvature tensors, $E_{\mu\nu}$, or $B_{\mu\nu}$, respectively, together with the spin vector $S^\mu$. Of the above operators, the quadrupole, octupole, and hexadecapole couplings should notably be taken into account up to the fourth PN (4PN) order [13,6,14]. These couplings explicitly read:

$$L_{ES^2} \equiv -\frac{C_{ES^2}}{2m}\frac{E_{\mu\nu}}{\sqrt{u^2}}S^\mu S^\nu, \tag{8}$$

$$L_{BS^3} \equiv -\frac{C_{BS^3}}{6m^2}\frac{D_\lambda B_{\mu\nu}}{\sqrt{u^2}}S^\mu S^\nu S^\lambda, \tag{9}$$

$$L_{ES^4} \equiv \frac{C_{ES^4}}{24m^3}\frac{D_\lambda D_\kappa E_{\mu\nu}}{\sqrt{u^2}}S^\mu S^\nu S^\lambda S^\kappa. \tag{10}$$

We recall that for the second EFT we need to remove the field modes at the orbital scale. To this end the field DOFs should be disentangled from the particle DOFs. This can only be attained if the gauge of the rotational variables is fixed in the action, as was put forward in [9]. We stress that as we work in an action approach the gauge of the rotational variables can be directly inserted at any stage. But first we need to switch to new rotational variables: the locally flat angular velocity with worldline Lorentz matrices, $\hat{\Omega}^{ab}_{\text{flat}} = \hat{\Lambda}^{Aa}\frac{d\hat{\Lambda}_A{}^b}{d\sigma}$, and the local spin, $\hat{S}_{ab} = \tilde{e}^{\mu}_a \tilde{e}^{\nu}_b \hat{S}_{\mu\nu}$. Then, using the Ricci rotation coefficients with the tetrad field, defined by $\omega_\mu{}^{ab} \equiv \tilde{e}^b{}_\nu D_\mu \tilde{e}^{a\nu}$, we can rewrite the minimal coupling term in Eq. 5 in the form [5,6]:

$$\frac{1}{2}\hat{S}_{\mu\nu}\hat{\Omega}^{\mu\nu} = \frac{1}{2}\hat{S}_{ab}\hat{\Omega}^{ab}_{\text{flat}} + \frac{1}{2}\hat{S}_{ab}\omega_\mu{}^{ab}u^\mu. \tag{11}$$

Now, before we integrate out the field modes at the orbital scale, we need to fix all gauges in the action, which also eliminates all unphysical DOFs from the action. To begin with, we apply the beneficial non-relativistic space+time Kaluza-Klein decomposition on the gravitational field, i.e. we switch to the non-relativistic gravitational (NRG) fields [15,16]. The tetrad field gauge is then accordingly fixed to Schwinger's time gauge with the NRG parametrization, and finally we fix the gauge of the rotational variables to a gauge, that is dubbed the "canonical" gauge.

## 3 Summary of formal results and applications

In conclusion, we have provided an EFT formulation for the essential case of spinning objects as the components of inspiralling compact binaries, which constitutes a challenging extension of the EFT of PN Gravity [6]. In particular, we have also provided spin-induced nonminimal couplings to all orders in spin for a gravitating spinning particle. Moreover, the equations of motion (EOMs), and Hamiltonians are simply derived from the resulting effective action of the composite object. The (physical) EOMs of both the position, and spin, take a simple form, and are obtained directly via a proper variation of the action. Notably, for the precession equations it should be stressed, that one makes an independent variation with respect to the spin, and to its conjugates, the Lorentz matrices [10,6]. In addition, the useful Hamiltonians are obtained in the standard manner similarly to the non-spinning case.

As for the implementation, we have completed in a series of works all of the interaction potentials, as well as their derivatives and observables, in the spinning sectors up to the 4PN order, on par with the PN accuracy recently attained in the generally simpler point mass sector. These potentials include: the linear-in-spin next-to-next-to-leading order (NNLO) spin-orbit at 3.5PN order [17], and NNLO spin1-spin2 at 4PN order [18,10], the NNLO spin-squared at 4PN order [14,19], and the LO cubic, and quartic in spin at 3.5PN, and 4PN orders, respectively [13].

Moreover, we have recently created a new public package, "EFTofPNG", for high precision computation in the EFT of PN Gravity (PNG), including spins [20]. The "EFTofPNG" package version 1.0 covers the point mass sector, and all the spin sectors, up to the 4PN order, and two-loop level. It is released as a public repository in GitHub, and can be found in the URL: "https://github.com/miche-levi/pncbc-eftofpng". The "EFTofPNG" package is self-contained, modular, and designed to be accessible to the classical Gravity community. Its final unit provides the full computation of derivatives of interest, and gauge invariant observables, and serves as a pipeline chain for the modeling of GW templates for the detectors.


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